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Modulation of coherence and polarization using nematic 90°-twist liquid-crystal spatial light modulators

C. Rickenstorff, E. Flores, M.A. Olvera-Santamaría and A.S. Ostrovsky
*Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Puebla,
Puebla 72000, México.
e-mail: 284700727@alumnos.fcfm.buap.mx*

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A technique for modulating the coherence and polarization of the electromagnetic field using 90°-twist liquid-crystal spatial light modulators (LC-SLMs) is proposed. The controlled change of the statistical properties of light is achieved through computer generated random signals applied to a pair of 90°-twist LC-SLMs arranged in an interferometric setup. Experimental results obtained using Holoeye LC2002 modulators show the efficiency of the proposed technique.

Keywords: Liquid crystal modulator; coherence; polarization.

Una técnica de modulación de coherencia y polarización del campo electromagnético que utiliza moduladores espaciales de luz en base de cristal líquido de tipo 90°-twist es propuesta. El cambio controlado de las propiedades estadísticas de la luz se lleva a cabo aplicando señales aleatorias generadas por computadora a dos 90°-twist LC-SLM colocadas en un arreglo interferométrico. La validez de la técnica se demuestra presentando los resultados experimentales usando el espécimen LC2002 de la marca Holoeye.

Descriptores: Modulador en base de cristal líquido; coherencia; polarización.

PACS: 42.25.Dd; 42.70.Df; 42.25.Kb

1. Introduction

Beginning from the mid-1980's the nematic liquid-crystal spatial light modulators LC-SLM have been used for amplitude or phase modulation of the optical field in many applications such as optical data processing, adaptive optics, real time holography, etc. Recently, with the development of the vector coherence theory several authors have shown that the LC-SLM is also able to modulate the coherence and polarization of the electromagnetic field [1-3]. However, the reported techniques are purely theoretical and presuppose the use of 0°-twist LC-SLMs that nowadays cost as much to 20,000 dls each [4]. Here we propose an alternative technique for modulating the coherence and polarization of light which uses widely available and more economic 90°-twist LC-SLM working in mostly phase mode. In order to obtain the desired result, two orthogonal 90°-twist LC-SLMs displaying a special random signal are placed at the opposite arms of a Mach-Zehnder interferometer. The efficiency of the proposed technique is demonstrated with experimental results using Holoeye LC2002 modulators.

2. Background

As well of the, the second order statistical properties of a random planar (primary or secondary) electromagnetic source can be completely described by the cross-spectral density matrix (for brevity we omit the explicit dependence of the considered quantities on frequency ν) given by the formula

$$\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)=\begin{pmatrix}\langle E_x^*(\mathbf{x}_1)E_x(\mathbf{x}_2)\rangle & \langle E_x^*(\mathbf{x}_1)E_y(\mathbf{x}_2)\rangle \\ \langle E_y^*(\mathbf{x}_1)E_x(\mathbf{x}_2)\rangle & \langle E_y^*(\mathbf{x}_1)E_y(\mathbf{x}_2)\rangle\end{pmatrix}, \quad (1)$$

where $E_x(\mathbf{x})$ and $E_y(\mathbf{x})$ are the orthogonal components of the electric field vector $\mathbf{E}(\mathbf{x})$, asterisk denotes the complex conjugate and the angle brackets denote the average over the statistical ensemble. Using this matrix, the fundamental statistical properties known as degree of coherence and degree of polarization of light are defined as

$$\mu(\mathbf{x}_1,\mathbf{x}_2)=\frac{\mathrm{Tr}\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)}{[\mathrm{Tr}\mathbf{W}(\mathbf{x}_1,\mathbf{x}_1)\mathrm{Tr}\mathbf{W}(\mathbf{x}_2,\mathbf{x}_2)]^{1/2}}, \quad (2)$$

$$P(\mathbf{x})=\left(1-\frac{4\mathrm{Det}\mathbf{W}(\mathbf{x},\mathbf{x})}{[\mathrm{Tr}\mathbf{W}(\mathbf{x},\mathbf{x})]^2}\right)^{1/2}, \quad (3)$$

