Decay width $Z_1 \rightarrow l\bar{l}$: effects of the little Higgs model

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In the framework of the Little Higgs Model (LHM), we calculate the decay widths $\Gamma(Z_1 \rightarrow l\bar{l})$ and $\Gamma_{inv}(Z_1 \rightarrow \nu\bar{\nu})$ with corrections of QED and QCD. We analyze this with recent data from LEP and compute the contribution of the model. We find that the deviations of the decay width of reactions $Z_1 \rightarrow l\bar{l}$ and $Z_1 \rightarrow \nu\bar{\nu}$ from its SM value are relatively large in the parameter space preferred by the electroweak precision data. Furthermore, with reasonable free parameter values, the absolute value of the relative correction parameter $\delta\Gamma/\Gamma_{SM}$ is of $15\% - 50\%$. The experimental measurement values could generate possible constraints on the free parameters of the LHM.

Keywords: Neutral currents; models beyond the standard model.

En el contexto del modelo Little Higgs (LHM), se calculan las anchuras de decaimiento $\Gamma(Z_1 \rightarrow l\bar{l})$ y $\Gamma_{inv}(Z_1 \rightarrow \nu\bar{\nu})$ con correcciones de QED y QCD. Se analiza esto con datos recientes de LEP y se calcula la contribución del modelo. Se encuentra que las desviaciones de la anchura de decaimiento de las reacciones $Z_1 \rightarrow l\bar{l}$ y $Z_1 \rightarrow \nu\bar{\nu}$ de su valor del modelo estándar son relativamente grandes en el espacio de parámetros preferido por los datos de precisión electrodébil. Además, con valores razonables de los parámetros libres, el valor absoluto del parámetro de corrección relativa $\delta\Gamma/\Gamma_{SM}$ es de $15\% - 50\%$. Los valores medidos experimentalmente pueden generar posibles restricciones sobre los parámetros libres del modelo LHM.

Descriptores: Corrientes neutras; modelos más allá del modelo estándar.

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1. Introduction

There are a number of scenarios for new physics beyond the Standard Model (SM) [1]. The most famous is the supersymmetric scenario. One of the principal motivations for physics beyond the Standard Model is resolving the hierarchy and fine-tuning problems between the electroweak scale and the Planck scale. Supersymmetric theories introduce an extended space-time symmetry and quadratically divergent quantum corrections are canceled due to the symmetry between the bosonic and fermionic partners. Technicolor theories introduce new strong dynamics at scales not much higher than the electroweak scale, thus deferring the hierarchy problem. TeV scale quantum gravity theories reinterpret the problem completely by lowering the fundamental Planck scale. A recently proposed alternative called the Little Higgs Model (LHM) [2–5] offers an alternative route to the solution of the hierarchy problem of the SM, reviving the idea that the Higgs doublet is a pseudo Goldstone boson of some global symmetry which is spontaneously broken at a TeV scale. The key feature of this type of model is that the Higgs boson is a pseudo-Goldstone boson of an approximate global symmetry which is spontaneously broken by a vev at a scale of a few TeV and is thus naturally light. In the LHM, a set of new heavy gauge bosons (A_2, Z_2, W_2) and a new heavy vector-like quark (T) are introduced which cancel the quadratic divergence induced by SM gauge boson loops and the top quark loop, respectively. The distinguishing characteristic of this model is the existence of these new particles and their couplings to the light Higgs. The measurement of these couplings and new particle effects might prove the existence of the little Higgs mechanism [6]. The global symmetry breaking scale is expected to be ≤ 10 TeV so the little Higgs model will be relevant for the hierarchy.

In this paper, assuming lepton universality, we calculate the decay width of the processes $Z_1 \rightarrow l\bar{l}$ and $Z_1 \rightarrow \nu\bar{\nu}$ in the LHM. When compared to the processes $Z_1 \rightarrow l\bar{l}$ and $Z_1 \rightarrow \nu\bar{\nu}$ in the SM, the process in the LHM receives the additional contribution arising from the vector and axial-vector couplings as well as the parameters of the LHM [7]. We find that with reasonable values of the free parameters, the deviation of the decay width $\delta\Gamma/\Gamma_{SM}$ from its SM is of $15\% - 50\%$. We also study the effects of the little Higgs model in the reactions $Z_1 \rightarrow l\bar{l}$ and $Z_1 \rightarrow \nu\bar{\nu}$. The leptonic Z_1 decays are free from the long distance effects and are thus clean.

Processes measured near the resonance have served to set bounds on the parameters of the model. Because this partial decay occurs in the resonance zone, the process is independent of the mass of the additional Z heavy gauge boson which appears in these kind of models. They also carry considerable information about the free parameters of the model used so it is therefore worthwhile to analyze these decay processes in the context of the new physics models.

This paper is organized as follows: In Sec. 2 we present the expressions for the decays widths $Z_1 \rightarrow l\bar{l}$ and $Z_1 \rightarrow \nu\bar{\nu}$ in the LHM. In Sec. 3 we present the numerical computation and, finally, we summarize our results in Sec. 4.

2. Width of $Z_1 \rightarrow l\bar{l}$ in the little Higgs model

The little Higgs model is based on a $SU(5)/SO(5)$ nonlinear sigma model. At the scale $\Lambda_s \sim 4\pi f$, the global

$SU(5)$ symmetry is broken into its subgroup $SO(5)$ via a vacuum condensate f, resulting in 14 Goldstone bosons. The effective field theory of these Goldstone bosons is parameterized by a non-linear sigma model with gauged symmetry $[SU(2)\times U(1)]^2$, spontaneously broken down to its diagonal subgroup $SU(2)\times U(1)$ identified as the SM electroweak gauge group. Four of these Goldstone boson are absorbed by the broken gauge generators, leaving 10 states that transform under the SM gauge group as a doublet H and a triplet Φ. This breaking scenario also gives rise to four massive gauge boson A_2, Z_2 and $W_2^{\pm}$, which might produce characteristic signatures in the present and future high energy collider experiments [7]. After electroweak symmetric breaking, all the light and heavy gauge bosons are obtained, namely, $A_1, Z_1, W_1^{\pm}$ of the SM and $A_2, Z_2, W_2^{\pm}$ of the LHM.

