On renormalizability of a non-linear abelian gauge model

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Considering that physical processes work as a group, a whole gauge procedure becomes necessary. In a previous work, we have developed this new approach for a classical non-linear abelian gauge model. At this work, one intends to understand the corresponding quantum extension through its renormalizability. For this, one studies Feynman graphs, quantum action principle, power counting procedure, Ward identities and primitively divergent graphs. Under this renormalization procedure one computes a non-linear whole abelian gauge model.

Keywords: Whole gauge symmetry; renormalizability; power counting; primitively divergent graphs; Ward identities.

Considerando que los procesos físicos actúan en grupo, un procedimiento de calibre de totalidad resulta necesario. En un trabajo anterior, hemos desarrollado este nuevo enfoque para um modelo abeliano no lineal clásico. En este trabajo intentamos comprender la extensión cuántica correspondiente mediante su renormalizabilidad. Para esto, estudiamos los gráficos de Feynman, principio de acción cuántica, procedimiento de contaje de potencias, identidades de Ward y gráficos primitivamente divergentes. Bajo este procedimiento de renormalización hacemos los cálculos para un modelo de calibre abeliano no-lineal.

Descriptores: Simetria de calibre de totalidad; renormalizabilidad; contaje de potencias; gráficos primitivamente divergentes; identidades de Ward.

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1. Introduction

Modern physics considers that the phenomena comprehension must be stipulated through the combination between a given concept supported by a determined symmetry. Under this context, this work is motivated to study the whole meaning through gauge invariance. It yields the so-called 'Non-linear abelian gauge model' which says that instead looking physics through ultimate elements one should systematize the phenomena under whole connections. Introduce through its fields network the interdependence meaning on the physical relationships.

Different origins based on Kaluza-Klein, supersymmetry, fibre bundle, σ-model formulations were studied for considering that such whole primordial notion can be configured through gauge theory [1]. As result, they are propitiating to introduce a fields set $\{A_{\mu I}\}$, transforming under a common gauge parameter

$$A_{\mu I}(x) \rightarrow A'_{\mu I}(x) = U A_{\mu I} U^{-1} + \frac{i}{g} \partial_\mu U \cdot U^{-1}, \quad (1)$$

where $I = 1, \ldots, N$.

The general organization of our article is: in Sec. 2, we present the general construction of our whole action; the Feynman rules are derived in Sec. 3; the analysis of the propagators and the field mixing is carried out in Sec. 4; in Sec. 5, we investigate the power-counting renormalizability; the Ward identities are worked out in Sec. 6 and, finally, Sec. 7 is devoted to our conclusions. An Appendix follows, where the primitively divergent diagrams of our model are depicted.

2. Whole lagrangian

In a previous work, one has established a non-linear abelian gauge invariant Lagrangian [2]. It is based on the Eq. (1) application for the abelian case. It brings the presence of N-potential fields $A_{\mu I}$ transforming under a common simple $U(1)$ group, where I means a flavour index. Physically, provides the antireductionism principle that says that nature acts together as a set $\{A_{\mu I}\}$. It also enlarges the gauge transformation for polynomial expressions $P_I(\alpha)$ on the gauge parameter [3,4],

$$A_{\mu I} \rightarrow A'_{\mu I} = A_{\mu I} + P'_I \partial_\mu \alpha. \quad (2)$$

Notice that Eq. (2) considers that each field transforms differently. However, simplicity leads us to study this whole gauge renormalizability by taking the case $P'_I(\alpha) = 1$. Thus we are going to consider from Eq. (2) two types of basis fields sets. They are called the so-called constructor basis $\{D_\mu, X_\mu^i\}$ where $2 \leq i \leq N$, and the physical basis $\{G_{\mu I}\}$ [5].

The constructor basis advantage is that it allows a more direct vision on gauge invariance. It redefines

$$D_\mu = \sum_I A_{\mu I}, \quad (3)$$

and

$$X_{\mu 1} = A_{\mu 1} - A_{\mu 2}, \quad X_{\mu(N-1)} = A_{\mu 1} - A_{\mu N}, \quad (4)$$

where, one gets the following transformation law

$$D_\mu \to D'_\mu = D_\mu + \partial_\mu \alpha, \quad X_{\mu i} \to X'_{\mu i} = X_{\mu i}, \quad (5)$$

that shows the presence of Maxwell plus Proca fields. The correspondent gauge invariant Lagrangian is

$$L = Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m_{ij} X_\mu{}^i X^{\mu j},$$

with

$$Z_{\mu\nu} = d D_{\mu\nu} + \alpha_i X_{\mu\nu}{}^i$$
$$+ \beta_i \Sigma_{\mu\nu}^i + \rho_i g_{\mu\nu} \Sigma_\alpha^{i\alpha} + \gamma_{ij} X_\mu{}^i X_\nu{}^j,$$

where

$$D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu,$$
$$X_{\mu\nu}{}^i = \partial_\mu X_\nu{}^i - \partial_\nu X_\mu{}^i,$$
$$\Sigma_{\mu\nu}^i = \partial_\mu X_\nu{}^i + \partial_\nu X_\mu{}^i. \quad (6)$$

