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Trajectory tracking for the chaotic pendulum using PI control law

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This paper presents the application of trajectory tracking using adaptive neural networks to the double chaotic pendulum. The controller structure proposed is composed by a neural identifier and a PI Control Law. Experimental results with the chaotic pendulum showed the usefulness of the proposed approach. To verify the analytical results, an example of a dynamical network is simulated and a theorem is proposed to ensure the tracking of the nonlinear system.

Keywords: Neural networks; trajectory tracking; adaptive control; Lyapunov function stability and PI control.

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1. Introduction

A double pendulum system is formed by one pendulum attached to another, as shown in Fig. 1. This is a physical system that can exhibit chaotic behavior.

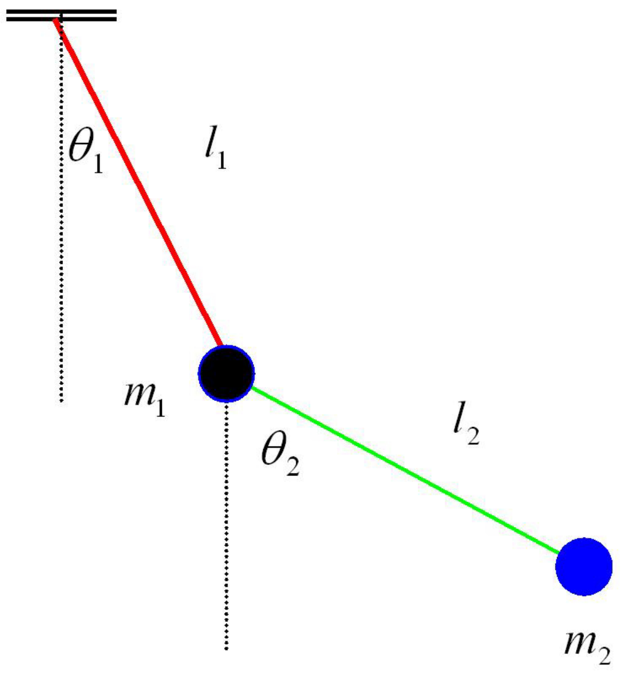

FIGURE 1. Chaotic Dynamical Systems.

We consider a double pendulum immersed in a gravitational field, where the masses m_1 and m_2 are tied to rigid wires with negligible mass and lengths l_1 and l_2, of link $q1$ and link $q2$ respectively.The nonlinear system that describes the dynamic of the chaotic pendulum will be called the Plant. The aim of this work is to force this system to follow a nonlinear reference system called the Reference. We propose a PI Control Law that guarantees the tracking error between the Plant and the Reference approaches to zero when $t \to \infty$. We verify this using a Lyapunov function. The problem of guaranteeing that the tracking error approaches to zero when $t \to \infty$ is the inverse problem of the previous work found in [12-14].

The PI control law was developed for a chaotic pendulum to allow trajectory tracking between the plant and the reference, a Duffing Equation. This is achieved by directly analyzing and describing the behavior of the system in terms of relevant parameters, which are the masses and lengths of the pendulums. For each case we analysed the graphs that show how angles behave as a function of time. Figures presented for a time of 200 seconds show the paths followed by each of the pendulums.

Artificial neural networks, computational models of the brain, are widely used on engineering applications due to their ability to estimate the relation between inputs and outputs from a learning process. Motivated by the seminal paper [1], there exists a continuously increasing interest in applying neural networks to identification and control of nonlinear systems. Most of these applications use feedforward structures [2,3]. Recently, recurrent neural networks are being developed; as an extension of the static neural networks capability to approximate nonlinear functions, recurrent neural networks can approximate nonlinear systems. They allow more efficient modelling of the underlying dynamical systems [4]. Three representative books [5,6] and [7] have reviewed the application of recurrent neural networks for nonlinear systems identification and control. In particular, [5] uses off-line learning, [6] analyzes adaptive identification and control by mean of on-line learning, where stability of the closed-loop system is established based on the Lyapunov function method. In Ref. 6, the trajectory tracking problem is reduced to a linear model following problem, with application to DC electric motors. In Ref. 7, analysis of Recurrent Neural Networks for identification, estimation and control are developed, with applications to chaos control, robotics and chemical processes.

