Perturbation method applied to a basic diode circuit

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Because of the exponential characteristic of silicon diodes, exact solutions cannot be established when operating point and transient analysis are computed. To overcome that problem, the present work proposes a perturbation method which allows obtaining approximated analytic expressions of diode-based circuits. Simulation results show that numerical solutions obtained by using the proposed method are similar to those reported in literature, with the advantage of not requiring a user-selected arbitrary expansion point. Additionally, the method does not use the Lambert function W, reducing the proposed solution complexity, which makes it suitable for engineering applications.

Keywords: Circuit analysis; nonlinear circuits perturbation method.

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1. Introduction

In general terms, to find exact solutions of operating point and transient analysis of circuits containing diodes is not possible, mainly because the exponential characteristic of diodes. However, it is possible to obtain expressions that allow modeling diode behavior by using approximate methods.

In Ref. 1, an approximate solution for transient response current in a serial circuit composed by an independent voltage source, an inductor, a resistance and a diode is proposed. Such solution have a good precision, but it requires a previous knowledge of the approximate steady state of the circuit in order to select an arbitrary expansion point. In Ref. 2, an explicit analytic expression for the DC current of a basic circuit containing a voltage source, a resistor, and a diode is achieved. Nevertheless, such expression is found in terms of Lambert W function, which increases the complexity making harder its application in engineering. In the same manner, in Ref. 1 two high-precision approximate analytical solutions for DC current in the diode circuit are proposed. What is more, to compute the first solution also requires to know a roughly approximate value for the expected DC current to select the expansion point, while the second solution requires the use of the Lambert W function mentioned before.

In this work, a perturbation method [3-9] which do not need an arbitrary expansion point or the use of Lambert function W, is proposed to obtain approximate solutions for DC and transient domains in diode-based circuits. Finally, some comparisons between the numerical approximations obtained using the proposed method, and the expressions reported in Ref. 1 and 2, will be shown.

2. Circuit analysis of a basic diode circuit

Figure 1(a) shows a circuit containing a independent voltage source (V), a resistor (R), an inductor (L), and a diode (D). The voltage drop at the diode is given by the expression

$$V_D = V_T \ln\left(\frac{i(t)}{I_s}+1\right), \qquad i(t)>0 \tag{1}$$

where I_s is the saturation current of the diode and V_T is the thermal voltage.

Now, we establish the nonlinear differential equation that describes the transient behavior for the circuit

$$\begin{aligned} &Ri(t) + L\frac{di(t)}{dt} + V_T \ln\left(\frac{i(t)}{I_s}+1\right) - V = 0,\\ &i(0) = A, \end{aligned} \tag{2}$$

where initial condition $i(0) = A$ stands for the the initial current circulating through inductor L in $t = 0$.

Equation (2) does not have analytic solution due to the natural logarithm term from the diode model. Hence, Ec.(1) can be approximated as

$$V_D = V_T \ln\left(\frac{i(t)}{I_s}\right) = V_T \ln(i(t)) - V_T \ln(I_s). \tag{3}$$

Next, reformulating (2) using (3)

$$\begin{aligned} &Ri(t) + L\frac{di(t)}{dt} + V_T \ln(i(t)) - V_T \ln(I_s) - V = 0,\\ &i(0) = A. \end{aligned} \tag{4}$$

From perturbation theory [3-11], the current can be expressed in function of a power series of V_T

$$\begin{aligned} i(t) &= i_0(t) + i_1(t)V_T \\ &+ i_2(t)V_T^2 + i_3(t)V_T^3 + i_4(t)V_T^4 + \cdots \end{aligned} \tag{5}$$

In order to keep solution simple, only the two first terms of (5) are substituted in (4). By regrouping terms

$$\begin{aligned} &R(i_0(t) + i_1(t)V_T) + L\left(\frac{di_0(t)}{dt} + \frac{di_1(t)}{dt}V_T\right) \\ &+ V_T \ln(i_0(t)) + V_T \ln\left(1 + \frac{i_1(t)}{i_0(t)}V_T\right) \\ &- V_T \ln(I_s) - V = 0, \end{aligned} \tag{6}$$

