RESEARCH Revista Mexicana de Física 60 (2014) 145–148 MARCH-APRIL 2014

The impact of time delay in the connectivity distribution of complex networks generated using the Barabási-Albert model

J. Acosta-Elías[a], P. D. Arjona-Villicaña[a], and L. Moldes-Navarro[b]
[a] *Universidad Autónoma de San Luis Potosí,*
Salvador Nava s/n, Zona Universitaria, CP 78290, San Luis Potosí, SLP, México,
e-mail: jacosta@uaslp.mx
[b] *Universitat Politècnica de Catalunya,*
Campus Nord UPC, Jordi Girona 1-3, 08034 Barcelona Spain.

Received 7 February 2012; accepted 22 January 2014

In the Barabási-Albert growth model for complex networks new nodes added to the network, obtain instant information from the entire network and employ preferential connectivity to select a node to establish a connection. In practice, information takes time to propagate from a sender to a receiver. We modify the Barabási-Albert model to include the time information takes to propagate between nodes. In the modified model a time delay is associated to the transmission of information and each new node must wait for a period of time to receive the network connectivity information. By adjusting this waiting time, different functional forms of the connectivity distribution are obtained. These connectivity distributions form a spectrum of functional forms which lie between two limiting cases: a power law distribution for large waiting times and an exponential distribution for short waiting times.

Keywords: Complex networks.

PACS: 89.75.-k; 64.60.aq

1. Introduction

Complex systems is an interdisciplinary field studying systems composed of many interacting parts and properties which cannot always be explained by simple random processes [1]. Examples of such systems include the Internet, condensed matter systems, ecosystems, finance markets, the brain, the immune system, granular materials, road traffic, insect colonies, bird flocking and the structure of human societies [1].

The interactions and properties of complex systems are often represented using a complex network [2]. These networks have become an interesting topic of research since the late 90's when it was discovered that there is a large number of complex networks with common topological properties [3, 4], despite having different origins and sizes or being at different stages of development. Among these topological properties, one of the most studied ones is the power law distribution of the vertex connectivity in these networks [5–10]. A power law distribution is represented by Eq. 1:

$$p(k) = ck^{-\gamma}, \tag{1}$$

where $p(k)$ is the probability that a node in the network has connectivity k, and γ is an exponent which represents the slope of the function in a log-log plot.

Barabási and Albert (BA) proposed a complex network growth model [11] with two main properties: node aggregation and preferential attachment. Aggregation means that new nodes connect to existing nodes in the network through m new links. Preferential attachment refers to the probability that an existing node will receive a new connection, which is directly proportional to the amount of links that this node already has and is given by Eq. 2:

$$\prod(k_i) = \frac{k_i}{\sum_j k_j}, \tag{2}$$

where $\prod(k_i)$ is the probability that existing node i receives a new connection and k_i is the connectivity of node i.

The exact connectivity distribution of the BA model may be obtained using various methods. This paper uses the Krapivsky solution [12], which is defined by Eq. 3:

$$p(k) = \frac{4n}{k(k+1)(k+2)}, \tag{3}$$

where n is the number of nodes in the network. The connectivity distribution of complex networks generated by the BA model has an exponent of $\gamma = 3$ [13]. However, in real systems gamma varies from 1.05 to 3.4 [3]. In order to emulate such systems, other processes have been incorporated into the BA model. Examples of such models are: models of systems that perform rewiring of their links at the same time as they grow [13]; models of systems in which nodes and links are created at different rates [14]; models where the probability $\prod(k_i)$ that existing node i receives a new connection is not only proportional to its degree k_i, but also to its age [15].

2. Time delay

In real networks, information travels from a source node to a destination node in a finite amount of time. However, the BA network growth model does not consider this time delay. Therefore, for systems in which this time delay is important, the BA model will not be appropriate.

In this paper we report the effect that delay has in propagating the information needed for the growth of a BA complex network. We argue that it is necessary to consider the time delays in the propagation of information in order to have an appropriate growth model of complex networks.

The BA model assumes that the delay to propagate information through the network is zero, that is: a new node n_i that needs to select an existing node n_x to which it will attach, chooses from all nodes present in the network. In other words, the new node n_i has a complete and updated view of the entire network at time t_i. This means that every new node that attaches itself to the network obtains information instantly about its entire topology.

Let us now consider what happens when the delay is not zero: each node that becomes attached to the network has to wait for a time t_w to receive connectivity information from the nodes in the network. If this time is short, the information will be obtained only from a subset of nodes, that is, each new node will have only a partial view of the network (see Fig. 1). However, if t_w is sufficiently long, information from all the nodes in the network could be obtained. Notice that in this case, it is also possible that information is old and inaccurate.

