On the mass flow rate from silos with lateral exit holes

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The mass flow rate, $\dot{m}$, associated with the lateral outflow of dry, cohesionless granular material through circular orifices of diameter D made in vertical walls of silos was measured experimentally in order to determine also the influence of the wall thickness of the silo, w. Geometrical arguments, based on the outflow happening, are given in order to have a general correlation for $\dot{m}$ embracing both quantities, D and w. The angle of repose appears to be an important characterization factor in these kinds of flows.

Keywords: Granular systems; granular flow; static sandpiles.

El flujo másico, $\dot{m}$, asociado con el flujo lateral de material granular seco sin cohesión, a través de orificos de diametro D, en paredes verticales de silos se midió experimentalmente para determinar también la influencia del grosor de pared del silo, w. Se presentan argumentos geométricos, basados en la existencia del flujo, para tener una correlación general de $\dot{m}$ que contenga a las cantidades D y w. El ángulo de reposo parece ser un factor importante en la caracterización de este tipo de flujos.

Descriptores: Sistemas granulados; flujo granular; apilamientos estáticos.

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1. Introduction

Since the first studies by Hagen in 1852 [1] it has been well known that the mass flow rate, $\dot{m}$, of the gravity flow of dry granular material emerging from the bottom exit of a silo scales essentially as $\dot{m} \sim \rho g^{1/2} D^{5/2}$ where $\dot{m}$ is the mass flow rate (grams/second), ρ is the bulk density, g is the acceleration due to gravity and D is the diameter of the circular orifice. This result contrasts with that occurring in liquids where the mass flow rate depends on the level of filling above the orifice. Moreover, in granular material a continuous flow occurs provided $D > 6d_g$, where d_g is the grain's diameter. After the fundamental study of Hagen was established, many researchers have proved the validity of his law, and slight modifications have also been proposed in order to improve the agreement with the experimental data. See, for instance [2-4].

Despite the enormous utility of Hagen's law only a few studies have conducted to tests its validity for the flow of grains from exit holes located in the vertical wall of a silo [5-10]. The aim of this work is to study experimentally the mass flow rate when the exit holes are located in the vertical walls of the silos for different orifice sizes, D, and several wall thicknesses, w. As can be seen afterwards, the thickness of the wall can be used to control the grains dosage but this flow can be arrested if w overcomes a critical value that depends also on the angle of repose the granular material.

In order to reach our goal, the division of this work is as follows: in next section we give a short review of experimental studies of the mass flow rate where the orifices were made in lateral walls. After, in Sec. 3, we report new experiments where the influence of D and w was studied. In Sec. 4 we propose, on the basis of our experimental results, a correlation that embraces both changes in D and w. In Sec. 5 we discuss some new results based on this new correlation and finally, in Sec. 6, we give the main conclusions of this work.

2. Previous works

To our knowledge, Bagrintsev and Koshkovskii [5] were the first researchers who studied experimentally the problem of the gravity driven lateral outflow of granular material in vertical cylindrical silos with vertical walls. They used oval and circular exit holes made in transparent plastic walls and in relation with the wall thickness uniquely observed that "the outflow capacity decreases as wall thickness increases". Later, other authors did experiments in silos with rectangular exit holes [6,9] and circular exit holes [7,8]. Bagrintsev and Koshkovskii [5] and Choudary and Kesava Rao [7] and Kumar Sharma and Kesava Rao [8] have found that apparently the better correlation among $\dot{m}$ and D was of the form $\dot{m} \sim \rho g^{1/2} D^{7/2}$, meanwhile Davies and Foye [6] and Sheldon and Durian [10] reported measurements of the mass flow rate in silos with lateral orifices that follows a relation of the

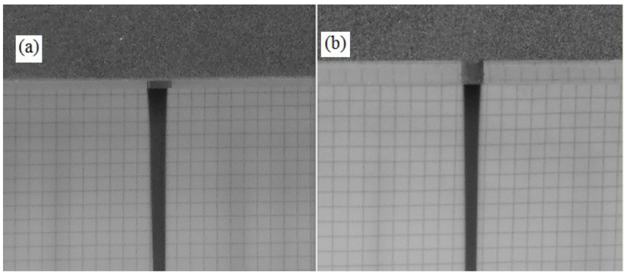

FIGURE 1. Snapshots of the outflow of granular material: (a) thin bottom wall and (b) thick bottom wall.