Control methods that are applicable to general nonlinear systems have been intensely developed since the early

1980's. Main approaches include, for example, the use of differential geometry theory [8]. Recently, the passivity approach has generated increasing interest for synthesizing control laws [9]. An important problem for these approaches is how to achieve robust nonlinear control in the presence of unmodelled dynamics and external disturbances. In this direction, there exists the so-called H_∞ nonlinear control approach [10]. One major difficulty with this approach, alongside its possible system structural instability, seems to be the requirement of solving some resulting partial differential equations. In order to alleviate this computational problem, the so-called inverse optimal control technique was recently developed, based on the input-to-state stability concept [11].

On the basis of the inverse optimal control approach, a control law for generating chaos in a recurrent neural network was designed in Ref. 12. In Ref. 13 and 14, this methodology was modified for stabilization and trajectory tracking of an unknown chaotic dynamical system, where the former is used to build an on-line model for the unknown plant and the latter, to ensure that the unknown plant tracks the reference trajectory. In this paper, we further improve the design by adequating it to systems with less inputs than states. The approach is based on the methodology developed in Ref. 13 and 14, in which the control law is optimal with respect to a well-defined Lyapunov function.

Robot manipulators present a practical challenge for control purposes due to the nonlinear and multivariable nature of their dynamical behavior. Motion control in joint space is the most fundamental task in robot control; it has motivated extensive research work in synthesizing different control methods such as fuzzy computed torque control [15], PI+PD fuzzy control [16] and static neural network control [17]. An important problem for developing control algorithms is that most robots models neglect practical aspects such as actuator dynamics, sensor noise, and friction, which, if are not considered in the design, may cause performance deterioration.

2. Modeling of the Plant

The unknown nonlinear plant is given as:

$$\dot{x}_p = F_p(x_p, u) \triangleq f_p(x_p) + g_p(x_p)u \tag{1}$$

Where x_p, $f_p \in R^n$, $u \in R^m$, $g_p \in R^{nxm}$. Both f_p and g_p are unknown, and we propose to model (1) by the neural network state space representation $\dot{x}=A(x)+W^*\Gamma_z(x)+\Omega u$, plus one more term modeling error.

We define the modeling error between the neural network and the plant by:

$$w_{\text{per}} = x - x_p \tag{2}$$

We assume the following hypothesis.

Hypothesis 1. (Objective of Modeling): Modeling error is exponentially stable, that is:

$$\dot{w}_{\text{per}} = -k w_{\text{per}} \tag{3}$$

In this work we consider $k = 1$, and now, from (2) we have $\dot{w}_{\text{per}} = \dot{x} - \dot{x}_p$ where: $\dot{x}_p = \dot{x} + w_{\text{per}}$

The unknown plant can be modeled as:

$$\dot{x}_p = \dot{x} + w_{per} = A(x) + W^*\Gamma_z(x) + w_{per} + \Omega u \tag{4}$$

W^* are the fixed weights but unknown from the neural network. They minimize the modeling error.

3. Trajectory Tracking

Theorem 1 *The unknown nonlinear system (1) modeled by (4), the on-line learning law*

$$tr\left\{ \dot{\widetilde{W}}^T \widetilde{W} \right\} = -e^T \widetilde{W}\sigma(x)$$

and the control law

$$\begin{aligned} u = \Omega^\dagger \bigg[& -\widehat{W}\Gamma(z(x) - z(x_p)) - (A+I)(x - x_p) \\ & + K_p e + K_i \int_0^t e(\tau)d\tau - \Upsilon\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right)e \\ & + f_r(x_r, u_r) - Ax_r - \widehat{W}\Gamma_z(x_r) - x_r + x_p \bigg] \end{aligned}$$

together ensure the trajectory tracking between the Plant and the Nonlinear Reference Signal $\dot{x}_r = f_r(x_r, u_r)$.

