

Ecological performance optimization of a thermoacoustic heat engine

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Ecological performance optimization of a generalized irreversible thermoacoustic heat engine with heat resistance, heat leakage, thermal relaxation and internal dissipation, in which heat transfer between the working fluid and heat reservoirs obeys a complex generalized heat transfer law $Q \propto \Delta(T)^n$ -where n is a complex-, is investigated in this paper. Both the real part and the imaginary part of the complex heat transfer exponent change the optimal ecological function versus efficiency relationship, quantitatively. The analytical formulas about the ecological function and thermal efficiency of the thermoacoustic heat engine are derived. Furthermore, the comparative analysis of the influences of various factors on optimal performance of the generalized irreversible thermoacoustic heat engine is carried out by detailed numerical examples. The optimal zone on the performance of the thermoacoustic heat engine is obtained by numerical analysis. The results obtained herein can provide some theoretical guidelines for the design of real thermoacoustic engines.

Keywords: Thermoacoustic heat engine; complex heat transfer exponent; ecological optimization; finite-time thermodynamics.

En este artículo se investigan la optimización del desempeño ecológico de un motor termoacústico irreversible generalizado con resistencia térmica, fugas de calor, relajación térmica y disipación interna, en el que la transferencia de calor entre el fluido de trabajo y el reservorio (recipiente) de calor obedece a una ley de transferencia de calor compleja generalizada [], donde n es complejo. La parte imaginaria y la parte real del exponente complejo de la transferencia de calor cambian cuantitativamente a la función ecológica óptima vs la relación de eficiencia. Se obtienen las fórmulas analíticas de la función ecológica y la eficiencia térmica del motor termoacústico. Asimismo, se hace un análisis comparativo de la influencia de varios factores cuando el desempeño del motor termoacústico irreversible generalizado es óptimo con ejemplos numéricos detallados. Se obtiene la zona óptima del desempeño del motor termoacústico por análisis numérico. Los resultados obtenidos proveen guías teóricas para el diseño de motores termoacústicos.

Descriptores: Motor térmico termoacústico; exponente complejo de transferencia de calor; optimización ecológica; termodinámica de tiempo finito.

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1. Introduction

Thermoacoustic engines (including prime mover and refrigerator) [1-4] are inherently simple, reliable and reasonably efficient, because of not having any moving parts or few moving parts and because they work with environmentally friendly fluids and materials. These advantages can meet well those requirements of the international ban on the production of CFC's, and of the discovery of high T_c superconductors, and the development of high speed and high density electronic circuits for active cryogenic cooling systems with sufficient availability, reliability, efficiency and feasibility for the environment. With this great potential, more and more engineers in the power and cryogenic engineering have been investigating the thermoacoustic engine.

Recently, Wu *et al.* [5-7] have studied the performance of generalized irreversible thermoacoustic engine (or cooler) cycle using the finite-time thermodynamics (FTT) [8-30]. Most of the previous works in the FTT have concentrated on power optimization, or the minimization of fixed cost for a heat engine. Another criterion for heat engines is the thermal-efficiency optimization that can be considered as the

variable-cost minimization. Alternatively, Angulo-Brown *et al.* [31] proposed the ecological criterion $E' = P' - T_L \sigma'$ for finite-time Carnot heat engines, where T_L is the temperature of the cold heat reservoir, P' is the power output and σ' is the entropy generation rate. Yan [32] showed that it might be more reasonable to use $E' = P' - T_0 \sigma'$ if the cold-reservoir temperature T_L is not equal to the environment temperature T_0 from the point of view of exergy analysis. The optimization of the ecological function represents a compromise between the power output P' and the loss power $T_0 \sigma'$, which is produced by entropy generation in the system and its surroundings. Furthermore, based on the point of view of exergy analysis, Chen *et al.* [33-36] provided a unified exergy-based ecological optimization objective for all of thermodynamic cycles, that is $E = A/\tau - T_0 \Delta S/\tau = A/\tau - T_0 \sigma$, where A is the exergy output of the cycle, T_0 is the environment temperature of the cycle, ΔS is the entropy generation of the cycle, τ is the cycle period, and σ is the entropy generation rate of the cycle. It represents the best compromise between the exergy output rate and the exergy loss rate (entropy generation rate) of the thermodynamic cycle. The ecological optimization has been carried out for endoreversible and irreversible