Considering that the physical masses are the poles of two point Green functions there is a physical basis which diagonalizes the transverse sector. For this, one has to introduce a matrix Ω [6] from where the physical basis $\{G_I\}$ is defined as

$$G_{\mu I} = \Omega_{I1}^{-1} D_\mu + \Omega_{Ii}^{-1} X_\mu^i. \quad (7)$$

Notice that the Ω matrix just depends on free parameters. The Ω matrix rotates the fields in order to obtain the propagators in terms of their physical poles. Now, writing the gauge transformation in terms of physical fields, one gets

$$G_{\mu I}(x) \to G'_{\mu I}(x) = G_{\mu I}(x) + \Omega_{I1}^{-1} \partial_\mu \alpha(x), \quad (8)$$

where every physical field being transformed becomes specified by a weight Ω_I^{-1} factor. Given that the model gauge invariance is naturally obtained through the constructor basis $\{D_\mu, X_{\mu i}\}$, its transition to the physical basis $\{G_{\mu I}\}$ is obtained from the Ω matrix invertible condition $\Omega_{IK}\Omega_{KJ}^{-1}=\delta_{IJ}$. Notice then, that the physical field $G_{\mu I}$ is that one associated to a particle with mass $m_I^2 \equiv m_{II}^2$, its corresponding propagator has the expression

$$\langle G_{\mu I} G_{\nu J} \rangle_T = \frac{1}{\Box + m_I^2} \delta_{IJ}.$$

In this way, one says that the transverse $G_{\mu I}$ field correspond to a particle with mass m_I [7].

Equation (8) yields the following transverse diagonalized gauge invariant Lagrangian

$$\mathcal{L}(G) = Z_{[\mu\nu]} Z^{[\mu\nu]} + Z_{(\mu\nu)} Z^{(\mu\nu)} - m_{II}^2 G_\mu{}^I G^{\mu I}$$
$$+ \xi_{IJ} \left(\partial_\mu G^{\mu I} \right) \left(\partial_\nu G^{\nu J} \right), \quad (9)$$

where the corresponding field strengths are written as

$$Z_{[\mu\nu]} = b_I G_{\mu\nu}{}^I + z_{[\mu\nu]},$$
$$Z_{(\mu\nu)} = \beta_I S_{\mu\nu}{}^I + \rho_I g_{\mu\nu} S_\alpha{}^{\alpha I} + z_{(\mu\nu)} + g_{\mu\nu} w_{(\alpha}{}^{\alpha)},$$

with

$$G_{\mu\nu}{}^I = \partial_\mu G_\nu{}^I - \partial_\nu G_\mu{}^I, \quad S_{\mu\nu}{}^I = \partial_\mu G_\nu{}^I + \partial_\nu G_\mu{}^I,$$
$$z_{[\mu\nu]} = \gamma_{[IJ]} G_\mu{}^I G_\nu{}^J, \quad z_{(\mu\nu)} = \gamma_{(IJ)} G_\mu{}^I G_\nu{}^J$$
$$w_{(\mu\nu)} = \tau_{(IJ)} G_\mu{}^I G_\nu{}^J. \quad (10)$$

Notice that coefficients b_I, β_I and so on are derived from original ones at Eq. (6).

Thus, in order to study the model renormalizability, we should rewrite Eq. (9) as $\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{int}$, where

$$\mathcal{L}_K = a_{(IJ)} \left(\partial_\mu G_\nu{}^I \right) \left(\partial^\mu G^{\nu J} \right) + b_{(IJ)} \left(\partial_\mu G_\nu{}^I \right) \left(\partial^\nu G^{\mu J} \right)$$
$$+ c_{(IJ)} \left(\partial_\mu G^{\mu I} \right) \left(\partial_\nu G^{\nu J} \right) - m_I^2 G_{\mu I} G^{\mu I}, \quad (11)$$

with

$$a_{(IJ)} = 2 b_I b_J + 2 \beta_I \beta_J, \quad b_{(IJ)} = -2 b_I b_J + 2 \beta_I \beta_J,$$
$$c_{(IJ)} = 4 \beta_I \rho_J + 4 \beta_J \rho_I + 16 \rho_I \rho_J, \quad (12)$$

and

$$\mathcal{L}_{int} = \mathcal{L}_{int}^{(3)} + \mathcal{L}_{int}^{(4)}, \quad (13)$$

where

$$\mathcal{L}_{int}^{(3)} = a_{IJK} \left(\partial_\mu G_\nu{}^I \right) G^{\mu J} G^{\nu K}$$
$$+ b_{I(JK)} \left(\partial_\mu G^{\mu I} \right) G_\nu{}^J G^{\nu K}, \quad (14)$$

$$\mathcal{L}_{int}^{(4)} = a_{(IJ)(KL)} G_\mu{}^I G^{\mu J} G_\nu{}^K G^{\nu L}$$
$$+ b_{(I(JK)L)} G_\mu{}^I G_\nu{}^J G^{\mu K} G^{\nu L}, \quad (15)$$

with

$$a_{IJK} = 4 b_I \gamma_{[JK]} + 4 \beta_I \gamma_{(JK)},$$
$$b_{I(JK)} = 4 \beta_I \tau_{(JK)} + 4 \rho_I \gamma_{(JK)} + 16 \rho_I \tau_{(JK)},$$
$$a_{(IJ)(KL)} = 2 \gamma_{(IJ)} \tau_{(KL)} + 4 \tau_{(IJ)} \tau_{(KL)},$$
$$b_{(I(JK)L)} = \gamma_{[IJ]} \gamma_{[KL]} + \gamma_{(IJ)} \gamma_{(KL)}. \quad (16)$$

