Spherical aberration correction using aspheric surfaces with an analytic-numerical method

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This method is a discrete case of the E. Wolf method for the design of an aspheric surface. Using the proposed method, the designer can select how many points (x, y) there will be on the entrance pupil at which the spherical aberration will be zero, by using the aspheric coefficients as degrees of freedom. For fitting the coordinates that correct the spherical aberration to an aspheric surface we solve a system of equations of the first degree. An optimisation procedure is not required because we use equations without approximations and with exact ray tracing. We obtained diffraction limited optical systems faster than the commercial programs.

Keywords: Aspherical surface; spherical aberration; aberration.

Este método es un caso discreto del método propuesto por E. Wolf para diseñar una superficie asférica. Con este método el diseñador puede seleccionar los puntos (x, y) en la pupila de entrada en los cuales la aberración esférica será cero, usando los coeficientes de asfericidad como grados de libertad. El ajuste de las coordenadas que corrigen la aberración esférica a una superficie asférica lo realizamos resolviendo un sistema de ecuaciones de primer grado. El procedimiento de optimización no es requerido por que usamos trazo de rayos exacto; con este método nosotros obtenemos sistemas limitados por difracción más rápido que con los programas comerciales.

Descriptores: Superficies asféricas; aberración esférica; aberración.

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1. Introduction

This method is a discrete case of the E. Wolf [1] method for the design of an aspheric surface. When the aspheric surface is the first or last surface of the system the solution is greatly simplified and he gives exact parametric equations. In both cases, the method needs to evaluate an integral and E. Wolf [1] proposes to evaluate this in several ways. He proposes a polynomial approximation but does not say how many terms are used to achieve it; also, if you use this approximation the solution is not exact. On the other hand, if a sufficient number of rays is traced from the object space to the space that precedes the correcting surface, then the integral might also be evaluated numerically; however, he did not say how many rays should be traced, nor what are the points (x, y) on the entrance pupil, which intersect the rays. Our method proposes the number and where the rays should be traced in order to correct the optical path difference (OPD).

Other methods have been proposed to analytically correct aberrations in the last surface, for example, Conrady [2] with the $(D - d)$ method of achromatisation. He calculates the radius of curvature of the last surface, knowing the desired value of D_{k-1} (Fig. 1) in the last element, to achieve achromatism in only one point on the exit pupil. Cordero-Davila *et al.* [3] deduced an equation for the conic constant of the last mirror of a two-mirror telescope, knowing the desired value of D_{k-1} (distance between mirrors along the marginal

ray), to achieve zero optical path difference (OPD) for only one point on the exit pupil. Castro-Ramos *et al.* [4] derived equations for the design of aplanatic microscope objectives of two conic mirrors. They found two equations of the second degree: one to correct spherical aberration and one for coma correction. These equations are exact and depend on D_{k-1} (distance between mirrors along marginal ray). The methods of Cordero-Davila, Conrady and Castro-Ramos depend of D_{k-1} and the correction is in only one point on the exit pupil. Our method proposes to make zero OPD in several points (x, y) on the pupil entrance and not just one.

Additionally, numerical methods have also been proposed. Romoly *et al.* [5] proposed a simple recursive method to determine the shape of the corrector plate for large telescopes without the use high-order polynomial coefficients, which lack precision in fitting the required shape of aspheric surface. We propose to fit the required shape by solving a system of equations of the first degree with high accuracy using the same number of aspherical coefficients and points of correction.

In the next section, we explain the condition for the design of optical systems that are free of spherical aberration. In Sec. 3, we explain the method using a general aspheric surface. Section 4 presents some examples in which we apply the developed methodology and finally, Sec. 5 offers conclusions.

2. Spherical aberration correction

The necessary condition to obtain a system with corrected spherical aberration is that both the paraxial and marginal optical paths must be equal [2,3,4], then from Fig. 1 we obtain

$$n_0 D_0 + n_1 D_1 + n_{k-1} D_{k-1} + n_k D_k$$
$$= d_0 + n d_1 + n_{k-1} d_{k-1} + n_k d_k. \tag{1}$$

where $n_{0,1...k}$ are the refractive indices of each medium, $d_{0,1...k}$ are the distances along the optical axes between surfaces, $D_{0,1...k}$ are the distances along the marginal rays between surfaces (Fig. 1).

From Fig. 1, we see that D_k is

$$D_k = \sqrt{(r_{k-1})^2 + (d_k - z_{k-1})^2}, \tag{2}$$

where $r_{k-1} = \sqrt{x_{k-1}^2 + y_{k-1}^2}$.

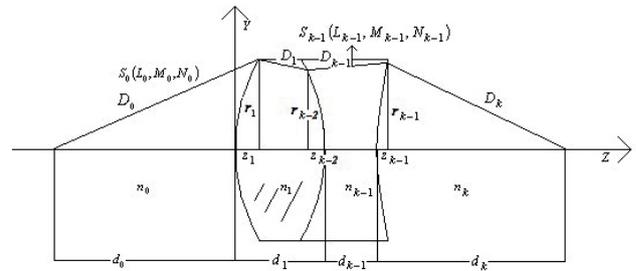The coordinates at the last surface for the marginal ray (S_{k-1}) are

FIGURE 1. Parameters used to correct the spherical aberration.

$$x_{k-1} = x_{k-2} + D_{k-1} L_{k-1}$$
$$y_{k-1} = y_{k-2} + D_{k-1} M_{k-1}$$
$$z_{k-1} = z_{k-2} - d_{k-1} + N_{k-1} D_{k-1}, \tag{3}$$

where $(L_{k-1}, M_{k-1}, N_{k-1},)$ are the direction cosines of the marginal ray S_{k-1} and $(r_{k-2 \text{ and } k-1}$ and $z_{k-2 \text{and} k-1})$ are the intersection coordinates between the marginal ray S_{k-1} and the last surfaces (Fig. 1).