respectively, where Tr stands for the trace and Det denotes the determinant. When an optical field characterized by $\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)$ traverses a screen with random transmittance described by the Jones matrix $\mathbf{T}(\mathbf{x})$, the transmitted cross-spectral density matrix is calculated with the formula

$$\mathbf{W}'(\mathbf{x}_1,\mathbf{x}_2)=\left\langle\mathbf{T}^\dagger(\mathbf{x}_1)\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{T}(\mathbf{x}_2)\right\rangle, \quad (4)$$

where the dagger denotes Hermitian conjugate and the angle brackets stand for the ensemble average. From Eqs. (2)-(4) one concludes that the degrees of coherence and polarization of the electromagnetic field are modified during the transmission through the random screen. Particularly, if the elements of $\mathbf{T}(\mathbf{x})$ are pure phase terms there are no energy losses in the modulated light.

3. System of two 90°-twist LC-SLM

The system for modulating the coherence and polarization of the electromagnetic field is shown in Fig. 1. In the arrangement, the orthogonal field components of the primary source

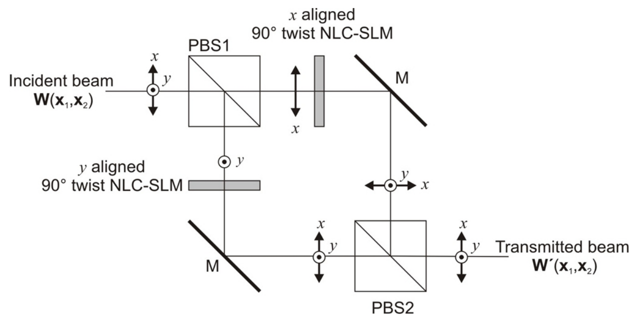

FIGURE 1. Schematic illustration of the technique for generating the partially coherent and partially polarized source: PBS1, PBS2, polarizing beam splitters; M, mirror. The bold-faced arrows denote polarization directions.

are separated by the polarizing beam splitter PBS1 and independently modulated by two 90°-twist LC-SLMs placed at the opposite arms of a Mach-Zehnder interferometer. Afterwards, the modulated field components are joined by the polarizing beam splitter PBS2 forming the secondary source. Upon close examination, each arm of the Mach-Zehnder interferometer is equivalent to a 90°-twist LC-SLM placed between two crossed polarizers. Lu and Saleh [5] demonstrated that such configuration yields nearly pure phase modulation of the incident field, provided that the LC cell is thick enough. Disregarding negligible changes in coherence and polarization due to transmission in free space and making use of the simplifications introduced in Ref. 5 the system of Fig. 1 is described by Jones matrix

$$\mathbf{T}(\mathbf{x})=\exp(-i\varphi_0)\begin{pmatrix} 0 & \exp[-i\varphi_2(\mathbf{x})] \\ \exp[-i\varphi_1(\mathbf{x})] & 0 \end{pmatrix}, \quad (5)$$

where φ_0 is a constant and $\varphi_{1(2)}(\mathbf{x})$ are computer generated signals applied to the LC-SLMs. The signals $\varphi_{1(2)}(\mathbf{x})$ are zero mean random variables with gaussian probability density

$$p[\varphi_{1(2)}(\mathbf{x})] = \frac{1}{\sqrt{2\pi}\sigma_\varphi}\exp\left[-\frac{\varphi_{1(2)}^2(\mathbf{x})}{2\sigma_\varphi^2}\right], \quad (6)$$

with variance $\left\langle\varphi^2(\mathbf{x})\right\rangle = \sigma_\varphi^2$ and cross correlation defined at two different points

$$\left\langle\varphi_{1(2)}(\mathbf{x}_1)\varphi_{1(2)}(\mathbf{x}_2)\right\rangle = \sigma_\varphi^2\exp\left[-\frac{\xi^2}{2\alpha_\varphi^2}\right], \quad (7)$$

where $\xi = |\mathbf{x}_1\text{-}\mathbf{x}_2|$ and α_φ is a positive constant related to the correlation width of $\varphi(\mathbf{x})$.