Now, by replacing the first term of the Taylor expansion of natural logarithm, we obtain

$$\begin{aligned} &R(i_0(t) + i_1(t)V_T) + L\left(\frac{di_0(t)}{dt} + \frac{di_1(t)}{dt}V_T\right) \\ &+ V_T \ln(i_0(t)) + V_T\frac{i_1(t)}{i_0(t)}V_T \\ &- V_T \ln(I_s) - V = 0, \end{aligned} \tag{7}$$

regrouping terms in function of V_T, and equating to zero each coefficient, the following system is formed

$$\begin{aligned} V_T^0 &: L\frac{i_0(t)}{dt} - V + Ri_0(t) = 0, i_0(0) = A, \\ V_T^1 &: L\frac{i_1(t)}{dt} + Ri_1(t) - \ln(I_s) + \ln(i_0(t)) \\ &\qquad = 0, i_1(0) = 0. \end{aligned} \tag{8}$$

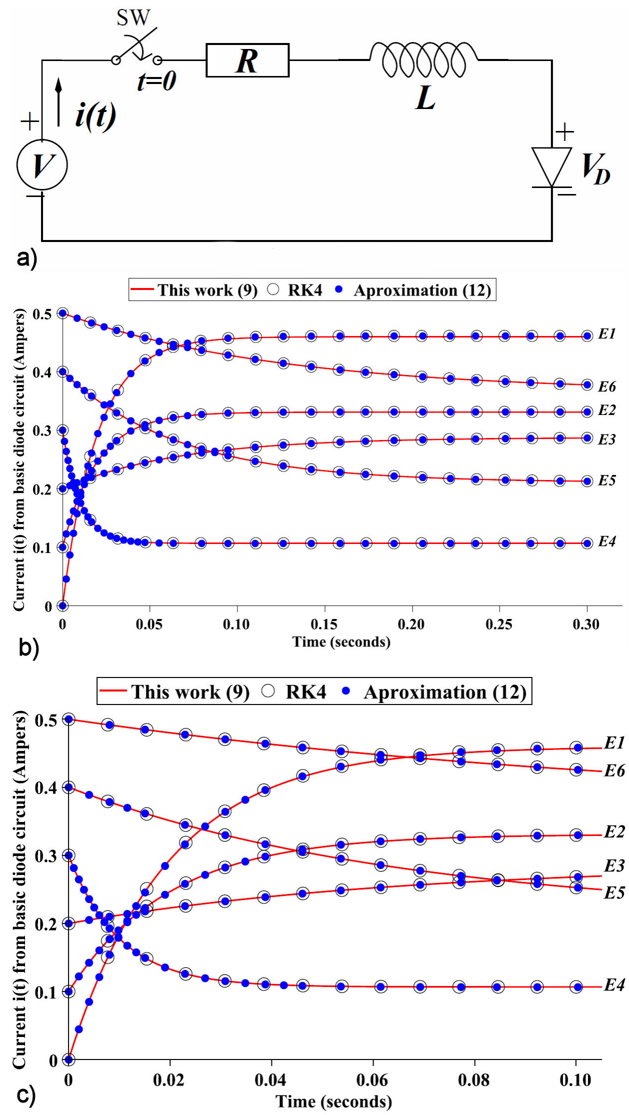

FIGURE 1. a) $t \geqslant 0$, $i(0) = A$. Basic diode circuit, b) Transient for six sets of parameters $E = [E_1, E_2\ , E_3, E_4, E_5, E_6]$, as it is shown at Tables I and II. Numerical solution RK4 for (2) (circles) and its approximate solutions (11) (solid line) and (13) (solid circles) and c) Zoom to (b). Time is in Seconds and current in Ampers.

Solving the system (8), we obtain

$$\begin{aligned} i_0(t) &= \frac{V}{R} + \exp\left(-\frac{Rt}{L}\right)\left(A - \frac{V}{R}\right), \\ i_1(t) &= \left[\frac{\ln(I_s)}{R}\exp\left(\frac{Rt}{L}\right) + G(t) + \left(\frac{1}{L} - \frac{RA}{VL}\right)t \right. \\ &\left. - \frac{\ln(I_s)}{R} + \frac{A}{V}\ln(A)\right]\exp\left(-\frac{Rt}{L}\right), \end{aligned} \tag{9}$$

where

$$\begin{aligned} G(t) &= \left(\frac{1}{R}\left[1 - \exp\left(\frac{Rt}{L}\right)\right] - \frac{A}{V}\right) \\ &\times \ln\left(\frac{1}{R}\left[V + (RA - V)\exp\left(-\frac{Rt}{L}\right)\right]\right). \end{aligned} \tag{10}$$