The masses of the new heavy gauge bosons in the LHM to the order of $\mathcal{O}(v^2/f^2)$ are given by following expressions [7]:

$$M_{A_1}^2 = 0, \tag{1}$$

$$M_{A_2}^2 = M_Z^2 s_W^2\left(\frac{f^2}{5s'^2c'^2v^2} - 1 + \frac{x_H c_W^2}{4s^2c^2s_W^2}\right), \tag{2}$$

$$M_{Z_1}^2 = M_Z^2\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{1}{4}(c^2-s^2)^2 + \frac{5}{4}(c'^2-s'^2)^2\right) + 8\frac{v'^2}{v^2}\right], \tag{3}$$

$$M_{Z_2}^2 = M_W^2\left(\frac{f^2}{s^2c^2v^2} - 1 - \frac{x_H s_W^2}{s'^2c'^2c_W^2}\right), \tag{4}$$

$$M_{W_1^{\pm}}^2 = M_W^2\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{1}{4}(c^2-s^2)^2\right) + 4\frac{v'^2}{v^2}\right], \tag{5}$$

$$M_{W_2^{\pm}}^2 = M_W^2\left(\frac{f^2}{s^2c^2v^2} - 1\right), \tag{6}$$

where M_Z and M_W are the SM gauge bosons masses and $c_W(s_W)$ denotes the cosine (sine) of the weak mixing angle, while $s, s'(c, c')$ represent the sine (cosine) of two mixing angles. Here, x_H characterizes the heavy gauge boson mixing and depends on the gauge couplings.

The couplings between neutral gauge bosons

$$V_i(V_i = Z_1, A_2, Z_2)$$

to a pair of fermions can be written in the form

$$-i\gamma\mu(g_V^{V_i f\bar{f}} + g_A^{V_i f\bar{f}}\gamma^5).$$

The couplings $g_V^{V_i f\bar{f}}$ and $g_A^{V_i f\bar{f}}$ also depend on the mixing parameters $s, s'(c, c')$ and the scale parameter f. The expressions for the couplings to a pair of leptons or anti-leptons in the LHM are [7]:

$$g_V^{Z_1 l\bar{l}} = -\frac{g}{2c_W}\left\{\left(-\frac{1}{2} + 2s_W^2\right) - \frac{v^2}{f^2}\left[-c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(2y_e - \frac{9}{5} + \frac{3}{2}c'^2\right)\right]\right\}, \tag{7}$$

$$g_A^{Z_1 l\bar{l}} = -\frac{g}{2c_W}\left\{\frac{1}{2} - \frac{v^2}{f^2}\left[c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(-\frac{1}{5} + \frac{1}{2}c'^2\right)\right]\right\}, \tag{8}$$

where f is the characteristic energy scale of the LHM [2, 8–10]. In the limit $f\to\infty$, the couplings of the SM are recovered.

The expression for the transition amplitude for the channel $Z_1 \to l\bar{l}$ $(l = e, \mu, \tau)$ is given by

$$M(Z_1\to l\bar{l}) = \bar{u}(l)\left[-i\gamma^{\mu}(g_V^{Z_1 l\bar{l}} + g_A^{Z_1 l\bar{l}}\gamma_5)\right]v(\bar{l})\varepsilon_{\mu}^{\lambda}(Z_L), \tag{9}$$

where $u(v)$ is the lepton (anti-lepton) spinor and $\varepsilon_{\mu}^{\lambda}$ is the Z_1 boson polarization vector.

Of the Eq. (9), the partial Z_1 decay width $\Gamma(Z_1\to l\bar{l})$, including QED and QCD corrections, is given by:

$$\Gamma(Z_1\to l\bar{l}) = \frac{G_F M_{Z_1}^3}{6\pi\sqrt{2}}\left[(\bar{g}_V^l)^2 + (\bar{g}_A^l)^2 - 2(\bar{g}_V^l)\frac{v^2}{f^2} \times\left(-c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(2y_e - \frac{9}{5} + \frac{3}{2}c'^2\right)\right) - 2(\bar{g}_A^l)\frac{v^2}{f^2}\left(c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(-\frac{1}{5} + \frac{1}{2}c'^2\right)\right)\right] \times(1 + \delta\rho + \delta\rho_l + \delta_{QED}). \tag{10}$$

The vector and axial-vector $Z_1 l\bar{l}$ couplings $\bar{g}_V^l$ and $\bar{g}_A^l$ compare one-loop and higher electroweak and internal QCD corrections through the form factors $\delta\rho_l$ and k_l, which can be written as:

$$\bar{g}_V^l = \sqrt{\rho_l}(\frac{1}{2} - 2sin^2\theta_{eff}^l), \qquad \bar{g}_A^l = \sqrt{\rho_l}(\frac{1}{2}), \tag{11}$$

with $\sin^2\theta_{eff}^l = k_l\sin^2\theta_W$. The term $\delta\rho$ is the deviation from the SM prediction for the ρ parameter $\rho = M_Z\cos\theta/M_W = 1 + \delta\rho$, taking into account contributions of the gauge group structure of LHM only. Considering Eqs. (3) and (5), the contribution to $\delta\rho$ is given by

$$\delta\rho \approx -\frac{v^2}{8f^2}\left[1 + 5(c'^2 - s'^2)^2\right]. \tag{12}$$

Also, δ_{QED} accounts for the final state photon radiation

$$\delta_{QED} = \frac{3\alpha(s)}{4\pi}Q^2, \tag{13}$$

where α is the QED coupling computed at the energy scale s, while Q is the lepton charge.