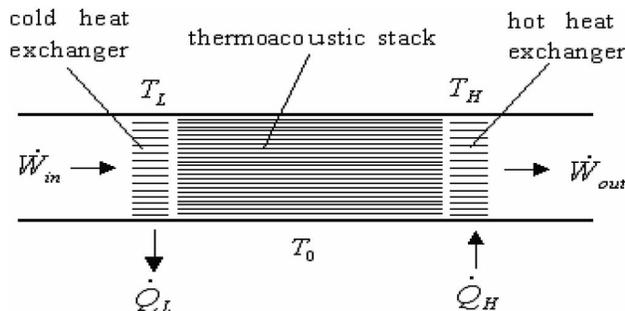


FIGURE 1. Energy flows in a thermoacoustic heat engine.

Carnot, Braysson, Brayton, Stirling and Ericsson heat engines, refrigerators, and heat pumps [37-54].

In the analysis of many papers assessing the influence of the heat transfer law on the ecological performance optimization of irreversible Carnot heat engines [45,46], the heat transfer exponent is assumed to be a real. But for thermoacoustic heat engines, the stack and two adjacent heat exchangers are the principle parts. While the acoustic wave carrying the working gas back and forth within these components, a longitudinal pressure oscillating in the sound channel induces a temperature oscillation in time with the angular frequency ω . In these circumstances, the gas temperature can be taken as complex. It results in a time-averaged heat exchange with complex exponent between the gas and the environment by hot and cold-side heat exchangers.

In this paper, the optimal ecological performance of a generalized irreversible thermoacoustic heat engine, with the losses of: heat resistance, heat leakage and internal irreversibility, in which the heat transfer between the working fluid and the heat reservoirs obeys a generalized heat transfer law $Q \propto \Delta(T^{n_1+n_2i})$, where n is a complex, is derived by taking an ecological optimization criterion as the objective. Numerical examples are provided to show the effects of complex heat transfer exponent, heat leakage and internal irreversibility on the optimal performance of the generalized irreversible thermoacoustic heat engine.

2. The model of thermoacoustic heat engine

The energy flow in a thermoacoustic heat engine is schematically illustrated in Fig. 1, where \dot{W}_{in} and \dot{W}_{out} are the flow of power inside the acoustic channel. To simulate the performance of a real thermoacoustic engine more realistically, the following assumptions are made for this model.

- (1) External irreversibility is caused by heat-transfers in the hot and cold-side of heat exchangers between the engine and its surrounding heat reservoirs. Because of the heat-transfers, the time average temperature (T_{H0} and T_{L0}) of the working fluid are different from the heat-reservoir temperatures (T_H and T_L). The second law of thermodynamics requires $T_H > T_{H0} > T_{L0} > T_L$.

As a result of thermoacoustic oscillation, the temperatures (T_{HC} and T_{LC}) of the working fluid can be expressed as complexes:

$$T_{HC} = T_{H0} + T_1 e^{i\omega t} \tag{1}$$

$$T_{LC} = T_{L0} + T_2 e^{i\omega t} \tag{2}$$

where T_1 and T_2 are the first-order acoustic quantities, and $i = \sqrt{-1}$. Here the reservoir temperatures (T_H and T_L) are assumed as real constants.

