Heating load, COP, exergy loss rate, exergy output rate and ecological optimizations for a class of generalized irreversible universal heat pump cycles

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The optimal performance of a class of generalized irreversible universal steady flow heat pump cycle model, which consists of two heat-absorbing branches, two heat-releasing branches and two irreversible adiabatic branches with the losses of heat-resistance, heat leakage and internal irreversibility is analyzed by using finite time thermodynamics. The analytical formulae about heating load, coefficient of performance (COP), exergy loss rate, exergy output rate and ecological function of the universal heat pump cycle are derived. Moreover, performance comparisons among maximum COP condition, a given exergy output rate condition and maximum ecological function condition are carried out by using numerical examples. It is shown that the ecological function objective is an excellent candidate objective with the ideal of an ecological and long-term goal. The effects of heat leakage and internal irreversibility on the cycle performance are discussed. The universal cycle model gives a unified description of seven heat pump cycles, and the results obtained include the performance characteristics of Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles with the losses of heat-resistance, heat leakage and internal irreversibility.

Keywords: Finite time thermodynamics; irreversible universal heat pump cycle; heating load; COP; exergy loss rate; exergy output rate; ecological function.

Se analiza el funcionamiento óptimo de una clase de modelo universal irreversible generalizado del ciclo de una bomba térmica de flujo constante, que consiste en dos ramas de absorción térmica, dos ramas de liberación de calor y dos ramas adiabáticas irreversibles con pérdidas de resistencia al calor, de salida del calor y de irrevocabilidad interna, usando la termodinámica de tiempos finitos. Se obtienen las fórmulas analíticas de la carga de calor, del coeficiente del funcionamiento (CF), del índice de pérdida de exergía, del índice de salida de exergía, así como de la función ecológica del ciclo universal de la bomba de calor. Por otra parte, se compara el funcionamiento de la condición máxima del CF dada una condición de razón de salida de exergía con la condición del máximo de la función ecológica usando ejemplos numéricos. Se demuestra que el objetivo de la función ecológica es un excelente de candidato para el ideal de una meta ecológica y de largo plazo. Se discuten los efectos del escape de calor y de la irreversibilidad interna sobre el funcionamiento del ciclo. El modelo universal del ciclo da una descripción unificada de siete ciclos de la bomba de calor, y los resultados obtenidos incluyen las características de funcionamiento de los ciclos de Brayton, de Otto, Diesel, Atkinson, Dual, Miller y Carnot para bombas térmicas con pérdidas de la resistencia al calor, del escape de calor y de irreversibilidad interna.

Descriptores: Termoninámica de tiempos finitos; ciclo irreversible universal de bomba térmica; carga térmica; tasa de pérdida de exergía; tasa de salida de exergía; función ecológica.

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1. Introduction

Recently, finite time thermodynamics (FTT) [1-9], as a powerful tool, has become the premier theory for analyzing and optimizing performance of various thermodynamic processes and cycles. FTT analysis and optimization for various heat pump cycles with objectives of heating load [10-23], coefficient of performance (COP) [10-23], heating load density [19,21], exergetic efficiency [20,23], and profit rate [24,25], were performed by some authors. Alternatively, in consideration of combining classical exergy concept [26] with FTT, Angulo-Brown [27] proposed the ecological criterion $E' = P - T_L\sigma$ for finite time Carnot heat engines, where T_L is the temperature of cold heat reservoir, P is the power output, and σ is the entropy generation rate. Yan [28] showed that it might be more reasonable to use $E = P - T_0\sigma$ if the cold-reservoir temperature T_L is not equal to the environment temperature T_0 from the viewpoint of exergy analysis. The optimization of the ecological function reflects a compromise between the power output P and the loss power $T_0\sigma$, which is produced by entropy generation in the system and its surroundings.

Moreover, on viewpoint of exergy analysis, Chen *et al.* [29] provided a unified exergy-based ecological optimization objective for all of thermodynamic cycles, that is:

$$E = A/\tau - T_0\Delta S/\tau = A/\tau - T_0\sigma \qquad (1)$$

where A is the exergy output of the cycle, T_0 is the environment temperature of the cycle, ΔS is the entropy generation of the cycle, τ is the period of the cycle, and σ is the entropy generation rate of the cycle. For heat pump cycles, the exergy output rate of the cycle is:

$$A/\tau = Q_1(1 - T_0/T_H) - Q_2(1 - T_0/T_L) \qquad (2)$$

where Q_1 is the rate of heat transfer released to the heat sink at T_H, and Q_2 is the rate of heat transfer supplied by the heat source at T_L. It can be seen from Eq. (1) that the best compromise between the exergy output rate and the exergy loss

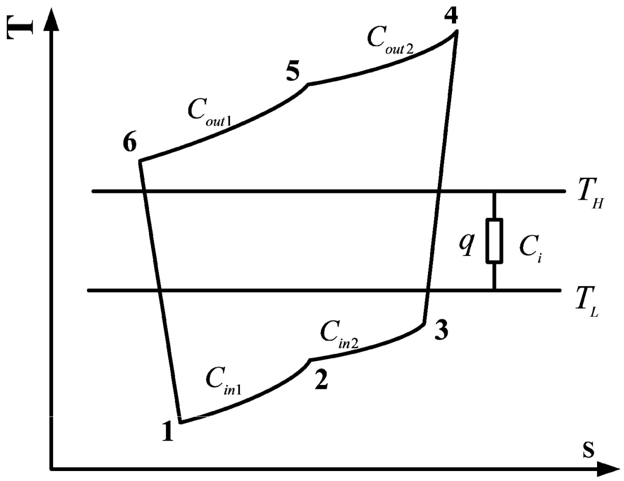

FIGURE 1. Irreversible universal heat pump cycle model.