Using the rotation symmetry ($r_{0,1,...k} = y_{0,1,...k}$) and substituting Eq. (3) into Eq. (2) we obtain

$$D_k = \sqrt{(y_{k-2} + M_{k-1} D_{k-1})^2 + (d_k - z_{k-2} + d_{k-1} - N_{k-1} D_{k-1})^2}, \tag{4}$$

and by substituting Eq. (4) into Eq. (1) and by squaring, we obtain a quadratic equation for D_{k-1}

$$a D_{k-1}^2 + b D_{k-1} + c = 0. \tag{5}$$

When the object is at a finite position, the coefficients of the second degree equation are calculated as

$$a = (n_k^2 - n_{k-1}^2), \tag{6a}$$

$$b = 2 \left[\begin{array}{c} n_k^2 y_{k-2} M_{k-1} - n_k^2 N_{k-1} (d_k - z_{k-2} + d_{k-1}) \\ + n_{k-1}(n_0 d_0 + n_1 d_1 + n_{k-1} d_{k-1} + n_k d_k - n_0 D_0 - n_1 D_1) \end{array} \right], \tag{6b}$$

$$c = \left[\begin{array}{c} n_k^2 y_{k-2}^2 + n_k^2 (d_k - z_{k-2} + d_{k-1})^2 \\ -(n_0 d_0 + n_1 d_1 + n_{k-1} d_{k-1} + n_k d_k - n_0 D_0 - n_1 D_1)^2 \end{array} \right], \tag{6c}$$

and when the object is at infinity, the coefficients are calculated as

$$a = (n_k^2 - n_{k-1}^2), \tag{7a}$$

$$b = 2 \left[\begin{array}{c} n_k^2 y_{k-2} M_{k-1} - n_k^2 N_{k-1} (d_k - z_{k-2} + d_{k-1}) \\ + n_{k-1}(n_1 d_1 + n_{k-1} d_{k-1} + n_k d_k - n_0 z_1 - n_1 D_1) \end{array} \right], \tag{7b}$$

$$c = \left[\begin{array}{c} n_k^2 y_{k-2}^2 + n_k^2 (d_k - z_{k-2} + d_{k-1})^2 \\ -(n_1 d_1 + n_{k-1} d_{k-1} + n_k d_k - n_0 z_1 - n_1 D_1)^2 \end{array} \right]. \tag{7c}$$

If the last surface is a mirror in air, the coefficient a is zero and in this case, only an equation of first grade should be solve to determine D_{k-1}.

From Fig. 1, we see that: D_0, D_1, D_{k-2}, d_0, d_1, d_{k-1}, d_k, y_1, y_{k-2}, z_{k-2}, M_{k-1} and N_{k-1} are exact parameters and we can determine these parameters by exact ray tracing. With these parameters we can calculate D_{k-1} using Eqs. (5), (6) and (7) and then, we calculate y_{k-1} and z_{k-1} using Eq. (3) to obtain an optical system free of spherical aberration.

3. General aspheric surface correction

We defined the general aspheric surface as

$$z_{\text{aspheric}} = z_{\text{spheric}} + a_1(x_{k-1}^2 + y_{k-1}^2)^2$$
$$+ a_2(x_{k-1}^2 + y_{k-1}^2)^3 + a_3(x_{k-1}^2 + y_{k-1}^2)^4 + \ldots, \quad (8)$$

where $z_{k-1=z_{\text{aspheric}}}$ that together with x_{k-1} and y_{k-1} are the coordinates at the last surface for the marginal ray and z_{spheric} is calculated with the axial curvature c_{k-1} and the same coordinates x_{k-1} and y_{k-1} as follows:

$$z_{\text{spheric}} = \frac{c_{k-1}(x_{k-1}^2 + y_{k-1}^2)}{1 + \sqrt{1 - c_{-1k}^2(x_{-1k}^2 + y_{-1k}^2)}}. \quad (9)$$

Using the rotation symmetry and solving Eq. (8) only for a_1, we obtain

$$a_1 = \frac{z_{\text{aspheric}} - z_{\text{spheric}}}{y_{k-1}^4}. \quad (10)$$

With this result, we have the spherical aberration correct for one point on the exit pupil. If we want to correct the spherical aberration for two points on the exit pupil, in the edge and the zonal spherical aberration; we need to solve the next equations system:

$$z_{\text{aspherica(edge)}} = z_{\text{spheric(edge)}}$$
$$+ a_1(y_{k-1(\text{edge})})^4 + a_2(y_{k-1(\text{edge})})^6$$

$$z_{\text{aspheric(zonal)}} = z_{\text{spheric(zonal)}}$$
$$+ a_1(y_{k-1(\text{zonal})})^4 + a_2(y_{k-1(\text{zonal})})^6. \quad (11)$$

