

Hall effect: the role of nonequilibrium charge carriers

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A new model of the Hall effect in the case of a bipolar semiconductor is present. Taking into account the nonequilibrium carriers, thermal generation and recombination processes assisted by traps (Shockley-Read model), the expressions for the electrochemical potential of electrons and holes, Hall field and Hall constant R_H are obtained. The dependence of these expressions of the distribution of the carriers along the direction of the Hall field in the case of intrinsic and extrinsic semiconductors is studied.

Keywords: Galvanomagnetic phenomena; bipolar semiconductors; recombination; Hall effect.

Presentamos un nuevo modelo del efecto Hall en el caso de semiconductores bipolares. Se toma en cuenta portadores fuera de equilibrio, procesos de generación y recombinación asistidos por trampas (modelo de Shockley-Read). Se obtienen expresiones para los potenciales electroquímicos de electrones y huecos, campo de Hall y constante de Hall R_H . Estudiamos la dependencia de estas expresiones de la distribución de portadores a lo largo de la dirección del campo Hall, en el caso de semiconductores intrínsecos y extrínsecos.

Descriptores: Fenómenos galvanomagnéticos; semiconductores bipolares; recombinación; efecto Hall.

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1. Introduction

The Hall effect has been used for a long time in the research of mechanisms of conduction. With the help of the Hall coefficient R_H , the type of carrier present in the semiconductor sample can be determined. The Hall voltage of a plate may be regarded as a signal carrying information. If the material properties and the device geometry are known, the Hall voltage can give information about the magnetic field H [1]. In this case the Hall device is applied as a magnetic sensor [2].

For the Hall effect, a conductor material is subjected to an electric $E = E_x$ and magnetic $H = H_y$ fields. An electron that travel with velocity \vec{v} experiment a force given by $e(\vec{v} \times \vec{H})/c$, the Lorentz force. This force produce a deflection of the electron in the direction perpendicular to the velocity \vec{v} and the magnetic field \vec{H} . If the semiconductor sample has the shape of a parallelepiped, some electrons can be deflected by the field \vec{H} to accumulate on the bottom or on the top face of the material. This space deviation of the charges generates the appearance of an electric field E . The process continues until the electric field balances the Lorentz force [3]. This process originates a voltage that can be measured through the surfaces of the sample. With the help of the voltage, the coefficient of Hall in the case of electrons ($R_H = -1/ne$) can be obtained [4].

The magnetic field acts differently on each of the carriers in the bipolar material, because each carrier has different energies. This implies that each of the electrons and holes will have different mobilities. Hence, carriers with energies higher than the average (hot electrons) will accumulate in the region of one of the crystal surfaces, and carriers with energies less than the average (cold electrons) will accumulate in the opposite surface. As a result, this redistribution generates

a transverse temperature gradient ∇T . This phenomenon is known as the Ettingshausen effect [5]. In this case we will have a thermoelectric field. The total electric field in this direction will include the electrical and temperature effects [6]. Very similar phenomena is discussed in review [7].

The simplicity of the Hall effect has led us to make some approximation of the problem. It generates some subtle pitfalls, which are present in every technique, but are not always studied or take into account. A simplification often made is to consider that effects due to finite breadth of the specimen can be neglected. These imply that the concentration of free charge carriers does not vary throughout the direction of the Hall field, $n - n_0 = p - p_0 = 0$ [5], the sample contains no free space charges [8]. As we will see this consideration is entirely correct, only when we have a very large recombination. On the other hand, the deviation of the equilibrium of the charge carriers (electrons and holes), give race to a concentration gradient along the z axis. This gradient generates a diffusion electric field. At this point other simplification is taken: $n - n_0 = p - p_0$, *i.e.*, the deviation of n and p is considered of the same order thereby ensures the space charge neutrality [9, 10]. But, in general case it is wrong (see [11]).

These problems are present in others studies, for example when a flows of a constant electric current through an anisotropic semiconductor, the carrier concentration deviates from the equilibrium value, producing regions of enrichment and depletion. In this case [12] considers only intrinsic semiconductor, *i.e.*, $n = p$.

When the system is in equilibrium we have the chemical potentials μ_n and μ_p for electrons and holes respectively, which are related by the equation $\mu_n + \mu_p = -\varepsilon_g$. When the system is far from equilibrium, the above relationship is no longer satisfied, *i.e.*, $\mu_n + \mu_p \neq -\varepsilon_g$. Two quasi Fermi levels

are generated, depending on the distribution of carriers along the z axis in the sample. In some references [2,4,7–9,11,13], it was neglected this dependence. When the system is in a nonequilibrium state, the authors [2, 4, 7–9, 11, 13] consider that the chemical potentials for electrons and holes are equal [10, 12], and that $n - n_0 = p - p_0$.