rate (entropy generation rate) of the thermodynamic cycles is achieved. The ecological optimization has been extended to endoreversible [20,30,31,34,35] and irreversible [23,32,33] Carnot [30-33] and Brayton [20,23,34,35] heat pump cycles

Generalization and unified description of thermodynamic cycle model is an important task of FTT research. The unified descriptions have been carried out for endoreversible [38-41] and irreversible [36,37,42], heat engines [36-38,40-42] and refrigerators [39] by taking the power and efficiency [36,37], profit rate [38,39] and ecological function [40-42] as optimization objectives. However, how about the unified description of heat pump cycles? Does a similar approach can also be applied to investigate the performance of heat pump cycles? To answer these questions, on the basis of irreversible Carnot heat pump cycle model [14-17,32,33], this paper will build a class of generalized irreversible universal steady flow heat pump cycle model consisting of two heat-absorbing branches, two heat-releasing branches and two adiabatic branches with the consideration of the losses of heat-resistance, heat leakage and internal irreversibility. The heating load, COP, exergy loss rate, exergy output rate and ecological function will be derived. Moreover, performance comparisons among maximum COP condition, a given exergy output rate condition and maximum ecological function condition will be carried out by using numerical examples. The results obtained include the heating load, COP, exergy loss rate, exergy output rate and ecological function characteristics of Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles with the losses of heat-resistance, heat leakage and internal irreversibility, which had been investigated in many literatures [10-20,22,30-34].

2. Cycle model

A universal heat pump cycle model with heat resistance, heat leakage and internal irreversibility coupled to two constant-temperature heat reservoirs T_H and T_L is shown in Fig. 1. The following assumptions are made for this model:

(1) It is assumed that the working fluid flows through the system in a quasi-steady fashion which has been done in many Refs. 13 to 25, 31 to 35, and 38 to 42. The cycle consists of two heat-absorbing branches with constant thermal capacities C_{in1} and C_{in2}, two heat- releasing branches with constant thermal capacities C_{out1} and C_{out2}, and two adiabatic branches. All six processes are irreversible.

(2) Because of the heat-transfer, the working-fluid temperatures are different from the reservoir temperatures.

(3) There exists a constant rate of bypass heat-leakage (q) from the heat-sink to the heat-source. This bypass heat leakage model was advanced first by Bejan [43,44]. Thus, one has $Q_H = Q_{H1} + Q_{H2} - q$ and $Q_L = Q_{L1} + Q_{L2} - q$, where $Q_{H1} + Q_{H2}$ is due to the driving force of $(T_5 - T_H)$ and $(T_4 - T_H)$, $Q_{L1} + Q_{L2}$ is due to the driving force of $(T_L - T_1)$ and $(T_L - T_2)$, Q_H is the rate of heat transfer released to the heat sink, *i.e.* the heating load of the cycle, and Q_L is the rate of heat transfer supplied by the heat source.

(4) A constant coefficient ϕ is introduced to characterize the additional internal miscellaneous irreversibility effects: $\phi = (Q_{H1} + Q_{H2})/(Q'_{H1} + Q'_{H2}) \geq 1$, where $Q_{H1} + Q_{H2}$ is the rate of heat-flow from the warm working-fluid to the heat-sink for the irreversible cycle model, while $Q'_{H1} + Q'_{H2}$ is that for the endoreversible cycle model with the only loss of heat-resistance.

To summarize, the generalized irreversible universal heat pump cycle model is characterized by the following two aspects:

(1) The different values of C_{in1}, C_{in2}, C_{out1} and C_{out2}. If C_{in1}, C_{in2}, C_{out1} and C_{out2} have different values, the cycle model can be changed into various special heat pump cycles.

(2) The different values of C_i and ϕ. If $q = 0$ and $\phi > 1$, the model is reduced to the irreversible universal heat pump cycle model with heat-resistance and internal irreversibility. If $q > 0$ and $\phi = 1$, the model is reduced to the irreversible universal heat pump cycle model with heat-resistance and heat-leakage losses. If $q = 0$ and $\phi = 1$, the model is reduced to the endoreversible universal heat pump cycle model with the only loss of heat-resistance.