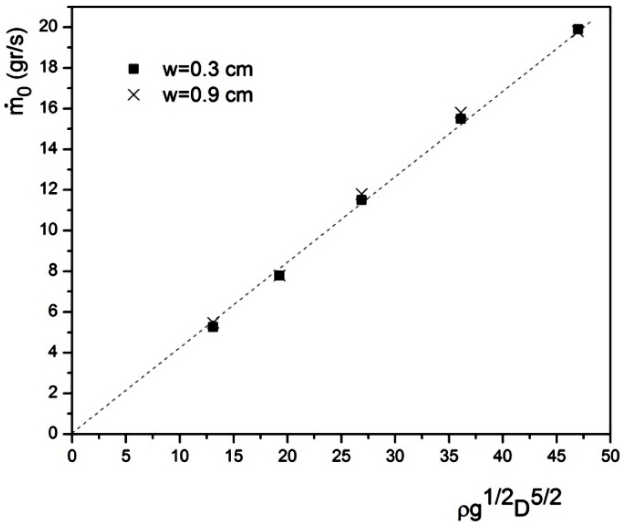

FIGURE 2. Plot of the mass flow rate through horizontal orifices, $\dot{m}_0$, as a function of $\rho g^{1/2} D^{5/2}$. Data fit the Hagen's law $\dot{m}_0 = 0.48\rho g^{1/2} D^{5/2}$. Error bars are of 4%.

the type $\dot{m} \sim \rho g^{1/2} D^{5/2}$ (it is important to comment that in some studies [5,6] D is essentially a hydraulic diameter). Nevertheless, none of the referred works have analyzed systematically the effect of the wall thickness on $\dot{m}$.

In order to reach a better understanding of the behavior of the mass flow rate in lateral circular exit holes we did a series of experiments, using circular orifices, where we mainly analyzed the influence of D and the wall thickness, w, on such a quantity. Clearly, the wall thickness will be important for the occurrence of the flow because if the silo wall is very thick the outflow of granular material will be arrested. Detailed experiments given in the next section allow quantify these and other facts.

3. Mass flow rate measurements

3.1. Bottom exit holes

It is well known that the wall thickness does not affect substantially the value of $\dot{m}_0$, the mass flow rate when the exit hole is located at the bottom of a silo. In Fig. 1 we show the jet of sand (mean diameter $d_g = 0.3$ mm and bulk density $\rho = 1.5$ gr/cm^3) that crosses through circular orifices in an

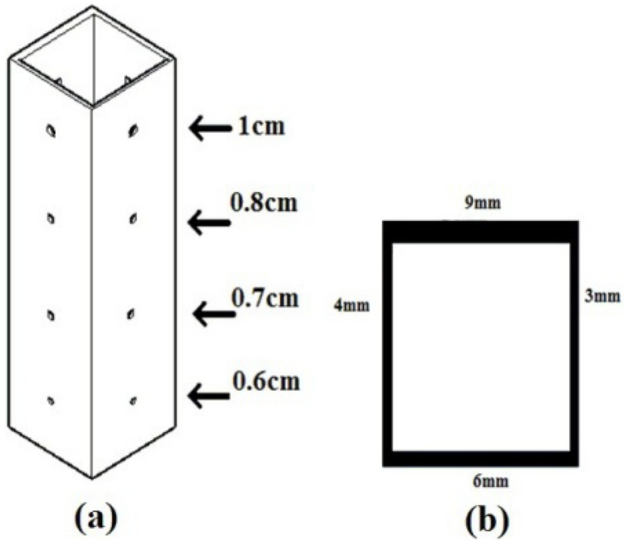

FIGURE 3. Depict of the silo used in experiments. (a) a schematic of the silo where the staggered exit holes are shown. Orifices of equal diameter are at the same height. (b) Top view of the silo showing the four different wall thicknesses.

acrylic-made box with a slim wall (Fig. 1(a)) and a bold wall (Fig. 1(b)). In Fig. 2 we show the experimental plot of $\dot{m}_0$, as a function of $\rho g^{1/2} D^{5/2}$, for two different wall thicknesses: $w = 0.3$ cm and $w = 0.9$ cm. In this figure we observe that both cases fit very well the Hagen's law, and thus the effect of w does not is appreciated. The relation that follows the straight line is