In semiconductors, in general one can see that $n \neq p$ which obliges us to work with two quasi-Fermi levels μ_n and μ_p , which are different. In this form we can give to a new quasineutrality condition, that it takes into account the distribution of carriers in the semiconductor samples, and the processes of volumetric and superficial recombination in the case of finite samples. With help of the nonequilibrium distribution of carriers, we determined the Hall coefficient, Hall field and we analyse the effects of the electrical potential as well as the origin of this. We are not consider the connection of the contacts to measure the Hall potential, but it is very important for making the measurements. This arises because usually the boundary conditions are formulated in the case of open circuits and free charge surfaces. Really it is necessary to work with appropriate boundary conditions that describe the current flow in the circuit, i.e, when the circuit is closed [14, 15].

2. Model description

Consider a bipolar semiconductor of parallelepiped form, length $2a$ along the x axis and $2b$ is the transverse length along the z axis. Additionally, we assume that the semiconductor is not degenerate and the carrier energy is given by $\varepsilon = p^2/2m_{n,p}^*$, where p is the quasimomentum of particles, $m_\beta^* = m_n^*, m_p^*$ are the electron and hole effective masses, ε is the carrier energy. The relations which govern the behaviour of electrons and holes in the presence of a longitudinal electric and transverse magnetic fields can be formulated as follows. The semiconductor is subjected to an external electric field E_x and a magnetic field H_y in the x and y directions respectively which do not vary over time. Then the hole and electron currents are given by [6]:

$$j_{\beta x} = e_\beta^2 I_{10}^\beta E_x - e_\beta^2 I_{20}^\beta E_z + e_\beta I_{20}^\beta \partial_z \mu_\beta(z), \quad (1)$$

$$j_{\beta z} = e_\beta^2 I_{20}^\beta E_x + e_\beta^2 I_{10}^\beta E_z - e_\beta I_{10}^\beta \partial_z \mu_\beta(z), \quad (2)$$

where $\beta = n, p$

$$I_{ik}^\beta = \frac{4\beta\omega_{H\beta}^{i-1}}{3\sqrt{\pi}m_\beta^*\theta^{5/2}} \int_0^\infty \frac{\varepsilon^{k+3/2} e^{-\varepsilon/\theta_\beta}}{\nu_\beta^i(\varepsilon) \left(1 + \left(\frac{\omega_{H\beta}}{\nu_\beta(\varepsilon)}\right)^2\right)} d\varepsilon. \quad (3)$$

Here, $e_\beta = e, -e$ are the charge for electrons and holes, $\omega_{H\beta} = eH/m_\beta^*c$ are the cyclotron frequency, $\theta_\beta = T_n, T_p$ are the electron and hole temperature,

$$\vec{E} = (E_x, 0, E_z) = (E_x^0, 0, -\nabla\varphi),$$

where φ is the electric potential, E_x^0 is the fixed external electric field and E_z is the electric field (the Hall field) variable

in the z direction, c is the velocity of light in vacuum. We work in the range of low electric field, in which the heating of electrons and holes is very small ($T_n \approx T_p \approx T$) [16]. This is achieved by assuming that the material is exposed at room temperature, and quickly dissipate the energy of the electrons and holes on the surface of the sample, so that the electrons and holes thermalize to the lattice temperature. In the presence of the unique relaxation mechanism of the electronic (hole) momentum, the frequency can be written as $\nu_\beta(\varepsilon) = \nu_{\beta 0}(T)(\varepsilon/T)^{-q_\beta}$ [17, 18], where q_β are the characteristic of relaxation mechanism.

We will consider that the magnetic field is weak $\omega_H/\nu_\beta \ll 1$, hence Eq. (3) can be written as:

$$I_{ik}^\beta \approx \frac{4\beta\omega_{H\beta}^{i-1}}{3\sqrt{\pi}m_\beta^* \nu_{\beta 0}^i(T)} \Gamma(iq_\beta + k + 5/2), \quad (4)$$

where $\Gamma(iq_\beta + k + 5/2)$ is the gamma function.