In order to obtain a prediction for the standard model partial Z_1 decay width into e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ we take the input parameters [11], $M_Z = 91.187$ GeV, $G = 1.16637\times10^{-5}$ GeV^{-2}, $\alpha(M_Z) = 1/128.95$ and

$\sin^2\theta_W = 0.22335$. Using the Zfitter package [12], these parameters can be used to obtain the form factors for the decays $Z_1 \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ which yield

$$\delta\rho_e, \delta\rho_\mu, \delta\rho_\tau = 0.00531, 0.00531, 0.00512, \tag{14}$$

and

$$\sin^2\theta_{W(eff)} = 0.2315, \tag{15}$$

translating into $\kappa_e, \kappa_\mu, \kappa_\tau = 1.0367, 1.0367, 1.0351$. Plugging these parameters into Eq. (10) together with the limit $f \to \infty$ and $\delta\rho \to 0$, we obtain the standard model prediction $\Gamma(Z_1 \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-) = 83.99, 83.99, 84.01$ MeV, respectively.

2.1. Width of $Z_1 \to \nu\bar{\nu}$ in the Little Higgs Model

The expressions for the couplings to a pair of neutrino or antineutrino in the LHM are the following [7]:

$$g_V^{Z_1\nu\bar{\nu}} = -\frac{g}{2c_W}\left\{\frac{1}{2} - \frac{v^2}{f^2}\left[c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(y_e - \frac{4}{5} + \frac{1}{2}c'^2\right)\right]\right\}, \tag{16}$$

$$g_A^{Z_1\nu\bar{\nu}} = -\frac{g}{2c_W}\left\{-\frac{1}{2} - \frac{v^2}{f^2}\left[-c_W x_Z^{W'} c/2s + \frac{s_W x_Z^{B'}}{s'c'}\left(-y_e + \frac{4}{5} - \frac{1}{2}c'^2\right)\right]\right\}. \tag{17}$$

In this case, the invisible Z_1 decay width Γ_{inv}, after receiving contributions from all neutrinos flavors is given by:

$$\Gamma(Z_1 \to \nu\bar{\nu}) = \frac{G_F M_{Z_1}^3}{12\pi\sqrt{2}}\left[1 + 4\frac{v^2}{f^2}\left(-\frac{c_W x_Z^{W'} c}{2s} + \frac{s_W x_Z^{B'}}{s'c'}\left(-y_e + \frac{4}{5} - \frac{1}{2}c'^2\right)\right)\right] \cdot (1 + \delta\rho + \delta\rho_\nu). \tag{18}$$

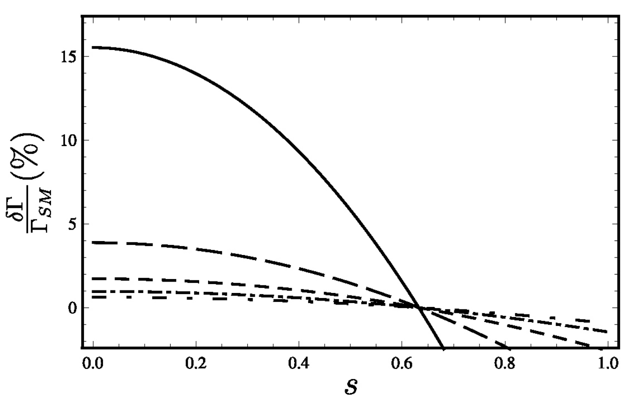

FIGURE 1. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the mixing parameter s for $c = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for f=2, 4, 6, 8, 10 TeV.

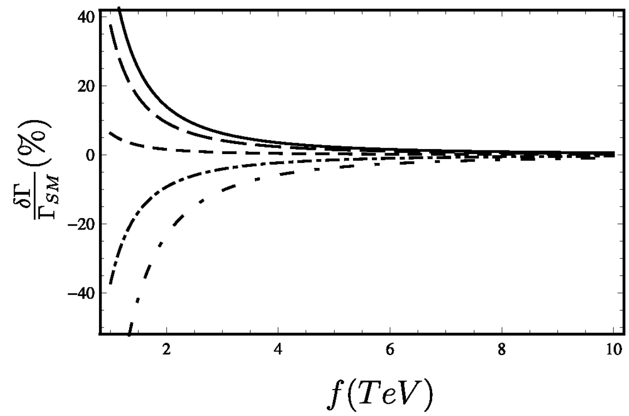

FIGURE 2. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter s. Starting from the top, the curves are for $s = 0.2, 0.4, 0.6, 0.8, 1$.

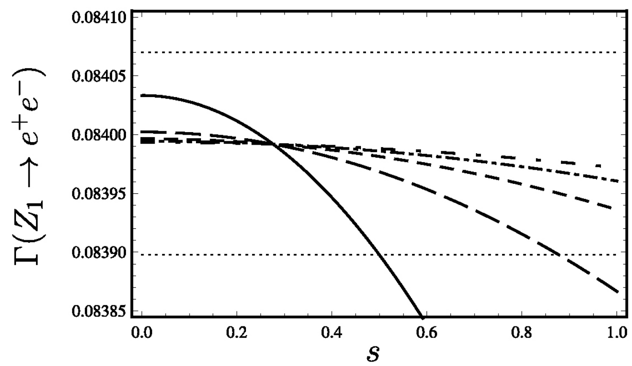

FIGURE 3. The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the mixing parameter s for $c = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for $f = 2, 4, 6, 8, 10$ TeV. The horizontal dotted lines denote the upper and lower values of $\Gamma^{exp}(Z_1 \to e^+e^-)$, respectively.