3. Feynman rules

Expanding in powers of $G_{\mu I}$, one gets for the effective action of the classical physical field $\Gamma(G)$. Physical interpretations are in general more explicit in momentum space. It yields,

$$\Gamma(G) = \sum_{n=1}^\infty \frac{i}{n!} \int \frac{d^4 k_1}{(2\pi)^4} \cdots \frac{d^4 k_{n-1}}{(2\pi)^4} \Gamma^{(n)}{}_{IJ...N}^{\mu\nu...\rho}(k_1; \ldots; k_n)$$
$$\times G_I(-k_1) \ldots G_N(-k_n), \quad (17)$$

where $k_1 + k_2 + \cdots + k_n = 0$.

An advantage of this formalism, written through the effective action Γ, relies on the fact that it allows an expansion in terms of the number of loops. This serie is written in terms of powers of the Planck constant,

$G_J(-p_2)$ $G_K(-p_3)$

$\Gamma^{(n)}(p_1, \ldots, p_n)$

$G_I(-p_1)$ $G_N(-p_N)$
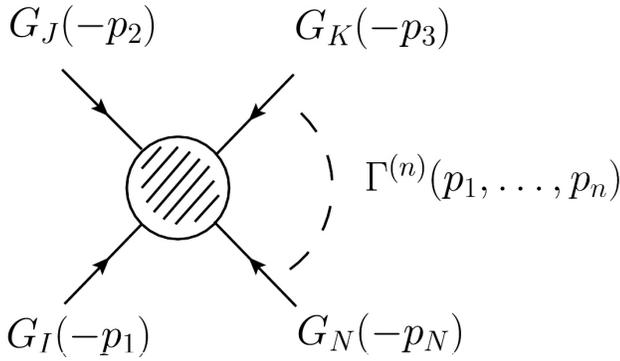
FIGURE 1. The convention to compute the effective action.

$$\Gamma = \sum_{L=0}^{\infty} \hbar^{L-1} \Gamma_{(L)}. \tag{18}$$

The conventions for the momentum flows are indicated in Fig. 1. Equations (11) and (13) show that propagators and vertices can be read off as coefficients of Fourier transform of the fields. This useful derivation of the Feynman rules is because at tree level, $\Gamma(G)$ means the classical action plus the external source terms. So, by taking the action,

$$S(G) = \int d^4x L[G_I, \partial G_I],$$

and making its Fourier transform, we can read off $\Gamma^{(n)\,\mu\nu\ldots\rho}_{IJK\ldots N}(k_1, \ldots, k_n)$ by adjusting the factors i and $(1/n!)$.

The quadratic part of the Lagrangian containing N-potential fields written in the constructor basis $V_\mu \equiv \{D_\mu, X_\mu^i\}$ is

$$L_K = \frac{1}{2} V_\mu^t \left[(K\Box + M^2) P_T^{\mu\nu} + (B\Box + M^2) P_L^{\mu\nu} \right] V_\nu. \tag{19}$$

Propagators are obtained from the block matrix inverse of the kinetic part. Writing in terms of the transverse and longitudinal projection operators

$$\langle V_{\mu I} V_{\nu J} \rangle_T = i \left[\frac{1}{Kk^2 - M^2} \right]^{-1}_{IJ} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right), \tag{20}$$

$$\langle V_{\mu I} V_{\nu J} \rangle_L = i \left[\frac{1}{Bk^2 - M^2} \right]^{-1}_{IJ} \frac{k_\mu k_\nu}{k^2}. \tag{21}$$

Observe that K, M^2, B are $N \times N$ real and symmetric matrices. Each matrix can be individually diagonalized by an orthogonal transformation. However, the diagonalization is not in general simultaneous.

For the vertices, the correspondence between the interacting terms that appear in the Lagrangian and the Feynman vertices is not one-to-one. The introduction of more potential fields in the same group enlarges the possibilities for playing with the Lorentz indices and also appear different possibilities for distributing the flavour indices. Thus it appears a kind of topology of gauge invariance where a determined graph incorporates different contributions from the Lagrangian terms.

4. Propagators

We should not prepare any interaction without first verifying about propagators. They represent the first contact between a field theory model and physics. Through then one reads off the external and internal quantum numbers carried by a given model. Their poles and residues inform about conditions for the model physicity, on the involved degrees of freedom, and on properties as the decay rate width. However the most crucial information is about the precondition for the associated physical entities be calculable: the renormalizability. Therefore for a given model not be rejected on its first stage it will need a health propagator. After this, when interaction be switched on, it should contain dimensionless coupling constants.