Let us write the distributions of chemical potentials, electron and hole concentrations in the following form,

$$\mu_\beta(z) = \mu_{\beta 0}(T) + \delta\mu_\beta(z),$$

$$\beta(z) = \beta_0(T) + \delta\beta(z),$$

where $\delta\mu_\beta(z)$ denotes the nonequilibrium chemical potentials for electrons and holes, $\delta\beta(z)$ are electron and hole nonequilibrium concentration, $\mu_{\beta 0}$ are the equilibrium electron and hole chemical potentials, β_0 are the equilibrium electron and hole concentrations. The equilibrium chemical potentials are related by the known expression [19],

$$\mu_{n0} = -\mu_{p0} - \varepsilon_g.$$

Electron and hole concentration in bipolar semiconductor are represented by the following expression [20],

$$\beta(z) = N_\beta(T) e^{\mu_\beta(T)/T}. \quad (5)$$

Here

$$N_\beta(T) = \frac{1}{4} (m_\beta T / 2\pi^3 \hbar^2)^{3/2}$$

are the electron and hole density of states. Expanding the equation we get

$$\begin{aligned} \beta(z) &= N_\beta(T) e^{\mu_{\beta 0}(T)/T} e^{\delta\mu_\beta(z)/T} \\ &\approx \beta_0 \left[1 + \frac{\delta\mu_\beta(z)}{T} \right]. \end{aligned}$$

It follows that

$$\delta\beta(z) = \frac{\beta_0}{T} \delta\mu_\beta(z). \quad (6)$$

Then, the hole and electron currents would be expressed as, (see Eqs. (1)-(2)):

$$j_{\beta x} = \sigma_{xx}^\beta E_x \mp \sigma_{xz}^\beta H E_z + D_{xz}^\beta H \partial_z \delta\beta(z), \quad (7)$$

$$j_{\beta z} = \pm \sigma_{zx}^\beta H E_x + \sigma_{zz}^\beta E_z \mp D_{zz}^\beta \partial_z \delta\beta(z), \quad (8)$$

with

$$\begin{aligned}\sigma_{xx}^{\beta} &= \frac{4e^2\beta}{3\sqrt{\pi}m_{\beta}^*\nu_{\beta 0}}\Gamma(5/2 + q_{\beta}), \\ \sigma_{xz}^{\beta} &= \frac{4e^3\beta}{3\sqrt{\pi}cm_{\beta}^*\nu_{\beta 0}^2}\Gamma(5/2 + 2q_{\beta}), \\ D_{xz}^{\beta} &= \frac{4e^2T}{3\sqrt{\pi}cm_{\beta}^*\nu_{\beta 0}^2}\Gamma(5/2 + 2q_{\beta}), \\ D_{zz}^{\beta} &= \frac{4eT}{3\sqrt{\pi}m_{\beta}^*\nu_{\beta 0}}\Gamma(5/2 + q_{\beta}), \\ \sigma_{zx}^{\beta} &= \sigma_{xz}^{\beta}, \quad \sigma_{zz}^{\beta} = \sigma_{xx}^{\beta}.\end{aligned}$$

3. Equations

The macroscopic description of the transport of non-equilibrium charge carriers is performed using the continuity equations for the electron and hole current densities and the Poisson Equation [21]. We consider the static case and the absence of the external generation of carriers (by light or other mechanism):

$$\nabla \cdot \vec{j}_{\beta} = \pm eR_{\beta}, \quad (9)$$

$$\nabla \cdot \vec{E} = \frac{4\pi}{\epsilon}\rho. \quad (10)$$

where ρ is the bulk charge density, ϵ is the material permittivity, R_{β} are the electron and hole recombination rates. Subtracting Eqs. (9), we obtain that

$$\nabla \cdot (\vec{j}_n + \vec{j}_p) - e(R_n - R_p) = 0. \quad (11)$$

The continuity equation for the total current $\vec{j} = \vec{j}_n + \vec{j}_p$ from the Maxwell's equations is

$$\nabla \cdot \vec{j} = 0. \quad (12)$$

Comparing Eqs. (11) and (12), we obtain

$$R_n - R_p = 0. \quad (13)$$

As it follows, the deviation of the concentration of the electrons trapped in the impurity level δn_t from the equilibrium value n_t^0 depends on the deviations of the electron and hole concentrations from their equilibrium values ($\delta n = n - n_0$, $\delta p = p - p_0$) through R_n and R_p . The recombination rates R_n and R_p are actually defined as the difference between the rates of capture of electrons and holes and the thermal generation [22]. It follows from Eq. (13) that $R_n = R_p = R$, the continuity equations for electrons and holes and Poisson equation take the form,

$$\nabla \cdot \vec{j}_{\beta} = \pm eR_{\beta} = \pm eR, \quad (14)$$

$$\nabla \cdot \delta \vec{E} = \frac{4\pi}{\epsilon}\delta\rho = \frac{4\pi e}{\epsilon}(\delta p - \delta n - \delta n_t). \quad (15)$$