Remark 2 $\Omega^\dagger$ *is the pseudo inverse in the sense of Moore–Penrose*

Proof. We proceed now to analyze the modeling error between the unknown plant modeled by (4) and the reference signal defined by:

$$\dot{x}_r = f_r(x_r, u_r), u_r \in R^n \tag{5}$$

For this purpose we define the modeling error between the plant and the reference signal by:

$$e = x_p - x_r \tag{6}$$

whose derivative with respect to time is

$$\dot{e} = \dot{x}_p - \dot{x}_r = A(x) + W^*\Gamma_z(x) + w_{\text{per}} + \Omega u - f_r(x_r, u_r) \tag{7}$$

Adding and subtracting to the right hand side of (7) the terms $\widehat{W}\Gamma_z(x_r)$, $\alpha_r(t, \widehat{W})$, Ae and taking into account that $w_{\text{per}} = x - x_p$, we have

$$\dot{e} = A(x) + W^*\Gamma_z(x) + x - x_p + \Omega u - f_r(x_r, u_r) \tag{7}$$

$$+ \widehat{W}\Gamma_z(x_r) - \widehat{W}\Gamma_z(x_r) + \Omega\alpha_r(t, \widehat{W}) \tag{8}$$

$$- \Omega\alpha_r(t, \widehat{W}) + Ae - Ae$$

$$\begin{aligned} \dot{e} = & Ae + W^*\Gamma_z(x) + \Omega u - f_r(x_r, u_r) \\ & + \widehat{W}\Gamma_z(x_r) + \Omega\alpha_r(t, \widehat{W}) - \widehat{W}\Gamma_z(x_r) \\ & - \Omega\alpha_r(t, \widehat{W}) - e - x_r - Ae + x + A(x) \end{aligned} \tag{8}$$

In this part, we consider the next supposition:

The neural network will follow the reference signal, even with the presence of disturbances, if:

$$Ax_r + \widehat{W}\Gamma_z(x_r) + x_r - x_p + \Omega\alpha_r(t,\widehat{W}) = f_r(x_r,u_r).$$

Then

$$\Omega\alpha_r(t,\widehat{W}) = f_r(x_r,u_r) - Ax_r - \widehat{W}\Gamma_z(x_r) - x_r + x_p \quad (9)$$

and we get

$$\dot{e} = Ae + W^*\Gamma_z(x) - \widehat{W}\Gamma_z(x_r) - Ae + (A+I)(x - x_r) + \Omega(u - \alpha_r(t,\widehat{W})) \quad (10)$$

where $\widehat{W}$ is the estimate of W^*.

Now, adding and subtracting in (10) the term $\widehat{W}\ \Gamma_z(x)$ we have:

$$\dot{e} = Ae + (W^* - \widehat{W})\Gamma_z(x) + \widehat{W}\Gamma(z(x) - z(x_r)) + (A+I)(x - x_r) - Ae + \Omega(u - \alpha_r(t,\widehat{W})) \quad (11)$$

We define,

$$\widetilde{W} = W^* - \widehat{W} \text{ and } \widetilde{u} = u - \alpha_r(t,\widehat{W}) \quad (12)$$

and replacing (12) in (11), we obtain

$$\dot{e} = Ae + \widetilde{W}\Gamma_z(x) + \widehat{W}\Gamma(z(x) - z(x_r)) + (A+I)(x - x_r) - Ae + \Omega\widetilde{u}$$

$$\dot{e} = Ae + \widetilde{W}\Gamma_z(x) + \widehat{W}\Gamma(z(x) - z(x_p) + z(x_p) - z(x_r)) + (A+I)(x - x_p + x_p - x_r) - Ae + \Omega\widetilde{u} \quad (13)$$

Now, we set:

$$\widetilde{u} = u_1 + u_2 \quad (14)$$

So, we define:

$$\Omega u_1 = -\widehat{W}\Gamma(z(x) - z(x_p)) - (A+I)(x - x_p) \quad (15)$$

and (13) it is reduced to:

$$\dot{e} = Ae + \widetilde{W}\Gamma_z(x) + \widehat{W}\Gamma(z(x_p) - z(x_r)) + (A+I)(x_p - x_r) - Ae + \Omega u_2$$