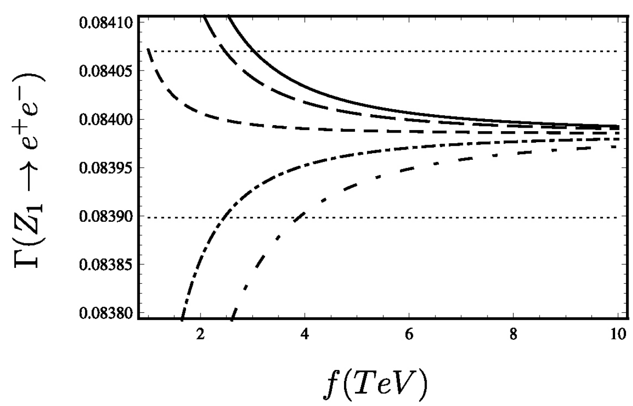

FIGURE 4. The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter s. Starting from the top, the curves are for $s = 0.2, 0.4, 0.6, 0.8, 1$. The horizontal dotted lines denote the upper and lower values of $\Gamma^{\rm exp}(Z_1 \to e^+e^-)$, respectively.

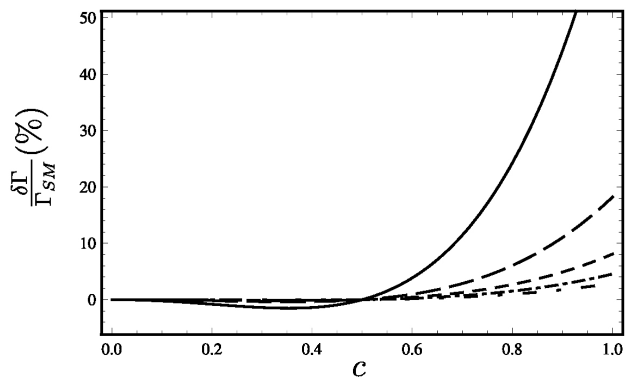

FIGURE 5. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the mixing parameter c for $s = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for f= 2, 4, 6, 8, 10 TeV.

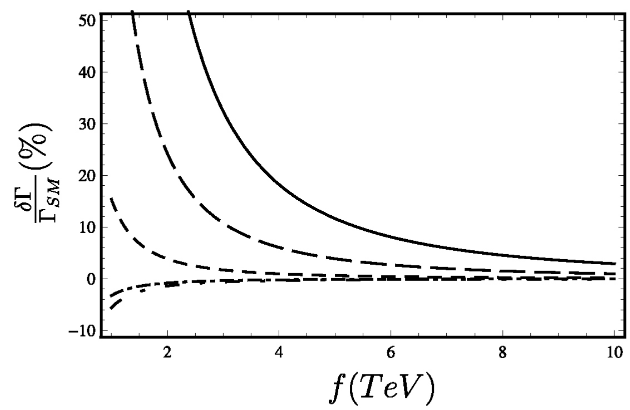

FIGURE 6. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter c. Starting from the top, the curves are for $c = 0.2, 0.4, 0.6, 0.8, 1$.

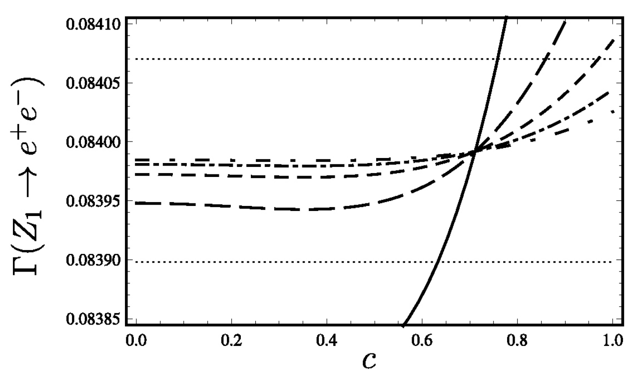

FIGURE 7. The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the mixing parameter c for $s = 0.5$ and different values of the scale of energy f. Starting from below, the curves are for $f = 2, 4, 6, 8, 10$ TeV. The horizontal dotted lines denote the upper and lower values of $\Gamma^{\exp}(Z_1 \to e^+e^-)$, respectively.

3. Results

In this section we present numerical results for the decay widths $\Gamma(Z_1 \to e^+e^-)$ and $\Gamma(Z_1 \to \nu\bar{\nu})$ in the context of the little Higgs model including QED and QCD corrections.

Our numerical results for the decay width $\Gamma(Z_1 \to e^+e^-)$ are summarized in Figs. 1-9. The relative correction $\delta\Gamma/\Gamma_{SM}$ is plotted in Fig. 1 as a function of the mixing parameters s for $c = 0.5$ and different values of scale energy f. $\delta\Gamma = \Gamma^{LHM} - \Gamma^{SM}$ and Γ^{SM} is the decay width predicted for the SM. We can see in this figure that the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ decreases when the mixing parameter s increases and is sensitive to the f energy scale. For $f = 2$ TeV, the absolute value of $\delta\Gamma/\Gamma_{SM}$ ranges from $2\% - 15\%$ in most of the parameter space limited by the electroweak precision data.

To see the dependence of relative correction on parameter f, we plot $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter $s = 0.2, 0.4, 0.6, 0.8, 1$ in Fig. 2. We can see that the absolute value of the relative correction decreases as f increases. The curves also demonstrate that the effect of the LHM is not sensitive to f in the range of $f \geq 6.5$ TeV. This is generally because, the extra contribution of the LHM to the decay width $\Gamma(Z_1 \to e^+e^-)$ is proportional to a factor of $1/f^2$. In this case, the absolute value of $\delta\Gamma/\Gamma_{SM}$ is in the range of 40% in most of the parameter space.

