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Quartic couplings, masses and thresholds in the basic extension of the Standard Model

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The Standard Model (SM), despite of being phenomenologically extremely successful, presents some fundamental questions, like the big differences in the values of the masses of the quarks (hierarchy of masses), and the possible generation of flavour changing neutral currents (inspired by the evidence about the oscillations of neutrinos). Hints how these questions might be answered may be obtained by the study of the Higgs sector of models beyond the standard model. The simplest extension of the SM known as the two-Higgs-doublet-model (2HDM) involves a second Higgs doublet and predicts the existence of five scalar particles: three neutral (A^0), (h^0, H^0) and two charged ($H^{\pm}$). In this paper we focus our attention on the basic results of the model, the masses of the five particles, and the theoretical constraints imposed by vacuum stability and the triviality principle. We address, on one side, the range of validity in the energy scale of the 2HDM by means of the renormalization group equations, and on the other, the consequences of a top/bottom Yukawa coupling unification, assuming that the hierarchy of the quark masses is atributted to the vacuum expectation values v_1 and v_2 of the Higgs fields and not to the Yukawa couplings.

Keywords: 2HDM Higgs masses; electroweak Higgs sector extensions; beyond standard model.

A pesar de que el modelo estándar (SM) es extremadamente exitoso fenomenológicamente, presenta algunos interrogantes fundamentales, tales como la gran diferencia en los valores de las masas de los quarks (jerarquía de masas) y la posible generación de corrientes débiles que cambien el sabor (inspirada en la evidencia de las oscilaciones de neutrinos). Algunos indicios de cómo estos interrogantes pueden ser contestados podrían ser obtenidos mediante el estudio del sector de Higgs de modelos más allá del modelo estándar. La extensión más simple del SM, conocida como el modelo estándar con dos dobletes de Higgs (2HDM) involucra un segundo doblete y predice la existencia de cinco partículas escalares: tres neutras (A^0), (h^0, H^0) y dos cargadas ($H^{\pm}$). En este trabajo enfocamos nuestra atención en los resultados básicos del modelo, las masas de las cinco partículas y las restricciones teóricas impuestas por las condiciones de estabilidad del vacío y el principio de trivialidad para determinar, por un lado el alcance en la escala de energías del 2HDM mediante las ecuaciones del grupo de renormalización, y por el otro, las consecuencias de una unificación de los acoplamientos de Yukawa de los sectores up/down de los quarks, asumiendo que la jerarquía de masas de los quarks es atribuíble, a los valores esperados en el vacío v_1 y v_2 de los campos de Higgs y no a los acoplamientos de Yukawa.

Descriptores: Masas Higgses 2HDM; sector Higgs electrodébil extendido; ampliación modelo estándar.

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1. Introduction

The Standard Model(SM) in high energy physics [1–3] has been remarkably successful in describing the properties of elementary particles, predicting the existence of the quarks c, t and b, and the third generation of leptons τ- ν_τ, the existence of the eight gluons, the weak bosons $W^{\pm}$, Z^0 and the Higgs boson [4, 5] before their discovery, predicting parity violating neutral-weak-currents, and in being consistent with all the experimental results [6, 7]. However, the SM falls short of being a complete theory of the fundamental interactions because of its lack of explanation of the probable unification of the fundamental interactions, the pattern and disparity of the particle masses (mass hierarchy), the origin of the CP violation in nature, the matter-antimatter asymmetry, the pattern of quark mixing, lepton mixing and the reason why there are 3 generations.

As a partial solution to confront these deficiencies, a large number of parameters must be put in "by hand" into the theory (rather than being derived from first principles), such as the three gauge couplings (g_1, g_2, g_3), nine fermionic masses (six quarks and three leptons), the Weinberg angle (θ_W), four quark-mixing parameters (CKM) and two more parameters in relation to the Higgs potential (μ and λ).

One of the most subtle aspects of the model is associated with the Higgs sector [8]. The Higgs field and its nonvanishing vacuum expectation value (vev) is the essential ingredient to carry out the spontaneous symmetry breaking (SSB) required to transform the hypothetical massless particles in the Lagrangian into the actual massive physical particles.

The extension of the SM with two Higgs doublets presents also the challenge that the quartic interactions between the scalar doublets are not theoretically determined. This model is widely studied. Recently, it has been reviewed in the very interesting and complete paper [9], where extensive references to the original literature may be found.

The general properties of the Higgs quartic potential have been studied in Refs. 10 and 11 on the basis of the Minkowski space structure of the 2HDM quartic potential. This analysis was oriented towards the possible topological structure of the Higgs potential that depends on the parameters λ_i of the potential and it did not address specific phenomenological problems. The connection of this analysis with the phenomenology is further complicated by the general nature of the considered transformations which also affect the kinetic part of the Higgs Lagrangian. Our analysis is compatible with the results of Refs. 10 to 12.

In this paper we consider this model mainly for three reasons. The first one is that the 2HDM has a much richer Higgs spectrum (3 neutral and 2 charged Higgses) and a different high energy behavior. This makes that a lower mass than in the SM Higgs is permitted. Another reason may be that a different pattern of hierarchy of the Yukawa couplings is possible, because of the presence of two independent vacuum expectation values of the Higgs fields (the importance of such analysis can be seen for example in the Higgs search scenarios, *e.g.*, see [13,14]). The third reason is that the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM) requires at least two Higgs doublets, so the Higgs sectors of the MSSM and the 2HDM are similar and the study of the 2HDM model may give important information on the properties of the Higgs sector in the MSSM (see for example Ref. 15).

The purpose of this work is to consider the masses of the five remnant particles that the model entails, and their dependence on the parameters λ_i and v_j. As the values of the quartic interactions λ_i between the scalar doublets are not theoretically determined, we propose several values for them inspired in some values for the Higgs masses or unification mechanism, given in the literature [16–25]. The values of the λ's that reproduce those masses, are constrained through stability and triviality principles. By using the proposed parameters we explore their energy dependence and the repercussion on the energy behaviour of the masses of the Higgses by numerically solving the renormalization group equations. Through the former results we obtain the region of validity of the model.

In Sec. 2, we introduce the Higgs potential for the 2HDM in a special parametrization, and the SSB for the normal vacuum conditions. In Sec. 3 we go over the Higgs mass matrix and its diagonalization results, mass eigestates and the mass spectrum. In Sec. 4 we classify the constraints for the quartic couplings derived from the mass formulas, from the vacuum stability principle and by imposing extreme stability conditions in which the lightest neutral Higgs boson is massles. In Sec. 5 we numerically solve the set of the renormalization group equations. Finally, Sec. 6 is devoted to the presentation of the results and the conclusions. In the appendix we present four tables related to the cases in which the model is valid until the electroweak unification scale.

2. The two-Higgs doublet model

In the SM the fermion masses arise, after the SSB, from the couplings between the fermions and a single Higgs doublet. The mass ratio of the b and t quark is of the order of $1/40$. To understand in a natural way the origin of this difference in the values of the masses of the third generation of quarks, one can assume the existence of a second Higgs-doublet in the Higgs sector of the SM. In this context one assumes that the quark b obtains its mass through the Φ_1 doublet and the quark t from another doublet Φ_2 (there are also other scenarios for the quark mass generation in the 2DHM but we will not be considering them here). In this way one can explain in a more natural way the hierarchy problem of the Yukawa couplings, as long as the free parameters of the new model acquire the appropriate values.