As the incident field is chosen a linearly polarized Gaussian beam characterized by the cross spectral density matrix

$$\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2) = E_0^2\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right) \times\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}, \quad (8)$$

where E_0 is the value of power spectrum at the origin of source plane, ε is the effective (rms) size of the source, and θ is the direction of polarization with respect to x axis. As seen from Eqs. (2) and (3), the beam described in Eq. (8) is completely coherent $\mu(\mathbf{x}_1,\mathbf{x}_2)$=1 and completely polarized $P(\mathbf{x})$=1, respectively. On substituting Eqs. (5) and (8) into Eq. (4), one finds that the modulated secondary source has the cross spectral density matrix

$$\mathbf{W}'(\mathbf{x}_1,\mathbf{x}_2) = E_0^2\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right) \times\begin{pmatrix} \sin^2\theta\left\langle\exp\{-i[\varphi_1(\mathbf{x}_2)-\varphi_1(\mathbf{x}_1)]\}\right\rangle & \cos\theta\sin\theta\left\langle\exp\{-i[\varphi_2(\mathbf{x}_2)-\varphi_1(\mathbf{x}_1)]\}\right\rangle \\ \cos\theta\sin\theta\left\langle\exp\{-i[\varphi_1(\mathbf{x}_2)-\varphi_2(\mathbf{x}_1)]\}\right\rangle & \cos^2\theta\left\langle\exp\{-i[\varphi_2(\mathbf{x}_2)-\varphi_2(\mathbf{x}_1)]\}\right\rangle \end{pmatrix}. \quad (9)$$

Based on the properties of function$\varphi(\mathbf{x})$ it can be shown [3] that

$$\left\langle\exp\left\{i[\varphi_{1(2)}(\mathbf{x}_2)\pm\varphi_{2(2)}(\mathbf{x}_1)]\right\}\right\rangle = \exp\left(-\sigma_\varphi^2\right), \quad (10)$$

$$\left\langle\exp\{-i[\varphi(\mathbf{x}_2)\pm\varphi(\mathbf{x}_1)]\}\right\rangle = \exp\left\{-\sigma_\varphi^2\left[1\pm\exp\left(\frac{\xi^2}{2\alpha_\varphi^2}\right)\right]\right\}. \quad (11)$$

In addition, to simplify the subsequent analysis, we assume that variance σ_φ of the control signal is large enough to accept the following approximations [1]

$$\exp\left(-\sigma_\varphi^2\right)\approx 0, \quad (12)$$

$$\exp\left\{-\sigma_\varphi^2\left[1-\exp\left(\frac{\xi^2}{2\alpha_\varphi^2}\right)\right]\right\}\approx\exp\left(-\frac{\xi^2}{2\eta_\varphi^2}\right), \quad (13)$$

where η_φ=α_φ/σ_φ. Finally, the transmitted cross spectral density matrix given by Eq. (9) has the form

$$\mathbf{W}'(\mathbf{x}_1,\mathbf{x}_2) = E_0^2\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right) \times\exp\left(-\frac{\xi^2}{2\eta_\varphi^2}\right)\begin{pmatrix} \sin^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix}, \quad (14)$$

and, using Eqs. (2) and (3), it is found that

$$\mu'(\xi) = \exp\left(-\frac{\xi^2}{2\eta_\varphi^2}\right), \qquad (15)$$

$$P'(\mathbf{x}) = |\cos 2\theta|. \qquad (16)$$

As can be seen from Eq. (16), the output degree of polarization depends solely on the angle of input polarization and varies from 0 to 1. Meanwhile, for a fixed angle θ of the primary source, the degree of coherence (15) is controlled with the proper choice of the computational variable η_φ of the modulation signal $\varphi(\mathbf{x})$.