According to the properties of working fluid and the theory of heat exchangers, the rate (q) of bypass heat-leakage from the heat-sink to the heat-source, the heat transfer rates (Q_{H1} and Q_{H2}) released to the heat sink and the heat transfer rates (Q_{L1} and Q_{L2}) supplied by heat source are, respectively, given by

$$q = C_i(T_H - T_L) \tag{3}$$

$$Q_{H1} = \dot{m}C_{out1}\,(T_5 - T_6) = \dot{m}C_{out1}E_{H1}\,(T_5 - T_H) \tag{4}$$

$$Q_{H2} = \dot{m}C_{out2}\,(T_4 - T_5) = \dot{m}C_{out2}E_{H2}\,(T_4 - T_H) \tag{5}$$

$$Q_{L1} = \dot{m}C_{in1}\,(T_2 - T_1) = \dot{m}C_{in1}E_{L1}\,(T_L - T_1) \tag{6}$$

$$Q_{L2} = \dot{m}C_{in2}\,(T_3 - T_2) = \dot{m}C_{in2}E_{L2}\,(T_L - T_2) \tag{7}$$

$$Q_H = Q_{H1} + Q_{H2} - q \tag{8}$$

$$Q_L = Q_{L1} + Q_{L2} - q \tag{9}$$

where $\dot{m}$ is mass flow rate of the working fluid, Q_H is the heating load of the cycle, E_{H1}, E_{H2}, E_{L1} and E_{L2} are the effectivenesses of the hot- and cold-side heat exchangers, and are defined as:

$$E_{H1} = 1 - \exp(-N_{H1}), \quad E_{H2} = 1 - \exp(-N_{H2})$$
$$E_{L1} = 1 - \exp(-N_{L1}), \quad E_{L2} = 1 - \exp(-N_{L2}) \tag{10}$$

where N_{H1}, N_{H2}, N_{L1} and N_{L2} are the numbers of heat transfer units of the hot- and cold- side heat exchangers, and are defined as:

$$N_{H1} = U_{H1}/(\dot{m}C_{out1}), \quad N_{H2} = U_{H2}/(\dot{m}C_{out2})$$
$$N_{L1} = U_{L1}/(\dot{m}C_{in1}), \quad N_{L2} = U_{L2}/(\dot{m}C_{in2}) \tag{11}$$

where U_{H1}, U_{H2}, U_{L1} and U_{L2} are the heat conductances, that is, the products of heat transfer coefficient α and heat transfer surface area F.

3. Performance analysis

Combining Eqs. (4) to (7) gives:

$$T_5 = (T_6 - E_{H1}T_H)\,/\,(1 - E_{H1}) \tag{12}$$

$$T_4 = \frac{[T_6 - T_H\,(E_{H1} + E_{H2}) + E_{H1}E_{H2}T_H]}{[(1 - E_{H1})\,(1 - E_{H2})]} \tag{13}$$

$$T_2 = (T_3 - E_{L2}T_L)/(1 - E_{L2}) \tag{14}$$

$$T_1 = \frac{[T_3 - T_L(E_{L1} + E_{L2}) + E_{L1}E_{L2}T_L]}{[(1 - E_{L1})(1 - E_{L2})]} \tag{15}$$

Consider the irreversible cycle $1-2-3-4-5-6-1$. Applying the second law of thermodynamics gives:

$$\phi = \frac{(Q_{H1} + Q_{H2})}{(Q'_{H1} + Q'_{H2})} = \frac{(C_{out1}\ln\frac{T_5}{T_6} + C_{out2}\ln\frac{T_4}{T_5})}{(C_{in1}\ln\frac{T_2}{T_1} + C_{in2}\ln\frac{T_3}{T_2})} \tag{16}$$

Equation (16) can be rewritten as:

$$T_2 = T_1 G \tag{17}$$

where

$$G = x^{\frac{C_{out1}}{\phi C_{in1}}} y^{-\frac{C_{in2}}{C_{in1}}} \times \left[\frac{T_6 - T_H\,(E_{H1} + E_{H2}) + E_{H1}E_{H2}T_H}{(1 - E_{H2})\,(T_6 - E_{H1}T_H)}\right]^{\frac{C_{out2}}{\phi C_{in1}}},$$

$x = T_5/T_6$ and $y = T_3/T_2$.

Combining Eqs. (14) to (17) gives:

$$T_1 = E_{L1}T_L/(G - 1 + E_{L1}) \tag{18}$$

$$T_2 = E_{L1}T_LG/(G - 1 + E_{L1}) \tag{19}$$

$$T_3 = yE_{L1}T_LG/(G - 1 + E_{L1}) \tag{20}$$

Combining Eqs. (4) to (7), (12), (13), and (18) to (20) gives the required power input of the cycle:

$$P = Q_H - Q_L = Q_{H1} + Q_{H2} - Q_{L1} - Q_{L2} = \dot{m}\frac{(T_6 - T_H)}{1 - E_{H1}}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - \frac{\dot{m}\,(G-1)\,T_L}{G - 1 + E_{L1}}\left[C_{in1}E_{L1} + C_{in2}E_{L2}\,(1 - E_{L1})\right] \tag{21}$$

Substituting Eqs. (3) to (5), (12), and (13) into (8) yields the heating load of the cycle:

$$Q_H = Q_{H1} + Q_{H2} - q = \dot{m}\frac{(T_6 - T_H)}{1 - E_{H1}}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - C_i\,(T_H - T_L) \tag{22}$$