In order to take the BRST renormalization programme to systematize the model we have to check the behavior of its corresponding propagators. They should be characterized in the form $(N(p)/p^2 - m^2)$, where $N(p)$ is a Lorentz covariant polynomial. This ensures a well defined perturbation theory in the sense that the Quantum Action Principle holds [8]. Then, as an immediate consequence, the counterterms are local and the model can be discussed to all orders of perturbation theory by means of Ward identities.

Three new aspects are developed by such non-linear abelian gauge model on propagator structure. They are an expression which contains various poles, the presence of mixing propagators between fields, and an associated transformation formula between propagators under a given field basis change. For showing such new aspects consider the $V_\mu \equiv \{D_\mu, X_{\mu i}\}$ constructor basis. Analyzing the transverse sector, as example, one gets following matricial expression:

$$\langle T V_\mu(x) V_\nu(y) \rangle_T = \frac{\text{cof}(\Box - K^{-1} M^2)}{\det(\Box - K^{-1} M^2)} K^{-1} P_{\mu\nu}^T, \tag{22}$$

where the numerator and denominator expressions are written as:

$$N = \sum_{p=0}^{n-1} b_p \mu^{2(p-1)} k^{2(n-p-1)}, \tag{23}$$

$$D = \sum_{p=0}^{n} a_p \mu^{2p} k^{2(n-p)}. \tag{24}$$

Notice as every propagator in Eq. (24) matrix is carrying N poles given by the determinant. This shows that has such extended gauge model the correspondence between propagators and poles is not necessarily univocal. Another fact showed is the presence of mixed propagators as $\langle T D_\mu(x) X_{\nu i}(y) \rangle$ and $\langle T X_{\mu i}(x) X_{\nu j}(y) \rangle$.

Now taking the basis transformation, $V_\mu = \Omega G_\mu$, one derives the following propagators covariance law [5]

$$\langle T(V_\mu V_\nu) \rangle = \Omega \langle G_\mu G_\nu \rangle \Omega^t, \tag{25}$$

which shows a rotation where poles are preserved (Ω matrix does not depend on momenta). However it also splits numerators with an expression depending on the parametrization basis. Consequently the respective residual matrix will transform as:

$$R_{GG}(k^2 = m^2) = \Omega^{-1} R_{VV}(k^2 = m^2)\Omega^{-1t}. \qquad (26)$$

An important consequence from (26) is about the residue sign under Ω transformation. Consider first the diagonal case. Given

$$R_{ij}^{VV}(k^2 = m^2) = \delta_{ij}, \qquad (27)$$

the residue sign is preserved as

$$R_{ij}^{GG}(k^2 = m^2) = \sum_l (\Omega_{ij}^{-1})^2 > 0. \qquad (28)$$

However this positivity invariance is not clear when mixed propagator are considered. It gives an expression:

$$R_{ij}^{GG}(k^2 = m^2) = \sum_l \Omega_{il}^{-1}\Omega_{jl}^{-1}, \qquad (29)$$

with an undetermined sign. A more general case analyzing about the residue sign through field transformations is when the initial field basis also contains non-diagonal propagators. Then, Eq. (26) informs that given a determined residue matrix the sign of its diagonal elements will not be necessarily preserved. We should notice that only the determinant sign of this residue matrix is invariant under the Ω transformation.

A correct power counting for Feynman integrals is a necessary ingredient for the perturbation theory be implemented. Thus we should initially study on the Eq. (22) asymptotic behavior. Considering the property where the parameters which build up the propagators coefficients can take any value without violating gauge invariance, it yields the general limit $(1/(k^2)^r)$ which verifies the expected result $[1/M^2]$ from dimensional analysis. The coefficient r would work as a kind of power of circumstances. This because it represents one type of choice on the model parameters values. For example, under certain circumstances of the theory parameters generate the following expression for Eq. (22):

$$\langle T(V_\mu(x)V_\nu(y))\rangle \sim \frac{b(\mu^2)^{r-1}}{a_0(k^2)^r + \cdots + a_n k^2}, \qquad (30)$$

where the UV and IR limits are $(1/(k^2)^r)$ and $(1/k^2)$ respectively. Thus, in principle, Eq. (24) reserve the possibility of containing good ultraviolet and infrared limits (ghosts are not relevant here).

As a particular case expression (22) can be rewritten through fraction parts decomposition. It gives:

$$\langle TV_\mu(x)V_\nu(y)\rangle_T = \frac{t_1}{k^2 - m_1^2} + \frac{t_1^*}{k^2 - m_1^2}$$
$$+ \frac{t_2}{k^2 - m_2^2} + \cdots + \frac{t_n}{k^2 - m_n^2}. \qquad (31)$$

Equation (31) is showing the propagator as a meromorphic function made of by isolated singularities which are eigenvalues of $K^{-1}M^2$ matrix and by analytic functions t's. One pole will be necessarily zero, the parameters $t_1, \ldots, t_n$ are dimensionless and depending on theory free coefficients and mass parameters. Observe that a given pole can be repeated in this expression since associated to a different t^*. The relevance of this fact is on the existence of equal masses associated to distinct residues and avoiding the undesired presence of double pole. Concluding either through Eq. (24) or by Eq. (31) we notice the desired propagator shape $(N(p)/p^2 - m^2)$ for the BRST systematic be followed. However it is crucial to avoid the presence of double pole. Observe that it allows the existence of equal masses but since they appear in Eq. (31) decomposition associated to distinct residues.