In general, if we want to correct the spherical aberration in more points on the exit pupil, it is better that we use a matrix form and then we have

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_n \end{bmatrix} = \begin{bmatrix} b_{11} & b_{21} & b_{31} & b_{41} & b_{51} & b_{n1} \\ b_{12} & b_{22} & b_{32} & b_{42} & b_{52} & b_{n2} \\ b_{13} & b_{23} & b_{33} & b_{43} & b_{53} & b_{n3} \\ b_{14} & b_{24} & b_{34} & b_{44} & b_{54} & b_{n4} \\ b_{15} & b_{25} & b_{35} & b_{45} & b_{55} & b_{n5} \\ b_{1n} & b_{2n} & b_{3n} & b_{4n} & b_{5n} & b_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_n \end{bmatrix}, \quad (12)$$

where $c_{1,2,3,\ldots,n}$ are the differences between z_{aspheric} and z_{spheric}, $b_{11,12,13,\ldots,nn}$ are the coordinates at the last surface for the marginal ray to the fourth power, sixth power, etc., and $a_{1,2,3,\ldots,n}$ are the coefficients of the general aspheric surface. Solving the equations system (12), we can determine the coefficients that correct the spherical aberration for m points (x, y) selected on the entrance pupil.

4. How many and which points should be corrected?

The spherical aberration of the wavefront for any optical system can be expressed as [6]:

$$W(0, y) = b_1(y^2)^2 + b_2(y^2)^3 + b_3(y^2)^4 + \ldots \quad (13)$$

Considering only f/numbers > 5, the spherical aberration can be represented only for the first and second terms and by combining these terms, the spherical aberration of the edge can be corrected as follows:

$$W(0, y) = b_1(y_m^2)^2 + b_2(y_m^2)^3 = 0. \quad (14)$$

Considering that y_m is the height of the ray at the edge of a pupil normalised to one, we have:

$$b_2 = -b_1. \quad (15)$$

If the spherical aberration of the edge is corrected, for example, see Fig. 4, the rays which pass through intermediate zones of the pupil lens are not corrected. This aberration is known as residual spherical aberration. Substituting Eq. (15) into Eq. (13) we derive Eq. (13) for two terms and by putting them equal to zero, we can determine the peak of the residual spherical aberration, as follows:

$$W(0, y) = b_1(y^2)^2 - b_1(y^2)^3 = 0$$
$$\frac{\partial W(0, y)}{\partial y} = 4b_1y^3 - 6b_1y^5 = 0$$
$$y^2 = \frac{2}{3} \quad (16)$$

The residual spherical aberration occurs when y is equal to the marginal y_m, multiplied by $\sqrt{2/3} = 0.8165$ and is called zonal spherical aberration. This analysis is similar to Kingslake's [7]. The difference is that the de focus term is not considered here. It is possible to correct the spherical aberration at the zonal $((y/y_m) = 0.8165)$ and marginal (y_m) heights on the pupil by using the first three terms of the expansion of the wavefront aberration given by Eq. (13) so

$$W(0, y) = b_1y_m^4 + b_2y_m^6 + b_3y_m^8 = 0$$
$$W(0, y) = b_1(0.8165 * y_m)^4$$
$$+ b_2(0.8165 * y_m)^6 + b_3(0.8165 * y_m)^8 = 0 \quad (17)$$

With the solution of the equations system (17), we derive Eq. (13) for three terms and putting them equal to zero, we can determine the peaks of the residual spherical aberration when the zonal and in the edge spherical aberrations are corrected, for example, see Fig. 5. The Optical Path Difference (OPD) curve has two peaks opposite above and below the 0.8165 zone. The zones with maximum and minimum residuals fall at values of y given by $y/y_m = 0.6210$ or 0.9297.

The points for the Kingslake analysis are $y/y_m = 1$, 0.8880, 0.7071, and 0.4597 and for our analysis $y/y_m = 1$, 0.9297, 0.8265, and 0.6210.

If we consider f/numbers < 1, we should correct the spherical aberration residual and its peaks fall at values $y = y_m(0.6210)$ or $y_m(0.9297)$. We use the fourth and fifth terms to correct these other y values and we can repeat the procedure to correct the spherical aberration of Eqs. (14),

(15) and (16) to four y. Using the solutions, we derive Eq. (13) to five terms and making them equal to zero, we can determine the peaks. The OPD curve has two peaks opposite above and two below the 0.6210 and 0.9297 zones. The zones with maximum and minimum residuals fall at values of y given by $y/y_m = 0.9738, 0.8773, 0.7181$ and 0.4435, for example, see Fig. 7. This analysis can continue because the expansion (13) is infinite.

The number of y values that must be corrected for each optical system depends on the optical system tolerances, for example, with one value of y $((1)(y_m))$, we correct lenses with f/numbers bigger than f/5, with two different values of $y((1)(y_m)$ and $(0.8165)(y_m))$, we correct lenses with f/numbers bigger than f/2 and with four different values of $y((1)(y_m), (0.8165)(y_m), (0.6210)(y_m), (0.9297)(y_m))$, we correct lenses with f/numbers bigger than f/1 but the designer decides the correction that is needed.