Combining Eqs. (21) with (22) gives the COP and the exergy loss rate ($T_0\sigma$) of the cycle:

$$\beta = \frac{Q_H}{P} = \frac{\dot{m}\frac{(T_6 - T_H)}{1 - E_{H1}}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - C_i\,(T_H - T_L)}{\frac{\dot{m}(T_6 - T_H)}{1 - E_{H1}}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - \frac{\dot{m}(G-1)T_L}{G - 1 + E_{L1}}\left[C_{in1}E_{L1} + C_{in2}E_{L2}\,(1 - E_{L1})\right]} \tag{23}$$

$$T_0\sigma = T_0\,(Q_H/T_H - Q_L/T_L) = \frac{\dot{m}\,T_0\,(T_6 - T_H)}{(1 - E_{H1})\,T_H}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - \frac{\dot{m}\,T_0\,(G-1)}{G - 1 + E_{L1}} \times \left[C_{in1}E_{L1} + C_{in2}E_{L2}\,(1 - E_{L1})\right] + C_iT_0\,(T_H - T_L)^2\,/\,(T_HT_L) \tag{24}$$

Combining Eqs. (2), (21), and (22) gives the exergy output rate of the cycle:

$$A/\tau = Q_H(1 - T_0/T_H) - Q_L(1 - T_0/T_L) = \frac{\dot{m}\,(T_H - T_0)\,(T_6 - T_H)}{(1 - E_{H1})\,T_H}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right]$$

$$+ \frac{\dot{m}\,(G - 1)\,(T_0 - T_L)}{G - 1 + E_{L1}}\left[C_{in1}E_{L1} + C_{in2}E_{L2}\,(1 - E_{L1})\right] - T_0C_i\,(T_H - T_L)^2/(T_HT_L) \quad (25)$$

Substituting Eqs. (24) and (25) into (1) yields the ecological function of the cycle:

$$E = \frac{\dot{m}\,(T_H - 2T_0)\,(T_6 - T_H)}{(1 - E_{H1})\,T_H}\left[C_{out1}E_{H1} + \frac{C_{out2}E_{H2}}{(1 - E_{H2})}\right] - 2T_0C_i\,(T_H - T_L)^2/(T_HT_L)$$

$$+ \frac{\dot{m}\,(G - 1)\,(2T_0 - T_L)}{G - 1 + E_{L1}}\left[C_{in1}E_{L1} + C_{in2}E_{L2}\,(1 - E_{L1})\right] \quad (26)$$

In order to make the cycle operate normally, state point 2 must be between state points 1 and 3, and state point 5 must be between state points 4 and 6. Therefore, the ranges of x and y are:

$$1 \le x \le \frac{T_6 - T_H\,(E_{H1} + E_{H2}) + E_{H1}E_{H2}T_H}{T_6\,(1 - E_{H1})\,(1 - E_{H2})} \quad (27)$$

$$1 \le y \le x^{\frac{C_{out1}}{\phi C_{in2}}}\left[\frac{T_6 - T_H\,(E_{H1} + E_{H2}) + E_{H1}E_{H2}T_H}{(1 - E_{H2})\,(T_6 - E_{H1}T_H)}\right]^{\frac{C_{out2}}{\phi C_{in2}}} \quad (28)$$

4. Discussion

Equations (22) to (26) are universal relations governing heating load, COP, exergy loss rate, exergy output rate and ecological function of the universal steady flow irreversible heat pump cycle model with consideration of heat transfer loss, heat leakage and internal irreversibility. If C_{in1}, C_{in2}, C_{out1}, C_{out2}, C_i and ϕ have different values, Eqs. (22) to (26) can become the corresponding analytical formulae for various special cycles with different kinds of losses.

(1) When $C_{out1} = C_{out2} = C_p$, $C_{in1} = C_{in2} = C_p$ and $E_{H1} = 0$, $E_{L2} = 0$, $x = y = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Brayton heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. Moreover, the different values of C_i and ϕ represent the different kinds of loss models of Brayton heat pump cycles [18-20,22,34]. Specially, if $C_i = 0$, the performance characteristics of the generalized irreversible Brayton heat pump cycle with constant-temperature heat reservoirs change to the performance characteristics of endoreversible Brayton heat pump cycle with the only loss of heat-resistance [18-20,34] and irreversible Brayton heat pump cycle with the losses of heat-resistance and internal irreversibility [22].

(2) When $C_{out1} = C_{out2} = C_V$, $C_{in1} = C_{in2} = C_V$ and $E_{H1} = 0$, $E_{L2} = 0$, $x = y = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Otto heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Otto heat pump cycle.

(3) When $C_{out1} = C_{out2} = C_p$, $C_{in1} = C_{in2} = C_V$ and $E_{H1} = 0$, $E_{L2} = 0$, $x = y = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Diesel heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Diesel heat pump cycle.

(4) When $C_{out1} = C_{out2} = C_V$, $C_{in1} = C_{in2} = C_p$ and $E_{H1} = 0$, $E_{L2} = 0$, $x = y = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Atkinson heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Atkinson heat pump cycle.