Considering that $e = x_p - x_r$, the last equation can be written as:

$$\dot{e} = (A+I)e + \widetilde{W}\Gamma_z(x) + \widehat{W}\Gamma(z(e + x_r) - z(x_r)) + \Omega u_2$$

$$\dot{e} = (A+I)e + \widetilde{W}\sigma(x) + \widehat{W}(\sigma(e + x_r) - \sigma(x_r)) + \Omega u_2$$

If $\phi(e) = \sigma(e + x_r) - \sigma(x_r)$, we get

$$\dot{e} = (A+I)e + \widetilde{W}\sigma(x) + \widehat{W}\phi(e) + \Omega u_2 \quad (16)$$

Now, the problem is to find the control law Ωu_2 that stabilizes the system (16). We will obtain the control law by using the Lyapunov methodology. In the next section we find the control law and continue the proof of Theorem (1).

4 Stability of the Tracking Error

Once (16) is obtained, we consider its stabilization in feedforward. We note $(e,\widehat{W})$=0, is an asymptotically stable equilibrium point of the undisturbed autonomous system($A = -\lambda I$ and $\lambda > 0$). For its stability, we propose the next PI control law:

$$\Omega u_2 = K_p e + K_i \int_0^t e(\tau)d\tau - \Upsilon\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right) e \quad (17)$$

The parameters K_p and K_i will be determined later, and $L_{\phi z}^2$ is the Lipschitz constant of ϕ_z, with $\Upsilon > 0$, [20].

We will show the feedback system is asymptotically stable. Replacing (17) in (16), then

$$\dot{e} = (A+I)e + \widetilde{W}\sigma(x) + \widehat{W}\phi(e) + K_p e + K_i \int_0^t e(\tau)d\tau - \Upsilon\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right) e \quad (18)$$

$$\dot{e} = -(\lambda - 1 - K_p)e + \widetilde{W}\sigma(x) + \widehat{W}\phi(e) + K_i \int_0^t e(\tau)d\tau - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right) e \quad (19)$$

and if

$$w = K_i \int_0^t e(\tau)d\tau,$$

then $\dot{w} = K_i\ e(\tau)d\tau$, we can rewrite (19) as:

$$\dot{e} = -(\lambda - 1 - K_p)e + \widetilde{W}\sigma(x) + \widehat{W}\phi(e) + w - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\widehat{W}\right\|^2 L_\phi^2\right) e \quad (20)$$

We will show the new state $(e,w)^T$ is asymptotically stable and the equilibrium point is $(e,w)^T = (0,0)^T$, when $\widetilde{W}\sigma(x_r) = 0$,which is taken as an external disturbance.

Let V be the candidate Lyapunov function [24,25] given by:

$$V = \frac{1}{2}(e^T,w^T)(e,w)^T + \frac{1}{2}tr\left\{\widetilde{W}^T\widetilde{W}\right\} \quad (21)$$

The time derivative of (21) along the trajectories of (20) is:

$$\dot{V} = (e^T,w^T)(\dot{e},\dot{w})^T + tr\left\{\dot{\widetilde{W}}^T\widetilde{W}\right\}$$

$$= e^T\dot{e} + w^T\dot{w} + tr\left\{\dot{\widetilde{W}}^T\widetilde{W}\right\} \quad (22)$$

$$\dot{V} = e^T \Bigg(- (\lambda - 1 - K_p)e + \widetilde{W}\sigma(x) + \widehat{W}\phi(e) + w - \gamma \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) \Bigg) e + w^T K_i e + tr \left\{ \dot{\tilde{W}}^T \tilde{W} \right\} \tag{23}$$

In this part, we select the next learning law of the neural network weights as in Ref. 6:

$$tr \left\{ \dot{\tilde{W}}^T \tilde{W} \right\} = -e^T \widetilde{W}\sigma(x) \tag{24}$$

Then (23) is reduced to

$$\dot{V} = -(\lambda - 1 - K_p)e^T e + e^T \widehat{W}\phi(e) + (1 + K_i)e^T w - \gamma \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e^T e \tag{25}$$

We apply the next inequality to the second term in the right hand side of (25)

$$x^T y \leq \frac{1}{2} x^T x + \frac{1}{2} y^T y \tag{26}$$

to get:

$$\dot{V} \leq - (\lambda - 1 - K_p)e^T e + \left(\frac{e^T e}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e^T e + (1 + K_i)e^T w - \gamma \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e^T e \tag{27}$$