$$\dot{m}_0 = a\rho g^{1/2} D^{5/2}, \quad (1)$$

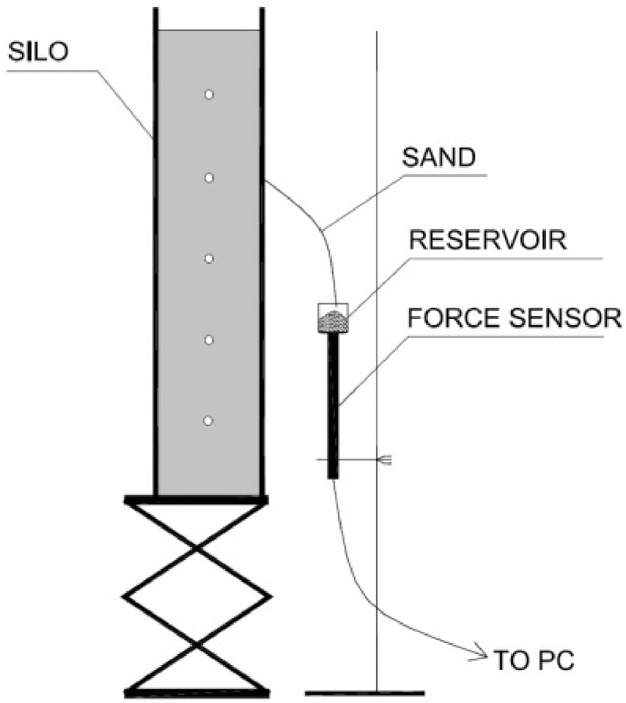

FIGURE 4. Schematic of the experimental setup to measure the mass flow rate of granular material through circular orifices in vertical walls.

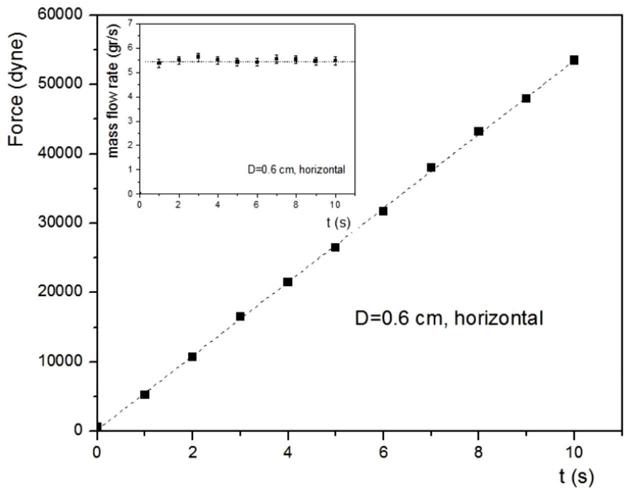

FIGURE 5. Plot of the weight of the granular material in the reservoir of the apparatus depicted in Fig. 4, F, as a function of time. In the inset is the plot of $\dot{m}_0$ as a function of time. Error bars are of 4%. Notice that the several measurements of $\dot{m}_0$ are around a constant (horizontal line).

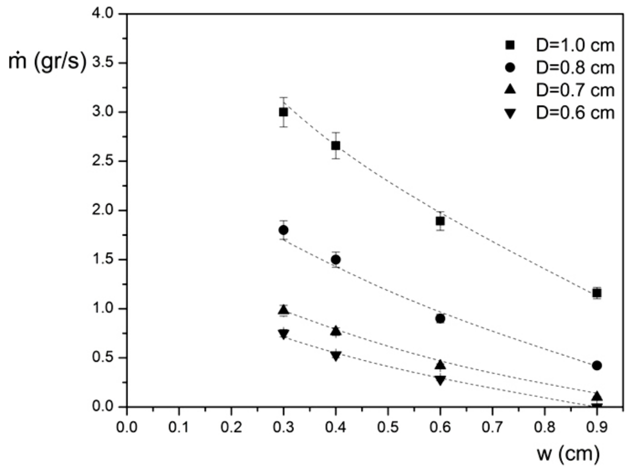

FIGURE 6. Plot of the mass flow rate, $\dot{m}$, as a function of the wall thickness w. Each fit to experimental data corresponding to different diameters was made by using the relation (2) which is discussed afterwards. Error bars are of 4%.