The Higgs sector of the 2HDM consists of two identical (hypercharge-one) scalar doublets Φ_1 and Φ_2. There are several proposals for the Higgs potential to describe the physical reality in the framework of the 2HDM [9,26,27]. The potential we consider in this paper is compatible with Ref. 28. It is such that the CP symmetry (charge-conjugation and parity) in the Higgs sector is conserved, the neutral-Higgs mediated flavor-changing neutral currents (FCNC) are suppressed in the leptonic sector, and in the quark-sector they are also forbidden by the GIM mechanism [29] in the one loop approximation. It is by far the most studied type II 2HDM, since it is the structure present in the Super Symmetric Models. In the Lagrangian $\mathcal{L}$ in which we leave out the leptonic terms,

$$\mathcal{L} = \mathcal{L}_{gf} + \mathcal{L}_{Kin} + \mathcal{L}_Y - V,$$

the $\mathcal{L}_{gf}$ and $\mathcal{L}_{Kin}$ correspond to kinetic parts of quarks and bosons and they contain the covariant derivatives that provide the interactions among the gauge bosons and the Higgs bosons. They also give rise, after the SSB, to the masses of the gauge bosons (mediators of the electroweak interactions). The fermion masses are generated, as follows, from the Yukawa couplings in $\mathcal{L}_Y$

$$\mathcal{L}_Y = \sum_{i.j} \left(g_{ij}^{(d)} \overline{\psi}_{Li} \Phi_1^c d_{Rj} + g_{ij}^{(u)} \overline{\psi}_{Li} \Phi_2 u_{Rj} \right) + \text{h.c.},$$

between the Higgs bosons and the quarks. In $\mathcal{L}_Y$, the $g_{ij}^{(u,d)}$ are the Yukawa coupling matrices in the Higgs basis. The superscripts (u,d) refer to the up and down sectors of quarks, respectively and the subscripts (L,R) correspond to the left handed doublets and right handed singlets in the quark sector. The explicit form of the Higgs doublets is given in the next section. In this paper, we will focus our attention on the potential V.

The Higgs potential V depends on seven real parameters μ_1^2, μ_2^2 and $\lambda_i \, (i = 1...,5)$ from which the five Higgs masses come up after the SSB. The most general renormalizable $SU(2) \times U(1)$ invariant Higgs potential, that preserves a CP and a Z_2 symmetry ($\Phi_1 \rightarrow \Phi_1,\ \Phi_2 \rightarrow -\Phi_2$) may be written as

$$\begin{aligned} V &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 \\ &+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\ &+ \frac{1}{2}\lambda_5 \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right]. \end{aligned} \tag{1}$$

For the sake of simplicity a special basis is introduced

$$A = \Phi_1^\dagger \Phi_1, \quad B = \Phi_2^\dagger \Phi_2, \quad C' = D'^\dagger = \Phi_1^\dagger \Phi_2.$$

In this basis

$$\begin{aligned} V &= \mu_1^2 A + \mu_2^2 B + \lambda_1 A^2 + \lambda_2 B^2 \\ &+ \lambda_3 AB + \lambda_4 C'D' + \frac{1}{2}\lambda_5 \left[C'^2 + D'^2\right]. \end{aligned}$$

The two Higgs doublets can be represented by eight real fields $\phi_i,\ i = 1, \ldots, 8$,

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}.$$

If charge is conserved and there is no CP violation in the Higgs sector, after the SSB, the non-vanishing vacuum expectation values (vevs) of the fields ϕ_3 and ϕ_7 are real, and the minimum occurs at

$$\langle \phi_3 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \phi_7 \rangle = \frac{v_2}{\sqrt{2}}, \tag{2}$$

hence

$$\langle A \rangle = \frac{1}{2}v_1^2, \quad \langle B \rangle = \frac{1}{2}v_2^2, \quad \langle C' \rangle = \langle D' \rangle = \frac{1}{2}v_1 v_2.$$

as mentioned before, v_1, and v_2 are real, positive and $v_1^2 + v_2^2 = v^2$. Experimentally, $v \approx 246$ GeV.

3. The mass matrix and the Higgs mass-eigenstates basis

As it is known, the conditions for the potential are obtained by minimizing

$$\left.\frac{\partial V}{\partial \phi_i}\right|_{\min} = 0,$$

and demanding that the matrix of the second derivatives at the minimum:

$$\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right|_{\min}$$

be positive definite. Note, that the choice of the vacuum expectation values given in Eq. (2) is compatible only with the neutral vacuum [10-12].

From the vanishing of the first derivatives at the minimum and after some simplifications two non trivial equations are obtained

$$\begin{aligned} \mu_1^2 + \lambda_1 v_1^2 + 2\lambda_T v_2^2 = 0 \ \text{ or } \ v_1 = 0, \\ \mu_2^2 + \lambda_2 v_2^2 + 2\lambda_T v_1^2 = 0 \ \text{ or } \ v_2 = 0, \end{aligned}$$

where

$$\lambda_T \equiv (\lambda_3 + \lambda_4 + \lambda_5)$$

and we discard the solutions with v_1 or v_2 equal to 0.

The mass matrix elements are obtained from the equation

$$M_{ij}^2 = \frac{1}{2} \left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\right|_{\phi_3 = \frac{v_1}{\sqrt{2}}, \phi_7 = \frac{v_2}{\sqrt{2}}}.$$

After the complete diagonalization, the mass spectrum becomes:

1. The mass eigenvalues for (H^0, h^0) are

$$\begin{aligned} M_{H^0,h^0}^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 \\ &\pm \sqrt{\left(\lambda_1 v_1^2 - \lambda_2 v_2^2\right)^2 + \left(v_1 v_2 \lambda_T\right)^2} > 0, \end{aligned} \tag{3}$$

2. The eigenvalues for the mass eigenstates $H^\pm$ and $G^\pm$ are

$$M_{G^\pm}^2 = 0\ , \quad M_{H^\pm}^2 = -\frac{1}{2}\left(\lambda_4 + \lambda_5\right) v^2 > 0, \tag{4}$$

3. Finally, the mass eigenvalues for G^0 and A^0 are

$$M_{G^0}^2 = 0, \quad M_{A^0}^2 = -\lambda_5 v^2 > 0, \tag{5}$$

The three massless Goldstone fields $G^\pm$ and G^0 become the longitudinal components of the gauge bosons $W^\pm$ and Z^0.