where $\vec{E} = \vec{E}^0 + \delta \vec{E}$, $\rho = \rho_0 + \delta\rho$. In the case of a semiconductor that contains a concentration N_t of impurity, generation and recombination processes are assisted by traps (Shockley–Read model). The recombination rates R_n and R_p can be calculated by (we are assuming that electron, hole and phonon have the same temperature) [23],

$$R_n = \alpha_n(T)n(N_t - n_t) - \alpha_n(T)n_1N_t, \quad (16)$$

$$R_p = \alpha_p(T)pn_t - \alpha_p(T)p_1(N_t - n_t). \quad (17)$$

Here, $\alpha_n(T)$ and $\alpha_p(T)$ are the electron and hole capture coefficients, n_1 (p_1) the electron (hole) concentration when the Fermi level matches the activation energy of the impurity. From $R_n = R_p$, we obtain the following expression for n_t ,

$$n_t = \frac{N_t(\alpha_n n + \alpha_p p_1)}{\alpha_n(n + n_1) + \alpha_p(p + p_1)}, \quad (18)$$

By substitution of Eq. (18) in Eq. (16) we obtain

$$R_n = R_p = R = \frac{\alpha_n \alpha_p N_t (np - n_1 p_1)}{\alpha_n(n + n_1) + \alpha_p(p + p_1)}. \quad (19)$$

Consider that the applied field is weak. In this case, the excess of the electron concentration ($\delta n_t = n_t - n_t^0$) on the level of impurities is reduced to,

$$\delta n_t = \frac{\alpha_n(N_t - n_t^0)\delta n + \alpha_p n_t^0 \delta p}{\alpha_n(n + n_1^0) + \alpha_p(p + p_1^0)}, \quad (20)$$

where n_t^0 , n_1^0 , and p_1^0 are the equilibrium values of the respective concentrations. From Eq. (16) or (17) we can obtain the recombination rate through the traps as follow [24]:

$$R = \frac{1}{\tau} \left[\left(\frac{N_t - n_t^0}{n + n_1^0} \right) \delta n + \left(\frac{n_t^0}{p + p_1^0} \right) \delta p \right], \quad (21)$$

with

$$\frac{1}{\tau} = \frac{\alpha_n \alpha_p (n_0 + n_1^0)(p_0 + p_1^0)}{\alpha_n(n + n_1^0) + \alpha_p(p + p_1^0)}. \quad (22)$$

4. Quasineutrality approximation

The concept of quasineutrality is basic in semiconductor device analysis and widely used in the literature on transport phenomena [22]. Traditionally it defined as $r_D^2 \ll l_D^2, d^2$; where r_D is the Debye radius, l_D the diffusion length, d the thickness of the sample. It can be readily seen from Poisson's equation,

$$-\nabla^2 \varphi = 4\pi\rho/\epsilon, \quad (23)$$

that $\delta\rho/\rho_0 = (r_D/l_D)^2$. If $(r_D/l_D)^2 \ll 1$, then $\delta\rho \rightarrow 0$. Under this condition the Poisson equation becomes an algebraic equation ($\delta\rho \approx 0$), that does not need boundary conditions. This algebraic equation established a relationship between the excess of carriers (electrons and holes) and make the Poisson equation redundant. From the condition $\delta\rho \approx 0$, the non-equilibrium charge carrier concentration δn and δp , are related by,

$$\delta\rho = e(\delta p - \delta n - \delta n_t) = 0. \quad (24)$$

It follows from the Eqs. (20) and (24) that,

$$\delta p = \zeta \delta n, \tag{25}$$

where

$$\zeta = \frac{\alpha_n(n_0 + n_1^0 + N_t - n_t^0) + \alpha_p(p_0 + p_1^0)}{\alpha_n(n_0 + n_1^0) + \alpha_p(p_0 + p_1^0 + n_t^0)}. \tag{26}$$

In this case,

$$R_n = R_p = R = \frac{\delta n}{\tau_1}, \tag{27}$$

where

$$\tau_1 = \frac{\tau}{\Lambda}, \tag{28}$$

and

$$\Lambda = \left(\frac{N_t - n_t^0}{n + n_1^0} \right) + \left(\frac{n_t^0}{p + p_1^0} \right) \zeta. \tag{29}$$