Another instruction contained in propagator is related to the BPHZL renormalization procedure where the infrared field dimension must be bigger or equal to the ultraviolet field dimension. We should emphasize that the importance of this method is due its validity for massless, massive and mixed cases. Then, considering that this non-linear abelian gauge model is the mixed case, it yields different ultraviolet and infrared dimensions for the involved fields. For UV case, one gets $d_{UV}^V = 2 - r$ where r is the ultraviolet asymptotic behavior. For IR case, $d_{IR}^V = 1, 2$ for massless and massive cases, respectively. Consequently, for $r \geq 1$ the existence of Green's functions for the counterterms Γ^c, is guaranteed by Lowenstein condition. Observe that this condition must be analyzed here for every propagator, separately.

5. Power counting analysis

A first aspect on the renormalizability program is to consider the power counting. By a mixture of topology and power counting one can understand where the divergences are [9]. Basically the Feynman diagrams superficial degree of divergence δ is the difference between the power momentum k in the numerator and denominator

$$\delta = dL - \sum_A \alpha_A I_A + V_3, \qquad (32)$$

where L means independent loop integrations, each providing in d dimensions, d powers of momenta. I_A means internal momenta, with lines

$$\langle G_{\mu I}\, G_{\nu J}\rangle \cong \int d^d k \frac{1}{(k^2)^{p_{IJ}}}$$

and

$$\langle \overline{\psi}\psi\rangle \cong \int d^d k \frac{1}{k},$$

which means for the vector fields propagators $\alpha_A = 2p_{IJ}$ and for the Dirac field $\alpha_A = 1$. V_3 means a vertex with 3 legs (one derivative).

The number of loops expressed in terms of vertices and internal lines is given by

$$L = \sum_A I_A - V + 1. \qquad (33)$$

Now, one should investigate on vertices. There are two types. A first one called V_3 which involves one derivative and second one V_4 without derivative. Considering V_3 there are three types: V_{3G_I} with three lines coming out from the same field $G_{\mu I}$, V_{2G_I,G_J} with two lines from the same field $G_{\mu I}$ and another one from a different field and V_{G_I,G_J,G_K} where the three fields are different. Thus,

$$V_{3,I} \equiv V_{3G_I} + \sum_I V_{2G_I,G_J} + \sum_{J,K} \frac{1}{2!} V_{G_I,G_J,G_K}. \qquad (34)$$

Considering V_4 one gets the following types of vertices: V_{4G_I}, V_{3G_I,G_J}, $V_{2G_I,2G_J}$, V_{2G_I,G_J,G_K}, V_{G_I,G_J,G_K,G_L}, $V_{G_I\overline{\psi}\psi}$. It gives

$$\begin{aligned} V_{4,I} &\equiv V_{4G_I} + V_{G_I,\overline{\psi}\psi} \\ &+ \sum_J \left(V_{3G_I,G_J} + V_{2G_I,2G_J} + V_{G_I,3G_J} \right) \\ &+ \sum_{J,K} \left(\frac{1}{2!} V_{2G_I,G_J,G_K} + V_{G_I,2G_J,G_K} \right) \\ &+ \sum_{J,K,L} \frac{1}{3!} V_{G_I,G_J,G_K,G_L}. \end{aligned} \qquad (35)$$

Substituting above expressions in (32), one obtains for four dimensions

$$\begin{aligned} \delta &= 4 - \sum_{I,J}(4 - 2p_{IJ})I_{G_I\,G_J} + 3I_{\overline{\psi}\psi} - 3\sum_I \bigg\{ V_{3G_I} \\ &+ \sum_J V_{2G_I,G_J} + \sum_{J,K} \frac{1}{2!} V_{G_IG_JG_K} \bigg\} - 4\sum_I \bigg\{ V_{4G_I} \\ &+ V_{G_I,\overline{\psi}\psi} + \sum_J \left(V_{3G_I,G_J} + V_{2G_I,2G_J} + V_{G_I,3G_J} \right) \\ &+ \frac{1}{2!} V_{2G_I,G_J,G_K} + V_{G_I,2G_J,G_K} \\ &+ \sum_{J,K,L} \frac{1}{3!} V_{G_I,G_J,G_K,G_L} \bigg\}. \end{aligned} \qquad (36)$$

A next step is to consider the topological relationships. For the spinorial field, it gives

$$2I_{\overline{\psi}\psi} + E_\psi = 2\sum_I V_{G_I\overline{\psi}\psi}. \qquad (37)$$

For the N-vector fields, one gets

$$\begin{aligned} 2I_{G_IG_I} &+ \sum_J I_{G_IG_J} + E_{G_I} = 4V_{4G_I} \\ &+ 3\left(\sum_J V_{3G_I,G_J} + V_{3G_I} \right) + \sum_{J,K,L} \frac{1}{3!} V_{G_I,G_J,G_K,G_L} \\ &+ 2\sum_J \left(V_{2G_I,2G_J} + V_{2G_I,G_J} + \frac{1}{2!}\sum_K V_{2G_I,G_J,G_K} \right) \\ &+ \sum_J \bigg\{ V_{G_I,3G_J} + V_{G_I} + \sum_{J,K} \left(V_{G_I,2G_J,G_K} \right. \\ &\left. + \frac{1}{2!} V_{G_I,G_J,G_K} \right) \bigg\} + V_{G_I\overline{\varphi}\varphi}. \end{aligned} \qquad (38)$$

