On the phenomenology underlying Taylor's hypothesis in atmospheric turbulence

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G.I. Taylor's hypothesis of transposition of turbulent statistics from the spatial to the temporal domain (and vice-versa) is usually explained in terms of smaller features being advected by a large-scale transport velocity, while intrinsic temporal velocity fluctuations are slower than the corresponding inertial terms, and turbulent velocity fluctuations remain small in comparison with the transport velocity. This formulation, widely known as "frozen turbulence", is undoubtedly correct in laboratory experiments where the stated conditions are being fulfilled, and perhaps in many natural settings. However, temporal structure functions of measured velocities in the atmospheric boundary layer during periods of higher transport velocities (tropical day time), when compared with periods of low activity (night time), show a very similar behavior, hereby raising the question whether the space–time similarity of turbulent fluctuations in terms of statistical moments is really due only to transport-like advection, or there might exist a different underlying phenomenology leading to the same result, and accounting for the behavior during low-advection periods. Based on the multifractality observed in the structure functions, the alternative explanation of a 4-D space-time multifractal field is suggested.

Keywords: Boundary layer processes; multifractal field; Taylor's hypothesis; turbulence.

La hipótesis de G.I. Taylor con respecto a la transposición de estadísticas del dominio espacial al dominio temporal (y viceversa) en turbulencia, se explica generalmente en términos de las estructuras turbulentas más pequeñas siendo arrastradas por una velocidad de transporte a escalas grandes, mientras que las fluctuaciones de velocidad intrínsicamente temporales sean más lentas que sus contrapartes inerciales y las fluctuaciones turbulentas de velocidad sean despreciables en comparación con la velocidad de transporte. Esta explicación, comunmente conocida como "turbulencia congelada", es sin duda correcta en el caso de aquellos experimentos de laboratorio donde se cumplen las condiciones enunciadas, así como en ciertos casos que ocurren en la naturaleza. Sin embargo, las funciones estructurales de las variaciones temporales de velocidad en la capa límite atmosférica durante períodos con velocidades de transporte más altas (mañanas tropicales), se muestran muy parecidas a las calculadas para períodos de baja intensidad de viento (noches), suscitando así la cuestión si realmente la similaridad espacio-temporal de los momentos estadísticos de las fluctuaciones turbulentas de velocidad se debe a una advección, o bien podría existir otra fenomenología subyacente que llevara al mismo resultado estadístico, pero que pudiese explicar también el mismo comportamiento durante los períodos de baja advección. Basados en la multifractalidad observada en las funciones estructurales, proponemos una explicación alternativa, involucrando un campo multifractal espacio-temporal.

Descriptores: Capa límite; campo multifractal; hipótesis de Taylor; turbulencia.

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1. Introduction

The existence of a universal behavior within the inertial subrange of fully-developed turbulence, in terms of velocity and pressure fields, as assumed in [1] and rigorously conjectured by [2], is expressed by a scaling of the velocity field with distance. Scaling is one of the fundamental transformations (such as space-time translations or Galilean transformations) that leave the Navier-Stokes dynamical equation

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\,\mathbf{v} = -\nabla p + \nu(\nabla\nabla)\,\mathbf{v} \qquad (1)$$

invariant, where $\mathbf{v}$ is the velocity, p the pressure, and ν the kinematic viscosity of the fluid, while t is the temporal coordinate, and ∇ the vector-operator $(\partial/\partial x, \partial/\partial y, \partial/\partial z)$, with $\mathbf{r} = (x, y, z)$ the spatial coordinates. If we transform, for some positive real λ, $\mathbf{r} \to \lambda\mathbf{r}$, $t \to \lambda^{1-h}t$, $\mathbf{v} \to \lambda^{h}\mathbf{v}$, and $p \to \lambda^{2h}p$, then from dimensional considerations, applying the Buckingham Product Theorem, we obtain $h = -1$. However, in the limit of high Reynolds numbers $Re{:=}\mathbf{r}\mathbf{v}/\nu{\to}\infty$, which is precisely the case of fully-developed turbulence, the viscous term becomes negligible, and h is not limited by dimensional reasons. Kolmogorov's statistical proportionality law $\Delta v^3\,(\Delta r) \propto |\Delta r|$ (see *e.g.* [3] for details) implies $h = 1/3$ for fully-developed, homogeneous turbulence. The power-law scaling of the velocity field with length can be phenomenologically explained by the energy cascading across scales (for the case of the atmosphere see [4,5]). Progressively refined cascade models [6,7] have been proposed in recent years to explain increasingly precise velocity measurements, reaching from simple monofractal scaling models to multifractal models with log-infinitely divisible [8,9], and in particular log-stable cascade generators [10,11].