As one can see, from the values of M_{A^0} and $M_{H^\pm}$, which do not depend explicitly on the parameters λ_i, $(i = 1, 2, 3)$, one could infer that there is a complete independence between the A^0, $H^\pm$ and the h^0, H^0, but this is not all true, in their energy scale dependence, as we shall see later. To add more information, we invert the former equations to express the quartic parameters in terms of the masses of the Higgs fields.

$$\begin{aligned} \lambda_1 &= \frac{1}{2v_1^2}\left(M_{H^0}^2 \cos^2\alpha + M_{h^0}^2 \sin^2\alpha\right), \\ \lambda_2 &= \frac{1}{2v_2^2}\left(M_{H^0}^2 \sin^2\alpha + M_{h^0}^2 \cos^2\alpha\right), \\ \lambda_3 &= \frac{\left(M_{H^0}^2 - M_{h^0}^2\right)}{2v_1 v_2}\sin 2\alpha + 2\frac{M_{H^\pm}^2}{v^2}, \\ \lambda_4 &= \frac{M_{A^0}^2 - 2M_{H^\pm}^2}{v^2}, \quad \lambda_5 = -\frac{M_{A^0}^2}{v^2}, \end{aligned} \tag{6}$$

where

$$\tan 2\alpha = \frac{\left(\lambda_3 + \lambda_4 + \lambda_5\right) v_1 v_2}{\left(\lambda_1 v_1^2 - \lambda_2 v_2^2\right)}, \quad -\frac{\pi}{2} < \alpha < \frac{\pi}{2}.$$

4. Vacuum stability constrains (VSC)

To make the discussion more transparent, in this section, let us introduce a different parametrization of the Higgs potential at its minimum . We introduce the parameters x_i defined as: $x_1 = v_1^2$, $x_2 = v_2^2$. The potential in Eq. (1) becomes

$$V = \frac{1}{2}V_2 + \frac{1}{4}V_4$$

where

$$V_2 = \mu_1^2 x_1 + \mu_2^2 x_2 \quad \text{and} \quad V_4 = \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_T x_1 x_2.$$

4.1. Bounds due to the positive mass-values

From previous results in Eqs. (3), (4), (5) and the mass positivity, one gets information for the allowed values of the λ_i parameters in agreement with [28] and [9]

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad (\lambda_4 + \lambda_5) < 0, \quad \lambda_5 < 0, \quad \lambda_4 < |\lambda_5|\,.$$

plus the implication from Eq. (3)

$$\lambda_1\lambda_2 > \frac{1}{4}(\lambda_3 + \lambda_4 + \lambda_5)^2\,.$$

and more precisely

$$\lambda_T + 2\sqrt{\lambda_1\lambda_2} \geq 0\,, \quad \lambda_3 + 2\sqrt{\lambda_1\lambda_2} \geq 0.$$

4.2. Massles Higgs boson

Another interesting case is when the lighter Higgs boson becomes massless, $M_{h_0} = 0$, which implies the condition

$$\lambda_T = -2\sqrt{\lambda_1\lambda_2}. \tag{7}$$

The general form of the Higgs quartic potential V_4 is

$$\begin{aligned} V_4 &= \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_T x_1 x_2 \\ &= \left(\sqrt{\lambda_1}x_1 - \sqrt{\lambda_2}x_2\right)^2 + \left(\lambda_T + 2\sqrt{\lambda_1\lambda_2}\right)x_1 x_2 \geq 0. \end{aligned}$$

From condition (7) it follows that the potential V_4 simplifies to

$$V_4 = \left(\sqrt{\lambda_1}x_1 - \sqrt{\lambda_2}x_2\right)^2.$$

Here we will consider two cases: the first one is when $V_4 = 0$, (Extreme condition) which implies

$$\sqrt{\frac{\lambda_1}{\lambda_2}} = \frac{x_2}{x_1} = \frac{v_2^2}{v_1^2} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{v_2^4}{v_1^4} = (\tan\beta)^4\,.$$

and

$$M_{H^0} = \begin{cases} (4\lambda_1\lambda_2)^{1/4}v, & \lambda_1 \neq \lambda_2, \quad v_1 \neq v_2, \\ (2\lambda)^{1/2}\,v, & \lambda_1 = \lambda_2 = \lambda, \quad v_1 = v_2 = v/\sqrt{2}. \end{cases}$$

In the general case, when $V_4 = \left(\sqrt{\lambda_1}x_1 - \sqrt{\lambda_2}x_2\right)^2 \neq 0$, which means that

$$\frac{\lambda_1}{\lambda_2} \neq (\tan\beta)^4\,, \quad \lambda_T = -2\sqrt{\lambda_1\lambda_2},$$

and the M_{H^0} becomes

$$M_{H^0} = \sqrt{2}\left(\lambda_1 v^2 + (\lambda_2 - \lambda_1)\,v_2^2\right)^{1/2}.$$

In both cases

$$M_{h^0} = 0, \quad M_{H^\pm} = \left(\frac{1}{2}\,|\lambda_4 + \lambda_5|\right)^{1/2} v,$$
$$M_{A^0} = |\lambda_5|^{1/2}\,v.$$

The problem of mass of the lighter Higgs boson in the 2HDM model remains open. The experimental limits on the Higgs mass are firm in the Standard Model [30]. However for the Minimal Supersymmetric Standard Model the experimental mass limits in search of neutral Higgs bosons obtained by LEP [31] are compatible with zero mass of the lighter Higgs boson.

On the theoretical side, the zero mass of the lighter Higgs boson implies the reduction of the number of parameters (see Eq. (7)) and this may be a signal of some additional symmetry, *e.g.*, the supersymmetric $SU(5)$ model contains naturally massless Higgs doublets [32]. The crucial test of such tree level symmetry would be radiative corrections to the Higgs mass and the stability of such a symmetry under such corrections.

Summarizing, the case of the massless neutral Higgs boson is an interesting possibility for the 2HDM model that is worth of mentioning and of further study.

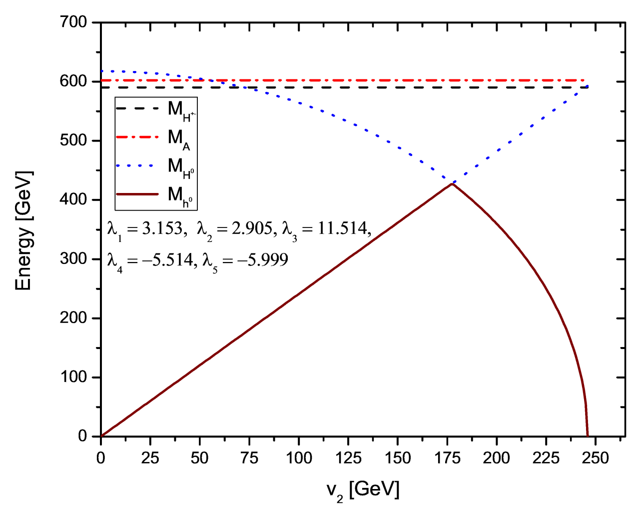

FIGURE 1. The v_2 dependence of the Higgs masses. $\tan\beta = 5$ corresponds to $v_2 = 248.9$.

4.3. Numerical evaluation of the Higgs masses in terms of $\tan\beta$

In general, according to Eqs. (3-5) the mass dependence on v_1, v_2 and v can be reformulated in terms of v and v_2. Therefore, with a fixed known v one can plot those masses in terms of v_2, and determine their $\tan\beta$ ($\tan\beta = v_2/\sqrt{v^2 - v_2^2}$) dependence, as shown in Fig. 1. One should notice that the charged and the CP odd Higgs masses remain constant (*i.e.*, they do not depend on $\tan\beta$, while the CP even neutral Higgs masses depend significantly on $\tan\beta$.

Recently, the bounds on the charged-Higgs mass have been studied extensively [17-24]. Taking into account some of the newest results, we will proceed to numerically evaluate the Higgs masses under different stability conditions in cases where $\lambda_1 = \lambda_2 = \lambda$ (one of the symmetries considered in Ref. 20) or $\lambda_1 \neq \lambda_2$, at the energy scale $E = M_t$, where M_t is the mass of the quark top.