In this section, the authors propose some points for the correction of spherical aberration; however, the method allows the designer to select any point on the pupil of the optical system.

5. Examples

5.1. Mirrors

Smith and Atchison [8] found analytically the equation to compute the conic constant and the radii of curvature of a mirror without spherical aberration, if the position of the object and the image are known:

$$k = \frac{4l'l}{(l' + l)^2} - 1 \quad \text{y} \quad r = \frac{2l'l}{(l' + l)}. \tag{18}$$

Where l is the object distance and l' is the image distance and the aperture stop is on the mirror.

We will compare the coordinates calculated with our method and the coordinates calculated with the parameters of Eq. (18) for only one surface. We consider the example where the distance between an object and mirror is -400 mm and between the mirror and the image is -133.334 mm. The diameter of the mirror is 200 mm.

Using Eq. (18), we have $r =$-200 mm and $k =$-0.25. Table I shows the surface coordinates calculated using Eq. (19), the conic constant and the radii of curvature calculated with Eq. (18). Also, Table I shows the surface coordinates calculated with our method.

$$z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (k + 1)c^2(x^2 + y^2)}} \tag{19}$$

Another very common example is that of a parabolic mirror ($k =$-1) with the object at the infinity. We consider a parabolic mirror with $r =$-200 and a diameter of 200 mm. Table II shows the surface coordinates calculated using Eq. (19) and also the surface coordinates calculated with our method.

In both cases, our method reproduces the same results as those obtained analytically for the points chosen.

TABLE I. Coordinates calculated with Eqs. (18) and (19) compared with the coordinates calculated with our method.

Points $(0, y)$ on the entrance pupil	Points $(0, y)$ on the mirror	Eq. (16) and (17) (z)	Our method (z)
(0,(100)(1))	(0, 94.201625)	-23.19341	-23.19341
(0,(100)(0.9297))	(0, 88.26454)	-20.245065	-20.245065
(0,(100)(0.8265))	(0, 79.29977)	-16.214063	-16.214063
(0,(100)(0.6210))	(0, 60.646956)	-9.359379	-9.359379

TABLE II. Coordinates calculated with Eq. (19) compared with the coordinates calculated with our method.

Points $(0, y)$ on the entrance pupil	Points $(0, y)$ on the mirror	Eq. (17) (z)	Our method (z)
$(0, (100)(1))$	$(0, 100)$	-25	-25
$(0, (100)(0.9297))$	$(0, 92.97)$	-21.608552	-21.608552
$(0, (100)(0.8265))$	$(0, 82.65)$	-17.077556	-17.077556
$(0, (100)(0.6210))$	$(0, 62.10)$	-9.641025	-9.641025

5.2. Gregorian telescope

The first example is an $f/10$ Gregorian telescope whose primary mirror is $f/1$ and spherical. Therefore, with a very large spherical aberration, the secondary mirror is aspheric and it is used to compensate the aberration of the primary mirror. The primary mirror diameter is 100 mm and the distance from the vertex of the primary mirror to the Gregorian focus is 50 mm (see Fig. 2).

We use this example to demonstrate that it is possible to compensate a very large spherical aberration by only using an aspheric surface. Similarly, we show with this example how the spherical aberration decreases when the number of aspheric coefficients increases.

5.2.1. First order design of Gregorian telescope

We use the equations of Malacara [9] for the first order design. First, we find the effective focal length of the telescope F and the primary mirror f_1 with following equations:

$$F = D_1 f_{\#\text{telescope}} \quad \text{and} \quad f_1 = D_1 f_{\#1}. \tag{20}$$

D_1 is the primary mirror diameter, $f_{\#\text{telescope}}$ is the f number of the telescope and $f_{\#1}$ is the f number of the primary mirror. The separation between the mirrors l, is calculated with the equation

$$l = \frac{f_1(F - s)}{f_1 + F}. \tag{21}$$

Also, we calculate the effective focal length of the secondary mirror as

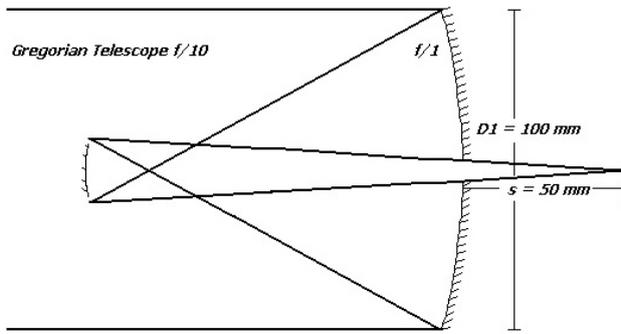$$f_2 = F\left(\frac{f_1(f_1 + s)}{f_1^2 - F^2}\right), \tag{22}$$

FIGURE 2. Parameters for design of a Gregorian telescope.

and we calculate the diameter of the secondary mirror using the following equation

$$D_2 = \frac{(f_1 - l)D_1}{f_1} \qquad (23)$$

and finally, we calculate the radii of curvature of the mirrors as follows:

$$r_1 = -2f_1 \quad \text{and} \quad r_2 = 2f_2. \qquad (24)$$

We show the paraxial parameters of a Gregorian telescope in Table 3

TABLE III. Paraxial parameters of the Gregorian telescope.