5. Boundary conditions

The continuity equations for electrons and holes have to be supplemented by appropriate boundary conditions [14]. Along the z direction, it follow that,

$$(j_{nz} + j_{pz})|_{z=\pm b} = 0. \tag{30}$$

Similarly to the case of volume recombination, the expression for surface recombination, can be rewritten as follow:

$$R_n^s = R_p^s = R_s = S_n \delta n + S_p \delta p. \tag{31}$$

From the quasineutrality approximation (25),

$$R_s = S \delta n, \tag{32}$$

with

$$S = S_n + S_p \zeta. \tag{33}$$

In this case, S takes the meaning of surface recombination velocity. The surfaces of the semiconductor sample, change their properties at distances of the order of Debye radius and becomes inhomogeneous, which explains the dependence of the rate of recombination R as a function of the coordinate z , thus

$$j_{nz}|_{z=\pm b} = \pm eS_{1,2} \delta n, \tag{34}$$

where S_1 and S_2 are the surface recombination velocity in $z = b$ and $z = -b$.

6. Solution of the equations

The continuity equations $\nabla \cdot \vec{j} = \nabla \cdot (\vec{j}_n + \vec{j}_p) = 0$ show, that $j_{nz} + j_{pz} = j_z = cte$. From the geometry of the Hall experiment and from the Eq. (30), we can see that no current flow in the z direction, then the constant is zero at all points of the sample, $j_{nz} + j_{pz} = j_z = 0$. With this condition, we obtain the Hall electric field $E_H = E_z$, and considering the quasineutrality condition ($\delta p = \zeta \delta n$), we get

$$E_z = - \left(\frac{\sigma_{xz}^n - \sigma_{xz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) H E_x + \left(\frac{D_{zz}^n - \zeta D_{zz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) \frac{\partial \delta n}{\partial z}. \tag{35}$$

Substituting this expression into Eq. (8), we obtain:

$$j_{nz} = \left(\frac{\sigma_{xx}^p \sigma_{xz}^n + \sigma_{xx}^n \sigma_{xz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) H E_x - \left(\frac{\sigma_{xx}^n D_{zz}^p \zeta + \sigma_{xx}^p D_{zz}^n}{\sigma_{xx}^n + \sigma_{xx}^p} \right) \frac{\partial \delta n}{\partial z}, \tag{36}$$

$$j_{pz} = -j_{nz}. \tag{37}$$

We can see that under the quasineutrality condition and the Eq. (37), we work only with one of the continuity equations for electrons or holes. By substitution of the Eq. (36) in Eq. (9), it can be rewritten as follows

$$\frac{\partial^2 \delta n}{\partial z^2} - \frac{\delta n}{l_D^2} = 0. \tag{38}$$

Thus

$$\frac{1}{l_D^2} = \frac{1}{\tau_1 D}, \tag{39}$$

and

$$\frac{1}{D} = \frac{e(\sigma_{xx}^n + \sigma_{xx}^p)}{\sigma_{xx}^n D_{zz}^p \zeta + \sigma_{xx}^p D_{zz}^n}. \tag{40}$$

The solution of the Eq. (38) is:

$$\delta n(z) = \gamma_1 e^{z/l_D} + \gamma_2 e^{-z/l_D}. \tag{41}$$

The constants γ_1 and γ_2 should be determined from boundary conditions (34). Assuming that surface recombination velocity on the surfaces of the semiconductor sample is the same, $S_1 = S_2 = S$, we obtain:

$$\gamma_1 = H E_x \tilde{\gamma},$$

$$\gamma_2 = -\gamma_1 = -H E_x \tilde{\gamma},$$

$$\tilde{\gamma} = \frac{\sigma_{xx}^n \sigma_{xz}^p + \sigma_{xx}^p \sigma_{xz}^n}{2(\sigma_{xx}^n + \sigma_{xx}^p) \left(eS \sinh\left(\frac{b}{l_D}\right) + \left(\frac{eD}{l_D}\right) \cosh\left(\frac{b}{l_D}\right) \right)}.$$

Thus, the variation of electron concentration along the sample (z direction) is:

$$\delta n(z) = 2H E_x \tilde{\gamma} \sinh(z/l_D). \tag{42}$$

Now we can find the Hall field E_z

$$E_z = - \left(\frac{\sigma_{xz}^n - \sigma_{xz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) H E_x + \left(\frac{D_{zz}^n - \zeta D_{zz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) H E_x (2\tilde{\gamma}/l_D) \cosh(z/l_D). \tag{43}$$