In Fig. 3 we show the dependence of the decay width $\Gamma(Z_1 \to e^+e^-)$ with respect to the mixing parameter s for $c = 0.5$ and different values of the scale of energy $f = 2, 4, 6, 8, 10$ TeV. To compare our calculation values with the experimental value $\Gamma^{exp} = (83.984 \pm 0.086)$ MeV and determine whether it puts new constraints on the LHM, we give $\Gamma^{exp}(Z_1 \to e^+e^-)$ in which the horizontal dotted lines indicate the upper and lower values, respectively. As seen in this figure, if the LHM prediction value for $\Gamma^{exp}(Z_1 \to e^+e^-)$ is in the range allowed by the LEP experiments, the mixing parameter s must be of the order $s = 0.5$ for $f = 2$ TeV and $s = 0.9$ for $f = 4$ TeV.

The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter $s = 0.2, 0.4, 0.6, 0.8, 1$ is presented in Fig. 4. The horizontal dotted lines denote the upper and lower values of $\Gamma^{exp} = (83.984 \pm 0.086)$ MeV, respectively. As seen in this figure, if the LHM prediction value for $\Gamma^{exp}(Z_1 \to e^+e^-)$ is in the range allowed by the LEP experiments, the scale parameter f must be of the order of $f = 1$ TeV for $s = 0.6$ and $f = 4$ TeV for $s = 1$.

The relative correction $\delta\Gamma/\Gamma_{SM}$ is plotted in Fig. 5 as a function of the mixing parameter c for $s = 0.5$ and different values of the scale energy f. In this figure we can see that the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ increases when the mixing parameter c increases and is sensitive to the scale of energy f. For $f = 2$ TeV, the absolute value of $\delta\Gamma/\Gamma_{SM}$ is in the range of 50% in most of the parameter space limited by the electroweak precision data. In Fig. 6, we

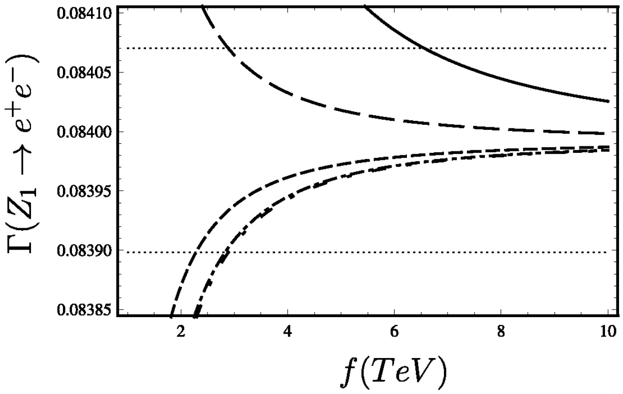

FIGURE 8. The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter c. Starting from top, the curves are for $c = 0.2, 0.4, 0.6, 0.8, 1$. The horizontal dotted lines denote the upper and lower values of $\Gamma^{\rm exp}(Z_1 \to e^+e^-)$, respectively.

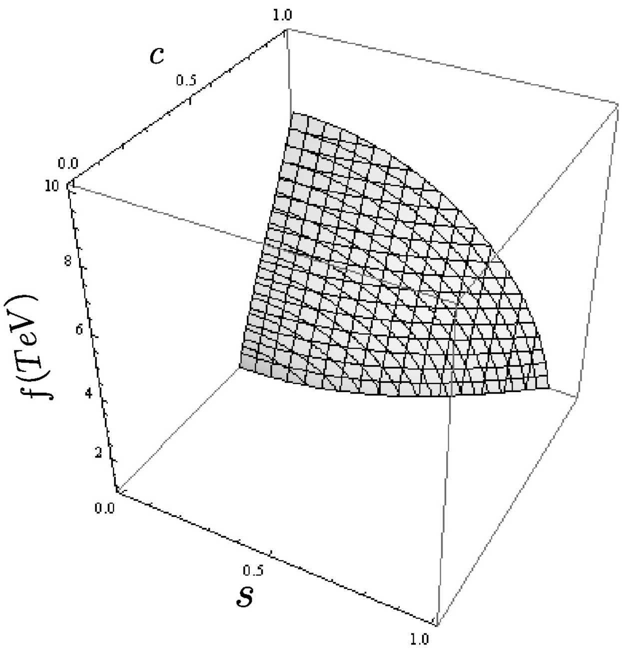

FIGURE 9. Possible values for s, c and f that can be developed as allowed by the decay width $\Gamma^{exp}(Z_1 \to e^+e^-)$ with 95% C.L.

present the relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $s = 0.5$ and different values of $c = 0.2, 0.4, 0.6, 0.8, 1$. Here it is shown that the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ decreases when the scale energy f increases and is sensitive to the mixing parameter c. For $c = 0.2$, the absolute value of $\delta\Gamma/\Gamma_{SM}$ ranges from $2\% - 50\%$.

In Fig. 7 we show the dependence of the decay width $\Gamma(Z_1 \to e^+e^-)$ with respect to the mixing parameter c for $s = 0.5$ and different values of the scale of energy $f = 2, 4, 6, 8, 10$ TeV. As seen in this figure, if the LHM prediction value for $\Gamma^{exp}(Z_1 \to e^+e^-)$ is in the range allowed by the LEP experiments, the mixing parameter c must be of the order $c = 0.6$ for $f = 2$ TeV and $c = 0.95$ for $f = 6$ TeV.