- (2) Consider that the heat transfer between the engine and its surroundings follows a generalized law $Q \propto \Delta(T^n)$, then

$$\dot{Q}'_{HC} = k_1 F_1 (T_H^n - T_{HC}^n) \text{sgn}(n_1) \tag{3}$$

$$\dot{Q}'_{LC} = k_2 F_2 (T_{LC}^n - T_L^n) \text{sgn}(n_1) \tag{4}$$

with sign function

$$\text{sgn}(n_1) = \begin{cases} 1 & n_1 > 0 \\ -1 & n_1 < 0 \end{cases} \tag{5}$$

where $n = n_1 + n_2i$ is a complex heat transfer exponent, k_1 is the generalized overall heat transfer coefficient and F_1 is the total heat transfer surface area of the hot-side heat exchanger, k_2 is the generalized overall heat transfer coefficient and F_2 is the total heat transfer surface area of the cold-side heat exchanger. Here the imaginary part n_2 of n indicates the relaxation of a heat transfer process. Defining $\dot{Q}_{HC} = \langle \dot{Q}'_{HC} \rangle_t$ and $\dot{Q}_{LC} = \langle \dot{Q}'_{LC} \rangle_t$ as the time average of \dot{Q}'_{HC} and \dot{Q}'_{LC} , respectively, Eqs. (3) and (4) can be rewritten as

$$\dot{Q}_{HC} = \frac{k_1 F_T}{1+f} (T_H^n - T_{H0}^n) \text{sgn}(n_1) \tag{6}$$

$$\dot{Q}_{LC} = \frac{k_2 F_T f}{1+f} (T_{L0}^n - T_L^n) \text{sgn}(n_1) \tag{7}$$

where $f = F_2/F_1$ and $F_T = F_1 + F_2$. Here, the total heat transfer surface area F_T of the two heat exchangers is assumed to be a constant.

- (3) There is a constant rate of heat leakage (q) from the heat source at the temperature T_H to heat sink at T_L such that

$$\dot{Q}_H = \dot{Q}_{HC} + q \tag{8}$$

$$\dot{Q}_L = \dot{Q}_{LC} + q \tag{9}$$

where \dot{Q}_H and \dot{Q}_L are the rates of total heat-transfer absorbed from the heat source and released to the heat sink.

- (4) Other than irreversibilities due to heat resistance between the working substance and the heat reservoirs, as well as the heat leakage between the heat reservoirs,

there are more irreversibilities such as friction, turbulence, and non-equilibrium activities inside the engine. Thus the power output produced by the irreversible thermoacoustic heat engine is less than that of the endoreversible thermoacoustic engine with the same heat input. In other words, the rate of heat flow (\dot{Q}_{LC}) from the cold working fluid to the heat sink for the irreversible thermoacoustic heat engine is larger than that of (\dot{Q}'_{LC}) the endoreversible thermoacoustic heat engine with the same heat input. A constant coefficient (ϕ) is introduced in the following expression to characterize the additional miscellaneous irreversible effects:

$$\phi = \dot{Q}_{LC}/\dot{Q}'_{LC} \geq 1 \tag{10}$$

The thermoacoustic heat engine being satisfied with above assumptions is called the generalized irreversible thermoacoustic heat engine with a complex heat transfer exponent.

3. Optimal characteristics

For an endoreversible thermoacoustic heat engine, the second law of thermodynamics requires

$$\dot{Q}'_{LC}/T_{L0} = \dot{Q}_{HC}/T_{H0} \tag{11}$$

Combining Eqs. (10) and (11) gives

$$\dot{Q}_{LC} = \phi x \dot{Q}_{HC} \tag{12}$$

where $x = T_{L0}/T_{H0}$ ($T_L/T_H \leq x \leq 1$) is the temperature ratio of the working fluid.

Combining Eqs. (6)-(12) yields

$$T_{H0}^n = \frac{k_1 f \phi x T_H^n + k_2 T_L^n}{k_2 x^n + k_1 x f \phi} \tag{13}$$

$$\dot{Q}_{HC} = \frac{k_1 f F_T (x^n T_H^n - T_L^n)}{(1+f)(x^n + \phi x f k_1/k_2)} \text{sgn}(n_1) \tag{14}$$

$$\dot{Q}_{LC} = \phi x \frac{k_1 f F_T (x^n T_H^n - T_L^n)}{(1+f)(x^n + \phi x f k_1/k_2)} \text{sgn}(n_1) \tag{15}$$