Surface	Effective focal length	Radii of curvature	Diameter	Separation
1	100 mm	-200 mm	100 mm	116.666
2	15.1515 mm	30.303 mm	16.667 mm	166.666

5.2.2. Exact design of Gregorian telescope

We must perform the exact ray tracing at points (x, y) selected on the entrance pupil until the penultimate surface; the results of this procedure are shown in Table IV.

M_0, N_0, M_1 and N_1 are the director cosines of the ray. Y_1 and Z_1 are the coordinates on the primary mirror. For the next step, we must apply Eqs. (5) and (7) to determine D_{k-1}. Subsequently, we calculate the last surface coordinates that correct the spherical aberration with Eq. (3); we show these coordinates in Table V.

Finally, we show the equation system from one to four coefficients and the changes in the telescope OPD and Strehl ratio with each coefficient; all OPD graphics will be computed with OSLO (Optics Software for Layout and Optimisation) [10].

5.2.2.1. One coefficient

We use the point $(0, Y_1 = (1)(y_m))$ on the entrance pupil (edge) to correct the spherical aberration. We solve Eq. (10) with the coordinates that correct the spherical aberration, as follows:

$$a_1 = \frac{1.572863 - 1.756004}{(-10.165674)^4} = -1.714908 x10^{-5}$$

Figure 3 shows the telescope OPD (Optical Path Different) without aspheric coefficient, only with two spherical mirrors; as you can see the spherical aberration is very large.

TABLE IV. Ray tracing parameters of the Gregorian telescope.

Ray Tracing Parameters	Points $(0,Y_1)$ on the entrance pupil $y_m = 50$			
M_0	0	0	0	0
N_0	1	1	1	1
Y_1	$Y_1 = (1)(y_m)$	$Y_1 = (0.93)(y_m)$	$Y_1 = (0.82)(y_m)$	$Y_1 = (0.621)(y_m)$
Z_1	-6.350833	-5.480720	-4.247605	-2.424957
M_1	-0.484123	-0.452257	-0.401292	-0.306735
N_1	-0.875	-0.891887	-0.91595	-0.951795

TABLE V. Coordinates of the last surface that corrects the spherical aberration of the Gregorian telescope.

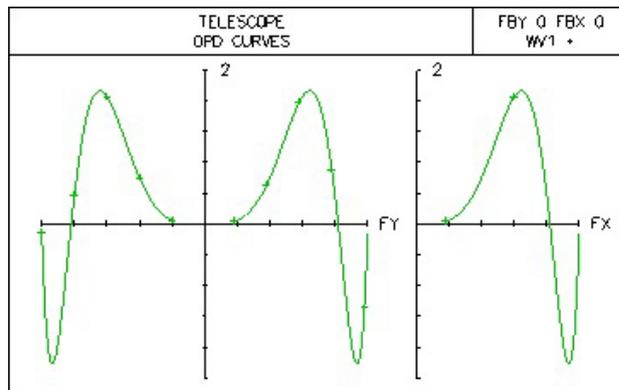| Coordinates that Corrects the Spherical Aberration	Points $(0,Y_1)$ on the entrance pupil			
	$Y_1 = (1)(y_m)$	$Y_1 = (0.93)(y_m)$	$Y_1 = (0.82)(y_m)$	$Y_1 = (0.621)(y_m)$
D_{k-1}	124.277682	123.197943	121.688106	119.499666
Y_2	-10.165674	-9.217172	-7.832513	-5.60476
$Z_2 = Z_{aspheric}$	1.572863	1.30724	0.958841	0.502539
$Z_{spheric}$	1.756004	1.435793	1.029742	0.52283
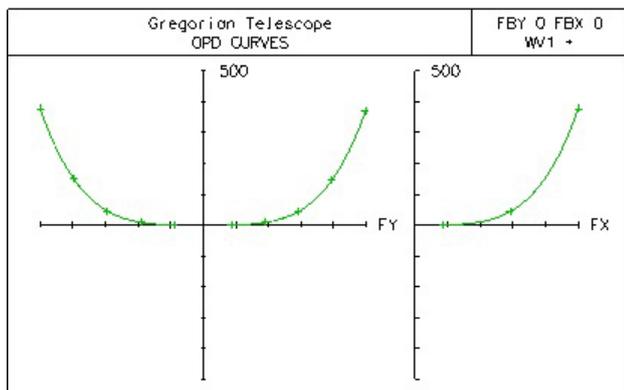
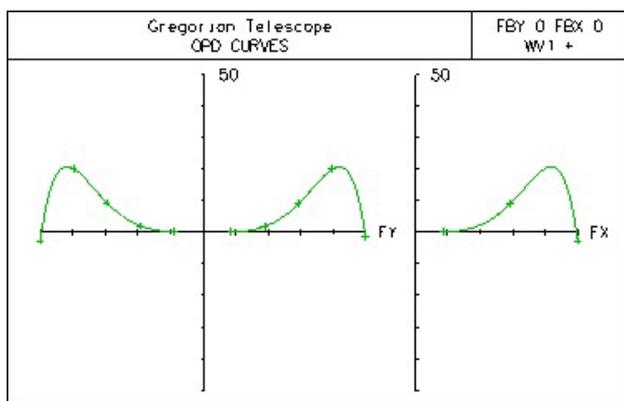FIGURE 3. Telescope OPD without aspheric coefficients.