The parameters in (27) are reduced to:

$$\dot{V} \leq -(\lambda - 1 - K_p)e^T e - (\Upsilon - 1) \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e^T e \tag{28}$$

In this part, if we choose $\lambda - 1 - K_p > 0$, and $\Upsilon - 1 > 0$, then $\dot{V} < 0, \forall\, e, w, \widehat{W} \neq 0$, the error tracking is asymptotically stable and it converges to zero for every $e \neq 0$. This means that the plant follows the reference asymptotically. Finally, the control law, which affects the plant and the neural network, is given by:

$$u = \Omega^\dagger [-\widehat{W}\Gamma(z(x) - z(x_p)) - (A + I)(x - x_p) + K_p e + K_i \int_0^t e(\tau) d\tau - \Upsilon \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e + f_r(x_r, u_r) - A x_r - \widehat{W}\Gamma_z(x_r) - x_r + x_p] \tag{29}$$

This control law gives asymptotic stability of error dynamics and thus ensures the tracking between the plant and the reference signal. ■

The results obtained can be confirmed by simulations and we show this in the next section.

From (28) we have

$$\dot{V} \leq -(\lambda - 1 - K_p)e^T e - (\Upsilon - 1) \times \left(\frac{1}{2} + \frac{1}{2} \left\| \widehat{W} \right\|^2 L_\phi^2 \right) e^T e < 0, \ \forall\ e \neq 0, \ \forall\ \widehat{W}$$

where V is decreasing and bounded from below by $V(0)$, and since

$$V = \frac{1}{2}(e^T, w^T)(e, w)^T + \frac{1}{2} tr \left\{ \widetilde{W}^T \tilde{W} \right\}$$

then we conclude that $e, \widetilde{W} \in L_1$; this means that the weights remain bounded.

4. Simulations

The neural network is modeled by the differential equation: $\dot{x} = Ax + W\sigma(x) + \Omega u$, with $A = -\lambda I$, $I \in R^{4x4}$, and $\lambda = 20$, W is estimated by using the learning law given in (24), $\sigma(x) = (\tanh(x_1), \tanh(x_2), ..., \tanh(x_n))^T$,

$$\Omega = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^T$$

and u is calculated by using (29).

The plant is stated in Refs. 20, 21, and it is given by:

$$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + (m_1 + m_2) g \sin\theta_1 = 0$$

$$m_2 l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) + m_2 l_2 \ddot{\theta}_2 - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + m_2 g \sin\theta_2 = 0$$

In Fig. 2 the trajectory of the Plane Phase for the Plant (Chaotic Pendulum) is shown in blue. The trajectory of the Plane Phase for the Reference (Duffing Equation) is shown in black.

The time evolution for the angles and applied torque are shown in Figs. 3-8. As can be seen, trajectory tracking is successfully obtained. We try to force this manipulator to track a reference signal given by the Duffing equation:

$$\ddot{x} - x + x^3 = 0.114 \cos(1.1t) \quad \text{with} \quad x(0) = 1, \quad \dot{x}(0) = 0.114 \tag{30}$$

In Figs. 3-6 the states trajectories are shown in blue for the Chaotic Pendulum and in black for the Duffing Equation dynamics.

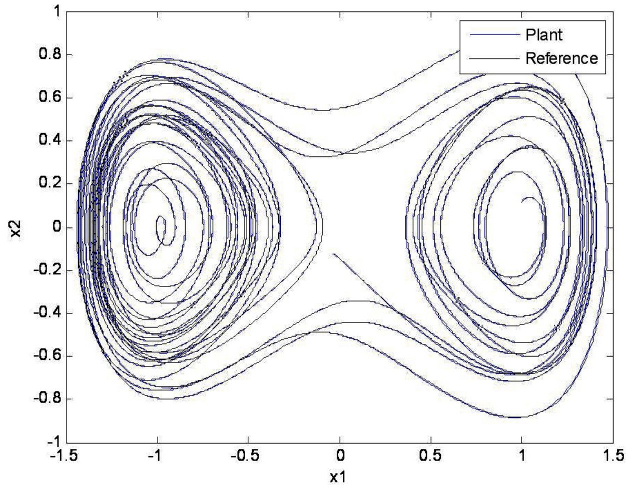

FIGURE 2. Plane phase portrait of the Plant and the Reference signal for the trajectory of Duffing equation.