The first law of thermodynamics gives that the power output, the efficiency and the entropy generation rate of the thermoacoustic heat engine are, respectively

$$P' = \dot{Q}_H - \dot{Q}_L = \dot{Q}_{HC} - \dot{Q}_{LC} \tag{16}$$

$$\eta' = P'/\dot{Q}_H = (\dot{Q}_{HC} - \dot{Q}_{LC})/(\dot{Q}_{HC} + q) \tag{17}$$

$$\begin{aligned} \sigma' &= \Delta S/\tau = \dot{Q}_L/T_L - \dot{Q}_H/T_H \\ &= (\dot{Q}_{LC} + q)/T_L - (\dot{Q}_{HC} + q)/T_H \end{aligned} \tag{18}$$

From Eqs. (14)-(18), one can obtain the complex power output (P'), the complex efficiency (η') and the complex entropy generation rate (σ') of the engine

$$P' = \dot{Q}_{HC} - \dot{Q}_{LC} = \frac{k_1 f F_T (1 - \phi x) [T_H^n - (T_L/x)^n]}{(1+f)(1 + \phi \delta f x^{1-n})} \text{sgn}(n_1) \tag{19}$$

$$\eta' = \frac{k_1 f F_T (1 - \phi x) [T_H^n - (T_L/x)^n]}{q(1+f)(1 + \phi \delta f x^{1-n}) + k_1 f F_T [T_H^n - (T_L/x)^n]} \text{sgn}(n_1) \tag{20}$$

$$\sigma' = (\phi x/T_L - 1/T_H) \frac{k_1 f F_T [T_H^n - (T_L/x)^n]}{(1+f)(1 + \phi \delta f x^{1-n})} \text{sgn}(n_1) - q(1/T_L - 1/T_H) \tag{21}$$

where $\delta = k_1/k_2$.

Substituting Eqs. (17) and (19) into ecological function $E' = P' - T_0 \sigma'$ yields

$$E' = [(1 + T_0/T_H) - \phi x (1 + T_0/T_L)] \frac{k_1 f F_T [T_H^n - (T_L/x)^n]}{(1+f)(1 + \phi \delta f x^{1-n})} \text{sgn}(n_1) + q(T_0/T_H - T_0/T_L) \tag{22}$$

From Eqs. (20) and (22), one can obtain the real parts of the efficiency and ecological function that are, respectively

$$\eta = R_e(\eta') = \frac{(1 - \phi x) [A_1 (A_1 + B_1) + A_2 (A_2 + B_2)]}{(A_1 + B_1)^2 + (A_2 + B_2)^2} \tag{23}$$

$$\begin{aligned} E &= R_e(E') = \frac{A_1 [1 + f \phi \delta x^{1-n_1} \cos(n_2 \ln x)] - A_2 f \phi \delta x^{1-n_1} \sin(n_2 \ln x)}{1 + 2\delta \phi f x^{1-n_1} \cos(n_2 \ln x) + f^2 \delta^2 \phi^2 x^{2(1-n_1)}} \\ &\times \frac{k_1 f F_T [(1 + T_0/T_H) - \phi x (1 + T_0/T_L)]}{1 + f} + q(T_0/T_H - T_0/T_L) \end{aligned} \tag{24}$$

where

$$B_1 = \frac{q(1+f)}{k_1 f F_T} [1 + \phi \delta f x^{1-n_1} \cos(n_2 \ln x)], \quad B_2 = -\frac{q(1+f) \phi \delta f x^{1-n_1} \sin(n_2 \ln x)}{k_1 f F_T},$$

$$A_1 = R_e [T_H^n - (T_L/x)^n] \operatorname{sgn}(n_1),$$

and $A_2 = I_m [T_H^n - (T_L/x)^n] \operatorname{sgn}(n_1)$, where $R_e(\cdot)$ and $I_m(\cdot)$ indicate the real and imaginary parts of the complex number.