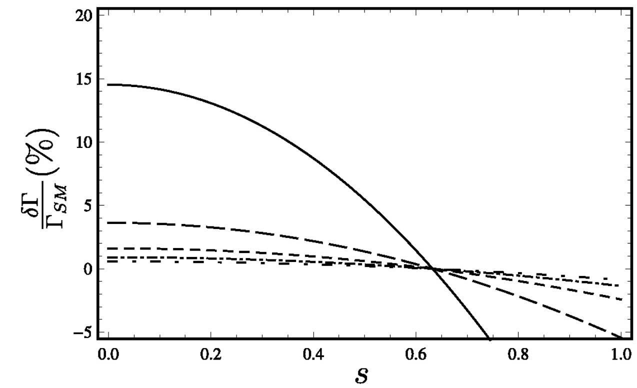

FIGURE 10. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the mixing parameter s for $c = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for $f = 2$, 4, 6, 8, 10 TeV.

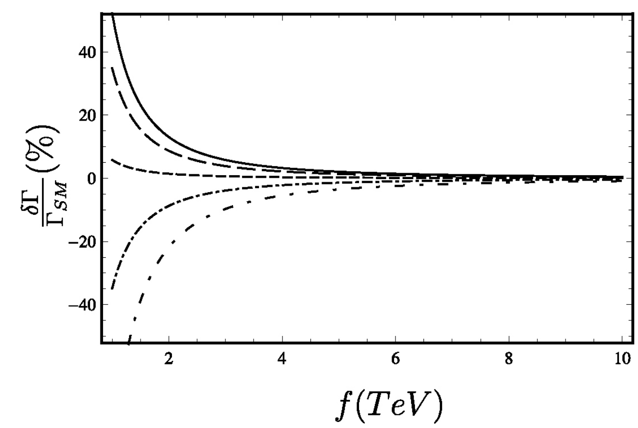

FIGURE 11. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter s. Starting from the top, the curves are for $s = 0.2, 0.4, 0.6, 0.8, 1$.

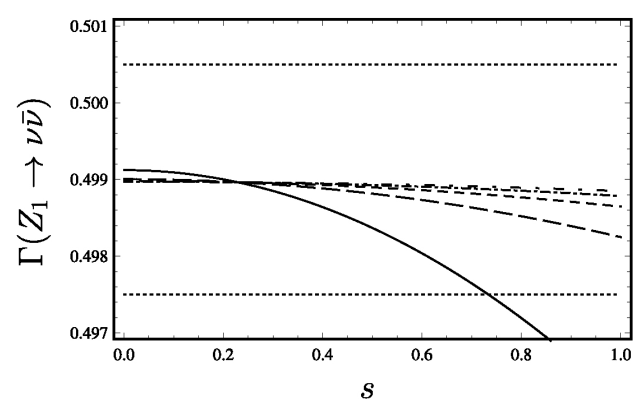

FIGURE 12. The decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ as a function of the mixing parameter s for $c = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for $f = 2, 4, 6, 8, 10$ TeV. The horizontal dotted lines denote the upper and lower values of $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$, respectively.

The decay width $\Gamma(Z_1 \to e^+e^-)$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter $c = 0.2, 0.4, 0.6, 0.8, 1$ is presented in Fig. 8. As shown, if the LHM prediction value for $\Gamma^{exp}(Z_1 \to e^+e^-)$ is in the range allowed by the LEP experiments, the scale parameter f must be of the order $f = 2.2$ TeV for $c = 0.6$ and $f = 6.5$ TeV for $c = 0.2$.

The graphic in Fig. 9 shows the allowed values for f, s and c that can be developed by the decay width $\Gamma^{\rm exp}$=(83.984 ± 0.086) MeV with 95% C.L. These possible values for f, s and c are in complete agreement with those reported in the literature.

The previous analysis and comments can readily be translated to the decay processes $Z_1 \to \mu^+\mu^-$ and $Z_1 \to \tau^+\tau^-$. From this we conclude that there are no significant changes with respect to the process $Z_1 \to e^+e^-$, which is consistent assuming lepton universality.

In the case of the invisible Z_1 decay width, we estimate the effects of the Little Higgs model. The procedure followed for the analysis is similar to that followed for the process $Z_1 \to e^+e^-$.

We summarize our results in Figs. 10 to 18, assuming lepton universality in Z_1 decay to neutrinos. As seen, the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ decreases when the mixing parameter s increases and is sensitive to the energy scale f. For $f = 2$ TeV, the absolute value of $\delta\Gamma/\Gamma_{SM}$ is in the range of $5\% - 15\%$ in most of the parameter space limited by the electroweak precision data.

Figure 11 shows the dependence of relative correction on the parameter f. $\delta\Gamma/\Gamma_{SM}$ is plotted as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter $s = 0.2, 0.4, 0.6, 0.8, 1$. We can see that the absolute value of the relative correction decreases as f increases. The curves also demonstrate that the effect of the LHM is not sensitive to f in the range of $f \geq 6$ TeV.