For example, consider a network which grows at a rate of one node per millisecond (ms), has a delay of 10 ms to transmit information between nodes and a diameter of 20 hops. In communication's networks a hop is a link between two nodes and corresponds, in graph theory, to an edge between two vertices. Then the information from the furthest nodes will take 400 ms to arrive (200 ms for the request and 200 ms for the answer). If a new node n_i waits 200 ms for the connectivity information it needs to decide to which node it should connect, it will only receive information from the nodes located 10 or less hops away from itself (10 hops for the request and 10 hops for the answer). It can be seen from this example that the impact of delay in the connectivity distribution depends on the diameter of the network in consideration.

Another issue that needs to be considered is that during this 200 ms period, the network underwent changes in its topological structure because of the attachment of 200 new nodes. This means node n_i will select an attachment node based on outdated information.

As a real world example consider the network formed by citations to scientific papers. In this example, articles correspond to the vertices or nodes in the network, and citations to the edges between the nodes. The growth of this network is carried out in the following way: A scientist, before writing a paper, tries to read the most recent papers on a subject. He cannot read all the papers from this subject. Each paper he reads has citations to other papers, so he selects some of the cited papers and reads them. This process ends when the researcher considers he has sufficient knowledge about the topic. If the time the scientist employs to read citations, t_w, is short, the overall view obtained will be incomplete. In contrast, if time t_w is sufficiently long, the view obtained should be more complete.

It is possible that at the moment of writing an article, somebody else might submit a paper on the same topic. The authors of the first text will not be able to read the submitted article and include it in their research or citations. Thus, authors necessarily cite older papers and not the newest ones.

This example shows how the time delays associated to information propagation through the network produce two important effects: The view obtained may be incomplete because there is not enough time to explore the complete network, and the view obtained is necessarily old outdated or in the past due to recent changes in the network's topology. In order to design better models which could approximate to real systems it is necessary to consider this behavior.

The main objective of this paper is to study the impact of delay in the connectivity distribution of networks generated with the BA model.

3. BA Growth Model with Delay

If the time delay that information needs to propagate is introduced to the BA model, each new node that becomes attached will need t_w time to obtain information about the current state of the network. The new node will use this information and preferential attachment to select an existing node to which it will attach.

The procedure previously described implies that, before connecting, new nodes do not have any knowledge about the state of the network. Therefore, the first step for a new node is to obtain information about the network connectivity, and the second step is to employ this information to choose the node to which it will connect.

Figure 1 shows a network in which the delay of every link is 10 ms. Dashed-line arrows represent data connectivity requests and solid-line arrows represent their answers. For example, if node n_i waits 50 ms, it will receive information from nodes n_x, n_2 and n_3. Information will not be received on time from nodes n_4 and n_5 because the data request takes 30 ms to propagate from n_i to n_4 or to n_5, and the answer takes another 30 ms to return. Note that in real networks these delays are variable, but for the sake of simplicity they are considered fixed in this example.

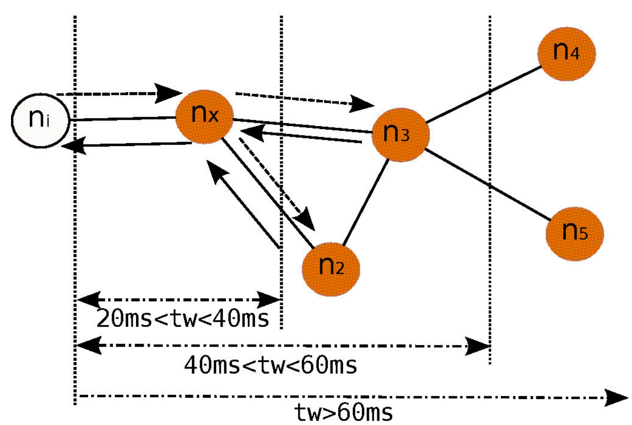

FIGURE 1. Impact of waiting time in the amount of topological information received by a new node (n_i).

The process described below may be used to simulate the growth of complex networks with delay:

1. A new node n_i randomly selects a node n_x which is already present in the network.

2. n_i sends a request for connectivity information to n_x and starts a count down timer, t_w.

3. n_x returns its answer to n_i.

4. n_x forwards n_i's request to the other nodes in the network.