Let us now select some special scenarios. In a first scenario, the M$_{H^0}$ will depend explicitly on v_2 and therefore on $\tan\beta$, as shown in Fig. 1, where we reproduce the values given in Refs. 17 and 18 at their best point $M_{H^\pm} = 608.8$ at $\tan\beta = 5$, considering $M_{H_A} = 621.7$ and assuming $\lambda_T = 0$. Using Eq. (6), we fix the λ's as follows

λ_1	λ_2	λ_3	λ_4	λ_5
3.15	2.91	11.51	-5.51	-6.0
	$\tan\beta$	M_{h^0}	M_{H^0}	v_2
	5.0	125	600.4	248.88

The running properties of the parameters and masses in this case will be analyzed in the following section.

In a second and third scenarios we assume that $\tan\beta = v_2/v_1 = m_t/m_b = 41.2$) and we focus our attention on the λ_i's ($i = 1, ..., 5$) that reproduce the values in Ref. 21 and 22, to explore the properties for the energy range of validity of the 2HDM, for smaller values for $M_{H^\pm}$. We obtain for the second scenario, according to Ref. 21, for $\tan\beta/M_{H^\pm} = 0.2$, and $\tan\beta = 41.2$ for $\{M_{H^\pm}, M_A\} = \{206.2, 206.2\}$, with $\lambda_T = -2\sqrt{\lambda_1\lambda_2} = -0.244$ and $(\lambda_1/\lambda_2)^{1/4} = 1.0$, the following values

λ_1	λ_2	λ_3	λ_4	λ_5
0.122	0.122	1.076	-0.66	-0.66
	$\tan\beta$	M_{h^0}	M_{H^0}	v_2
	41.2	0	125.4	253.7

Finally, the third scenario is an interesting one, due to its RGE behavior, for a still lower value of $M_{H^\pm}$ in the interval [80,150] GeV fixed in Ref. 22, when $\{M_{H^\pm}, M_A\} \approx \{129.4, 131.9\}$, $(\lambda_1/\lambda_2) = 1.0$ and $\lambda_T + 2\sqrt{\lambda_1\lambda_2} > 0$

λ_1	λ_2	λ_3	λ_4	λ_5
0.18	0.18	0.3	-0.25	-0.27
	$\tan\beta$	M_{h^0}	M_{H^0}	v_2
	41.2	2.9	152.26	253.7

5. Triviality constrains.

5.1. Renormalization group equations

In this section we explore the asymptotic behavior of the parameters in the model, and their relations, through the Renormalization Group Equations (RGE) [26,33,34]. The RGE are a powerful tool to determine by the triviality principle, the energy bounds of the parameters and the validity of the model. In order to proceed in this way, to numerically evaluate the energy dependence of the λ_i quartic couplings, it is necessary to consider the RGE of all the parameters, i.e., the couplings g_1, g_2, g_3 of the gauge group $U(1) \times SU(2) \times SU(3)$, the vacuum expectation values v_1, v_2, and the Yukawa couplings of the top and the down quark sectors g_t and g_d, respectively [35–37].

The RGE determine the dependence of the coupling constants and other parameters of the Lagrangian on t, defined as $t = \ln(E/M_t)$, where E is the renormalization point energy. The RGE for the gauge couplings g_1, g_2, g_3 are:

$$\frac{dg_i}{dt} = \frac{1}{(4\pi)^2} b_i g_i^3 \quad (i = 1, 2, 3),$$

where $b_i = (21/5, -2, -7)$. The RGE for the Yukawa couplings of the top and bottom quarks g_t, g_b are

$$\frac{dg_t}{dt} = \frac{1}{(4\pi)^2}\left(\frac{9}{2}g_t^2 + \frac{1}{2}g_b^2 - (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\right) g_t,$$
$$\frac{dg_b}{dt} = \frac{1}{(4\pi)^2}\left(\frac{9}{2}g_b^2 + \frac{1}{2}g_t^2 - (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\right) g_b,$$

and for the vacuum expectation values v_1 and v_2

$$\frac{d}{dt}v_1 = \frac{1}{(4\pi)^2}\left[-3g_t^2 + ((9/20)\,g_1^2 + (9/4)\,g_2^2)\right] v_1,$$
$$\frac{d}{dt}v_2 = \frac{1}{(4\pi)^2}\left[-3g_b^2 + ((9/20)\,g_1^2 + (9/4)\,g_2^2)\right] v_2,$$

In the equations for the quartic couplings we do include the quark Yukawa contributions of both sectors.

$$\frac{d\lambda_1}{dt} = \frac{1}{16\pi^2}\left\{24(\lambda_1)^2 - 3\lambda_1\left[3g^2 + (g')^2 - 4g_t^2\right] + 2(\lambda_3)^2 + (\lambda_4)^2 + (\lambda_5)^2 + 2\lambda_3\lambda_4 + \frac{3}{8}\left(g^2 + (g')^2\right)^2 + \frac{3}{4}g^4 - 6g_t^4\right\},$$
$$\frac{d\lambda_2}{dt} = \frac{1}{16\pi^2}\left\{24(\lambda_2)^2 - 3\lambda_2\left[3g^2 + (g')^2 - 4g_b^2\right] + 2(\lambda_3)^2 + (\lambda_4)^2 + (\lambda_5)^2 + 2\lambda_3\lambda_4 + \frac{3}{8}\left[(g')^2 + g^2\right]^2 + \frac{3}{4}g^4 - 6g_b^4\right\},$$
$$\frac{d\lambda_3}{dt} = \frac{1}{16\pi^2}\left\{4(\lambda_3)^2 + 4(3\lambda_3 + \lambda_4)(\lambda_1 + \lambda_2) - 3\lambda_3\left[3g^2 + (g')^2 - 2\left(g_t^2 + g_b^2\right)\right] + 2(\lambda_4)^2\right.$$

$$+2\left(\lambda_5\right)^2+\frac{3}{4}\left[g^2-\left(g'\right)^2\right]^2+\frac{3}{2}g^4-12g_t^2g_b^2\Big\},$$

$$\frac{d\lambda_4}{dt}=\frac{1}{16\pi^2}\left\{4\left(\lambda_4\right)^2+4\lambda_4\left(\lambda_1+\lambda_2+2\lambda_3\right)-3\lambda_4\left[3g^2+\left(g'\right)^2-2\left(g_t^2+g_b^2\right)\right]+8\left(\lambda_5\right)^2+3g^2\left(g'\right)^2+12g_t^2g_b^2\right\}$$

$$\frac{d\lambda_5}{dt}=\frac{1}{16\pi^2}\lambda_5\left\{4\left(\lambda_1+\lambda_2+2\lambda_3+3\lambda_4\right)-3\left[3g^2+\left(g'\right)^2-2\left(g_t^2+g_b^2\right)\right]\right\}.$$