Using (5) and (9), we obtain the following first order expression for the transient current

$$
\begin{aligned}
i(t) &= i_0(t) + i_1(t)V_T, \\
&= \frac{V}{R} + \exp\left(-\frac{Rt}{L}\right)\left(A - \frac{V}{R}\right) \\
&+ V_T\left[\frac{\ln(I_s)}{R}\exp\left(\frac{Rt}{L}\right) + G(t) + \left(\frac{1}{L} - \frac{RA}{VL}\right)t\right. \\
&\left. - \frac{\ln(I_s)}{R} + \frac{A}{V}\ln(A)\right]\exp\left(-\frac{Rt}{L}\right), \qquad (11)
\end{aligned}
$$

The expression for steady state current (DC current) can be obtained calculating the limit (11) as

$$
\begin{aligned}
i_{dc} &= \lim_{t\to\infty} i(t) = \frac{1}{R}\left(V + V_T\ln\left(\frac{I_sR}{V}\right)\right), \\
V &> 0. \qquad (12)
\end{aligned}
$$

3. Another approximations

In Ref. 1, the transient response for the same circuit was calculated, given the result

$$
i(t) = i_{dc} + (A - i_{dc})\exp\left(-\frac{(RB + V_T)t}{BL}\right), \qquad (13)
$$

where the DC current i_{dc} is

$$
\begin{aligned}
i_{dc} &= \frac{B(V - V_T\ln(B/I_s) + V_T)}{RB + V_T}, \\
V &\geq V_T\ln(B/I_s) - V_T. \qquad (14)
\end{aligned}
$$

where B is the is the current at the expansion point. After several algebraic manipulations of (14), the need of an expansion point for B [1] is not longer required, resulting

$$
i_{dc} = I_s\exp\left(\frac{V}{V_T} - W\left(\frac{I_sR}{V_T}\exp\left(\frac{V}{V_T}\right)\right)\right), \qquad (15)
$$

where W represents the Lambert W function [12-22].

TABLE I. Transient (2) RK4 and its approximate solutions (11) and (13); DC exact solution (16) and its approximations (12), (14) and (15); for 3 different sets of E =[V (Volts), R (Ω), L (H), A (Ampers)].

Transient	Time (Sec)	E_1=[3,5,0.1,0]	E_2=[4,10,0.2,0.1]	E_3=[5,15,1,0.2]
This work (11)	0.00	0.0000000000	0.1000000000	0.2000000000
RK4	0.00	0.0000000000	0.1000000000	0.2000000000
(13) [1]	0.00	0.0000000000	0.1000000000	0.2000000000
This work (11)	0.02	0.2940427361	0.2469713801	0.2228947813
RK4	0.02	0.2948394355	0.2471752991	0.2229078118
(13) [1]	0.02	0.2932367519	0.2466090191	0.2227823932
This work (11)	0.05	0.4235142569	0.3124639698	0.2465129412
RK4	0.05	0.4247266709	0.3128637547	0.2465683401
(13) [1]	0.05	0.4242469063	0.3125006693	0.2463499002
This work (11)	0.10	0.4568994517	0.3294586538	0.2683204993
RK4	0.10	0.4582435276	0.3299344440	0.2684483344
(13) [1]	0.10	0.4581825403	0.3297197849	0.2681875195
This work (11)	0.15	0.4595585919	0.3308233101	0.2785541171
RK4	0.15	0.4609175439	0.3313083307	0.2787337170
(13) [1]	0.15	0.4608970616	0.3311150649	0.2784762497
This work (11)	0.30	0.4597884740	0.3309423605	0.2866721322
RK4	0.30	0.4611492802	0.3314284941	0.2869128044
(13) [1]	0.30	0.4611329550	0.3312380295	0.2866836894

DC	E_1=[3,5,0.1,0]	E_2=[4,10,0.2,0.1]	E_3=[5,15,1,0.2]
This work (12)	0.459788589306	0.330942421957	0.287609148788
Exact (16) [2]	0.461149372833	0.331428537242	0.287861896561
Aprox. (14) [1]	0.461133075751	0.331238094926	0.287642323377
Aprox. (15) [1]	0.461149372834	0.331428537241	0.287861896560

As in Ref. 2, an exact expression for the current in the circuit shown in Fig. 1(a) is formulated

$$i_{dc} = -I_s + \frac{V_T}{R} W\left(\frac{I_s R}{V_T}\exp\left(\frac{V + I_s R}{V_T}\right)\right). \qquad (16)$$