G.I. Taylor [12] hypothesized that second-order statistics can be rescaled from space to time and vice-versa, a property routinely used in experimental turbulence (see *e.g.* [3]). The original explanation put forth by Taylor for this hypothesis is a so-called "frozen" field (large-scale movement advecting small-scale features, which in the particular case of the atmosphere is associated with large-scale circulation carrying along smaller eddies, at higher velocities than their own relaxation speed). While the hypothesis itself has been widely confirmed experimentally, the underlying phenomenology explaining the hypothesis may not necessarily be limited to the "frozen" turbulence scenario. As a matter of fact, in the absence of a clearly identifiable large-scale current (such as the one created *e.g.* in a wind tunnel) the presence of eddies at all scales makes it a non-trivial issue whether a unique advection velocity can be employed at all, as discussed *e.g.* in [13]. In [14], a model is proposed, where the advection velocity is adjusted as a function of the relative intensity of turbulent fluctuations, in an attempt to determine the extent to which intertwined large-scale features are able to "transport" smaller-scale eddies. The model has been indirectly confirmed by [15].

What has prompted the existence of this work is the fact that the Taylorian space-time transposition does not seem to break down as the mean velocity tends to 0, as it should in the case of a purely advection-led transposition. We argue that different physical mechanisms (or a combination thereof) give rise to the Taylorian behavior under different circumstances, which explains the "survival" of the mentioned behavior as the mean velocity tends to 0, and could also be interpreted as an alternative phenomenology explaining the adjustments proposed in [15].

2. Velocity data

Turbulent velocity fields were measured at a deforested site in the state of Rondônia, Brazil ($10°45'$ S, $62°\ 22'$ W) during the wet-season months of January and February 1999. The land is now used as pasture and is dominated by short grasses (Brachiaria brizantha), about 25 cm tall. Isolated indigenous trees are scattered throughout the landscape. The measurements were obtained as part of the NASA TRMM-LBA (Tropical Rainfall Measuring Mission – Large scale Biosphere-Atmosphere) project in the Amazonia. To measure the three components of the wind speed, a sonic anemometer (Solent A1002R, Gill Instruments, Lymington, UK) was deployed on a tower at 6 m above the surface. The 10-m tower was also instrumented with other sensors that allowed to measure variables such as water vapor concentration, air temperature profiles, short-wave and long-wave radiation. The sonic anemometer recorded wind speed and virtual temperature at a frequency of 10 Hz. These fast-response data were obtained via data acquisition and electronic signal condition systems (model SCXI 2400, National Instruments, Austin, TX), which were interfaced with a computer. A detailed description of the data set can be found in [16].

We observed that during daytime the absolute values of wind velocity components (Fig. 1, top) exhibit the expected increase, due mainly to the thermal forcing. We use these two different regimes (day- vs. night-time) to compare the particularities of the Taylorian behavior in each case. Let us recall that the justification of the "frozen" field lies precisely in a relatively low intensity of turbulent fluctuations with respect to the transport velocity generated by the largest scales (which can be considered a "mean flow" velocity with respect to smaller scales). But no such transport velocity justifies the hypothesis during night hours, when mean velocities are low (mostly, less than 1 m/s), and a mean transport velocity virtually nonexistent.