The former equations are non linear, coupled, ordinary differential equations whose solution provides the information about the renormalization point energy dependence of the masses of the five Higgs particles of the 2HDM. To numerically solve the RGE, the initial or final conditions for the parameters have to be previously chosen. In order to do so we use Ref. 6. The range of values, we take, for the energy and the variable t are $E=\{M_t,E_u\}=\{173.2,1.234\cdot 10^{13}\}$, $t=\{0,t_u=25\}$, respectively (see Table I for the correspondence between t and the energy E in GeVs), where M_t stands for the mass of the quark top and E_u corresponds to the electroweak unification energy where $g_1(E_t)=g_2(E_t)$. The gauge couplings $(g_1,g_2,g_3)_{E=M_t}\simeq(0.4627,0.6466,1.2367,)$ are obtained using the following relations

$$g_1(M_t)=\sqrt{5/3}g_e/\cos\theta_W,\quad g_2(M_t)=g_e/\sin\theta_W,$$
$$g_3(M_t)=\sqrt{4\pi\alpha_s(M_t)},\quad \alpha_e(M_t)=g_e^2/4\pi=1/(127.9),$$

where θ_W is the Weinberg angle where $\sin^2\theta_W(M_t)$=0.235 and $\alpha_s=0.1217$. The vev standard value that arises from

$$v=2M_z/\sqrt{g_2^2+g_e^2},$$

is $v(M_t)=253.81$ GeV at $M_z=91.19$ GeV.
In order to specify more rigorously the energy limits for the quartic couplings, we have numerically solved the RGE for the gauge group couplings g_1, g_2, g_3, (Fig.2), the vacuum expectation values v_1, v_2, and the top and the down quark Yukawa couplings g_t and g_d, under the following assumptions:

- The heaviest quark masses are related with the vevs v_1 and v_2 and the Yukawa couplings $g^{(u)}$ and $g^{(d)}$

$$M_t=\frac{v_2}{\sqrt{2}}g_t,\quad M_b=\frac{v_1}{\sqrt{2}}g_b,\quad \tan\beta=\frac{v_2}{v_1}.$$

- The gauge bosons masses are related with the gauge couplings g' and g

$$M_W=\frac{1}{2}vg,\quad M_Z=\frac{M_W}{\cos\theta_W}=\frac{1}{2}v\sqrt{g^2+(g')^2},$$

where θ_W is the Weinberg angle and e the electron charge

$$e=g\sin\theta_W=g'\cos\theta_W.$$

- Unification of the Yukawa couplings at $E=M_t$ or at E_u, *i.e.*, $g_b=g_t$, and $\tan\beta=M_t/M_b$.

It is interesting to explore now, the energy bounds of the 2DHM, through the running of the quartic couplings which determine the mass values of the Higgses. In the first scenario considered in the previous section, when $M_{H^\pm}=609$ GeV, $M_A=621.7$ GeV, the range of validity of the model is short $M_t<E<592.2$ GeV *i.e.*, $0<t<1.23$ as can be seen in Figs. 3. Here the new physics would appear at a very low energy. The second scenario is depicted at Figs. 4, the model presents here an intermediate range of validity $0<t\lesssim 18$. Now we will rather focus our attention on the cases where we can explore the universality of the Yukawa couplings and its unification, to study the mass-hierarchy problem. In this case, as can be seen in Figs. 5-8, the 2HDM is valid in the whole range of energies, this means $M_t<E<E_u$ where E_u is the electroweak unification energy.

In Fig. 5 we observe a very slow dependence of the quartic couplings and the Higgs masses on the renormalization point energy. The model is characterized by rather small values of the quartic couplings and the value of $\tan\beta$ such that it permits the unification of the Yukawa couplings of the up and down quarks $g_t=g_b$. In Figs. 6, 7 and 8 we show the

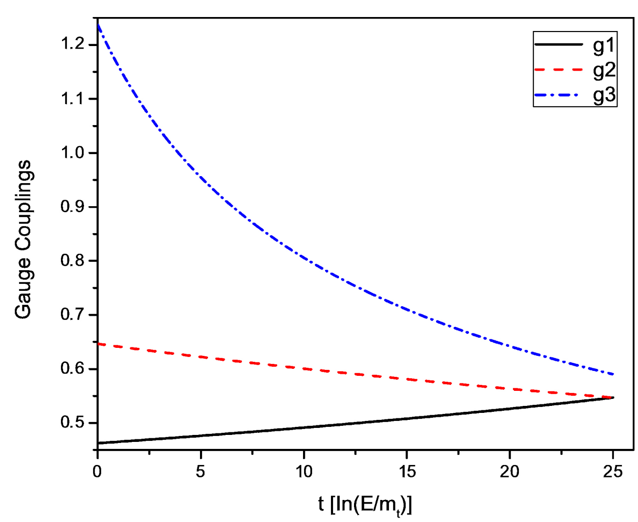

FIGURE 2. The energy dependence of the gauge couplings in the 2HDM.

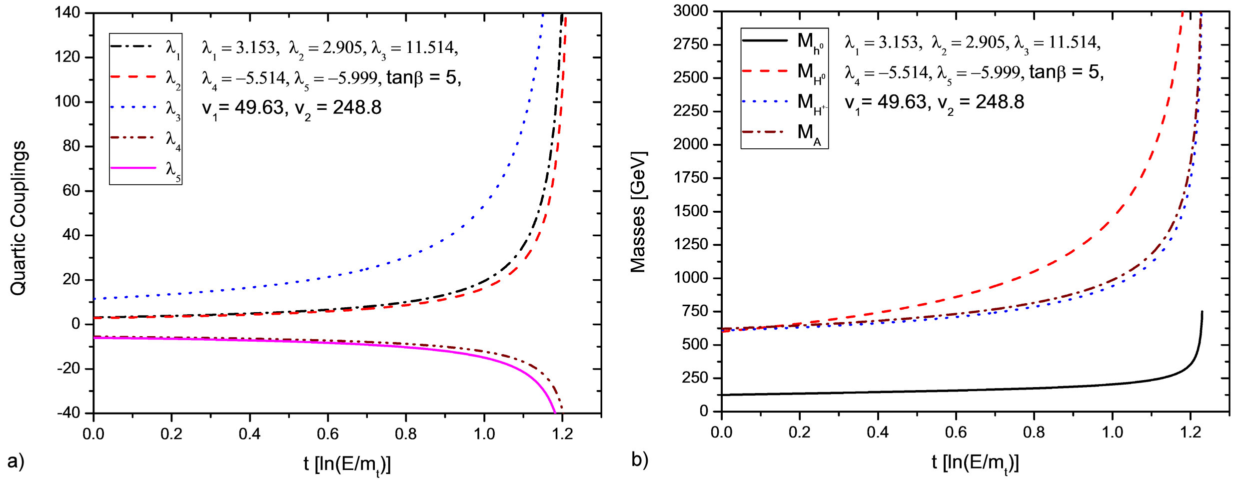

FIGURE 3. The energy dependence of the quartic couplings and the Higgs masses, in the first scenario with $\tan\beta = 5.0$.