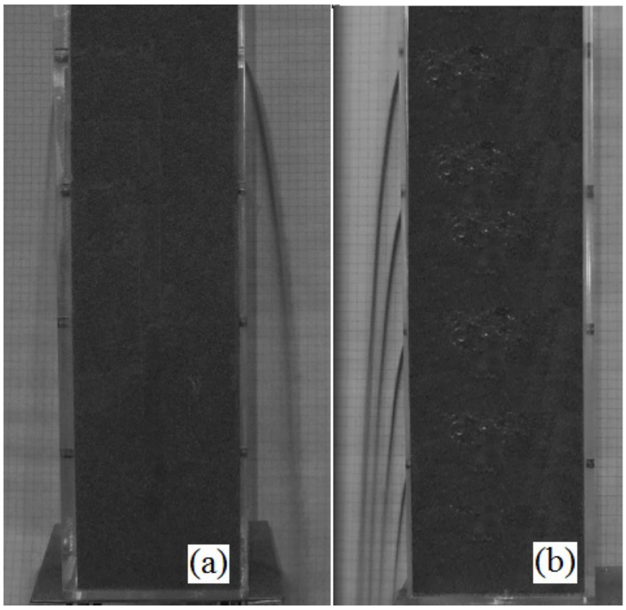

FIGURE 7. Snapshot of the outflow of sand: in (a) we show the effect of the wall thickness on the mass flow rate because the two holes have the same diameter but in the thinner wall the flow is strongest. (b) in the staggered holes the mass flow rate increases due to the diameter of the holes in a same wall increase.

where the dimensionless constant has the value $a = 0.48$. Afterwards we will discuss more about our measurement procedure of $\dot{m}$ and its accuracy degree.

3.2. Lateral exit holes

In order to reveal the influence of the wall thickness on the mass flow rate in silos with vertical walls, experiments were made upon an acrylic box, 50 cm height and 10×10 cm^2 inner cross-section, as the one shown in Fig. 3. In Fig. 3(a) the positions of the staggered orifices of different diameter D are sketched. As is sketched, orifices were made at the middle part of each wall. Diameters of the exit holes were: $D = 0.6$ cm at a height $H = 5$ cm from the bottom, $D = 0.7$ cm at $H = 15$ cm, $D = 0.8$ cm at $H = 25$ cm, and $D = 1.0$ at $H = 35$ cm. The circular orifices were made in each wall of the silo; in experiments four different wall thicknesses were used : $w = 0.3$, 0.4, 0.6 and 0.9 cm. In Fig. 3(b) we show a top view of the silo with the four different wall thicknesses. Thus, an exit hole of a given diameter is at the same height H in each wall.

A schematic of the experimental procedure employed to get the mass flow rates is shown in Fig. 4: a reservoir attached to a force sensor model Pasco CI-6537 with a resolution of 0.03 N was located close to the wall and data of weights were acquired each 1 s for each orifice in each wall, *i.e.*, in each experimental run only one orifice was opened.

To understand our measurement procedure is crucial to quantify the involved forces in the fall of sand. In quantitative terms, suppose that the sand is dropping at a steady rate of $\dot{m}$ gr/s and that it takes t_1 seconds for the sand particles to hit the reservoir. The velocity of sand hitting the compartment would be essentially $v = gt$ (in the simplest case we have neglected the initial velocity of grains just at the exit of the hole). In an infinitesimal time, dt, the change in momentum experienced by the reservoir would be $v\dot{m}dt$, so that the impulsive force experienced by the reservoir would be $F = v\dot{m}dt/dt = \dot{m}v = \dot{m}gt_1$. This force corresponds to a stepwise increase in the weight in the reservoir and for a time of measurement t_m, longer than t_1, will occur a succession points as that shown in plot of Fig. 5. Finally, we can plot $\dot{m}$ as a function of time. See inset of Fig. 5. There, is appreciated that $\dot{m}$ is near a constant. Thus, actually the time-average of $\dot{m}$ is the quantity that we report as the mass flow rate. For example, the mean value of this quantity was computed from ten independent measurements for each couple of values (D, w). It is important to comment that no dependence on the level of filling of the silos was detected in our mass flow rate measurements.

In Fig. 6 we show the plot of $\dot{m}$ vs w for different diameters. It is noticed that for a fixed value of D there is a strong dependence of the mass flow rate on the wall thickness w. In some cases, for instance, when $D = 0.6$ cm in each wall, the outflow is arrested if $w \simeq 0.9$. In Fig. 6 there are a set of non linear fits which will be discussed in the next section on the basis of a simple geometrical correlation.

4. A geometrical correlation

From the results reported in plot of Fig. 6 it is clear that the wall thickness w also modulates $\dot{m}$. As an illustration, in Fig. 7(a) we show visually such a behavior: in a thin wall the outflow is strongest than in a thick wall, meanwhile in Fig. 7(b) it is observed that the flow is strongest as the height increases because the diameter of the staggered holes also increases.