3. Background, analysis and results

Let us consider the scaling of the Fourier energy spectrum $E(k)$ with respect to the wavenumber k:

$$E(k) \propto k^{\beta}. \tag{2}$$

The planetary-scale $\beta = -3$ range is widely believed to be the result of a two-dimensional direct (large scale to small scale) enstrophy (=vorticity^2/2) cascade [17], dimensionally agreeing with Kraichnan two-dimensional turbulence theory [18]. This also agrees with the observational fact that at the planetary scale, the Coriolis force, together with the juxtaposition of oceans and continents, and landscape features, induce a large-scale vorticity which subsequently cascades down to smaller scales. This, in turn, argues in favor of a frozen-field explanation of Taylor's hypothesis at the planetary range of scales, given the mentioned regularity of global circulation, dominated by the induced large-scale vorticity.

It is a non-trivial problem to determine around what scale atmospheric turbulence changes from a 2-D to a 3-D behavior. This is particularly the case since thermal energy injection and two-phase effects in the atmospheric boundary layer naturally occur around the range of scales, where fluid elements geometrically undergo the transformation from a 3-D to a 2-D configuration. This geometrically-based occurrence has an equally interesting dynamical counterpart: the energy injection at the above-mentioned scales causes a 2-D inverse energy cascade (from the transition scales to larger scales), according to Kraichnan's theory [18], as well as a 3-D direct energy cascade (from those scales to smaller scales), in agreement with Kolmogorov's classical 3-D isotropic turbulence scaling hypothesis [2]. But then again, both of these cascades have a $\beta = -5/3$ spectral exponent, which makes them spectrally indistinguishable, and calls for more refined scaling analyses to define the junction between the 2-D inverse and the 3-D direct energy cascade. A relevant tool for such an analysis is offered by the structure function ζ of absolute values of velocity increments, defined from

$$|\Delta v(\Delta t)|^p \propto \Delta t^{\zeta(p)}, \tag{3}$$

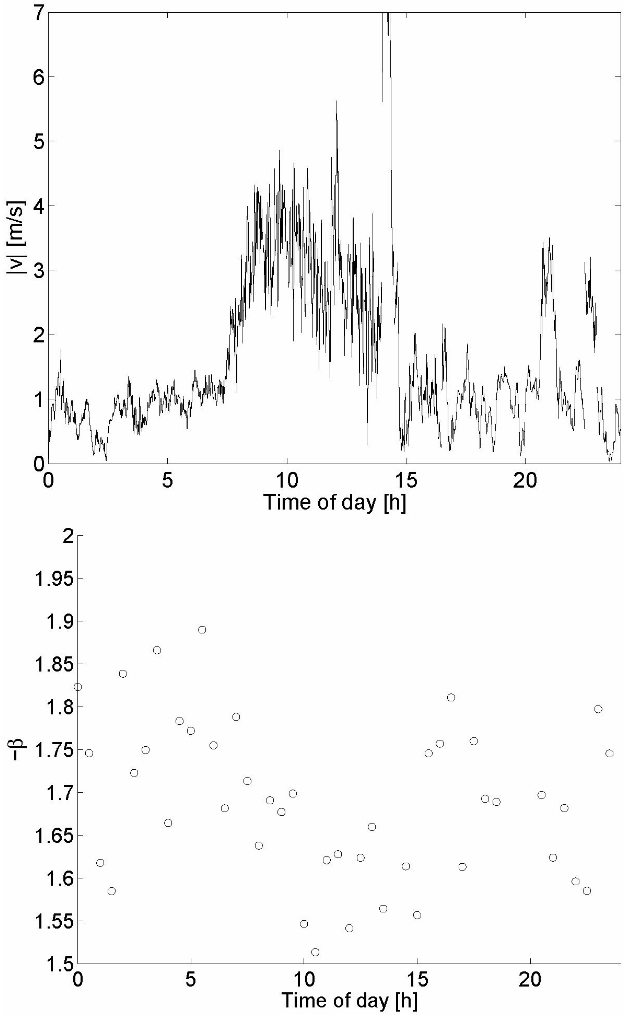

FIGURE 1. Top: Absolute value of wind speed during January 30, 1999. Bottom: Fourier power spectrum estimation at different hours of that same day.