From this result we obtain the value of the potential $\delta\varphi(z)$, which can be calculated from the equation $E_z = -\partial_z \varphi(z)$, integrating from $(-b, z)$ and assuming that $\varphi(-b) = 0$,

$$\delta\varphi(z) = \left(\frac{\sigma_{xz}^n - \sigma_{xz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) (z + b) H E_x - \left(\frac{D_{zz}^n - \zeta D_{zz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right) 2H E_x \tilde{\gamma} (\sinh(b/l_D) + \sinh(z/l_D)).$$

The Hall coefficient $R_H = E_z/Hj_x$ is given by:

$$R_H = -\frac{\sigma_{xz}^n - \sigma_{xz}^p}{(\sigma_{xx}^n + \sigma_{xx}^p)^2} + \left(\frac{D_{zz}^n - \zeta D_{zz}^p}{(\sigma_{xx}^n + \sigma_{xx}^p)^2} \right) (2\tilde{\gamma}/l_D) \cosh(z/l_D), \quad (44)$$

Notice that $R_H = R_H(z)$, thus it is not possible to define R_H as a constant. Another aspect to consider is the dependence of R_H on the amount $\zeta = \zeta(n_0, p_0, N_t, n_t^0, n_1^0, p_1^0)$, which connect with the equilibrium electrons and holes concentrations, as well as the impurity centres (traps) presents in the sample.

7. Results and Discussion

With the results presented above we can see in detail some special cases, when we have a finite and infinite lifetime of recombination, and study the intrinsic and extrinsic semiconductors:

i) Strong Recombination: This case is obtained when the volumetric recombination processes are strong. This happens when the lifetime is small $\tau \rightarrow 0$ ($d = 2b \gg l_D \gg r_D \rightarrow 0$) or the surface recombination velocity is large, $S \rightarrow \infty$. In this case the concentration of electrons $\delta n(z) \rightarrow 0$. The electrochemical potential is given by,

$$\delta\tilde{\varphi}_\beta(z) = \xi(z + b)HE_x, \quad (45)$$

where

$$\xi = \frac{e}{cm_p^* \nu_{p0}} \left[\frac{b^2 n_0 \Gamma(\frac{5}{2} + 2q_n) - p_0 \Gamma(\frac{5}{2} + 2q_p)}{b n_0 \Gamma(\frac{5}{2} + q_n) + p_0 \Gamma(\frac{5}{2} + q_p)} \right],$$

here $b = m_p^* \nu_{p0} / m_n^* \nu_{n0}$. Notice that the electrochemical potential is equal to the potential difference, *i.e.*, $\delta\tilde{\varphi}_n(z) = \delta\tilde{\varphi}_p(z) = \delta\varphi(z)$. The electrochemical potential is not affected by the presence of recombination processes because that processes are very fast. The electric potential depends solely on the accumulation of charge on the surface of the sample, which causes the polarization of the sample. On the other hand the Hall coefficient R_H , takes the form:

$$R_H = \frac{\sigma_{xz}^p - \sigma_{xz}^n}{(\sigma_{xx}^n + \sigma_{xx}^p)^2} = \frac{3\sqrt{\pi} [p_0 \Gamma(\frac{5}{2} + 2q_p) - b^2 n_0 \Gamma(\frac{5}{2} + 2q_n)]}{4ec [p_0 \Gamma(\frac{5}{2} + q_p) + b n_0 \Gamma(\frac{5}{2} + q_n)]^2}. \quad (46)$$

We see that $\delta n = \delta p = 0$, and recover the classic case for the Hall coefficient.

If the dominant scattering mechanism is by acoustic vibrations for electrons and holes ($q_n = q_p = -1/2$), we get

$$R_H = (3\pi/8ec)((p_0 - b^2 n_0)/(p_0 + b n_0)^2). \quad (47)$$

From this result we can apply two cases: extrinsic and intrinsic semiconductors.

i.a) Intrinsic Semiconductors ($n_0 \approx p_0$). In this case there are no impurities concentration ($N_t \approx 0$) and the only viable recombination mechanism is band-band, the quasineutrality condition (25), change by $\delta n \approx \delta p$. In this case $\zeta \approx 1$ and ($n_0 = p_0 = n_i$), we obtained:

$$R_H = \frac{3\sqrt{\pi} [\Gamma(\frac{5}{2} + 2q_p) - b^2 \Gamma(\frac{5}{2} + 2q_n)]}{4ecn_i [\Gamma(\frac{5}{2} + q_p) + b \Gamma(\frac{5}{2} + q_n)]^2}. \quad (48)$$