The decay width $Z_1 \to \nu\bar{\nu}$ with respect to the mixing parameter s for $c = 0.5$ and different values of the scale of energy $f = 2, 4, 6, 8, 10$ TeV is presented in Fig. 12. In this figure the horizontal dotted lines denote the upper and lower values of $\Gamma_{\rm inv}^{\rm exp} = (499 \pm 1.5)$ MeV, respectively. As shown,

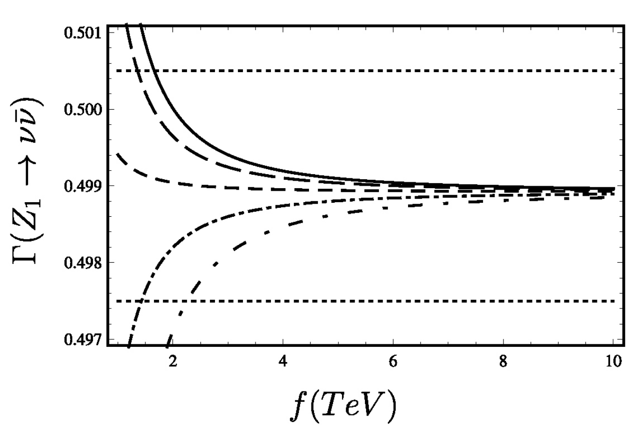

FIGURE 13. The decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ as a function of the scale of energy f for $c = 0.5$ and different values of the mixing parameter s. Starting from the top, the curves are for $s = 0.2, 0.4, 0.6, 0.8, 1$. The horizontal dotted lines denote the upper and lower values of $\Gamma_{\rm inv}^{\rm exp}(Z_1 \to \nu\bar{\nu})$, respectively.

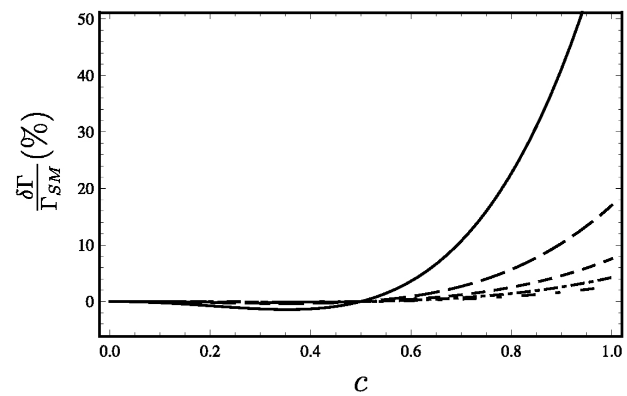

FIGURE 14. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the mixing parameter c for $s = 0.5$ and different values of the scale of energy f. Starting from the top, the curves are for $f = 2$, 4, 6, 8, 10 TeV.

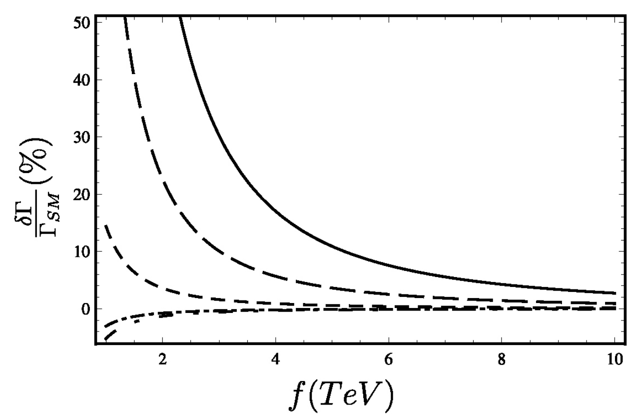

FIGURE 15. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter c. Starting from the top, the curves are for $c = 0.2, 0.4, 0.6, 0.8, 1$.

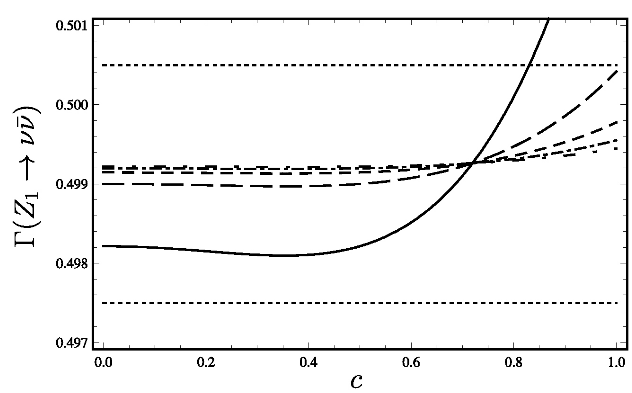

FIGURE 16. The decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ as a function of the mixing parameter c for $s = 0.5$ and different values of the scale of energy f. Starting from below, the curves are for $f = 2$, 4, 6, 8, 10 TeV. The horizontal dotted lines denote the upper and lower values of $\Gamma_{\rm inv}^{\rm exp}(Z_1 \to \nu\bar{\nu})$, respectively.

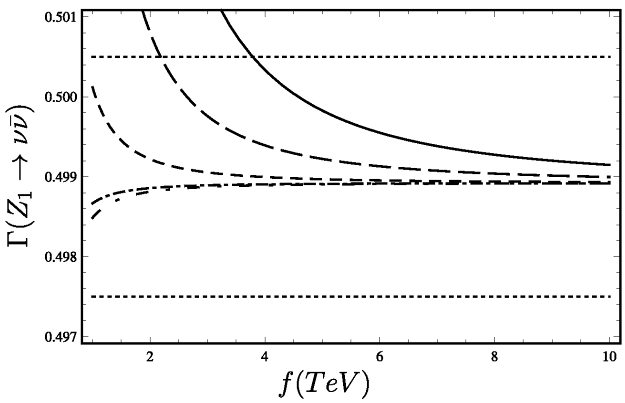

FIGURE 17. The decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter c. Starting from top, the curves are for $c = 0.2$, 0.4, 0.6, 0.8, 1. The horizontal dotted lines denote the upper and lower values of $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$, respectively.