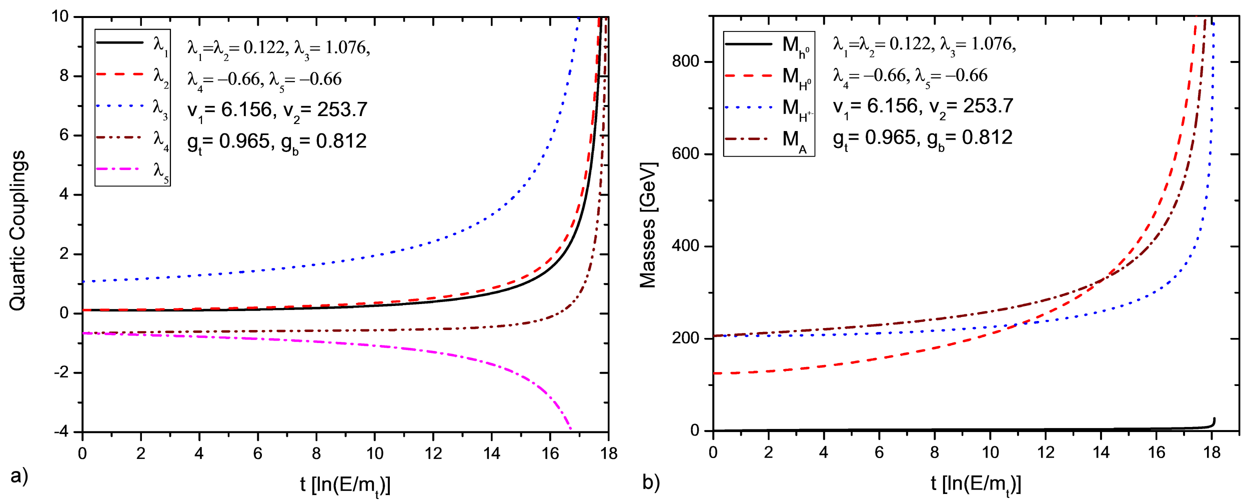

FIGURE 4. The energy dependence of the quartic couplings and the Higgs masses, in the second scenario with $\tan\beta = 41.2$ (2D) case.

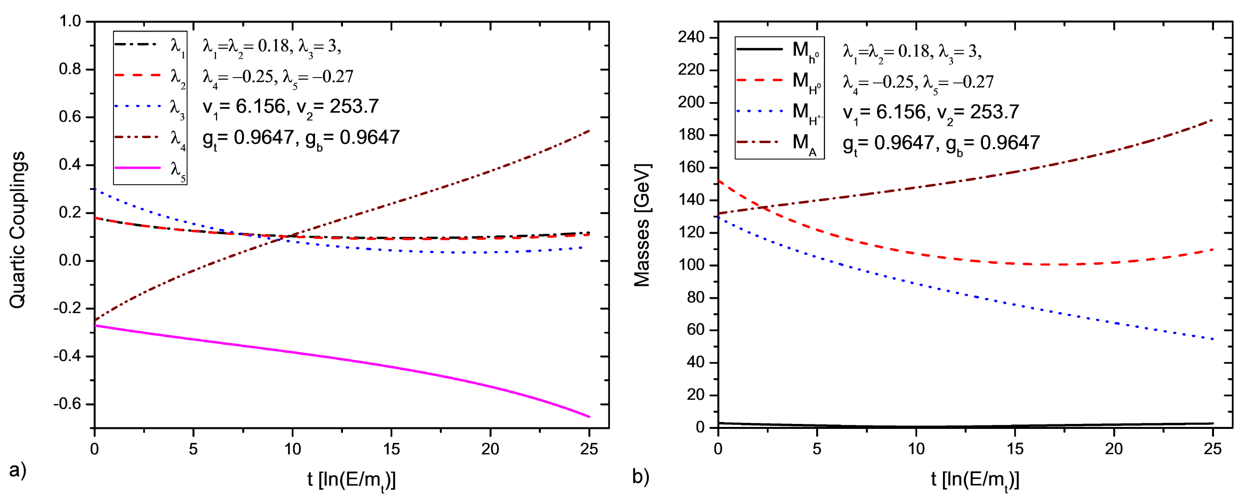

FIGURE 5. The energy dependence of the quartic couplings and the Higgs masses, in the third scenario with $\tan\beta = 41.2$.

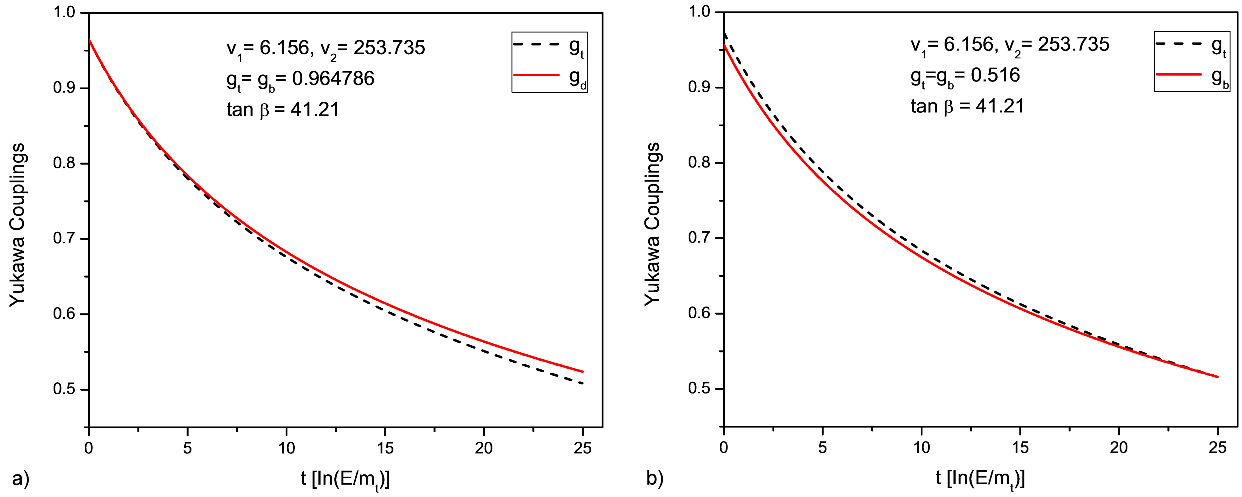

FIGURE 6. The first figure shows the unification of the Yukawa couplings at low energy corresponding to the Fig. 11, and the second figure shows the unification of the Yukawa couplings at high energy corresponding to the Fig. 12.