FIGURE 4. Telescope OPD with one aspheric coefficient in the pupil edge.

Figure 4 shows the telescope OPD with one aspheric coefficient. As can be seen, there is one pupil point with zero spherical aberration in the edge.

The changes with only one coefficient are very significant but the correction is not complete because the Strehl ratio of the telescope is 0.006707; therefore, we need to correct the spherical aberration residual.

5.2.2.2. Two coefficients

We use two points $(0, Y_1=(1)(y_m))$ and $(0, Y_1=(0.82)(y_m))$ on the entrance pupil to correct the spherical aberration. We solve the equations system (11), with the coordinates of Table V, as follows:

$$1.572862 = 1.756003$$
$$+ a_1(-10.165673)^4 + a_2(-10.165673)^6$$
$$0.95884 = 1.029742$$
$$+ a_1(-7.832513)^4 + a_2(-7.832513)^6$$

The solutions are $a_1 = -2.130735 \times 10^{-5}$ and $a_2 = 4.023808 \times 10^{-8}$. Figure 5 shows the telescope OPD with two aspheric coefficients. As can be seen, there are two pupil points with zero spherical aberration.

FIGURE 5. Telescope OPD with two aspheric coefficients.

The correction with two aspheric coefficients is better than that with one aspheric coefficient but the correction is not complete because the Strehl ratio of the telescope is 0.02555. Therefore, we need additional aspheric coefficients and thus, we choose other pupil points according to the analysis of section four.

5.2.2.3. Three coefficients

Now we use points $(0, Y_1 = (1)(y_m))$, $(0, Y_1 = (0.82)(y_m))$ and $(0, Y_1= (0.621)(y_m))$ on the entrance pupil to correct the spherical aberration. We solve the equations system (12), with the coordinates of Table V, as follows:

$$1.572862 = 1.756003 + a_1(-10.165673)^4$$
$$+ a_2(-10.165673)^6 + a_3(-10.165673)^8$$
$$0.95884 = 1.029742 + a_1(-7.832513)^4$$
$$+ a_2(-7.832513)^6 + a_3(-7.832513)^8$$
$$0.502539 = 0.522830 + a_1(-5.604760)^4$$
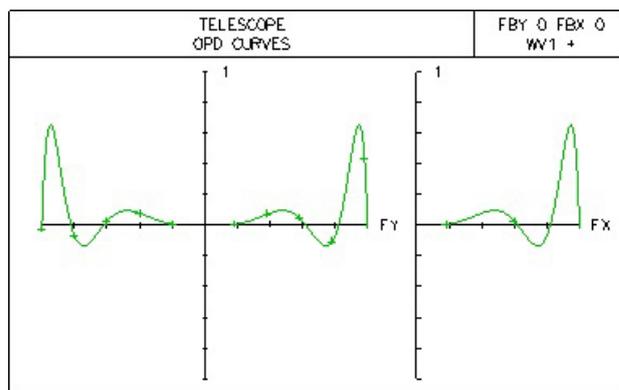$$+ a_2(-5.604760)^6 + a_3(-5.604760)^8$$

FIGURE 6. Telescope OPD with three aspheric coefficients.

The solutions are

$$a_1 = -2.283420 \times 10^{-5}, \quad a_2 = 7.990124 \times 10^{-8}$$

$$\text{and} \quad a_3 = -2.408364 \times 10^{-10}.$$

Figure 6 shows the telescope OPD with three aspheric coefficients. As can be seen, there are three pupil points with zero spherical aberration.

However, the correction with three coefficients is still not complete because the Strehl ratio of the telescope is 0.285879. We need other aspheric coefficients and thus, we must choose other pupil points according to the analysis of section four.

5.2.2.4. Four coefficients

We use four points $(0, Y_1 = (1)(y_m))$, $(0, Y_1 = (0.93)(y_m))$, $(0, Y_1 = (0.82)(y_m))$ and $(0, Y_1 = (0.621)(y_m))$ on the entrance pupil to correct the spherical aberration. We solve the equations system (12) with the coordinates of Table V, as follows:

$$1.572862 = 1.756003$$
$$+ a_1(-10.165673)^4 + a_2(-10.165673)^6$$
$$+ a_3(-10.165673)^8 + a_4(-10.165673)^{10}$$
$$1.307240 = 1.435793$$
$$+ a_1(-9.217172)^4 + a_2(-9.217172)^6$$
$$+ a_3(-9.217172)^8 + a_4(-9.217172)^{10}$$

$$0.95884 = 1.029742$$
$$+ a_1(-7.832513)^4 + a_2(-7.832513)^6$$
$$+ a_3(-7.832513)^8 + a_4(-7.832513)^{10}$$
$$0.502539 = 0.522830$$
$$+ a_1(-5.604760)^4 + a_2(-5.604760)^6$$
$$+ a_3(-5.604760)^8 + a_4(-5.604760)^{10}$$

The solutions are

$$a_1 = -2.306354 \times 10^{-5}, \quad a_2 = 9.315968 \times 10^{-8},$$

$$a_3 = -4.666647 \times 10^{-10} \quad \text{and} \quad 1.151583 \times 10^{-12}.$$

Figure 7 shows the telescope OPD with four aspheric coefficients. As can be seen, there are four pupil points with zero spherical aberration. Figure 7 also shows the difference between those points proposed by Kingslake [7] and our points.

