State space second order filter estimation

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The second order stochastic filter is based on difference models with uncorrelated innovation conditions structured in state space having stationary properties through a surface with bounded drift around the mean value. This allows building recursive estimation without generality lost and basic properties over the stochastic state space surface with unknown gains viewed as a black-box scheme. The spatial region generated gave an approximation to real parametres set with a sufficient convergence rate in a probability sense. The results were applied in adaptive identification states with a high convergence rate, observed in the functional error described illustratively in simulations. This technique was developed over the smooth slide surface having advantages over other traditional filters.

Keywords: State space estimation; least squares method; instrumental variable; second probability moment convergence rate.

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1. Introduction

A system in a physic sense is involved with noise viewed as a black-box scheme, wherein its excitations and the output measures are directly affected. But these sampled signals propose information allowing the selection or the building of a differential stochastic model that commonly is a mathematical approximation between the evolution measures considering its excitations. Sampled signals directly correspond to a finite difference. This order represents the number of delays. Thus, the first corresponds in differences with a delay and so on [1].

The estimation is a basic tool considered in the parametres description. Where it is used?

An example is in Digital systems Adaptive Control Theory (*ACT*) using the estimation techniques that adjust the unknown gains into the controller according to a reference model with observable states. This mechanism is known as Digital Filter Estimation (*DFE*) affecting the control action gains before the feedback system ends modifying the parametres set with respect to an objective reference. Therefore, the Proportional Integrate and Difference (*PID*) discrete control action is calculated using other techniques, such as Butterworth tools, without changes through time. In spite of this, the gains are adjusted based on the experience designers.

Independently of the parametres estimation techniques, the input perturbations affect the convergence rate requiring adaptive considerations. The signal error is applied in the adaptive estimation in some sense as a feedback mechanism maximizing the convergence rate. Therefore, the filter result is known as an Adaptive Filter Estimation (*AFE*).

The control law action commonly operates in a PC using discrete platform calculus. In this case, the *AFE* is expressed in finite differences simplifying the digital charge using a recursive technique [1] as: Least Square Method (LSM) [2], Instrumental Variable (IV) [3], Forgetting Factor (FF) [4], and the Kalman Filter (KF) [4] therefore the gradient minimizes the convergence rate making an implementation without losing its stability. The reference model parametres estimation has some of the following conditions:

$a)$ *Stationary* input-output rate selecting the specific filter [2] as LSM, IV, FF needing the pseudo-inverse method [3] with an innovative correlation matrix [4]. These were described in terms of a linear stochastic model as $Ay_k + \xi_k = u_k$, with uncertainty around the equilibrium solution [5]. The input and output vectors $y_k, \xi_k, u_k \in (R, \Im(R), P)_k$ are mixed with internal gain A into the black-box Auto-Regressive Moving Average (ARMA (1, 1)) description. In all cases the stochastic gradient was applied to the second probability moment. These filters are off-line optimal [6] and their simulations show that the functional error converges exponentially to an equilibrium point in affine base feedback loops [7]. The on-line estimation in the Single-Input Single-Output (SISO) case is developed recursively bounded by a time interval with respect to the natural frequency reference output model [8-10].

$b)$ *Non-stationary* conditions [11]. The filters use the variances matrix with maximum likelihood estimation and have good distribution convergence with non-stationary responses [12,13]. The spatial time-delay estimation for known signals is considered having high changes with marginal stable conditions, and in spite of the filter based on the invariance principle, converges in a probability sense, with a good distribution description.