Maximizing η and E with respect to f by setting $d\eta/df = 0$ or $dE/df = 0$ in Eqs. (23) and (24) yields the same optimal ratio of heat-exchanger area (f_{opt})

$$f = f_{opt} = \frac{1}{4}(b - \sqrt{8y + b^2 - 4c}) + \frac{1}{2} \left[\frac{1}{4} (b - \sqrt{8y + b^2 - 4c})^2 - 4 \left(y - \frac{by - d}{\sqrt{8y + b^2 - 4c}} \right) \right]^{0.5} \quad (25)$$

where

$$y = \left\{ -\frac{e}{2} + \left[\left(\frac{e}{2} \right)^2 - \left(\frac{c^2}{36} \right)^3 \right]^{0.5} \right\}^{1/3} + \left\{ -\frac{e}{2} - \left[\left(\frac{e}{2} \right)^2 - \left(\frac{c^2}{36} \right)^3 \right]^{0.5} \right\}^{1/3} + \frac{c}{6} \quad (26)$$

$$b = \frac{2A_1 x^{n_1-1}}{A_1 \cos(n_2 \ln x) - A_2 \sin(n_2 \ln x)} \quad (27)$$

$$c = \frac{2A_1 x^{2n_1-2} \cos(n_2 \ln x) + A_1 \phi \delta x^{n_1-1}}{\phi \delta [A_1 \cos(n_2 \ln x) - A_2 \sin(n_2 \ln x)]} - \frac{x^{2n_1-1}}{(\phi \delta)^2} - \frac{2x^{n_1-1} \cos(n_2 \ln x)}{\phi \delta} \quad (28)$$

$$d = -2x^{2n_1-2}/(\phi \delta)^2 \quad (29)$$

$$e = \frac{e_1 c}{2} - \frac{c^3}{108} - \frac{A_1^2 e_1 x^{2n_1-2}}{2 [A_1 \cos(n_2 \ln x) - A_2 \sin(n_2 \ln x)]^2} - \frac{x^{4n_1-4}}{2(\phi \delta)^4} \quad (30)$$

$$e_1 = \frac{A_1 x^{3n_1-3}}{(\phi \delta)^2 [A_1 \cos(n_2 \ln x) - A_2 \sin(n_2 \ln x)]} \quad (31)$$

Substituting Eq. (25) into Eqs. (23) and (24), respectively, yields the optimal thermal efficiency and ecological function in the following forms:

$$\eta = \left\{ \frac{(1 - \phi x) [A_1 (A_1 + B_1) + A_2 (A_2 + B_2)]}{(A_1 + B_1)^2 + (A_2 + B_2)^2} \right\}_{f=f_{opt}} \quad (32)$$

$$E = \frac{A_1 [1 + f_{opt} \phi \delta x^{1-n_1} \cos(n_2 \ln x)] - A_2 f_{opt} \phi \delta x^{1-n_1} \sin(n_2 \ln x)}{1 + 2\delta \phi f_{opt} x^{1-n_1} \cos(n_2 \ln x) + f_{opt}^2 \delta^2 \phi^2 x^{2(1-n_1)}} \times \frac{k_1 f_{opt} F_T [(1 + T_0/T_H) - \phi x (1 + T_0/T_L)]}{1 + f_{opt}} + q (T_0/T_H - T_0/T_L) \quad (33)$$

The parameter equation defined by Eqs. (32) and (33) gives the fundamental relationship between the optimal ecological function versus thermal efficiency consisting of the interim variable.

Maximizing E with respect to x by setting $dE/dx = 0$ in Eq. (33) can yield the optimal temperature ratio x_{opt} and the maximum ecological function E_{max} of the thermoacoustic heat engine. The corresponding efficiency η_E can be obtained by substituting the optimal temperature ratio into Eq. (32).

4. Discussions

If $\phi = 1$ and $q \neq 0$, Eqs. (32) and (33) become:

$$\eta = \left\{ \frac{(1-x)[A_1(A_1+B_1) + A_2(A_2+B_2)]}{(A_1+B_1)^2 + (A_2+B_2)^2} \right\}_{f=f_{opt}} \quad (34)$$

$$E = \frac{A_1 [1 + f_{opt} \delta x^{1-n_1} \cos(n_2 \ln x)] - A_2 f_{opt} \delta x^{1-n_1} \sin(n_2 \ln x)}{1 + 2\delta f_{opt} x^{1-n_1} \cos(n_2 \ln x) + f_{opt}^2 \delta^2 x^{2(1-n_1)}} \\ \times \frac{k_1 f_{opt} F_T [(1 + T_0/T_H) - x(1 + T_0/T_L)]}{1 + f_{opt}} + q(T_0/T_H - T_0/T_L) \quad (35)$$

Equations (34) and (35) are the relationship between the efficiency and the ecological function of the irreversible thermoacoustic heat engine with heat resistances and heat leakage losses.