(5) When $C_{out1} = C_V$, $C_{out2} = C_p$, $C_{in1} = C_{in2} = C_V$ and $E_{H1} \ne 0$, $E_{H2} \ne 0$, $E_{L2} = 0$, $y = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Dual heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. If

$E_{H1} = 0$, $E_{L2} = 0$ and $x = y = 1$ further, the Dual heat pump cycle becomes the Diesel heat pump cycle; and if $E_{H2} = 0$, $E_{L2} = 0$ and $y = 1$ further, the Dual heat pump cycle becomes the Otto heat pump cycle. In this case, the range of x is

$$1 \le x \le \frac{T_6 - T_H (E_{H1} + E_{H2}) + E_{H1} E_{H2} T_H}{T_6 (1 - E_{H1})(1 - E_{H2})} = \frac{T_4}{T_6} \quad (29)$$

and the value of x is given by:

$$x = \frac{T_5}{T_6} = \frac{(T_6 - E_{H1} T_H)}{(1 - E_{H1}) T_6} \quad (30)$$

Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Dual heat pump cycle.

(6) When $C_{out1} = C_{out2} = C_V$, $C_{in1} = C_p$, $C_{in2} = C_V$ and $E_{H1} = 0$, $E_{L1} \neq 0$, $E_{L2} \neq 0$, $x = 1$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Miller heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility. If $E_{H1} = 0$, $E_{L2} = 0$ and $x = y = 1$ further, the Miller heat pump cycle becomes the Atkinson heat pump cycle; and if $E_{H1} = 0$, $E_{L1} = 0$ and $x = 1$ further, the Miller heat pump cycle becomes the Otto heat pump cycle. In this case, the range of y is:

$$1 \le y \le \left[\frac{T_6 - T_H E_{H2}}{(1 - E_{H2}) T_6}\right]^{\frac{1}{\phi}} \quad (31)$$

Combining Eqs. (6), (7) and (17) yields the value of

$$y : \left[\frac{T_6 - T_H E_{H2}}{(1 - E_{H2}) T_6}\right]^{\frac{1}{\phi k}} = \frac{E_{L2}(1 - E_{L1}) y^{\frac{1}{k}}}{E_{L2} - E_{L1}(y - 1 + E_{L2})} \quad (32)$$

Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Miller heat pump cycle.

(7) When $C_{out1} = C_{out2} = C_{in1} = C_{in2} \to \infty$, Eqs. (22) to (26) can be simplified into the performance characteristics of the generalized irreversible steady flow Carnot heat pump cycle with the losses of heat-resistance, heat leakage and internal irreversibility [15-17,32,33]. Moreover, the different values of C_i and ϕ represent the different kinds of loss models of the Carnot heat pump cycle [10-18,30-33]. Specially, if $C_i = 0$ and $\phi = 1$, the performance characteristics of the generalized irreversible Carnot heat pump cycle change to the performance characteristics of the endoreversible Carnot heat pump cycle [10-13,18,30,31].

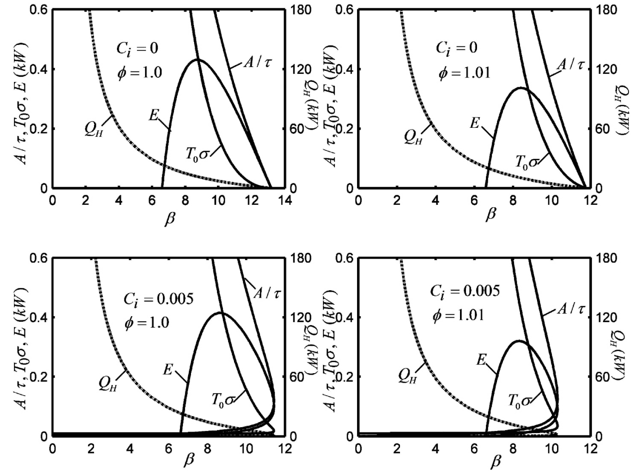

FIGURE 2. Performance characteristics of Brayton heat pump cycle with different loss terms.

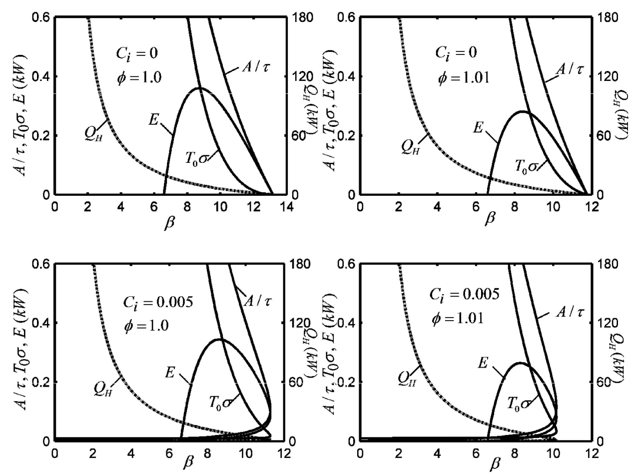

FIGURE 3. Performance characteristics of Otto heat pump cycle with different loss terms.