The filter estimation is used in:

1. Electrical rotor-position requiring specific velocity using a filter estimation technique $P(k-1) = E\left\{Y(k-1)V(k-1)^T\right\}$ affecting the control law amplitude voltages.
2. The filter estimation based on the traditional LSM [14]. Multi-Input Multi-Output Orthogonal Frequencies-Division with Multi-Access (MIMO-OFDMA) system having offset up-link non-stationary frequencies adjusting their gains dynamically using LSM [15]. The results were limited to short smooth conditions [16].
3. The Signal-to-Noise Ratio (SNR) was seen as an answer to the black-box system having non-stationary conditions. The estimation considered was based on a gradient technique and the results had a good distribution convergence [16]. The Azimuth and Elevation Variation (AEV) in short-range and low-flying air routes were predicted with a LSM state space parametres estimation method [17].
4. The 3D affine motion estimation problem used two cameras via observations as a single feature point. The nine rotational parametres, the three translational parametres, and the 3D position were estimated using LSM state space parametres estimation. A closed-loop non-linear observer was developed for the affine motion problem [18].

Different estimation techniques were resumed observing black-box system output stationary or non-stationary conditions. In the present paper, considering the Auto Regressive Moving Average ARMA (2, 1) model as a black-box output evolution described in state space form is solved using the vector estimation with the traditional inverse technique instead of the pseudo-inverse solution.

2. Development

The extended ARMA (2,1) as a SISO black-box answer is described in (1)

$$y(k) = ay(k-1) + by(k-2) + c\omega(k) \tag{1}$$

Having a complete description with $\{y_k\} \in R_{(-1,1)}$, $\{\omega(k)\} \in N(\mu, \sigma^2 < \infty)$ (1) [19].

Theorem 1. *The system (1) with respect to a reference signal is viewed as a black-box scheme and has a state space estimator described in (2).*

$$\hat{A}(k) = \left(E\left\{X(k)V(k)^T\right\} - cE\left\{W(k)V(k)^T\right\}\right)\left(E\left\{Y(k)V(k)^T\right\}\right)^+ \tag{2}$$

Matrix parametres $A(k)$ has the form (3). *The vectors $X(k)$, $Y(k)$, $W(k)$ are given by $X(k) = [x_{1,k}\ x_{2,k}\ x_{3,k}]^T$, $Y(k) = [x_{1,k-1}\ x_{2,k-1}\ x_{3,k-1}]^T$, $W(k) = [0\ 0\ \omega_k]$, respectively. The instrumental variable $V(k) \in R^{[3\times1]}$ in agreement to* [20] *calculates the gain matrix* (3).

$$\hat{A}(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hat{b}(k) & \hat{a}(k) & -1 \end{bmatrix} \tag{3}$$

Proof. (See Appendix)

Theorem 2. *The recursive estimation based on (2) is described in (4).*

$$\hat{A}(k) = \hat{A}(k-1)Q(k-1)Q(k)^+ + S(k). \tag{4}$$

Proof. (See Appendix)

3. Simulation

The simulation results considered an ARMA (2, 1) in (1) in a SISO condition having the identification form (5).

$$\hat{y}(k) = \hat{a}(k)y(k-1) + \hat{b}(k)y(k-2) + c\omega(k) \tag{5}$$

A periodic input considered in (1) and estimated based on (2) and applied in (5) show the results in Fig. 1. The parametres $\hat{a}$, $\hat{b}$ estimated with respect to periodic output reference model are shown in Fig. 2.

Figure 3 shows the identification functional error trace, with recursive form (6)

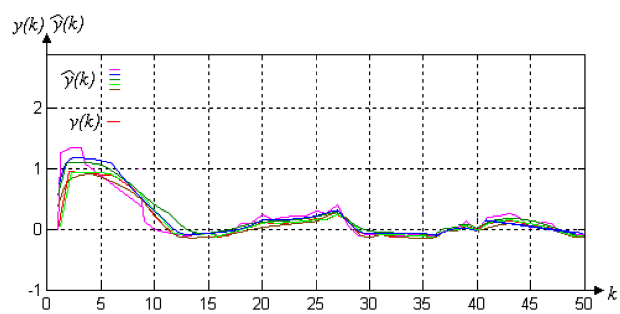

FIGURE 1. The identified input running five times with respect to the reference model in red.