All these results allow us to propose the next model. As can be seen in Fig. 8, we show schematically the lateral view of the zone where there is a hole of size D in a vertical wall of a silo (8(a)). In Fig. 8(c) is more evident that always there is a *natural* angle of wall, α, which can be defined as $\alpha = \arctan(D/w)$. Meanwhile, in this same figure is observed that if there is no flow due to the wall thickness is wide enough, the granular material maintained there will attains its angle of repose, θ_r. Thus, an outflow is kept as long as $\alpha > \theta_r$ (see Fig. 8(a) and 8(b)) and conversely the outflow should be arrested if $\theta_r \geq \alpha$. Consequently, the mass flow rate itself must be proportional to $(\alpha - \theta_r)$, *i.e.*, $\dot{m} \sim (\alpha - \theta_r)$.

Another important feature to get a general relation for $\dot{m}$ is that the mass flow rate through vertical holes is a fraction of $\dot{m}_0$ (the mass flow rate through bottom holes) [10,11]. Consequently, the mass flow rate dependence on D and w would be a relation of the form

$$\dot{m} = c\dot{m}_0 \left[\arctan\left(\frac{D}{w}\right) - \theta_r\right], \tag{2}$$

where c is a dimensionless fitting parameter.

In order to show if the Eq. (2) is a correct correlation, in Fig. 9 we have plotted $\dot{m}$ as a function of $\dot{m}_0 [\alpha - \theta_r]$, where $\dot{m}_0$ is given by Eq. (1) and also was measured. Here, we used the experimentally measured angle of repose $\theta_r = 33^\circ = 0.57$ rad. This angle was measured by using the circular heap method [12].

The direct comparison between $\dot{m}$ and $\dot{m}_0 [\alpha - \theta_r]$ yields a linear relation of the form

$$\dot{m} = 0.23\dot{m}_0 \left[\arctan\left(\frac{D}{w}\right) - 0.57\right], \tag{3}$$

and the different sets of data (taken from Fig. 6) fit very well this straight line (in our experiments the value of c was $c = 0.23$, but, as in the case of the Hagen's law, this proportionality factor must be determined depending on the different materials, grains and silo walls, involved in the flow).

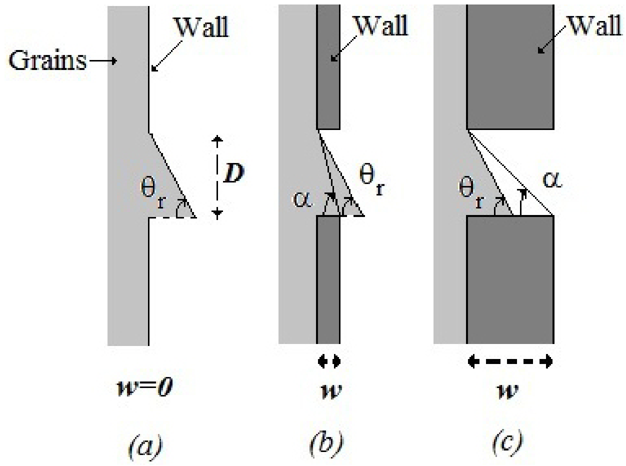

FIGURE 8. Schematic lateral view of the exit hole in a vertical silo. In (a) the wall thickness is ideally null, meanwhile in (b) the wall thickness satisfies that $\alpha > \theta_r$ and a granular outflow proportional to $(\alpha - \theta_r)$ occurs. Finally, in (c) the wall thickness produces the condition $\alpha < \theta_r$ and consequently there is no flow. The granular flow is arrested in such a form that the slope of the granular material in the exit hole is featured by the angle of repose.

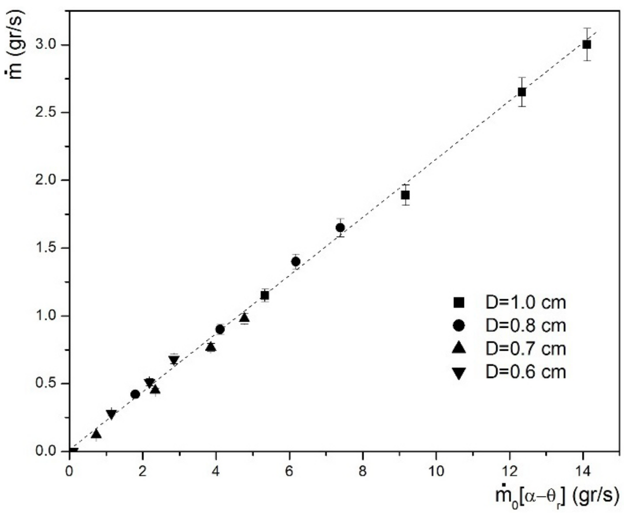

FIGURE 9. Plot of the mass flow rate $\dot{m}$, as a function of $\dot{m}_0 [\alpha - \theta_r]$. Data fit the straight line and produces the relation $\dot{m} = 0.23\dot{m}_0 [\alpha - \theta_r] = 0.23\dot{m}_0 [\arctan(D/w) - \theta_r]$. Error bars are of 4%.