If $\phi > 1$ and $q = 0$, Eqs. (32) and (33) become:

$$\eta = \left\{ \frac{(1-\phi x)[A_1(A_1+B_1) + A_2(A_2+B_2)]}{(A_1+B_1)^2 + (A_2+B_2)^2} \right\}_{f=f_{opt}, q=0} \quad (36)$$

$$E = \frac{A_1 [1 + f_{opt} \phi \delta x^{1-n_1} \cos(n_2 \ln x)] - A_2 f_{opt} \phi \delta x^{1-n_1} \sin(n_2 \ln x)}{1 + 2\delta \phi f_{opt} x^{1-n_1} \cos(n_2 \ln x) + f_{opt}^2 \delta^2 \phi^2 x^{2(1-n_1)}} \\ \times \frac{k_1 f_{opt} F_T [(1 + T_0/T_H) - \phi x(1 + T_0/T_L)]}{1 + f_{opt}} \quad (37)$$

Equations (36) and (37) are the relationship between the efficiency and the ecological function of the irreversible thermoacoustic heat engine with heat resistance and internal irreversibility losses.

If $\phi = 1$ and $q = 0$, Eqs. (32) and (33) become:

$$\eta = \left\{ \frac{(1-x)[A_1(A_1+B_1) + A_2(A_2+B_2)]}{(A_1+B_1)^2 + (A_2+B_2)^2} \right\}_{f=f_{opt}, q=0} \quad (38)$$

$$E = \frac{A_1 [1 + f_{opt} \delta x^{1-n_1} \cos(n_2 \ln x)] - A_2 f_{opt} \delta x^{1-n_1} \sin(n_2 \ln x)}{1 + 2\delta f_{opt} x^{1-n_1} \cos(n_2 \ln x) + f_{opt}^2 \delta^2 x^{2(1-n_1)}} \\ \times \frac{k_1 f_{opt} F_T [(1 + T_0/T_H) - x(1 + T_0/T_L)]}{1 + f_{opt}} \quad (39)$$

Equations (38) and (39) are the relationship between the efficiency and the ecological function of the endoreversible thermoacoustic heat engine.

5. Numerical examples

To illustrate the preceding analysis, numerical examples are provided. In the calculations, it is set, that $T_H=1200$ K, $T_L=400$ K, $T_0=298.15$ K; $k_1=k_2$; $\phi=1.0, 1.1, 1.2$; $q=C_i(T_H^n-T_L^n)$ (same as Ref. 35) and $C_i=0.0, 0.02$ kW/K; C_i is the thermal conductance inside the thermoacoustic heat engine.

Figures 2 and 3 show the effects of the heat leakage, the internal irreversibility losses and the heat transfer exponent on the relationship between the ecological function and

thermal efficiency. One can see that for all heat transfer laws, the influences of the internal irreversibility losses and the heat leakage on the relationship between the ecological function and efficiency are obviously different: The ecological function E decreases along with increasing of the internal irreversibility ϕ , but the curves of $E - \eta$ are not changeable; the heat leakage affects strongly the relationship between the ecological and efficiency functions, the curves of $E - \eta$ are