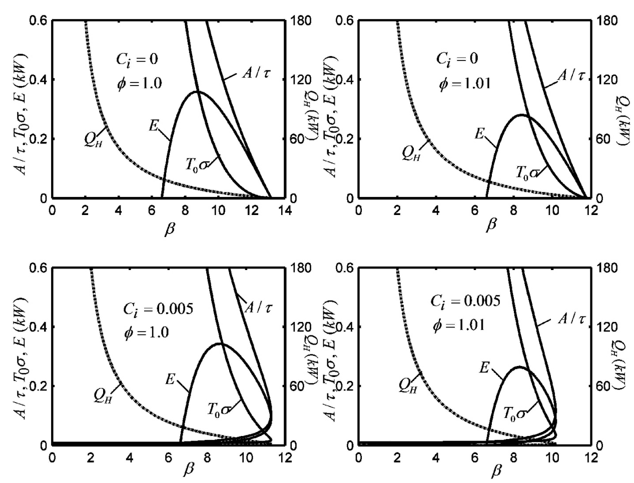

FIGURE 4. Performance characteristics of Diesel heat pump cycle with different loss terms.

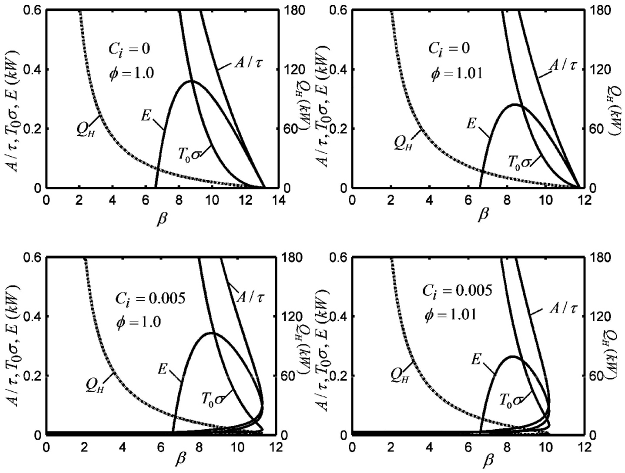

FIGURE 5. Performance characteristics of Atkinson heat pump cycle with different loss terms.

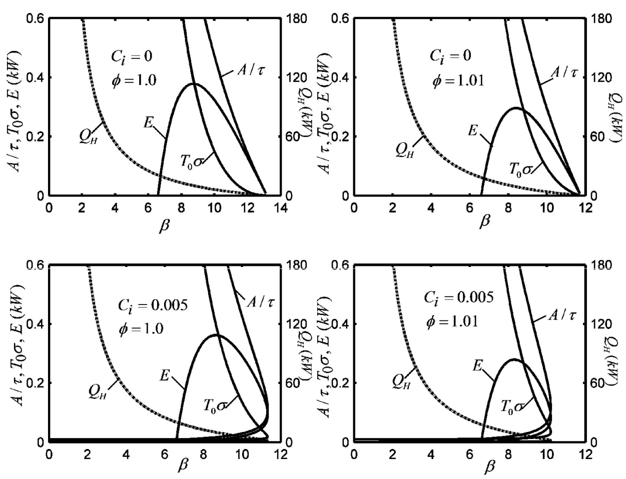

FIGURE 6. Performance characteristics of Dual heat pump cycle with different loss terms.

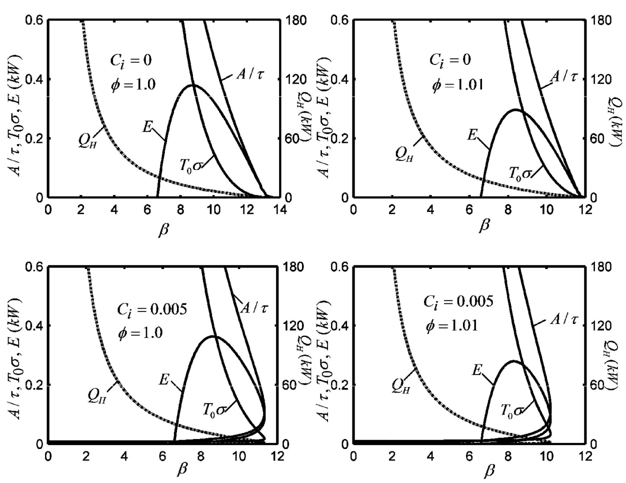

FIGURE 7. Performance characteristics of Miller heat pump cycle with different loss terms.

5. Numerical examples

To illustrate the preceding analysis, air is chosen as the working fluid of the universal steady flow heat pump cycle, and numerical examples are provided. In the calculations, it is set that T_H=290.0 K $T_L=T_0=268.0$ K, C_V=0.7165 kJ/(kg·K), k=1.4 C_p=1.0031 kJ/(kg·K), and $\dot{m}$=1.1165 kg/s. The effectivenesses of the heat exchangers are set at E_{H1}=0, E_{L2}=0, and $E_{H2}=E_{L1}$=0.9 for Brayton, Otto, Diesel and Atkinson cycles; $E_{H1}=E_{H2}=E_{L1}$=0.9, E_{L2}=0 for Dual cycle, as well as E_{H1}=0 and $E_{H2}=E_{L1}=E_{L2}$=0.9 for Miller cycle. The temperature of the working fluid T_6 is a variable and its reasonable value is greater than T_H