5. Each answer that n_x receives comes from other nodes present in the network and are forwarded to n_i.

6. Once t_w finishes, n_i selects a node n_y. Here, n_y belongs to a subset of nodes from which n_i received information while timer t_w was running. Therefore, t_w determines the number of nodes from which n_i received information.

The process described above will be repeated from Step 1 to Step 6 for $i = 3, 4, 5...n - 1$ where n is the final size of the network.

4. Numerical Simulations

Experiments were carried out using the ns-2 network simulator [16]. Ns-2 is a discrete events simulator able to consider how packages (information) propagate in a network and also the effects of delay. In these experiments networks were grown up to a maximum size of 1,000 nodes because ns-2 requires a large amount of memory and cpu cycles when simulations are packet-intensive, as it is in our case.

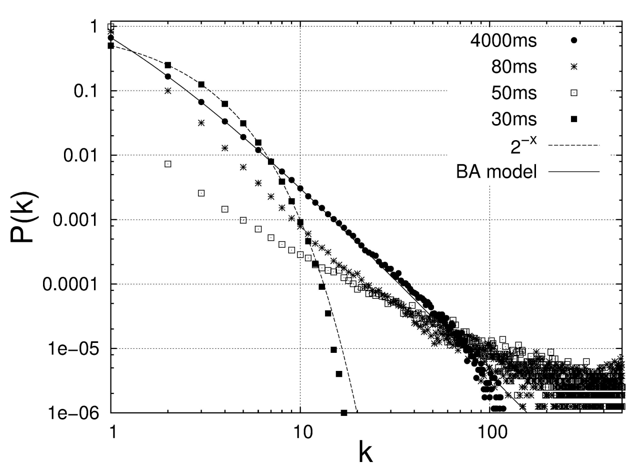

FIGURE 2. The connectivity distribution of a network generated by ns-2 using the BA model. All links have a 10 ms delay and 1 Mbps bandwidth. The growth speed take values from a random range from 1 ms to 4000 ms. When $t_w = 4000$ ms the connectivity distribution matches the BA model distribution (Eq. 3). In contrast, if $t_w = 30$ ms the connectivity distribution decays exponentially.

The experiments carried out are described in the following subsections and their results are displayed in Fig. 2. For all these experiments the node-birth rate, v_b, is random and uniformly varied between 0.001 seconds and 4 seconds and t_w is the time that new nodes will wait for topological information before selecting a node to attach to. Each experiment has a different value for t_w. The links that joined the nodes were undirected and had a 1 Mbps data rate with a 10 ms delay.

4.1. Experiment 1

In this experiment, t_w was set to 4 seconds. Given the links' delay properties and the final network diameter, t_w is long enough for each new node to receive connectivity information from all the nodes in the network before selecting a node. As a result, the connectivity distribution obtained in this experiment is similar to the distribution produced by the BA model, as can be seen by the solid line in Fig. 2. It seems that the connectivity distribution is not affected by the variable node-birth rate.

4.2. Experiment 2

For this experiment t_w was set to 80 ms.

As the network grows its diameter increases. When the diameter is still less than 40 ms ($t_w/2$) new nodes receive information from the entire network, meaning that all nodes have non-zero probability of being chosen. However, when the diameter is greater than 40 ms new nodes cannot receive information from the whole network. Instead, they only receive information from nodes whose distance is less than 40 ms. Additionally, a fraction of this information is 4 0ms old and the topology of the network may have changed during this time. In other words, a fraction of the information given to a new node can be obsolete at the time of selecting an existing node to connect.

From Fig. 2 one can see that the connectivity distribution It can be seen in Fig. 2 that the distribution connectivity for this experiment follows a slightly concave line with respect to the abscissa axis. This graph shows that the percentage of nodes with one link has increased with respect to the BA model.

4.3. Experiment 3

This experiment is similar to the one in Sec. 4.2., but now t_w has been set to 50 ms and, therefore, the information obtained by new nodes has to come from a smaller subset of nodes than the one in Experiment 2. This subset includes all nodes that can be reached in less than 25 ms (50 ms/2). Figure 2 shows that the connectivity distribution for this experiment is a concave line which is more pronounced than that for the previous experiment.

4.4. Experiment 4

In this experiment t_w was set to 30 ms. Since information requests from the next node take at least 20 ms to arrive (10 ms for the request and 10 ms for the answer), it is not possible to receive information from nodes that are two or more hops away from the requesting node: it would take at least 20 ms for the request to arrive and a further 20 ms for the reply to come back. Therefore, new nodes in this experiment can only choose to connect to their corresponding attachment node, which was originally selected at random. Thus, the model effectively behaves like a random growth model.