We see from Fig. 7 that the mean difference is that the points suggested in this work have P-V 0.06531 λ and RMS 0.02058 λ and that the Kingslake points have P-V 0.212 λ and RMS 0.05354 λ. The correction with four coefficients is complete because the Strehl ratio of the telescope is 0.9853 with four points suggested and for Kingslake's points, the Strehl ratio of the telescope is 0.9280 (Fig. 8).

The points suggested in this work are slightly better than Kingslake's points but both are diffraction limited.
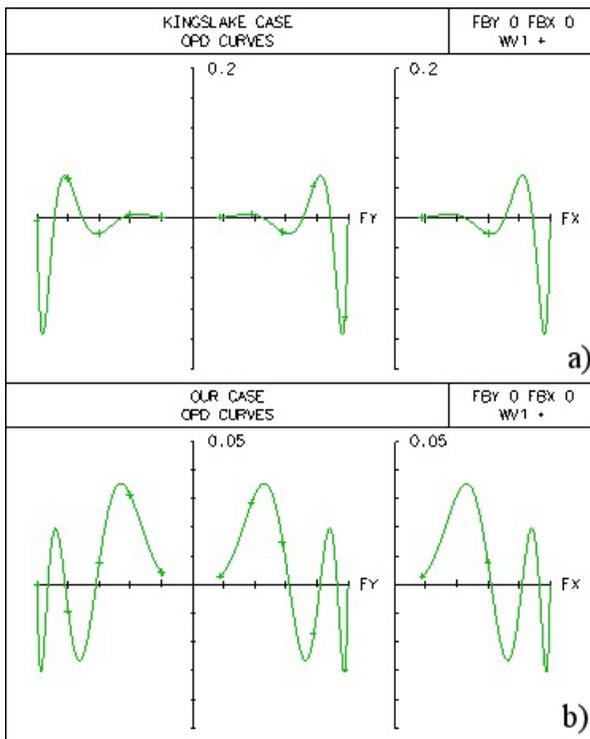

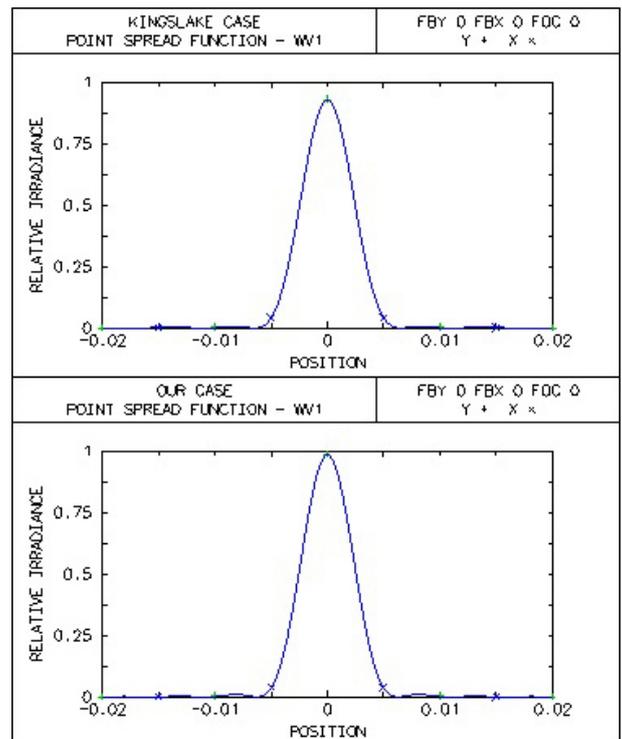FIGURE 7. Telescope OPD with four aspheric coefficients.

FIGURE 8. Telescope PSF with four aspheric coefficients.

TABLE VI. First order parameters of the lens $f/1$.

Surface	Radius	Thickness	Aperture Radius	Glass
Object		400 mm		Air
1	70.07 mm	29.011 mm	50 mm	BK-7
2	-169.131 mm	118.589 mm	50 mm	Air
Image				Air

TABLE VII. Coefficients calculated to compensate the spherical aberration of the lens $f/1$.

Coefficients	Points $(0,Y_1)$ on the entrance pupil $y_m = 50$ mm	Value
a_1	$Y_1 = (1)(y_m)$	7.671046×10^{-7}
a_2	$Y_1 = (0.93)(y_m)$	$-5.373468 \times 10^{-11}$
a_3	$Y_1 = (0.82)(y_m)$	7.179357×10^{-15}
a_4	$Y_1 = (0.621)(y_m)$	-5.69022×10^{-19}

5.3. Lens f/1

The second example is a single lens f/l with 100 mm of effective focal length and the object is 400 mm from the lens. The next table shows the first order parameters.

The first surface is spherical and the second surface will be aspheric and it is used to compensate the spherical aberration. In this case, we use four aspheric coefficients to compensate the spherical aberration. As can be seen in Fig. 9, there are four pupil points with zero spherical aberration. In Table VII we show the aspheric coefficients of the second surface.

The correction with four coefficients is complete because the Strehl ratio of the telescope is 0.9728.