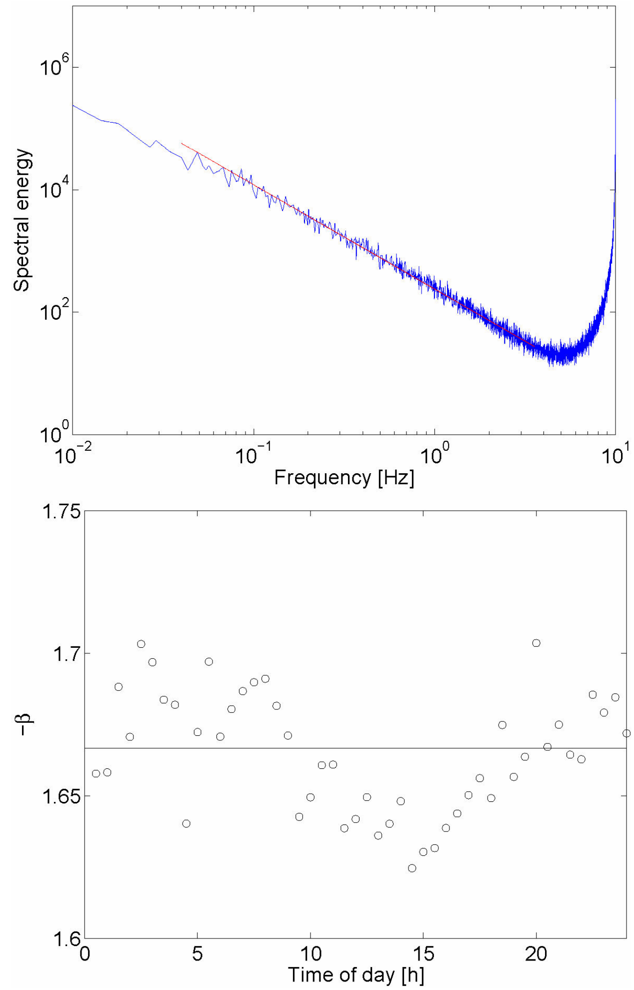

FIGURE 2. Top: Ensemble Fourier power spectrum estimation at midday across the daily realizations, during the whole period of the experiment (the exponent estimation line is marked in red). Bottom: ensemble estimations of the Fourier power-spectral exponent at different hours, across the daily realizations, for the whole January-February period of the experiment.

where $\Delta v(\Delta t) := v(t+\Delta t) - v(t)$ and p represents the order of statistical moments for which $\zeta(p)$ is defined (note that ζ, as defined, reflects the scaling law of the absolute values of velocity increments with the time intervals over which they were measured, and may be different of the structure function of signed velocity increments, for those values of p where the latter exists). In [17] it was found that no part of the $-5/3$ power spectrum is compatible with 2-D turbulence, based on the fact that the second-order structure functions $\zeta(2)$ of the velocities do not agree with the corresponding 2-D isotropic relationship. Similarly, [17] computed the outer scale of the 3-D cascade from the convergence point of the scaling statistical moments of different orders, and found that it lies at synoptic scales. This would imply that the whole $-5/3$ power spectrum scaling range should be interpreted as a 3-D energy inertial scaling range, and that the so-called "mesoscale gap" does not actually play any role [19] in the qualitative behavior of atmospheric turbulence.