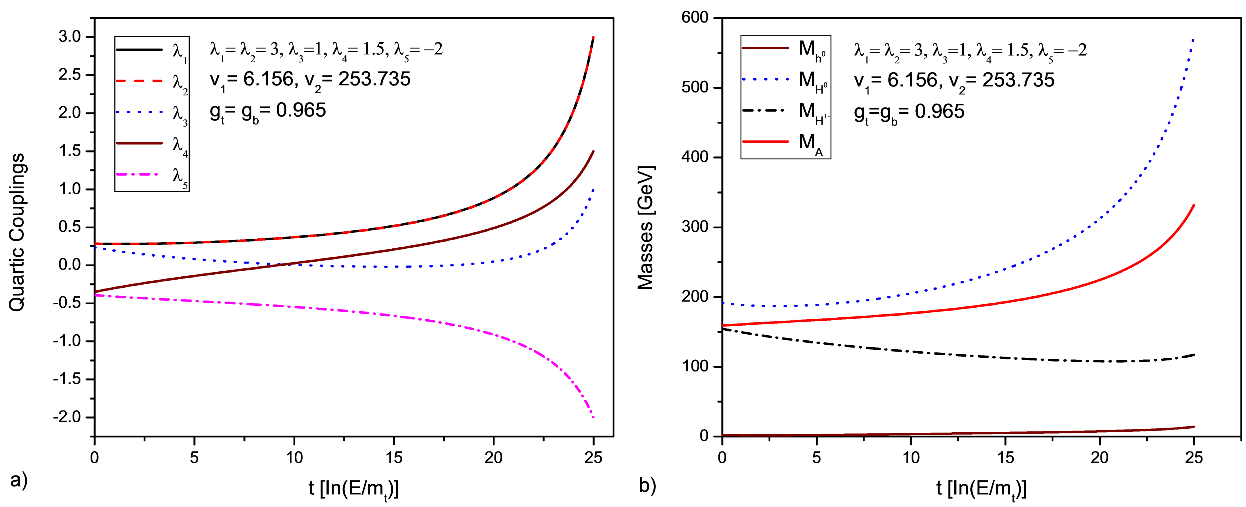

FIGURE 7. The energy dependence of the quartic couplings and the Higgs masses in the $\tan\beta = 41.2$ case with Yukawa couplings $g_t = g_b$ at low energy and $\lambda_1 = \lambda_2$ at high energy.

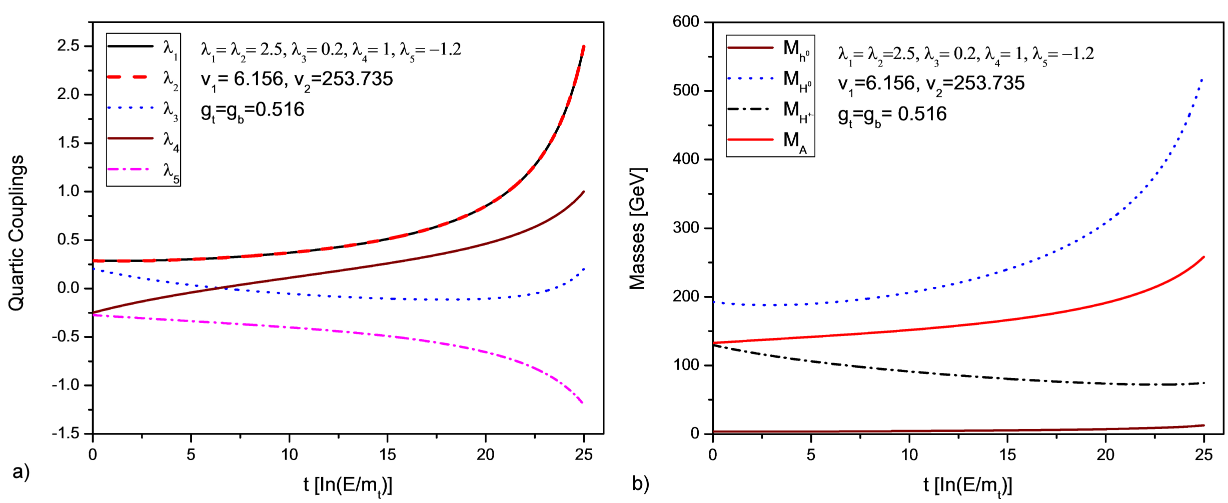

FIGURE 8. The energy dependence of the quartic couplings and the Higgs masses in the $\tan\beta = 41.2$ case with equal Yukawa couplings $g_t = g_b$ and $\lambda_1 = \lambda_2$ at high energy.

evolution of the Yukawa couplings, quartic couplings and the Higgs masses for the case when the Yukawa couplings are unified. In the first plot of the Fig. 6 we assume that they are unified at low energy and in the second plot of the Fig. 6 they are unified at high energy. The evolution of the quartic couplings and Higgs masses are similar in both cases. As a complement, in the Appendix, we present several tables with data associated with figures Figs. 5-8 for the running of the masses and couplings. In Table II, related to Fig. 5, we consider an additional column refered to a 7 TeV energy in which the $M_{H^0} = 127.7$. This value is very close to the one reported in Refs. 4 and 5 as an evidence of the existence of the SM Higgs Particle.

6. Results and conclusions

With the aim to explore the Higgs mass content of the 2HDM extension of the standard model, among the different forms of the Lagrangian describing the same physical reality, we have chosen a specific one, in which the vacuum expectation values of both Higgs fields are real, and for simplicity also preserving the CP symmetry.

We present, the analytical expressions for the masses of the five predicted physical Higgs particles in terms of the λ_i parameters. For completeness, we consider the λ_i's in terms of those masses, with which one can verify convincingly some of the imposed restrictions on them. We have also verified, through the mass formulas, a set of constraints to be satisfied by the scalar parameters that determine the couplings and self-couplings of the Higgs fields.

We have considered the condition on the Higgs potential for $M_{h^0} = 0$ and analyzed using the renormalization group method the validity of the model in three scenarios. We performed a numerical analyses of the flow of the λ_i's and masses as governed by the one-loop RGEs, in case when the Yukawa couplings for the t quark mass is related to v_2 and the b quark mass is related to v_1 and located the energy where the Landau pole emerges in those cases.

As many authors base their calculations on symmetry conditions, such as $\lambda_1 = \lambda_2$ or impose, for simplicity, specific and particular values for $\tan\beta$, in a phenomenological study of special events, it is important to analyze the consequences and limitations of such assumptions and conclusions. We tried at least partially to address this problem. We have considered here symmetries in the λ_i parameters, unification of the Yukawa couplings at low energy E_0 (M_t scale) or high energy E_u (weak-unification scale), hierarchy of the quark masses and determined the energy range of validity of the model which depends on the values of the Higgs masses. The main symmetry considered here is the unification of the Yukawa couplings. It seems this symmetry makes the Higgs sector very stable as can be seen in Fig. 5, preserves the unitarity conditions Ref. 23 and gives at $E = 7$ TeV the Higgs mass $M_{H^0} = 127.65$ GeV. See Table II.

From our analysis, one can observe that lower values of the charged Higgs mass $M_{H^\pm}$ lead to a larger range of validity of the model and the new physics is shifted to higher energies.

The most important result of this paper is the derivation of restrictions for the quartic couplings of the Higgs potential. The bounds on the couplings are obtained from physical conditions that have to be fulfilled by the physically consistent theory and they include the positivity of the squares of the Higgs masses (Eqs. (3-5)). Next we consider a restriction obtained from the assumption on the values the Higgs masses. A very interesting case follows from the condition in Eq. (7), from which it follows that vanishing of the mass the lighter Higgs boson at the tree level is compatible with the phenomenology of the 2HDM. It is remarkable that this result depends only on the quartic part of the Higgs potential. For such a scenario, if there are two Higgs doublets, the recently discovered neutral Higgs boson at LHC [38] would be the heavier one.

In summary, the results in this paper may be a basis for further investigation in relation to the behavior and energy dependent characteristics of the Higgs particles and we believe that the results of this paper shed new light on physics of the Higgs sector.