5.4. Cemented doublet f/2

The final example is a cemented doublet $f/2$ with 100 mm of effective focal length and the object is at infinity. The following Table shows the first order parameters.
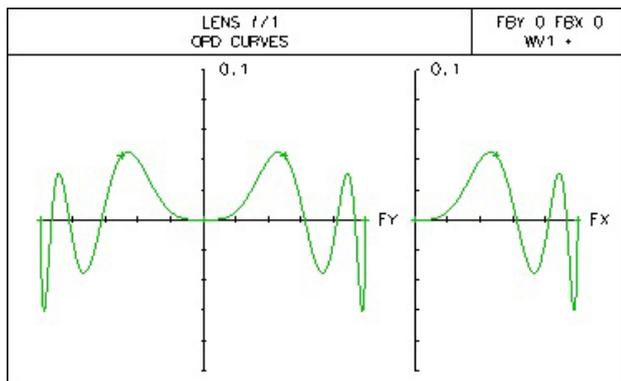

FIGURE 9. Lens $f/1$OPD with the object in finite position.

TABLE VIII. First order parameters of the doublet $f/2$.

Surface	Radius	Thickness	Aperture Radius	Glass
Object		∞		Air
1	67.799 mm	16.031mm	25 mm	BK-7
2	-38.396 mm	3.0 mm	25 mm	F-2
3	-116.331 mm	90.990 mm	25 mm	Air
Image				Air

TABLE XIX. Coefficients calculated to compensate the spherical aberration of the doublet $f/2$.

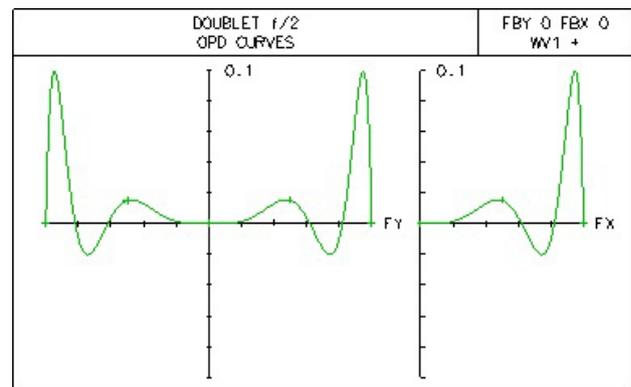| Coefficients	Points $(0,Y_1)$ on the entrance pupil $y_m=25$ mm	Value
a_1	$Y_1 = (1)(y_m)$	1.855043×10^{-7}
a_2	$Y_1 = (0.82)(y_m)$	$-3.077739 \times 10^{-10}$
a_3	$Y_1 = (0.621)(y_m)$	$-1.782926 \times 10^{-13}$

FIGURE 10. Cemented doublet $f/2$ OPD with the object at the infinity..

The first and second surfaces are spherical and the third surface will be aspheric and it is used to compensate the spherical aberration. In this case, we use three aspheric coefficients to compensate the spherical aberration. As can be seen in Fig. 10, there are three pupil points with zero spherical aberration. In Table XIX we show the aspheric coefficients of the third surface.

The correction with four coefficients is complete because the Strehl ratio of the telescope is 0.9587.

If the conjugates are changed, we can use the first surface to correct the spherical aberration by applying this method.

6. Conclusions

We present an analytic-numerical method to compensate the spherical aberration by using one aspheric surface on the last surface of the optical system. The calculations of the aspheric coefficients only require solving a system of first degree equations; therefore, this method is a quick and simple procedure by which to obtain the solution. The method can

be applied from one surface until n number of surfaces but the last surface must be an aspheric surface.

As the equations are not approximate, the process of optimisation is not required. With the examples, we have demonstrated that with an appropriate number of aspheric coefficients, it is possible to obtain diffraction limited optical systems. We also show how the value of the spherical aberration changes when the number of aspheric coefficients increases. Thus, we can choose the correction degree that we need and we have proposed some positions on the pupil entrance where the spherical aberration correction is better.

1. E. Wolf, *Proc. Phys. Soc.* **61** (1948) 494-503.

2. R. Kingslake, *Lens Design Fundamentals* (Academic Press, Inc., New York, 1978) p. 93.

3. A. Cordero-Davila, S. Vazquez-Montiel, A. Cornejo-Rodriguez and O. Cardona-Nuñes, "*A relation between the conic constants of two mirror telescope*", 16th Congress of the International Commission for optics: (Optics as a Key to High Technology, Proc. SPIE, 1993). p. 159.

4. J. Castro-Ramos, A. Cordero-Davila, S. Vazquez-Montiel and D. Gale, *Appl. Opt.* **37** (1998) 5195.

5. A. Romoli, A. Zuccaro Marchi, L. Gambicorti and F. Simonetti, *Appl. Opt.* **49** (2010) 2898.

6. Robert R. Shannon, *The art and science of optical design,* (Cambridge University Press, USA, 1997). p. 196.

7. R. Kingslake, *Lens Design Fundamentals* (Academic Press, Inc., New York, 1978). p. 114.

8. G. Smith and D.A. Atchison, *The eye and visual instruments*, (Cambridge, New York, 1997). p. 120.

9. Malacara D. and Z. Malacara., *Handbook of lens design*, (Marcel Dekker Inc., New York, 1994).

10. Lambda Research Corporation, "*OSLO Optics Software for Layout and Optimization*", (Optics Reference, Version 6.1, Littleton, MA, USA, 2001).