It appears therefore that a 3-D direct energy cascade of spectral exponent -5/3 in space is responsible, transposed through advection, for the spectra that we obtain from point-velocity measurements. However, as argued earlier, the advective transposition should break down as the advection velocity tends to 0. Dimensional considerations, based on the Buckingham Product Theorem (just as for the other cases cited above), give in this case $E(\omega) \propto \omega^{-2}\epsilon$, where ω is the respective frequency for which the spectral density is being calculated, and ϵ the specific energy dissipation rate. It is interesting to observe that the same value of the spectral exponent has been found in [21], in a similar context, where the scaling arises in fully developed free-convective turbulence of helium gas heated from below. However, let us note that the atmosphere offers a unique case study in this respect, distinguished by the fact that its huge scale allows for considerable Reynolds numbers to be attained, with very low intensities of "external" flow, a setting that cannot be reproduced in the laboratory. In [22] it is noted for example that in typical experimental conditions for atmospheric turbulence,

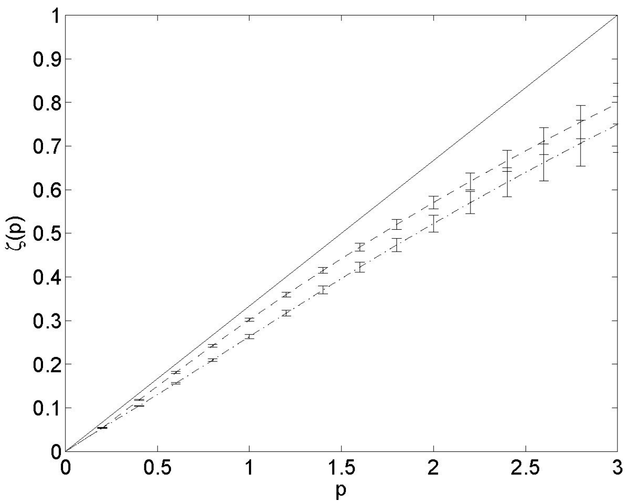

FIGURE 3. Ensemble estimation of the structure function ζ of temporal point-velocity fluctuations during January – February 1999, for the time windows 12:00 to 12:30 a.m. (dash-dotted line), and 12:00 to 12:30 p.m. (dashed line), respectively. The solid line represents the $1/3$-slope values according to Kolmogorov [2].

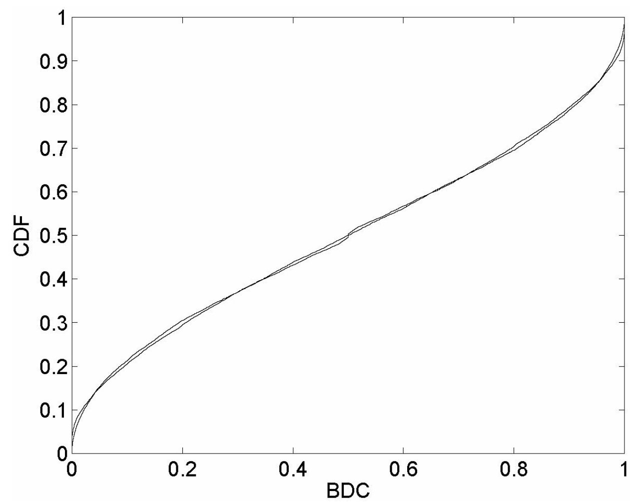

FIGURE 4. Probability-unbiased estimation of the cumulative distribution function (CDF) of breakdown coefficients (BDC) of temporal point-velocity fluctuations on January 30, 1999, between 1:00 and 1:30 a.m., and between 12:00 and 12:30 p.m., respectively.

the ratio between the relaxation speed of smaller eddies and the velocity of the "mean wind" is of the order of 0.2 to 0.5. In the present data set we found values up to 0.7. Figure 1, bottom, shows an example of the daily power-spectral exponents, estimated from point-velocity time series, and one can appreciate (comparing with the top figure) that during daytime, with high advection velocities, the estimated exponents are grouped around the -5/3 value, whereas at night, a shift towards the -2 value can be observed. The ensemble power spectra (an example in Fig. 2, top) lead to exponent estimations that exhibit the same tendencies; however, less pronounced due to ensemble averaging (Fig. 2, bottom).

In order to better understand what is actually occurring in the case of low advection velocities, we compare the time-domain ζ functions, defined as in (3), between daytime and night-time (Fig. 3). The $\zeta(p)$ are estimated from the linearity of the logarithmic plots of $|\Delta v(\Delta t)|^p$ vs. Δt.