Putting Eqs. (36), (37) and (38) together, one obtains for the model superficial degree of divergence:

$$\begin{aligned} \delta &= 4 - \frac{3}{2}E_\psi - \sum_I E_{G_I} \\ &+ \sum_I 2(1 - p_{II})I_{G_IG_I} + \sum_{IJ}(1 - p_{IJ})I_{G_IG_J}. \end{aligned} \qquad (39)$$

Equation (39) shows that even containing massive vector field the model preserves a health power counting without requiring coupling conserved currents as in the usual Proca case [10]. Then, notice that for the usual cases where $p_{II}=1$, we find

$$\delta = 4 - \frac{3}{2}E_\psi - \sum_I E_{G_I}, \qquad (40)$$

where Eq. (40) proves the existence of a finite number of primitively divergent graphs. It yields a finite number of counterterms. Thus, for a given number of external lines, no matter to what order of perturbation theory we go, the superficial degree of divergence remains the same. In a more rigorous treatment we have to worry about the momenta in some subdiagram going to infinity with other momenta held fixed. At Appendix, we list for one loop the corresponding primitively divergent graphs.

6. Ward identities

Another important aspect in order to analyse the renormalizability is on the Ward identity [11]. In the usual electrodynamics its importance is to show that the scalar component of the photon field does not receive radiative correction. This means that the spin-zero part remains frozen. Our objective here will be to derive the Ward identities for this non-linear abelian model and to take similar consideration.

Considering the $U(1)$-gauge transformations in the set of fields $\{G_{\mu I}, \psi, \overline{\psi}\}$,

$$\delta G_{\mu I} = \Omega_{I1}^{-1} \partial_\mu \alpha, \quad \delta\psi = i\alpha\psi, \quad \delta\overline{\psi} = -i\alpha\psi, \qquad (41)$$

and the corresponding Green's functions funcional generator

$$Z[J_I, \xi, \overline{\xi}] = \int \mathcal{D}G_{\mu I} \mathcal{D}\psi \mathcal{D}\overline{\psi} e^{iS}, \qquad (42)$$

where

$$S = \int d^4 x \left[\frac{-1}{2\alpha} (\sigma_I \partial_\mu G^{\mu I})^2 + J_{\mu I} G^{\mu I} + \overline{\xi}\psi + \overline{\psi}\xi \right]. \qquad (43)$$

One derives the following expressions in terms of $Z[J_I, \xi, \overline{\xi}]$, $W[J_I, \xi, \overline{\xi}]$ and $\Gamma(G_{\mu I}, \psi, \overline{\psi})$:

$$\left[\frac{i}{\alpha} \sigma_I \sigma_J \Omega_{I1} \partial_\mu \frac{\delta}{\delta J_{\mu,J}} \right.$$
$$\left. - \Omega_{I1} \partial_\mu J_I^\mu + g \left(\overline{\xi} \frac{\delta}{\delta \overline{\xi}} - \xi \frac{\delta}{\delta \xi} \right) \right] Z = 0, \qquad (44)$$

$$\left[\frac{i}{\alpha} \sigma_I \sigma_J \Omega_{I1} \Box \partial_\mu \frac{\delta}{\delta J_{\mu J}} \right.$$
$$\left. + i\Omega_{I1} \partial_\mu J_I^\mu Z^{-1} + g \left(\overline{\xi} \frac{\delta}{\delta \overline{\xi}} - \xi \frac{\delta}{\delta \xi} \right) \right] W = 0, \qquad (45)$$

$$\left[\frac{i}{\alpha} \sigma_I \sigma_J \Omega_{I1} \partial_\mu G_I^\mu \right.$$
$$\left. - i\Omega_{I1} \partial_\mu \frac{\delta\Gamma}{\delta G_{\mu I}} + g \left(\overline{\psi} \frac{\delta\Gamma}{\delta \overline{\psi}} - \psi \frac{\delta\Gamma}{\delta \psi} \right) \right] = 0. \qquad (46)$$

Remember the relationship $W = -i \ln Z$ and

$$\Gamma(G_{\mu I}, \psi, \overline{\psi}) = W - \int d^4 x (J_{\mu I} G^{\mu I} + \overline{\xi}\psi + \overline{\psi}\xi).$$

Deriving the last equation with respect to $G_{\mu I}$ and taking fields equal zero, and the relationship $\sigma_I \Omega_{I1} = 1$, it results the following system of equations

$$\Omega_{I1} \partial^\mu \Gamma_{\mu\nu,IJ} = \Delta_{\nu J}, \qquad (47)$$

where

$$\Gamma_{\mu\nu,IJ} \equiv \frac{\delta^2 \Gamma}{\delta G_{\mu I} \, \delta G_{\nu J}}, \quad \Delta_{\nu I} = \frac{\sigma_I}{\alpha} \partial_\nu \delta(x - y). \qquad (48)$$