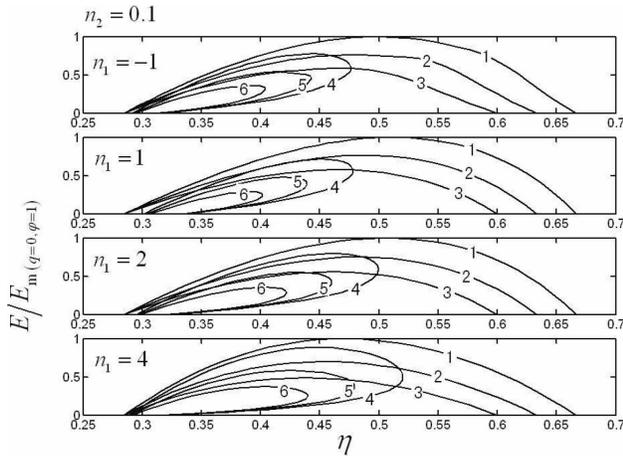


FIGURE 2. $E - \eta$ performance characteristic for the thermoacoustic engine with $n_2 = 0.1$: 1. $\varphi = 1, C_i = 0$; 2. $\varphi = 1.1, C_i = 0$; 3. $\varphi = 1.2, C_i = 0$; 4. $\varphi = 1, C_i > 0$; 5. $\varphi = 1.1, C_i > 0$; 6. $\varphi = 1.2, C_i > 0$.

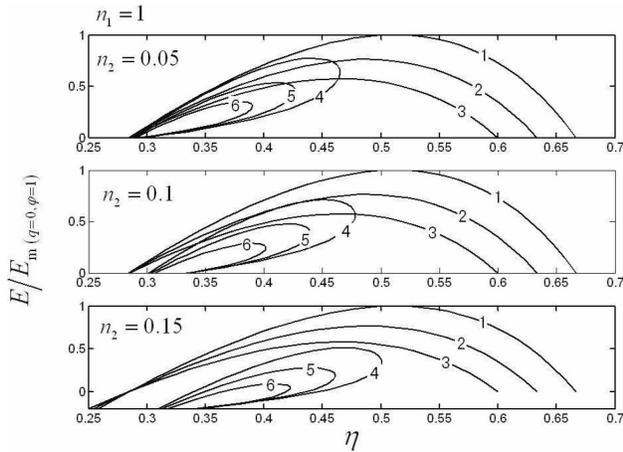


FIGURE 3. $E - \eta$ performance characteristic for the thermoacoustic engine with $n_1 = 1$: 1. $\varphi = 1, C_i = 0$; 2. $\varphi = 1.1, C_i = 0$; 3. $\varphi = 1.2, C_i = 0$; 4. $\varphi = 1, C_i > 0$; 5. $\varphi = 1.1, C_i > 0$; 6. $\varphi = 1.2, C_i > 0$.

parabolic-like, in the case of $q = 0$, while the curves are loop-shaped, in the case of $q \neq 0$.

From Figs. 2 and 3, one can also see that both the real part n_1 and the imaginary part n_2 of the complex heat transfer exponent n don't change the parabolic-like or the loop-shaped forms of the curves of $E - \eta$. Figure 2 illustrates that when the imaginary part $n_2 = 0.1$ is fixed, the corresponding efficiency η_E at the maximum ecological function decreases with the increase of absolute value of the real part n_1 . The reason is that the power output is sensitive to the temperature; when the absolute value of the real part n_1 increases, it sacrifices a little part of the temperature ratio and decreases the thermal efficiency to some extent, but increases the power output to a great extent, which is induced by the increases of the temperature differences between the heat exchangers and the working fluid. Figure 3 shows that when the real part $n_1 = 1$ is fixed, the maximum ecological function decreases

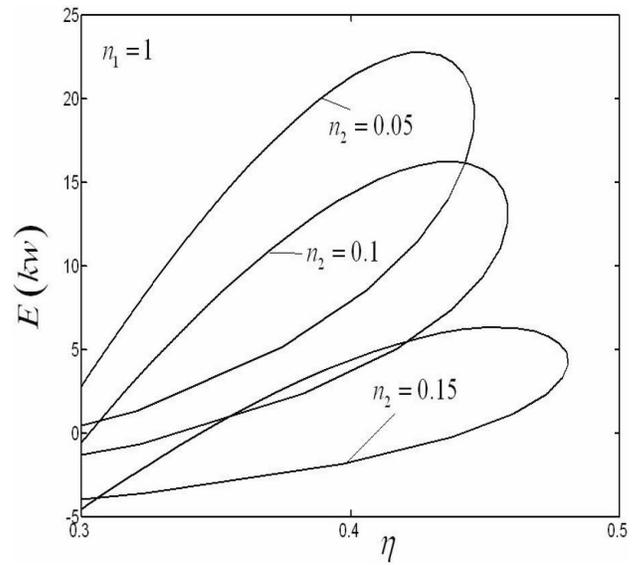


FIGURE 4. Optimal ecological function versus efficiency with $n_1 = 1, n_2 = 0.05, n_2 = 0.1$ and $n_2 = 0.15$.