If the dominant scattering mechanism is by acoustic vibrations for electrons and holes ($q_n = q_p = -1/2$), we get the classic result,

$$R_H = \frac{3\pi}{8ecn_i} \left[\frac{1 - b}{1 + b} \right]. \quad (49)$$

i.b) Extrinsic Semiconductors ($n_0 \gg p_0$). The potential difference (Hall potential) is given by:

$$\delta\varphi_\beta(z) = \frac{e}{cm_n^* \nu_{n0}} \left[\frac{\Gamma(\frac{5}{2} + 2q_n)}{\Gamma(\frac{5}{2} + q_n)} \right] (z + b)HE_x.$$

In this case the Hall coefficient is:

$$R_H = -\frac{3\sqrt{\pi} \Gamma(5/2 + 2q_n)}{4ecn_0 \Gamma^2(5/2 + q_n)}. \quad (50)$$

Depending on the dominant dispersion process we obtain the classical results. If the scattering mechanism does not depend on energy ($q_n = 0$), then $R_H = -1/n_0 ec$. If the dominant scattering mechanism is by acoustic vibrations ($q_n = -1/2$) we obtained $R_H = -3\pi/8ecn_0$.

ii) Weak Recombination: This case is obtained when the volumetric recombination processes are weak (this happens when the lifetime is great $\tau \rightarrow \infty$, $l_D \gg d = 2b \gg r_D$), and the surface recombination is weak, $S \rightarrow 0$. The concentration of non-equilibrium electrons is:

$$\delta n(z) = \left[\frac{\sigma_{xx}^n \sigma_{xz}^p + \sigma_{xx}^p \sigma_{xz}^n}{\sigma_{xx}^p D_{zz}^n + \zeta \sigma_{xx}^n D_{zz}^p} \right] HE_x z, \quad (51)$$

In the point $z = 0$, the electronic concentration is zero and increase reaching its maximum value at the surface of the semiconductor sample $z = \pm b$. Similar situation we have with the concentration of holes. This behaviour will be reflected in the form of the potential difference,

$$\delta\varphi(z) = \left[\frac{\sigma_{xz}^n - \sigma_{xz}^p}{\sigma_{xx}^n + \sigma_{xx}^p} \right] (z + b)HE_x - \left[\frac{(D_{zz}^n - \zeta D_{zz}^p)(\sigma_{xx}^n \sigma_{xz}^p + \sigma_{xx}^p \sigma_{xz}^n)}{(\sigma_{xx}^n + \sigma_{xx}^p)(\sigma_{xx}^p D_{zz}^n + \zeta \sigma_{xx}^n D_{zz}^p)} \right] (z + b)HE_x,$$

Notice that in this case the potential difference is affected by the variation of the electron concentration $\delta n(z)$, and being different from the classical results. This expression includes explicitly the term ζ , which takes into account the physical material characteristics, *i.e.*, concentration of traps present in the material, the activation energy, as well as the recombination coefficients α_n and α_p . This is a big difference with all results presented in the literature, which excludes this dependence [9, 11, 13]. As we have a variation in concentration two

quasi-Fermi levels exist, which reflected in the electrochemical potentials for electrons and holes ($\delta\tilde{\varphi}_n(z) \neq \delta\tilde{\varphi}_p(z)$). The Hall coefficient dependence on the term ζ , as we see below

$$R_H = -\frac{\sigma_{xz}^n - \sigma_{xz}^p}{(\sigma_{xx}^n + \sigma_{xx}^p)^2} + \left[\frac{(D_{zz}^n - \zeta D_{zz}^p)(\sigma_{xx}^n \sigma_{xz}^p + \sigma_{xx}^p \sigma_{xz}^n)}{(\sigma_{xx}^n + \sigma_{xx}^p)^2 (\sigma_{xx}^p D_{zz}^n + \zeta \sigma_{xx}^n D_{zz}^p)} \right] \quad (52)$$

It is important to note that when the recombination processes are weak and only when $\zeta = 1$ it can be reproduce of the result presents in [9, 13] for the Hall effect. If the dominant scattering mechanism is by acoustic vibrations for electrons and holes ($q_n = q_p = -1/2$), we obtain:

$$R_H = \frac{3\pi}{8ec} \left[\frac{p_0 - \mathfrak{b}n_0}{(\mathfrak{b}n_0 + p_0)(n_0 + p_0)} \right]. \quad (53)$$

ii.a) Intrinsic Semiconductors ($n_0 \approx p_0$). In this case there are no impurities concentration ($N_t \approx 0$) and the only viable recombination mechanism is band-band. The quasineutrality condition (25), change to $\delta n \approx \delta p$, with $\zeta \approx 1$, and

$$\delta n(z) = \left[\frac{\Gamma(\frac{5}{2} + 2q_n)}{\Gamma(\frac{5}{2} + q_n)} + \frac{1}{\mathfrak{b}} \frac{\Gamma(\frac{5}{2} + 2q_p)}{\Gamma(\frac{5}{2} + q_p)} \right] \frac{e^2 n_0 H E_x z}{2cT m_n \nu_{n0}},$$