Before solving for the physical basis $\{G_{\mu I}\}$, we are going to study first for the constructor basis $\{D_\mu, X_{\mu I}\}$. It yields

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta D_\mu(x) \delta D_\nu(y)} = \partial_\mu \frac{\delta^2 \Gamma_{Free}}{\delta D_\mu(x) \delta D_\nu(y)},$$

$$\partial_\mu \frac{\delta^2 \Gamma}{\delta D_\mu(x) \delta X_{\nu i}(y)} = \partial_\mu \frac{\delta^2 \Gamma_{Free}}{\delta D_\mu(x) \delta X_{\nu i}(y)}, \qquad (49)$$

where the above equations are showing that the D_μ field longitudinal part does not receive quantic corrections. Given such information one can study it for the physical basis $\{G_{\mu I}\}$. It says that the longitudinal part of just one field can be frozen. In order to proof such assumption, let us reconsider Eq. (48)

$$\Gamma_{\mu\nu,IK} = \frac{1}{\Omega_{K1}} \Delta_I, \quad K \text{ fixed}, \quad 1 \leq I \leq N, \qquad (50)$$

and the ansatz

$$G_{\mu I} = \Omega_{K1}^{-1} \delta_{KI} D_\mu + \Omega_{Ii}^{-1} X_\mu^i. \qquad (51)$$

Considering the corresponding effective action

$$\Gamma_{\text{Free}} = \int d^4 x \frac{1}{2} G_{\mu I} \left[(A\Box + D)_{IJ} \eta^{\mu\nu} + B_{IJ} \partial^\mu \partial^\nu \right] G_{\nu I},$$

where A, B (kinetic matrices) and D (mass matrix) are $N \times N$ matrices determined from Eqs. (19) and (7). It yields

$$\partial_\mu \Gamma_{\text{Free},IJ}^{\mu\nu} = \left[(A + B)_{IJ} \partial_\nu + D_{IJ} \partial_\nu \right] \delta(x - y).$$

Taking the identity $\Omega_{I1} d_{IJ} = 0$, one gets

$$\partial_\nu \delta(x - y) = \left[\sum_J \Omega_{I1} (a + b)_{IJ} \right]^{-1}$$
$$\times \left[\sum_J \Omega_{I1} \partial_\mu \Gamma_{Free,IJ}^{\mu\nu} \right]. \qquad (52)$$

In order to understand this equation, let us study it for $N = 2$. Substituting Eq. (52) in Eq. (48),

$$\partial^\mu \Gamma_{\mu\nu,11} = \frac{\beta_1}{\alpha \Omega_{11}} \left[(a + b)_{11} + (a + b)_{12} \right]^{-1}$$
$$\times \partial^\mu \left[\Gamma_{\mu\nu,11}^{Free} + \Gamma_{\mu\nu,12}^{Free} \right],$$

$$\partial^\mu \Gamma_{\mu\nu,12} = \frac{\beta_2}{\alpha \Omega_{11}} \left[(a + b)_{11} + (a + b)_{12} \right]^{-1}$$
$$\times \partial^\mu \left[\Gamma_{\mu\nu,11}^{Free} + \Gamma_{\mu\nu,12}^{Free} \right].$$

Thus the generalization for N-fields is immediate. Choosing a field $G_{\mu K}$ depending on D_μ field as Eq. (51) shows, one derives

$$\partial^\mu \Gamma_{\mu\nu,IK} = \frac{\sigma_I}{\alpha \Omega_{K1}} \left[\sum_J [(a + b)_{KJ}]^{-1} \right]$$
$$\times \left[\sum_J \partial^\mu \Gamma_{\mu\nu,IJ}^{Free} \right]. \qquad (53)$$

Equations (53) are establishing that the N-propagators involving the field $G_{\mu K}$ are linear combinations on the effective free action. This result guarantees that the longitudinal part of the propagator $G_{\mu K}$ do not suffer radiative corrections.

7. Conclusion

A first step for a model consistency is on its renormalizability. This works shows that such non-linear abelian gauge model is compatible with the Quantum Action Principle, power counting, finite number of counterterms and provided with a Ward identity that freezes the longitudinal part from one of the N-involved fields. Ward identity guarantees that even for such abelian generalization a massless photon is preserved when it interacts with electron and positron and also with another vector fields. In a forthcoming work we are going to derive relationships between counterterms [12].

The proposal conveyed in this paper opens up a new possible venue to approach gravity in its quantum version. Massive gravitons, which show up whenever higher curvature powers, dynamical torsion or a cosmological constant are added up to the Einstein-Hilbert action, usually appear with a Planckean mass. More recently, we have understood that quantum gravity effects and possibly new massive gravitons may emerge at a much lower energy scale, at the TeV order.

Our approach, based upon the introduction of new families of gauge potentials, may be extended to the non-abelian case [13]. Adopting the viewpoint that gravity admits an Yang-Mills formulation associated to the Lorentz group, we are able to improve the ultraviolet behaviour of the graviton propagator along with the appearance of massive gravitons at a desirable scale (TeV for example), which may be fixed by the free parameters we are able to bring into the action by means of the extra families of gauge potentials, according to what is presented in Secs. 2 and 3.