Appendix: Tables with the initial data for the figures

TABLE I. Energy scale

t	Energy [GeV]
0	1.731×10^{2}
0.5	2.85393×10^{2}
1	4.70535×10^{2}
3	3.47681×10^{3}
3.7	7.00143×10^{3}
6	6.98335×10^{4}
9	1.402×10^{6}
12	2.817×10^{7}
15	5.658×10^{8}
18	1.136×10^{10}
21	2.282×10^{11}
22	6.205×10^{11}
23	1.686×10^{12}
24	4.585×10^{12}
25	1.246×10^{13}

TABLE II. Data for Fig. 5.

	E[GeV]=173.1	E[GeV]=7000	E[GeV]=1.2×10^{13}
$\tan\beta$	41.21	41.21	41.02
v_1	6.16	5.97	5.71
v_2	253.73	245.87	234.47
v	253.81	245.94	234.53
g_t	0.965	0.817	0.508
g_b	0.965	0.820	0.524
λ_1	0.18	0.135	0.123
λ_2	0.18	0.135	0.115
λ_3	0.3	0.19	0.095
λ_4	-0.25	-0.087	0.56
λ_5	-0.27	-0.32	-0.675
m_{h^0}	2.92	1.93	2.8
m_{H^0}	152.26	127.65	112.27
m_A	129.42	110.20	55.84
$m_{H^\pm}$	131.88	138.01	192.64

TABLE III. Data for Fig. 7 and first plot of Fig. 6.

	E[GeV]=173.1	E[GeV]=1.2×10^{13}
$\tan\beta$	41.21	41.02
v_1	6.156	5.715
v_2	253.74	234.465
v	253.81	234.534
g_t	0.9647	0.5082
g_b	0.9647	0.5238
λ_1	0.2849	3
λ_2	0.2852	3
λ_3	0.2347	1
λ_4	-0.3487	-1.5
λ_5	-0.393	-2
m_{h^0}	2.1252	13.9518
m_{H^0}	191.684	574.32
m_A	159.115	331.682
$m_{H^\pm}$	154.569	117.267

TABLE IV. Data for Fig. 8 and second plot of Fig. 6.

	E[GeV] $= 173.1$	E[GeV] $= 1.2 \times 10^{13}$
$\tan\beta$	41.21	41.43
v_1	6.156	5.6869
v_2	253.735	235.641
v	253.81	235.709
g_t	0.9729	0.516
g_b	0.9567	0.516
λ_1	0.2915	2.5
λ_2	0.2876	2.5
λ_3	0.2043	0.2
λ_4	-0.2498	1
λ_5	-0.2735	-1.2
m_{h^0}	3.9232	12.7164
m_{H^0}	192.458	526.909
m_A	132.749	258.207
$m_{H^\pm}$	129.838	74.5378

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1. S. Glashow, *Nucl. Phys.* **22** (1961) 579.
2. S. Weinberg, *Phys. Rev. Lett.* **19**(1967) 1264.
3. A. Salam, *Proceedings of the Nobel Symposium*, edited by N. Svartholm, Vol. **8**, (Almquist and Wiksell. Stockholm, 1968).
4. ATLAS Collaboration, *Phys. Lett B* **716** (2012) 1–29.
5. CMS Collaboration, *Phys. Lett B* **716** (2012) 30–61.
6. J. Beringer *et al*. (Particle Data Group), *Phys. Rev. D* **86** (2012) 010001.
7. S. Kraml *et al*, CERN-CPNSH Workshop, May 2004 - Dec. 2005. Edited by Sabine Kraml, (GENEVA 2006), arXiv:hep-ph/0608079.
8. P. W. Higgs, *Phys. Lett.* **12** (1964) 132 .

9. G. C. Branco, *et al*., *Theory and phenomenology of two-Higgs-doublet models*, Physics Reports (2012), doi: 10.1016/j.physrep. 2012.02.002. arXiv:1106.0034v3 [hep-ph]
10. I.P. Ivanov, *Phys. Lett.* **B632** (2006) 360.
11. I.P. Ivanov, *Phys. Rev D* **75** (2007) 035001.
12. I.P. Ivanov, *Phys. Rev D* **77** (2008) 015017.
13. David López-Val and Joan Solà, *Phys. Rev. D* **81**, (2010) 113005.
14. Joan Solà, David López-Val, arXiv:1107.1305 [hep-ph].
15. Apostolos Pilaftsis, Carlos E.M. Wagner, *Nucl. Phys*. **553** (1999) 3.
16. J.L. Díaz Cruz and A. Rosado, *Rev. Mex. Fis.* **53** (2007) 396.
17. F. Mahmoudi and O. Stål, *Phys. Rev. D* **81** (2010) 035015, arXiv:0907.1791 [hep-ph].
18. F. Mahmoudi, O. Stål, *Phys. Rev. D* **81** 035016. arXiv:0907.1791 [hep-ph].
19. N. Barros *et al*, *Proceedings of Science: Prospects for Charged Higgs Discovery at Colliders*, (16-19 September 2008, Uppsala, Sweden), arXiv:0906.5453v2 [hep-ph] 4 Sep 2009.
20. M. Maniatis, A. von Manteuffel, O. Nachtmann, and F. Nagel, *Eur. Phys. J. C.* **48** (2006) 805. [hep-ph/0605184].
21. M. Rozanska, O. Brein, *International Workshop on Prospects For Charged Higgs Discovery At Colliders* (CHARGED 2010)/20.
22. M. Hashemi, *The European Physical Journal C* **72** (2012) 1994. arXiv:1109.5356 [hep-ph]
23. J. Horejsi and M. Kladiva, *Eur. Phys. J. C* **46** (2006) 81 [hep-ph/0510154].
24. R. Guedes, S. Kanemura, S. Moretti, R. Santos, and K. Yagyu, arxiv:1102.3791 [hep-ph].
25. S. Kanemura,T. Kubota, and E. Takasugi, *Phys. Lett. B* **313** (1993) 155 [hep-ph/9303263].
26. K. Inoue, A. Kakuto and Y. Nakano, *Prog. Theor. Phys*. **63** (1980) 234.
27. J. F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs hunter´s guide* (Addison-Wesley, Reading, MA, 1990).
28. D. Kominis and R. Sekhar Chivukula, *Phys. Lett. B* **304** (1993) 152.
29. S. L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev. D* **2** (1977) 1285.
30. G. Bernardi, M. Carena and T. Junk, J. Beringer *et al*. (Particle Data Group), *Phys. Rev. D* **86** (2012) 010001.
31. The LEP collaborations ALEPH, DELPHI, L3 and OPAL, The LEP Working Group for Higgs Boson Searches, *Eur. Phys. J. C* **47** (2006) 547.
32. A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, *Phys. Lett. B* **115** (1982) 380–384.
33. P. M. Ferreira, L. Lavoura, and J. P. Silva, *Phys. Lett. B* **688** (2010) 341. [arXiv:1001.2561 [hep-ph]].
34. H. E. Haber and R. Hempfling, *Phys. Rev. D* **48** (1993) 4280 [hep-ph/9307201].
35. H. Arason *et al*., *Phys. Rev. D* **46** (1992) 3945.
36. C. R. Das, M. K. Parida, *Eur. Phys. J. C* **20** (2001) 121.
37. G. Cvetic, S. S. Hwang, and C. S. Kim, *Phys. Rev. D* **58** (1998) 116003.
38. ATLAS Collaboration, *Phys. Lett. B* **716** (2012) 1-29; CMS Collaboration, *Phys. Lett. B* **716** (2012) 30-61.