Figures 2-7 show the effect of heat leakage (q) and internal irreversibility (ϕ) on the performance characteristics for Brayton, Otto, Diesel Atkinson Dual and Miller heat pump cycles. It can be seen from these figures that heat leakage and internal irreversibility make the COP, exergy output rate and ecological function decrease and exergy loss rate increases for each typical heat pump cycle for the fixed heating load. When there is no heat leakage, *i.e.* $C_i = 0$ the heating load, exergy loss rate and exergy output rate decrease with the increase of COP while the curve of ecological function versus COP is a parabolic-like one. It shows that there exists a maximum ecological function ($E_{\max}$), and the corresponding heating load, COP, exergy loss rate and exergy output rate at the maximum ecological function are $Q_{H,\,E}$, β_E, $T_0\sigma_E$ and A_E/τ, respectively. When there is heat leakage, *i.e.* $C_i = 0.005$ kW/K, the curves of the heating load, exergy loss rate and exergy output rate versus COP are parabolic-like ones. There exists a maximum COP ($\beta_{\max}$), and the corresponding heating load, exergy loss rate and exergy output rate at maximum COP are $Q_{H,\,\beta}$, $T_0\sigma_\beta$ and A_β/τ, respectively. The curve of the ecological function versus COP characteristic is loop-shaped one. That is, there not only exists a maximum ecological function ($E_{\max}$) with the corresponding COP (β_E), but also exists a maximum COP ($\beta_{\max}$) with the corresponding ecological function (E_β). Therefore, the influence of q is different from the influence of ϕ on the performance of the universal heat pump cycle. The internal irreversibility only reduces the performance of the universal heat pump cycle, *i.e.*, only has influence quantitatively; the heat leakage changes the curves of the ecological function versus COP characteristic from a parabolic-like one into a loop-shaped one, *i.e.*, has the influence both quantitatively and qualitatively.

Figure 8 shows the $A/\tau - \beta$, $T_0\Delta S - \beta$, $E - \beta$ and $Q_H - \beta$ characteristics of the generalized irreversible Brayton heat pump cycle with C_i=0.005 kW/K and ϕ=1.01. The other cycles have similar performance characteristics. For this numerical example, the maximum COP is β_{max}= 10.301, and the corresponding exergy output rate, exergy loss rate, ecological function and heating load are A_β/τ=0.125 kW, $T_0\sigma_\beta$=0.035 kW, E_β=0.090 kW and $Q_{H,\beta}$=1.645 kW, respectively. The maximum ecological

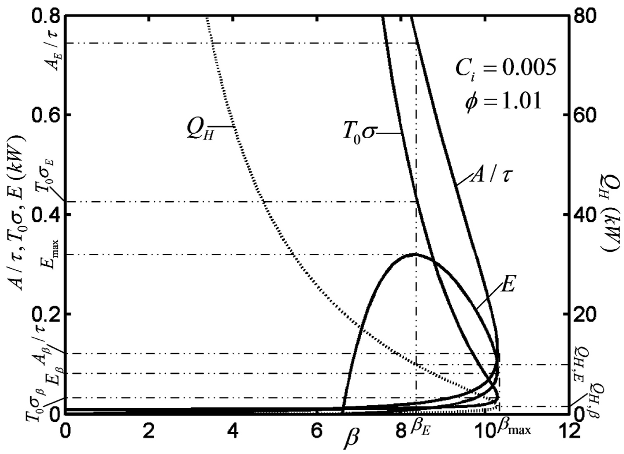

FIGURE 8. $A/\tau - \beta$, $T_0\sigma - \beta$, $E - \beta$ and $Q_H - \beta$ characteristics for the generalized irreversible Brayton heat pump cycle.

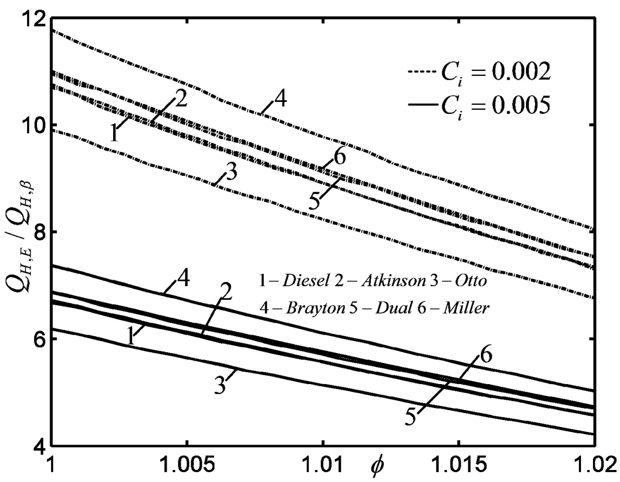

FIGURE 9. Effects of heat leakage on $Q_{H,E}/Q_{H,\beta} - \phi$ characteristics for six heat pump cycles.