Appendix

Primitively divergent graphs

We list for one loop the corresponding superficial degree of divergence δ for each primitively divergent graph. Variables X, Y, etc. correspond to propagators powers as $\langle G_{\mu I}\, G_{\mu J}\rangle = \dfrac{1}{(k^2 - m^2)^X}$.

$$\delta = 3 - 2X$$
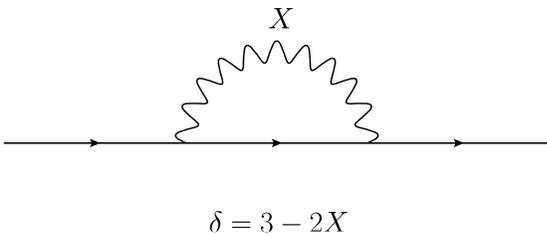
FIGURE 2. Fermion self-energy.

(a) $\delta = 6 - 2X - 2Y$ (b) $\delta = 2$
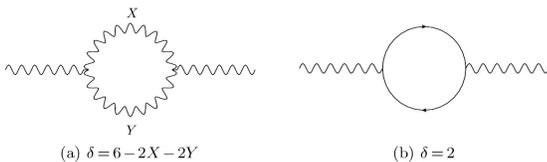
FIGURE 3. Vectorial fields self-energy.

(a) $\delta = 7 - 2X - 2Y - 2Z$ (b) $\delta = 5 - 2X - 2Y$ (c) $\delta = 1$
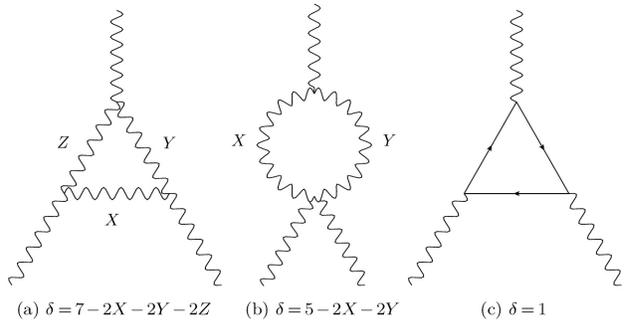
FIGURE 4. Vectorial fields annihilation.

(a) $\delta = 2 - 2X$ (b) $\delta = 4 - 2X - 2Y$
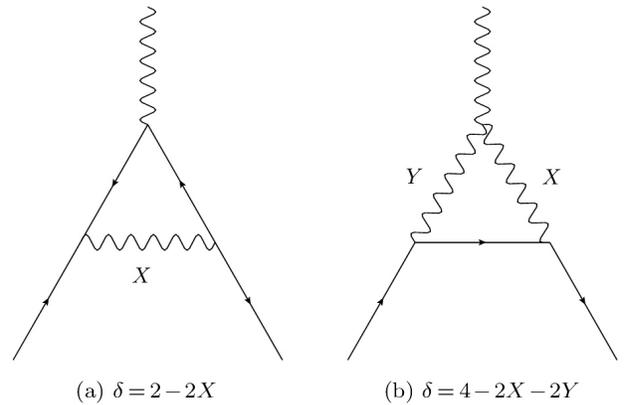
FIGURE 5. Fermion-antifermion pair annihilation.

(a) $\delta = 8 - 2X - 2Y - 2Z - 2W$ (b) $\delta = 0$

(c) $\delta = 6 - 2X - 2Y - 2Z$ (d) $\delta = 4 - 2X - 2Y$
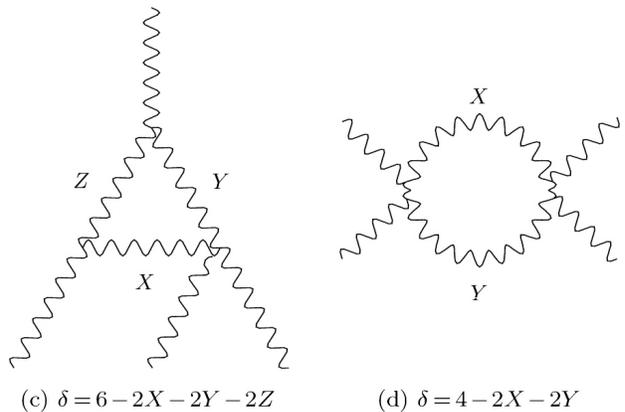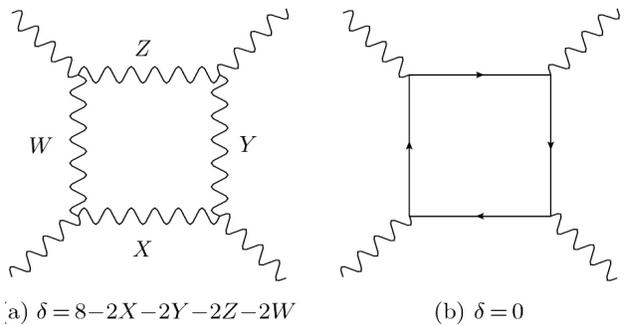
FIGURE 6. Vectorial fields scattering.

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