4. Experiments and results

The arrangement for generating and characterizing the secondary source is shown in Fig. 2. In the setup, the second Mach-Zehnder interferometer with the translating pinholes reproduces the Young experiment [6] used for to measure the elements of matrix $\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2)$. In our case, the elements of matrix (14) are determined through the interference of *xx* and *yy* components of the modulated beam selected by polarizers P_1 and P_2. Finally, observing the visibility of the fringes at the exit of the beam splitter BS, the elements of the cross spectral density matrix are determined and substituted in Eqs. (2) and (3). Particularly, the 90°-twist LC-SLMs used in the experiment employed another polarizer-analyser configuration for to achieve the phase mostly mode, because they were not wide enough as required in Ref. 5.

As the primary source it was used a linearly polarized He-Ne laser (Spectra-Physics model 117A, λ=633 nm, 4.5 mW) whose polarization angle θ was controlled by means of a rotary stage. As the 90°-twist LC-SLM we used the Holoeye LC2002 model with resolution of 800$\times$600 pixels (32 μm square in size) and 256 grey level signal display. The control

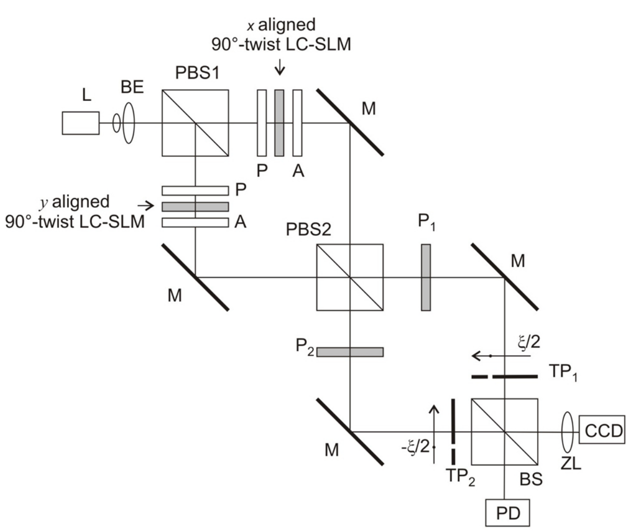

FIGURE 2. Experimental setup: L, laser; BE, beam expander; ZL, zoom-lens; PD, photodiode; P, polarizer; A, analyser; BS, beam splitter; TP, translating pinhole; P_1, P_2, polarizers. Other notations are the same as in Fig. 1.

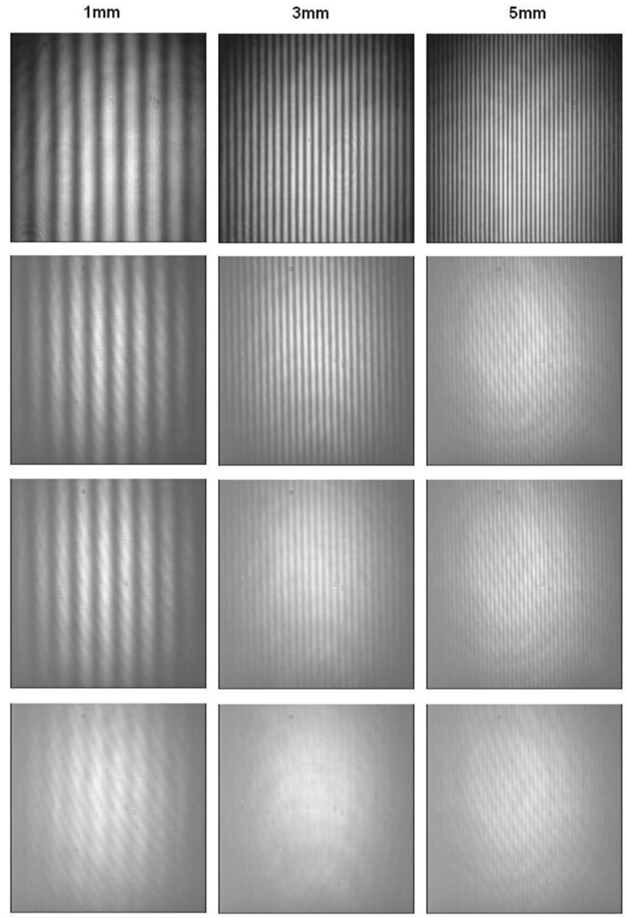

FIGURE 3. Interference fringes observed during coherence measurements at ξ = 1, 3 and 5 mm. From top to bottom row: primary source; secondary source η_φ=3, η_φ=2 and η_φ=1, respectively.

of the LC-SLMs was realized independently by two computers generating the random signal with the required Gaussian statistics.