The interesting feature to observe here is the deviation of the time-domain $\zeta\,(p)$ functions from the straight $p/3$ line. This is a behavior characteristic of multifractality (let us recall that the definition of multifractality as the nonlinearity of $\zeta\,(p)$ functions stems from the existence of a continuous range of Hölder exponents in the measure); see also [20]. This deviation is considerably higher during day time than during night time, a fact that has to be connected to the larger day-time velocities (which, in turn, relate to a deeper inertial subrange). In any case, both cases show an unmistakable multifractal behavior, and consequently should be regarded as different realizations of multifractal cascades. As multifractal cascades are trivially non-ergodic in their structure functions (both mathematically and physically: real-life cascading processes approach structure-function ergodicity over domains that largely exceed the scale where the cascades are generated, which arguably allows the process to contain several realizations of an ensemble), an operator that ensures ergodicity over multifractal cascades should be employed for the purpose of further investigating the generator characteristics of the observed cascades. Such operators are the breakdown coefficients (BDCs), defined for a measure μ on the Borel subsets of some interval of the real line as:

$$b\,(I_1, I_2) = \mu\,(I_1)/\,\mu\,(I_2)\,, \tag{4}$$

where $I_1 \subseteq I_2 \subset \mathbf{R}$ are two intervals of the real line. Aside from their ergodicity, the relevance of breakdown coefficients for scale-invariant processes in general, and for multifractal measures in particular, lies in the fact that their probability distribution function only depends on the quotient of the lengths of the intervals I_1 and I_2 (for an overview of properties, as well as generalizations, see [23,24]). In our case, the probability distribution functions of BDCs for daytime and night-time (Fig. 4) confirm the likelihood of a multifractal structure of the data, by means of their asymptotes in 0 and 1 (see [25]), and most importantly, show a striking resemblance between each other.

This evidence suggests the identity of the multifractal energy cascade kernel between day and night. Given the fact that during the night, the turbulent intensity dominates the mean velocity, *i.e.*, point-measurements in Eulerian coordinates are at least to some extent reflecting an intrinsic temporal variability; whereas, during the day a "frozen field"-type of advection makes point-measurements reflect mainly the spatial variability, we can conclude from the similarity of the day- and night-time energy cascades that a 4-D space-time multifractal model is appropriate for explaining the observed features. In a Taylorian setting, this is to say that the turbulence-intensity correction to the transport velocity is a fraction (reflecting the importance of turbulent intensity vs. mean flow) of the $\zeta\,(p)^{th}$ root of the ratio between the spatial and the temporal structure functions. Let us note that in the case of a space-time anisotropy of the structure functions, the transport velocity correction should be a function of the order p of the moment for which the transposed statistics are evaluated; otherwise, for isotropic scaling between space and time, the correction should be the same for all orders of moments (and obviously identical to the one used for the "classical" Taylorian second-moment transposition).

4. Conclusions and perspectives

Employing a straightforward thread of thought (of which a number of elements were previously known), we offer evidence in favor of a 4-D space-time multifractal model of turbulence, using data from atmospheric turbulence, where relatively high Reynolds numbers can be reached without a considerable mean flow. The multifractal intrinsic temporal component determines the transposition of statistics between space and time at low mean flows (with respect to the turbulent intensity), much in the same way as the advection velocity does at high mean flows. The final conclusion on whether this model is in fact correct will have to rely on simultaneous (albeit expensive) multi-anemometer measurements in the space-time domain. If the 4-D multifractality is confirmed, an additional bonus of such a field study would be an answer to whether there exists an anisotropy between the spatial scaling and the temporal scaling. Such a model could offer the correct description of processes where atmospheric small-scale turbulence is involved, such as cloud drop nucleation, raindrop growth and size distribution under different atmospheric conditions, and others. A theoretical perspective for the to-be 4-D multifractality would be the phenomenological description of the intrinsic temporal multifractal component.

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