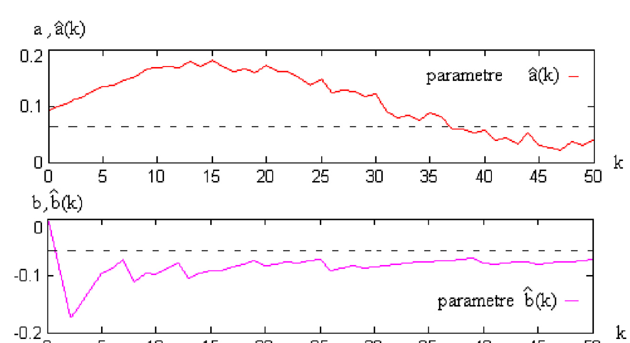

FIGURE 2. The parametres $\hat{a}$, $\hat{b}$ with respect to a periodic output reference model.

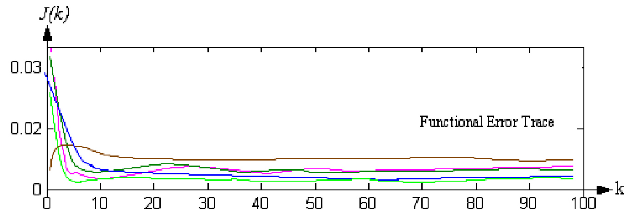

FIGURE 3. Functional error $J(k)$ trace with respect to five periodic output reference model.

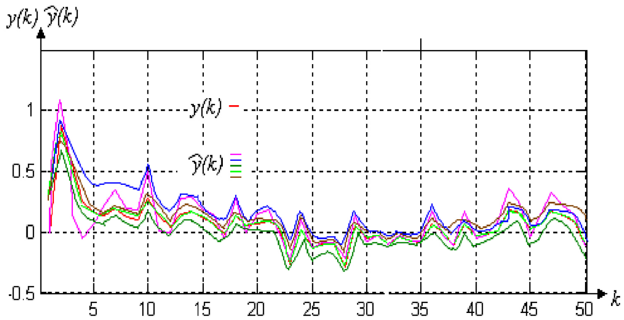

FIGURE 4. A periodic signal with slow perturbation in red and identified five times using the filter estimation proposed.

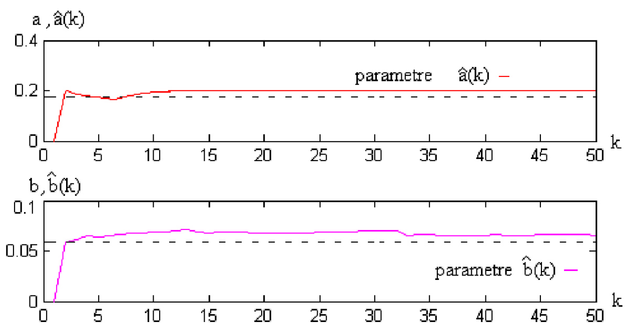

FIGURE 5. Parametres $\hat{a}(k)$ and $\hat{b}(k)$.

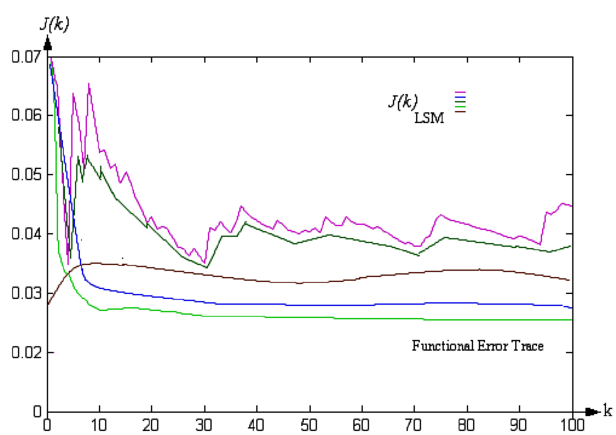

FIGURE 6. LSM Functional Error Estimation for five stochastic signals.

$$J(k) = \frac{1}{k}\left(e(k)^2 + (k-1)J(k-1)\right) \tag{6}$$

and $e(k) = y(k) - \hat{y}(k)$.