The Hall constant in the case when the dominant scattering mechanism is by acoustic vibrations for electrons and holes ($q_n = q_p = -1/2$), is:

$$R_H = \frac{3\pi}{16ecn_i} \left[\frac{1 - \mathfrak{b}}{1 + \mathfrak{b}} \right]. \quad (54)$$

ii.b) Extrinsic Semiconductors ($n_0 \gg p_0$).

The concentration of non-equilibrium electrons is:

$$\delta n(z) = \left[\frac{\mathfrak{b}\Gamma(\frac{5}{2} + 2q_n)}{\Gamma(\frac{5}{2} + q_n)} + \frac{\Gamma(\frac{5}{2} + 2q_p)}{\Gamma(\frac{5}{2} + q_p)} \right] \frac{e^2 p_0 H E_x z}{2cT m_p \nu_{p0} \zeta},$$

In this case we have $\zeta = \zeta(n_0, p_0, N_t, n_t^0, n_1^0, p_1^0)$ thus we consider two cases, i) low temperature, where $N_t \sim n_t^0 \gg n_0 \sim n_1^0 \gg p_0 \sim p_0$, consequently $\zeta \rightarrow 1$, ii) high temperature, where $n_1^0 \gg N_t \sim n_0 \gg p_0 \sim n_t^0 \gg p_1^0$, thus $\zeta \rightarrow 1$ too. The similarity of the limits is due to the following: considering that the recombination coefficients for electrons and holes are of the same order, in the first case the concentration of trapping centres dominate over other material concentrations and the low temperature reduce the transition of electrons from the valence band to the conduction band. In the

second case the concentration of electrons in the conduction band dominates in the semiconductors materials, additionally the high temperature stimulate the generation of electrons.

The electric potential is given by

$$\delta\varphi(z) = \frac{e(z+b)}{cm_n^* \nu_{n0}} \left[\frac{\Gamma(\frac{5}{2} + 2q_n)}{\Gamma(\frac{5}{2} + q_n)} \right] H E_x, \quad (55)$$

and the Hall coefficient is

$$R_H = -\frac{3\sqrt{\pi}}{4ecn_0} \frac{\Gamma(5/2 + 2q_n)}{\Gamma^2(5/2 + q_n)}. \quad (56)$$

From the above results we see that the potential difference and the Hall coefficient are identical to the case of strong recombination for extrinsic semiconductors. This can be explained by saying due to the high concentration of electrons in the sample, the effects of a redistribution of carriers in the potential are negligible, only it is important the redistribution of charge on the surface of the sample.

8. Conclusion

When the recombination is strong the Hall potential, Hall field and Hall coefficient do not depend on the distribution of the nonequilibrium carriers, and the trapping centres. We can say that independent of the amount of impurities presents in the sample they do not affect the measurement of the Hall potential. The source of the Hall potential is due to accumulation of carriers on the surface of the sample.

If the recombination is weak, the Hall coefficient, the Hall field, as well as the potential is modified by the presence of the distribution of the nonequilibrium carriers $\delta n(z)$, that includes explicitly the dependence of the concentration of impurities.

In the case of extrinsic semiconductors, the Hall constant does not depend on the distribution of the nonequilibrium carriers in cases of strong and weak recombination. The measured potential does not depend on the presence of the redistribution of the nonequilibrium carriers along the z axis, it only depend on the concentration of carriers in the surface of the semiconductor sample.

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1. It is practice in the literature to write H in a Lorentz force ($\vec{F} = e\vec{v} \times \vec{H}/c$), whenever $\vec{H} \equiv \vec{B}$. For those cases where $\vec{H} \neq \vec{B}$, it would be necessary to replace \vec{H} with \vec{B} in the Lorentz force and in the ensuing theory.

2. Edward Ramsden, Hall-Effect Sensors (Newnes-Elsevier, USA, 2006).

3. If we take an electron, the Lorentz force will not be balanced across the Hall field, $e\vec{v}H/c \neq eE_z$, because the velocity of

- electrons depends on the energy of the carrier $v(\varepsilon)$. In this case it is necessary to work with the average velocity of the electrons $\langle v(\varepsilon) \rangle$, which give equality in the Lorenz force $e \langle v(\varepsilon) \rangle H/c = eE_z$.
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