Giving all these results, we can conclude that Eq. (2) is a universal curve and that, essentially, in this work we would needed of a unique experiment to fix $\dot{m}_0$ and then determine the main parameter of such equation (c), because the other point is the value $\dot{m}_0 [\alpha - \theta_r] = 0$. Giving two points, always it is possible to depict the corresponding straight line, as the one shown in Fig. 9.

5. Other results

The results of the previous Section give us the confident to explore another important consequences of Eq. (2). The first one is that Eq. (2) allows us to determine the critical value, w_c, for which the outflow will be arrested, as was discussed in Sec. 3. Graphically, the outflow will be arrested when $\dot{m}_0 [\alpha - \theta_r] = 0$ or if $\alpha = \theta_r$. See Fig. 9. Equation (2) also allows found the critical value of w for the arrest, as a function of D and θ_r, it is

$$w_c = \frac{D}{\tan\theta_r}. \quad (4)$$

By using the four values of D used in the experiments and using that $\theta_r = 33^\circ$ we have verified that this relation predicts the precise thickness wall for which the outflow will be arrested.

A second one consequence takes in to account that $\dot{m}_0 = a\rho g^{1/2} D^{5/2}$, and by using it in Eq. (2), we have that

$$\dot{m} = c'\rho g^{1/2} D^{5/2} \left[\arctan\left(D/w\right) - \theta_r\right], \quad (5)$$

where the dimensionless parameter c' here takes the value $c' = ac = 0.11$. The Eq. (5) is the explicit formula of the mass flow rate with simultaneous dependence on D and w. An expansion in series of $(D/w) < 1$ transforms Eq. (5) into

$$\dot{m} = c'\rho g^{1/2} \left[\frac{1}{w} D^{\frac{7}{2}} - \theta_r D^{\frac{5}{2}} + O\left(D^{\frac{11}{2}}\right)\right]. \quad (6)$$

In Eq. (6) the term proportional to $D^{7/2}$ will be dominant if the wall thickness satisfies the condition

$$\frac{D}{\theta_r} > w, \quad (7)$$

this condition is commonly fulfilled when the wall thickness is small respect to the hole diameter. Due to it, experiments of Bagrintsev and Koshkovskii [5] and Kesava Rao [7,8], where were used very thin walls, have shown that apparently the better correlation among $\dot{m}$ and D takes the form $\dot{m} = c'\rho g^{1/2} D^{7/2}/w$.

6. Conclusions

In this work we studied experimentally the problem of the mass flow rate of granular material through circular orifices in vertical walls of silos. Specifically, we have studied the dependence of $\dot{m}$ on D and w by using well characterized beach sand. To our knowledge, this is the first time that a systematic study of the effect of the wall thickness on the mass flow rate has been done. Our results show that Eq. (2), based in geometrical arguments, describes very well the influence of w and D on the mass flow rate, which can be considered a general formula including both quantities. In such an equation the role of the angle of repose is fundamental to describe the occurrence of the outflow of grains and its arrest. We gave also evidence that the use of Eq. (2) only requires to measure $\dot{m}_0$ (if α and θ_r are known) to depict the corresponding straight line. Moreover, when the wall thickness is small respect to the hole diameters, the explicit dependence on D yields that $\dot{m} = c'\rho g^{1/2} D^{\frac{7}{2}}/w$; this later result has been reported previously in some experiments of vertical orifices [5,7,8] and is obtained straightforward from our model. Finally, some studies have shown that the discharge rates from bottom exit decrease with the increase in particle size [13]. In such a case, instead the term $D^{5/2}$ in Eq. (1), it has been introduced the annular zone effect through the term $(D - kd_g)^{5/2}$ (where k is a constant); however, this assumption and the existence of many different orifice shapes requires the realization of more studies in order to have a general formula valid in a wide range of practical configurations. Work along these lines is in progress.

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