4. Numerical Simulation and Discussion

It was considered for all numerical examples that $I_s = $ 1E-12 A and $V_T = 25.85$ mV just as reported in Ref. 1. Some representative points from the circuit transient analysis, for six sets of parameters $E = [E_1, E_2, E_3, E_4, E_5, E_6]$, are shown in Table I and Table II. Previously reported numerical solutions, such as the approximation (13) reported in Ref. 1 (Employing $B = 0.5$ as an expansion point), and the numerical curve obtained by using a fourth order Range-Kutta numerical method (RK4), are compared with the approximation obtained in this work (11). Therefore, Table I and Table II show the DC steady current value obtained by using the proposed approximation (12), the exact solutions (16) obtained by [2], and the approximations (14) and (15) reported by [1]. It can be noticed that expression (15) is the most exact, followed by (14) and (12), which have similar precision. For instance, for the case E_6, relative errors of DC analysis are 8.2E-12, 1.5E-4, and 3.1E-4 for (15), (14), and (12), respectively. Besides, from both Tables, it can be observed that the proposed approximation (11) achieved a good accuracy, similar to the obtained by (13). For instance, for case E_6, the average relative error of transient analysis for the selected points is 5.7E-5 and 1.9E-5 for (11) and (13) respectively. However, (11) and (12) have the advantage of not requiring any arbitrary expansion point B, which implies to know the approximate value for DC current in the circuit (see Fig. 1(a)). For that reason, the proposed method provides a more general analytic approximation for both transient current and DC expression.

In addition, Eq. (12), besides its simplicity, is just expressed in terms of exponentials and natural logarithms, and the Lambert W function is not required as in the exact (16) and approximate expressions (15), which reduces its complexity and makes it suitable for engineering applications. Finally, Figs. 1(b) and 1(c) are a graphical comparison of numerical data presented in Table I and Table II, where it can be observed the high accuracy of the proposed approximations. Based on those results, further works can be focused

TABLE II. Transient (2) RK4 and its approximate solutions (11) and (13); DC exact solution (16) and its approximations (12), (14) and (15); for 3 different sets of E =[V (Volts), R (Ω), L (H), A (Ampers)].

Transient	Time (Sec)	E_4=[6,50,0.5,0.3]	E_5=[7,30,2,0.4]	E_6=[8,20,2.5,0.5]
This work (11)	0.00	0.3000000000	0.4000000000	0.5000000000
RK4	0.00	0.3000000000	0.4000000000	0.5000000000
(13) [1]	0.00	0.3000000000	0.4000000000	0.5000000000
This work (11)	0.02	0.1327916582	0.3508506734	0.4800694461
RK4	0.02	0.1328176289	0.3508526347	0.4800704783
(13) [1]	0.02	0.1326159952	0.3508437863	0.4800704512
This work (11)	0.05	0.1080892378	0.2999834489	0.4555697682
RK4	0.05	0.1081446676	0.2999938148	0.4555755750
(13) [1]	0.05	0.1077772223	0.2999615267	0.4555749149
This work (11)	0.10	0.1068193756	0.2528235002	0.4258116764
RK4	0.10	0.1068791939	0.2528539440	0.4258306310
(13) [1]	0.10	0.1064887143	0.2527677354	0.4258266654
This work (11)	0.15	0.1068109940	0.2305930709	0.4058814021
RK4	0.15	0.1068708896	0.2306427324	0.4059161317
(13) [1]	0.15	0.1064800771	0.2305037616	0.4059064256
This work (11)	0.30	0.1068109383	0.2128518485	0.3776080977
RK4	0.30	0.1068708682	0.2129315062	0.3776843143
(13) [1]	0.30	0.1064800188	0.2127081260	0.3776537615

DC	E_4=[6,50,0.5,0.3]	E_5=[7,30,2,0.4]	E_6=[8,20,2.5,0.5]
This work (12)	0.106810938331	0.210778575971	0.365471210979
Exact (16) [2]	0.106870843487	0.210865816323	0.365587482915
Aprox. (14) [1]	0.106480018836	0.210620562523	0.365530401850
Aprox. (15) [1]	0.106870843487	0.210865816323	0.365587482912

to find an equivalent circuit based on Ec. (12), which allows the analysis of larger circuit in DC domain. In the same fashion, applications of Ecs. (11) and (12) can be extended to explore their possibilities of being used to calculate other analytical characteristics, such as transitory power consumption, symbolic small signal analysis [23] and symbolic sensitivity analysis, among many others.

5. Conclusions

In this work, two approximate solutions for a basic diode, both in DC domain and transient analysis, were proposed. Several simulations results allowed us to conclude that proposed approximations have a similar accuracy in comparison which similar works reported in literature, but with a reduced complexity derived from the fact of not requiring: a previously known expansion arbitrary point and the use of the Lambert function W. Finally, the obtained expression for the steady state current or DC current is easier to use than other solutions reported in recent literature, with no significant loss in accuracy.

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