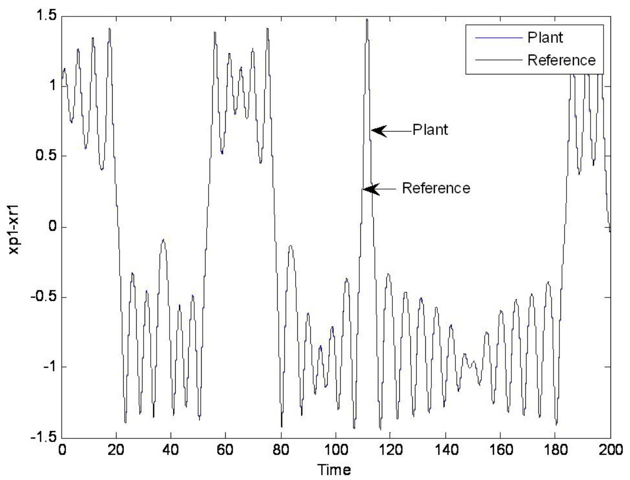

FIGURE 3. Time evolution for the angular position $q1$ (rad) of link 1.

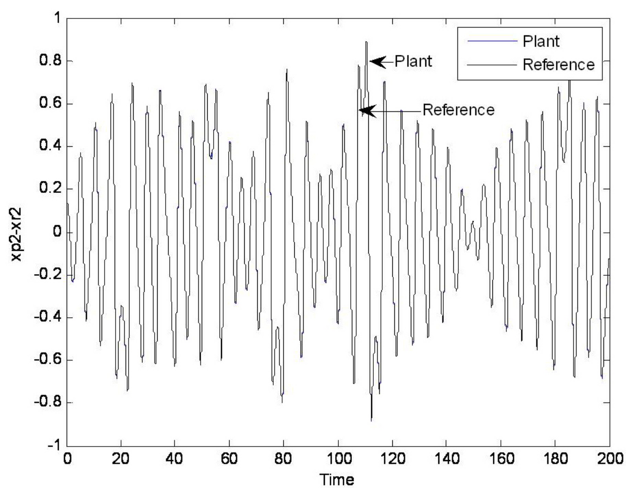

FIGURE 4. Time evolution for the angular position $q2$ (rad) of link 2.

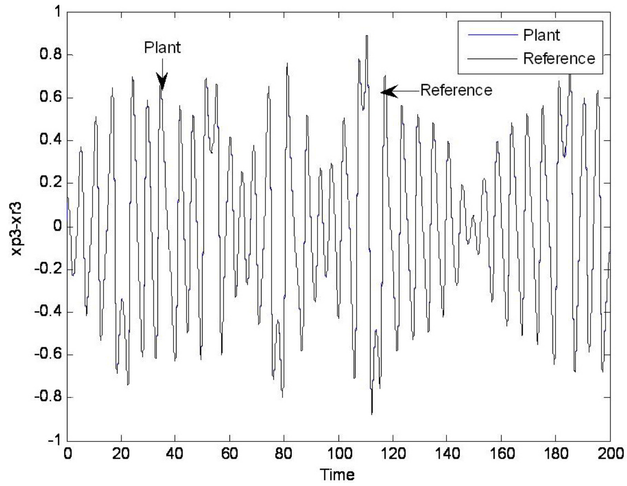

FIGURE 5. Time evolution for the angular velocity of link 1 $q1$ (rad/seg).