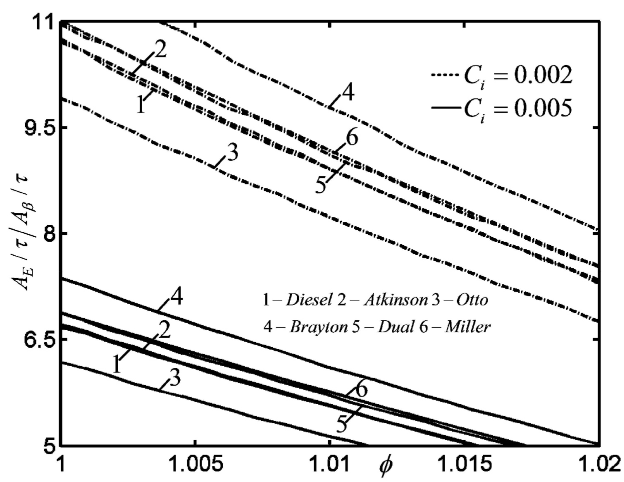

FIGURE 10. Effects of heat leakage on $(A_E/\tau)/(A_\beta/\tau) - \phi$ characteristics for six heat pump cycles.

function is $E_{\max}$=0.321 kW, and the corresponding exergy output rate, exergy loss rate, heating load and COP are $A_E/\tau = 0.770$ kW, $T_0\sigma_E = 0.449$ kW, $Q_{H,E} = 10.143$ kW and $\beta_E = 8.324$, respectively. Comparing the maximum COP condition with the maximum ecological function condition, the exergy output rate decreases by 83.78%, the ecological function decreases by 71.96% and the heating load decreases by 83.78% at the maximum COP condition, though the COP increases by 19.19% and exergy loss rate decreases by 92.20%. If the Brayton heat pump operates at a new condition where $A/\tau = 0.901$ kW, comparing the new condition with the maximum ecological function condition the COP decreases by 4.26%, the ecological function decreases by 2.69% and the exergy loss rate increases by 31.29% at the new condition though both the exergy output rate and heating load increase by 17.13%

Figures 9 and 10 show the effects of heat leakage (C_i) on $Q_{H,E}/Q_{H,\beta}$ versus ϕ and $(A_E/\tau)/(A_\beta/\tau)$ versus ϕ characteristics for six heat pump cycles, respectively. Figure 9 indicates that the heating load increases sharply when the work condition is selected at the maximum ecological function, which is important when the heating load is required to some extent. Meanwhile, $Q_{H,E}/Q_{H,\beta}$ decreases with the increase of C_i for the fixed ϕ; $Q_{H,E}/Q_{H,\beta}$ decreases with the increase of ϕ when C_i is taken into account. Moreover, the Brayton heat pump cycle has the maximum value of $Q_{H,E}/Q_{H,\beta}$ among the six typical heat pump cycles for the fixed C_i and ϕ, and the Otto heat pump cycle has the minimum one. $(A_E/\tau)/(A_\beta/\tau)$ versus ϕ characteristic is similar with $Q_{H,E}/Q_{H,\beta}$ versus ϕ characteristic as shown in Fig. 10.

From Figs. 2-10 one can see that under condition that the heating load is required to some extent, the optimization of the exergy-based ecological function makes the larger decrease of the exergy loss rate and the improvement of the COP with the cost of a little amount of exergy output rate. Therefore, the optimization of the exergy-based ecological function shows a compromise between the exergy output rate and the exergy loss rate and also shows a compromise between the heating load and the COP, and it represents a new energy utilization mode which is effective and long-term.

6. Conclusion

Based on the irreversible Carnot heat pump cycle model [14-17,32,33], a generalized irreversible universal steady flow heat pump cycle model is established in this paper. The universal heat pump cycle consists of two heat-absorbing branches, two heat-releasing branches and two irreversible adiabatic branches, and with the losses of heat-resistance, heat leakage and internal irreversibility. The heating load, exergy loss rate, exergy output rate and ecological function versus COP for Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles are analyzed by numerical examples. The optimization of the exergy-based ecological function makes the larger reduction of the exergy loss rate and the improvement of the COP with the cost of

Nomenclature	
A	exergy output of the cycle (kJ)
C_i	heat leakage coefficient (kW/K)
C_p	constant pressure specific heat [kW/(kg · K)]
C_V	constant volume specific heat [kW/(kg · K)]
E	ecological function of the cycle (kW)/ effectiveness of the heat exchanger
F	heat transfer surface area (m^2)
k	ratio of the specific heats
$\dot{m}$	mass flow rate (kg/s)
N	number of heat transfer units
P	power input of the cycle (kW)
Q	rate of heat transfer (kW)
q	heat leakage rate (kW)
T	temperature (K)
U	heat conductance (kW/K)
x	temperature ratio of the working fluid
y	temperature ratio of the working fluid
Greek symbols	
α	heat-transfer coefficient [kW /(K $\cdot m^2$)]
ϕ	internal irreversibility coefficient
β	coefficient of performance (COP)
τ	period of the cycle (s)
ΔS	entropy generation of the cycle (kJ/K)
σ	entropy generation rate of the cycle (kW/K)
Subscripts	
E	ecological
$H, H1, H2$	hot side/heat sink
$in1, in2$	input
$L, L1, L2$	cold side/heat source
max	maximum
$out1, out2$	output
β	coefficient of performance
0	ambient
1, 2, 3, 4, 5, 6	state points of the model cycle

a little amount of exergy output rate. Thus, the ecological criterion shows a compromise between the exergy output rate and the exergy loss rate and also shows a compromise between the heating load and the COP. It is believed that the ecological function objective is an excellent candidate objective compared with exergy output rate, exergy loss rate, heating load and COP with the ideal of ecological and long-term goal, and it is beneficial to the utilization of energy. The results obtained include the performance characteristics of generalized irreversible steady flow Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles and can provide some significant guidelines for the optimal design and operation of the real heat pumps.

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