In the second experiment $\{\omega(k)\} \in N(\mu, \sigma^2 < \infty)$ the periodic function now has a short perturbation. The identifier describes a random sequence of the black-box output observable signal, bounded by an interval region, shown as an evolution time system in Fig. 4.

Figure 5 shows the parametres estimation with slow reference output perturbations.

Figure 6 shows the traditional LSM MatLab model and five state space functional errors trace.

4. Conclusion

The state space description was used and the recursive state space parametres estimation considered the black-box system answer as a first stochastic differences order and a second grade model. The new system has a matrix form with unknown matrix parameters. The stochastic gradient was used and the second probability moment allows building the matrix estimation. A recursive technique was developed considering stationary conditions between input and output signals. The simulation results with smooth evolutive conditions gave a satisfactory estimation for two cases running five times, both with bounded perturbations. Therefore the estimation affects in an adaptive form the control law action or an identification scheme, observed in Figs. 3 and 4, with a high convergence rate to the ideal signal viewed in Fig. 6.

Appendix

Proof (Theorem 1): *The model described in (1) considering that $x_{1,k} = x_{1,k-1'}$, $x_{2,k} = x_{2,k-1'}$, $x_{3,k} = y_k - x_{3,k-1}$ in state space is (7)*

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & a & -1 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \omega(k) \tag{7}$$

In vector form (7) symbolically is described in (8).

$$X(k) = AY(k) + cW(k) \tag{8}$$

The second probability moment using the instrumental variable has the form (9).

$$E\left\{X(k)V(k)^T\right\} = \hat{A}(k)E\left\{Y(k)V(k)^T\right\} + cE\left\{W(k)V(k)^T\right\} \tag{9}$$

The matrix parametres is described symbolically in (10)

$$\hat{A}(k) = E\left[\left\{X(k)V(k)^T\right\} - cE\left\{W(k)V(k)^T\right\}\right]\left(E\left\{Y(k)V(k)^T\right\}\right)^+ \tag{10}$$

And it was developed in (2).

Proof (Theorem 2): The system (1) with parametres estimation (2) symbolically has the form (11):

$$A(k) = P(k)Q(k)^+ \tag{11}$$

The matrices considered in (11) are defined in (12) and (13)

$$P(k) := E\left[\left\{X(k)V(k)^T\right\} - cE\left\{W(k)V(k)^T\right\}\right] \quad (12)$$

$$Q(k) := E\left\{Y(k)V(k)^T\right\} \quad (13)$$

With stationary conditions of (11) symbolically the matrix estimation has the form (14) one step later.

$$\hat{A}(k-1) = \frac{Q(k-1)}{P(k-1)} \quad (14)$$

The recursive form of (12) is described in (15).

$$P(k) = \frac{k-1}{k}P(k-1) + \frac{1}{k}\left(X(k-1)V(k-1)^T\right) - cW(k-1)V(k) - 1^T \quad (15)$$

Considering (14) in (15) multiplying by $Q(k-1)/Q(k-1)$ to $P(k-1)$ is described in (16)

$$P(k) = \frac{k-1}{k}P(k-1)\left\lfloor\frac{Q(k-1)}{Q(k-1)}\right\rfloor + \frac{1}{k} \times \left(X(k-1)V(k-1)^T\right) - cW(k-1)V(k) - 1^T \quad (16)$$

With (14) applied in (16), the $P(k)$ recursively is (17)

$$P(k) = \frac{k-1}{k}\hat{A}(k-1)Q(k-1) + \frac{1}{k} \times \left(X(k-1)V(k-1)^T\right) - cW(k-1)V(k) - 1^T \quad (17)$$

And (17) in (11) is described in (18)

$$\hat{A}(k) = \frac{k-1}{k}\hat{A}(k-1)\frac{Q(k-1)}{Q(k)} + \frac{1}{kQ(k)}\left(X(k-1)V(k-1)^T\right) - \frac{c}{Q(k)}W(k-1)V(k) - 1^T \quad (18)$$

And (18) corresponds to (4).

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