When a network grows by adding nodes randomly, as it happens in this case, the connectivity distribution decays as an exponential [17], as it is shown in Fig. 2.

4.5. Discussion

From Fig. 2, it can be seen that when t_w is long enough, new nodes will receive information from all the network and, therefore, all nodes in the network will have non-zero probability of receiving a new link causing the distribution to decay following the same connectivity distribution at the BA model (see Eq. 3).

In contrast, when t_w is not long enough, n_i will have a partial view of the network topology which includes a PV subset of nodes. Figure 2 shows that in this case, the number of nodes with just one link increases and nodes with two or more links diminishes. t_w values of 50 ms and 80 ms have been also included to demonstrate variations in this behavior. Our hypothesis is that this happens because most nodes not in PV ($\overline{PV}$) have just one link and this causes connectivity one nodes to increase and the other connectivities to decrease with respect to the power law distribution. This behavior reverses for large connectivities as can be seen in Fig. 2 when $k > 100$.

When t_w is less than the required time to obtain topological information from other nodes in the system, new nodes will only receive information from the immediate node (see Fig. 1) and will, in fact, attach randomly. This causes the connectivity distribution to follow an exponential curve. Figure 2 shows this case for $t_w = 30$ ms.

5. Conclusions

In this paper, the impact that different values of time delays have in the connectivity distribution of a modified version of the Barabási-Albert network growth model is reported. When the delay that information needs to propagate is considered in the BA model, each node is added to the network must wait a time, t_w, to receive information about the network's topology. A set of connectivity distributions were obtained for different values of t_w. At the extreme case when t_w is long enough to receive the full topological information from the network, the connectivity distribution follows the BA model; but if t_w is too short and it does not allow to receive any topological information, the distribution is exponential.

There are many factors present in real networks which could affect the growth and topology of the final system, and in this paper we have only considered one: delay. Therefore, we would like to continue this study by including some of the effects that delays could have on properties of complex networks and their processes, for example: rewiring, stale nodes, etc.

6. Acknowledgments

The authors would like to thank the Universidad Autónoma de San Luis Potosí, the Universidad Politécnica de Catalunya and Mexico's CONACYT for its support.

1. M. E. J. Newman, *Am. J. Phys*. **79** (2011) 800.
2. A.-L. Barabási, *Science* **325** (2009) 412.
3. Reká Albert and Albert-Lazlo Barabási, *Rev. Mod. Phys*. **74** (2002) 47.
4. S. N. Dorogovtsev and A. V. Goltsev, *Rev. Mod. Phys* **80** (2008) 1275.
5. H. Jeong, B. Tombor, R. Albert, Z. N. Oltvai, and A.-L. Barabási, *Nature* **407** (2000) 651.
6. Barabási, R. Albert, H. Jeong, and G. Bianconi, *Science* **287** (2000) 2115.
7. A.L. Barabási, H. Jeong, Z. Néda, E. Ravasz, A. Schubert, and T. Vicsek, *Physica A* **311** (2002) 590.
8. M. Faloutsos, P. Faloutsos, and C. Faloutsos, *in proceeding of ACM SIGCOMM* (Boston Sep 1999) 251.
9. B. A. Huberman and L. A. Adamic, *Nature* **410** (1999) 131.
10. V. M. Eguiluz, D. R. Chialvo, G. A. Cecchi, M. Baliki, and A. V. Apkarian, *Phys. Rev. Lett.* **94** (2005) 018102.
11. A.-L. Barabási and R. Albert, *Science* **286** (1999) 509.
12. P. L. Krapivsky, S. Redner, and F. Leyvraz, *Phys. Rev. Lett.* **85** (2000) 4629.
13. Reká Albert and A.-L. Barabási, *Phys. Rev. Lett.* **85** (2000) 5234.
14. S. N. Dorogovtsev and J. F. F. Mendes, *Phys. Rev. E* **63** (2001) 025101.
15. S. N. Dorogovtsev and J. F. F. Mendes, *Phys. Rev. E* **62** (2000) 1842.
16. Teerawat Issariyakul and Ekram Hossain, *Introduction to Network Simulator NS2*(Springuer, New York, 2009) Vol. 1.
17. S. N. Dorogovtsev and J. F. F. Mendes, *Evolution of Networks: From Biological Nets to the Internet and WWW* (Oxford University Press, Oxford, 16 January 2003.) pp. 112–114.