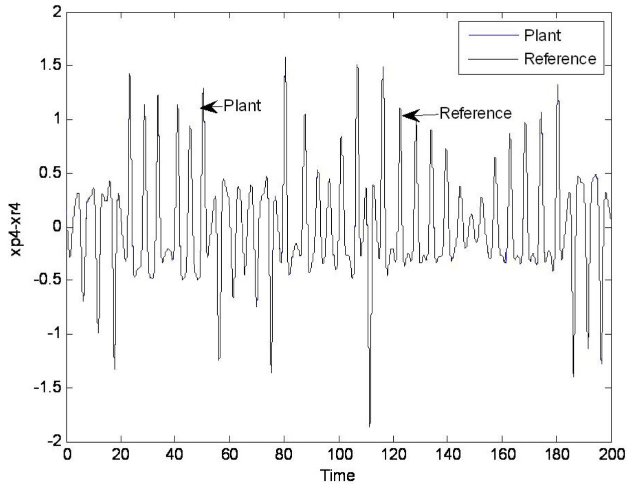

FIGURE 6. Time evolution for the angular velocity of link 2 $q2$ (rad/seg).

Note: In the previous figures we showed the Chaotic Pendulum trajectories as well as the reference signal they should follow.

In Figs. 7-8 we show the torque applied to the links in the caothic pendulum.

We can see that the Recurrent Neural Controller ensures rapid convergence of the system outputs to the reference trajectory. The controller is robust [22] in presence of disturbances applied to the system. Another important issue of this approach related to other neural controllers, is that most neural controllers are based on indirect control, first the neural network identifies the unknown system and when the identification error is small enough, the control is applied. In our approach, direct control is considered, the learning laws for the neural networks depend explicitly of the tracking error instead of the identification error. This approach results in faster response of the system.

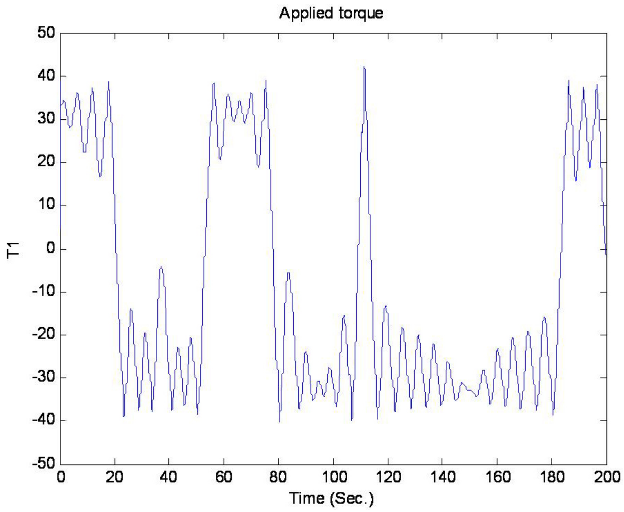

FIGURE 7. Torque (Nm) applied to link $q1$.

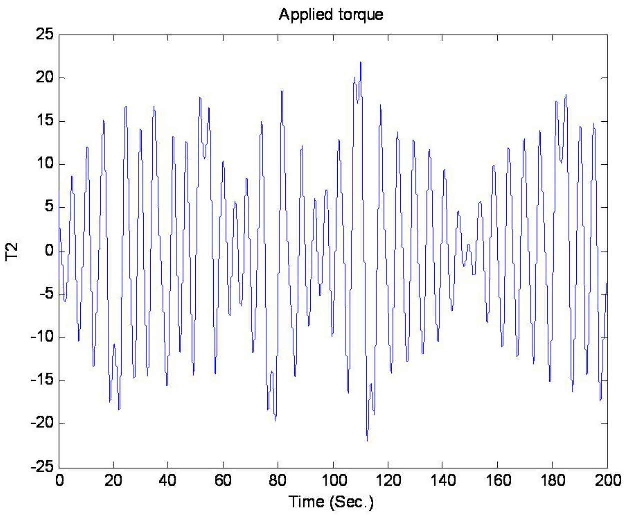

FIGURE 8. Torque (Nm) applied to link $q2$.

5. Conclusions

We have extended the adaptive recurrent neural control previously developed in Refs. 13, 14 and 18 for trajectory tracking control problem in order to consider less inputs than states. Stability of the tracking error is analized via Lyapunov control functions and the control law is obtained based on the PI approach. A Chaotic pendulum model with friction terms and unknown external disturbances is used to verify the design for trajectory tracking, with satisfactory performance. Research along this line will continue to implement the control algorithm in real time and to further test it in a laboratory environment (see [19,23]).

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