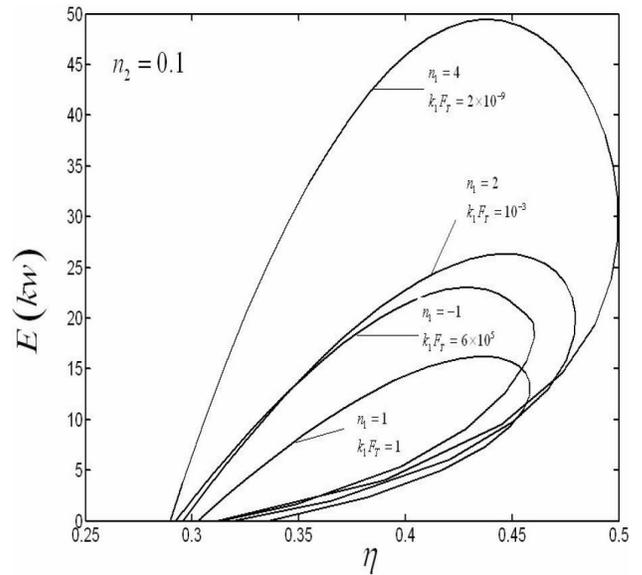


FIGURE 5. Optimal ecological function versus efficiency with $n_2 = 0.1, n_1 = -1, n_1 = 1, n_1 = 2$ and $n_1 = 4$

when the imaginary part n_2 increases, and the imaginary part n_2 of the complex heat transfer exponent n indicates energy dissipation.

The effects of complex exponent $n = n_1 + in_2$ on the optimal ecological function versus efficiency characteristics with $T_H = 1200$ K, $T_L = 400$ K, $T_0 = 298.15$ K, $\delta = 1$, $q = 16$ W, and $\varphi = 1.05$ are shown in Figs. 4 and 5. They show that E versus η characteristics of a generalized irreversible thermoacoustic heat engine with a complex heat transfer exponent is a loop-shaped curve. For all n_1 and n_2 , $E = E_{\max}$ when $\eta = \eta_0$ and $\eta = \eta_{\max}$ when $E = E_0$. For example, when $n_1 = 1$, the E bound (E_{\max}) corresponding to $n_2 = 0.05, 0.10$ and 0.15 are 22.7562, 16.2284 and

6.3245 (kW), respectively, and the maximum thermal efficiency η_{\max} corresponding to $n_2 = 0.05, 0.10$ and 0.15 are 0.4459, 0.1583 and 0.4808, respectively.

The optimization criteria of the thermoacoustic heat engine can be obtained from parameters E_{\max} , E_0 , η_{\max} and η_0 as follows:

$$E_0 \leq E \leq E_{\max} \quad \text{and} \quad \eta_0 \leq \eta \leq \eta_{\max} \quad (40)$$

6. Conclusion

The optimal ecological performance of a generalized irreversible thermoacoustic heat engine with the losses of heat-resistance, heat leakage and internal irreversibility, in which the heat transfer between the working fluid and the heat reservoirs obeys a generalized heat transfer law $Q \propto \Delta(T)^n$ where n is complex is derived by taking an ecological optimization criterion as the objective. The heat transfer exponent for a thermoacoustic heat engine must be a complex number due to the thermal relaxation induced by the thermo-

acoustic oscillation. The effects of the complex heat transfer exponent on the optimal performance for a thermoacoustic heat engine are analyzed by numerical examples. The optimal zone of the thermoacoustic engine with a complex heat transfer exponent is obtained. The results obtained herein are helpful for the selection of the optimal mode of operation of the real thermoacoustic heat engines.

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