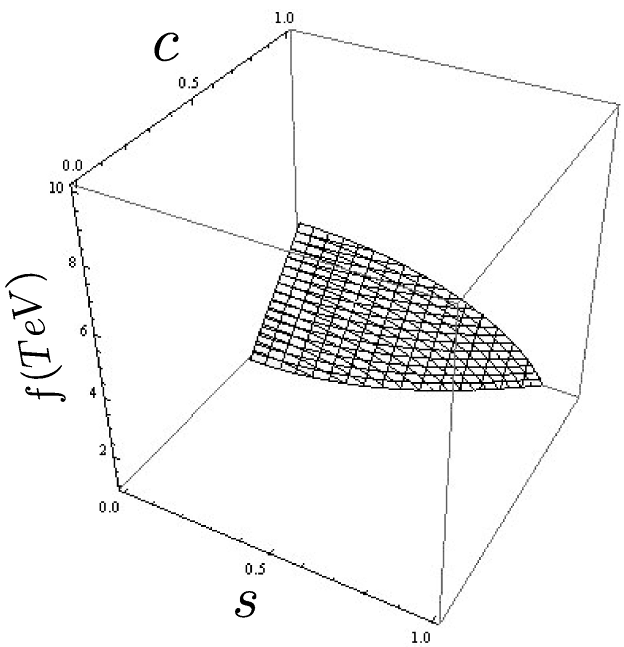

FIGURE 18. Possible values for s, c and f that can be developed as allowed by the decay width $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$ with 95% C.L.

if the LHM prediction value for $\Gamma^{\rm exp}(Z_1 \to \nu\bar{\nu})$ is in the range allowed by the LEP experiments, the mixing parameter s must be of the order $s = 0.7$ for $f = 2$ TeV.

In Fig. 13 we show the dependence of the decay width $\Gamma_{\rm inv}(Z_1 \to \nu\bar{\nu})$ with respect to the scale of energy f for $c = 0.5$ and different values of the mixing parameter s=$0.2, 0.4, 0.6, 0.8, 1$. This figure demonstrates that if the LHM prediction value for $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$ is in the range allowed by the LEP experiments, the scale parameter f must be of the order of $f = 1.5$ TeV for $s = 0.4$ and $f = 2.5$ TeV for $s = 1$.

To see the dependence of relative correction on the parameter of mixing c, we plot $\delta\Gamma/\Gamma_{SM}$ as a function of c for $s = 0.5$ and different values of the scale of energy $f = 2, 4, 6, 8, 10$ TeV in Fig. 14. Here we can see that the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ increases when the mixing parameter c increases and is sensitive to the f energy scale. For $f = 2$ TeV, the absolute value of $\delta\Gamma/\Gamma_{SM}$ is in the range of 50% in most of the parameter space limited by the electroweak precision data. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy f for $s = 0.5$ and different values of $c = 0.2, 0.4, 0.6, 0.8, 1$ is presented in Fig. 15. In this figure we can see that the absolute value of the relative correction $\delta\Gamma/\Gamma_{SM}$ decreases when the scale energy f increases and is sensitive to the mixing parameter c. For $c = 0.2$, the absolute value of $\delta\Gamma/\Gamma_{SM}$ is in the range of $2\% - 50\%$.

In Fig. 16 we show the dependence of the decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ with respect to the mixing parameter c for $s = 0.5$ and different values of the scale of energy $f = 2$, 4, 6, 8, 10 TeV. As seen in this figure, if the LHM prediction value for $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$ is in the range allowed by the LEP experiments, the mixing parameter c must be of the order $c = 0.8$ for $f = 2$ TeV and $c = 1$ for $f = 4$ TeV. The decay width $\Gamma(Z_1 \to \nu\bar{\nu})$ as a function of the scale of energy f for $s = 0.5$ and different values of the mixing parameter $c = 0.2, 0.4, 0.6, 0.8, 1$ is presented in Fig. 17. As shown, if the LHM prediction value for $\Gamma^{\rm exp}_{\rm inv}(Z_1 \to \nu\bar{\nu})$ is in the range allowed by the LEP experiments, the scale parameter f must be of the order $f = 2.3$ TeV for $c = 0.4$ and $f = 3.8$ TeV for $c = 0.2$.

Finally, the graphic in Fig. 18 shows the allowed values for f, s and c that can be developed allowed by the decay width $\Gamma^{\rm exp}_{\rm inv} = (499 \pm 1.5)$ MeV with 95% C.L.. We see that the values for f, s and c are in complete agreement with those reported in the literature.

4. Conclusions

Because it can solve the hierarchy problem, little Higgs model is a promising alternative model of new physics beyond the standard model. Among the various little Higgs models, model [7] is one of the simplest and most phenomenologically viable models. The distinguishing feature of this model is the existence of the new scalars, the new gauge bosons, and the vector-like top quark. These new particles contribute to the experimental observables which could provide some clues to the existence of the little Higgs model. In this paper, we analyze the effects of the little Higgs model including the QED and QCD corrections on the decay widths $\Gamma(Z_1 \to e^+e^-)$ and $\Gamma_{inv}(Z_1 \to \nu\bar{\nu})$, respectively.

The SM gauge boson Z_1 is now abundantly produced at the LHC and will be as well at the future high energy linear e^+e^- collider experiments. It is possible to examine its properties with unprecedented precision. We calculate the decay width correction of the little Higgs model of the processes $\Gamma(Z_1 \to e^+e^-)$ and $\Gamma_{inv}(Z_1 \to \nu\bar{\nu})$. We find that the correction is significant even when we consider the constraint of electroweak precision data on the parameters. In the favorable parameter space, the absolute value of the relative correction parameter $\delta\Gamma/\Gamma_{SM}$ for both processes is $15\% - 50\%$. We conclude that future experiments at the ILC could determine the effects on the $\Gamma(Z_1 \to e^+e^-)$ and $\Gamma_{inv}(Z_1 \to \nu\bar{\nu})$ decay widths contributed by the LHM in a given parameter space or put more stringent constraints on the LHM parameters. In addition, these results have never been reported in the literature before and could be of relevance for the scientific community.

Acknowledgments

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