We realized two sets of experiments. In the first one the degree of polarization was determined for different values of the input polarization angle θ, the position ξ =0 and the value $\eta_\varphi \approx 1$ of the control signal. In the second experiment we measured the degree of coherence for θ = π/4 and different positions ξ of pinholes, varying parameter η_φ of the control signal. To realize the measurements of the degree of coherence we used two pinholes with diameter of 200 μm mounted on motorized translation stages.

The interference patterns obtained during the coherence measurements are shown in Fig. 3. In the figure it can be seen that the primary source has maximum contrast (visibility) while it diminishes proportionally as a function of parameter η_φ of the control signal and the pinhole separation in the secondary source.

On the other hand, the experimental curves of the degree of coherence and the degree of polarization are shown in Fig. 4. During the measurements it was noticed that LC2002 specimen was not able to provide pure phase modulation in contraposition to model (5). However, it was observed that

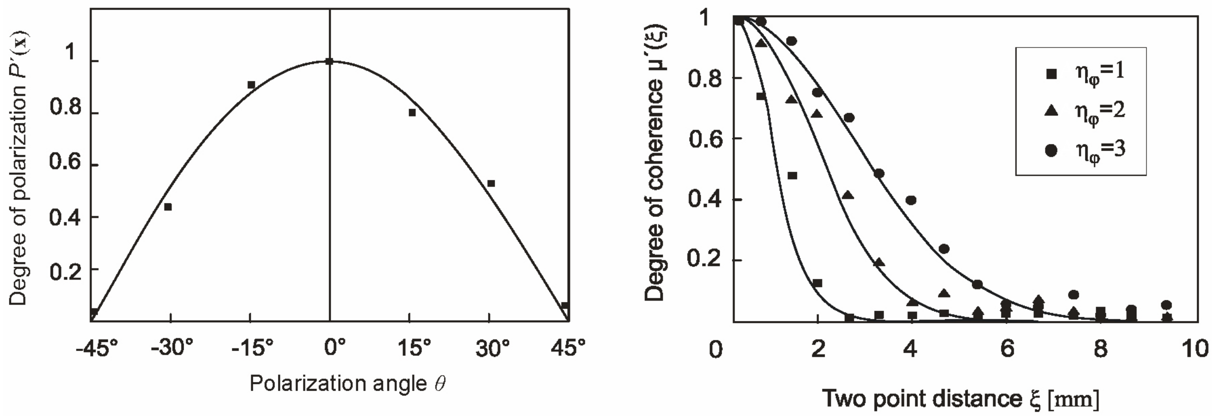

FIGURE 4. Left: measurements of polarization degree for $\eta_\varphi \approx 1$. Right: measurements of the degree of coherence for $\theta = \pi/4$ and different values of parameter η_φ.

this situation did not affect appreciably the theoretical predictions.

5. Conclusions

In this work a novel technique for modulation signal $\varphi(x)$ the degree of coherence and the degree of polarization of an electromagnetic beam using widely available 90°-twist LC-SLMs was presented. In the case of a linearly polarized Gaussian beam, the obtained degree of polarization depended solely on the angle of input polarization, while the output degree of coherence was controlled with the proper choice of the computational variable η_φ of the control signal $\varphi(x)$. Although the 90°-twist LC-SLM LC2002 did not provide pure phase modulation, the expected coherence and polarization curves were not affected significantly, demonstrating that this fact was not so critic rather than the displayed signal on the LC-SLMs was random.

In future works we plan to overcome this situation changing the wavelength of the source or using another LC-SLM. Particularly, the LC2002 Holoeye device achieved coherence modulation close to the range from 0 to 1.

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