Improved method for extracting the equivalent circuit elements of a CRLH-TL unit cell

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This paper presents an improved method to extract the parameters of an equivalent circuit model of a CRLH-TL unit cell. This method is based on the analysis of the unit cell equivalent circuit admittance and impedance frequency dependence. This procedure is validated using both, electromagnetic simulation data, and measured scattering parameters. The proposed methodology is easy, and has a better performance than the existing method.

Keywords: CRLH-TL unit cell; equivalent circuit; parameters extraction.

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1. Introduction

The Composite Right/Left Handed Transmission Lines (CRLH-TL) structures, especially the unit cell, have originated a growing interest in the microwave field due to its unique features such as the size reduction of microwave components.

A CRLH-TL structure is the microstrip implementation of the metamaterial concept introduced by [1-2], and it has been used in many microwave applications such as couplers [3, 4], power dividers [5], antennas [6], filters [7], and amplifiers [8]. The unit cell of the CRLH-TL structure consists of an interdigital capacitor and a stub inductor shorted to ground plane with a via hole.

For microwave Computer-Aided Design (CAD) it is essential to have accurate model parameter values, that can be defined using an electrical equivalent circuit. Currently, the classical method to extract the parameter values, is the one proposed by Caloz *et al.* [9]; it is based on the analysis of admittance and impedance of each element of the unit cell. They consider a π-network for the interdigital capacitor, and a T-network for the stub inductor, and develop a procedure based on functions that depend on frequency and partial derivatives. However, the use of partial differential equations presents uncertainty in the circuit frequency response, therefore the parameters extraction has to be performed where the variations in the frequency range are relatively low.

In this work, an improved CRLH-TL unit cell equivalent circuit parameters extraction method is proposed, which is based on the analysis of the unit cell equivalent circuit admittance and impedance frequency dependence, where the equivalent circuit admittance and impedance are expressed as linear functions. The parameter identification is easier than the method described in Ref. 9, as it is seen in the following sections.

2. Proposed Parameter Extraction Method

2.1. CRLH-TL Unit Cell Model

Figure 1(a) shows a CRLH-TL unit cell. Its equivalent circuit model is shown in Fig. 1(b), where Z_s represents the interdigital capacitor, modeled by a right-handed inductance L_R in series with a left-handed capacitance C_L, and Y_p represents the stub inductor shorted to ground plane with a via hole, modeled by a right-handed capacitance C_R in parallel with a left-handed inductance L_L.

The equivalent circuit impedance can be written as follows

$$Z_T = \begin{bmatrix} Z_s + \frac{1}{Y_p} & \frac{1}{Y_p} \\ \frac{1}{Y_p} & \frac{1}{Y_p} \end{bmatrix} \tag{1}$$

From Eq. (1) we define

$$Z_{T_{11}} = Z_s + \frac{1}{Y_p} \tag{2}$$

$$Z_{T_{12}} = \frac{1}{Y_p} \tag{3}$$

from which

$$Z_s = Z_{T_{11}} - Z_{T_{12}}, \tag{4}$$

and from Eq. (3)

$$Y_p = \frac{1}{Z_{T_{12}}}. \tag{5}$$

2.2. Parameter Extraction

From the Fig. 1(b), Z_s and Y_p are given by

$$Z_s = j\left[\omega L_R - \frac{1}{\omega C_L}\right] \tag{6}$$

$$Y_p = j\left[\omega C_R - \frac{1}{\omega L_L}\right] \tag{7}$$

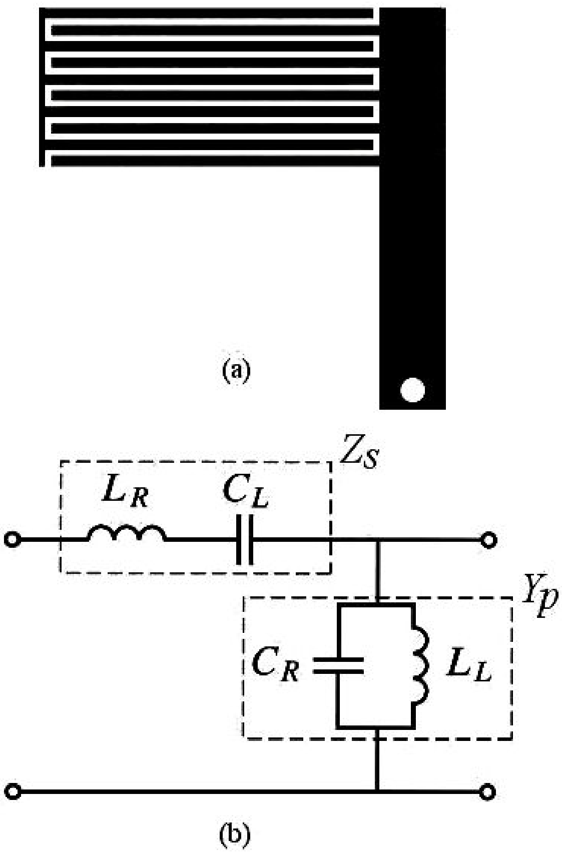

FIGURE 1. a) Unit cell of a microstrip CRLH-TL. b) Equivalent circuit of a unit cell.

or, equivalently

$$\mathrm{Im}\{Z_s\} = \omega L_R - \frac{1}{\omega C_L} \tag{8}$$

$$\mathrm{Im}\{Y_p\} = \omega C_R - \frac{1}{\omega L_L} \tag{9}$$

Multiplying (8) and (9) by ω, we have

$$\omega\ \mathrm{Im}\{Z_s\} = \omega^2 L_R - \frac{1}{C_L} \tag{10}$$

$$\omega\ \mathrm{Im}\{Y_p\} = \omega^2 C_R - \frac{1}{L_L}. \tag{11}$$

The extraction of parameters L_R, C_L, C_R, and L_L are obtained by applying a linear regression (least-squares fitting) to Eqs. (10), and (11).

2.3. Parameter Optimization

Once the parameters L_R, C_L, C_R, and L_L are calculated, an optimization algorithm is applied to get a more accurate model that describes the CRLH-TL unit cell. This is done by minimizing an objective function expressed as follows

$$H = \frac{1}{m}\sum_{k=1}^{m}\left[\frac{|S_{ij}^T(f_k)| - |S_{ij}^E(f_k)|}{|S_{ij}^T(f_k)|}\right]^2 \tag{12}$$

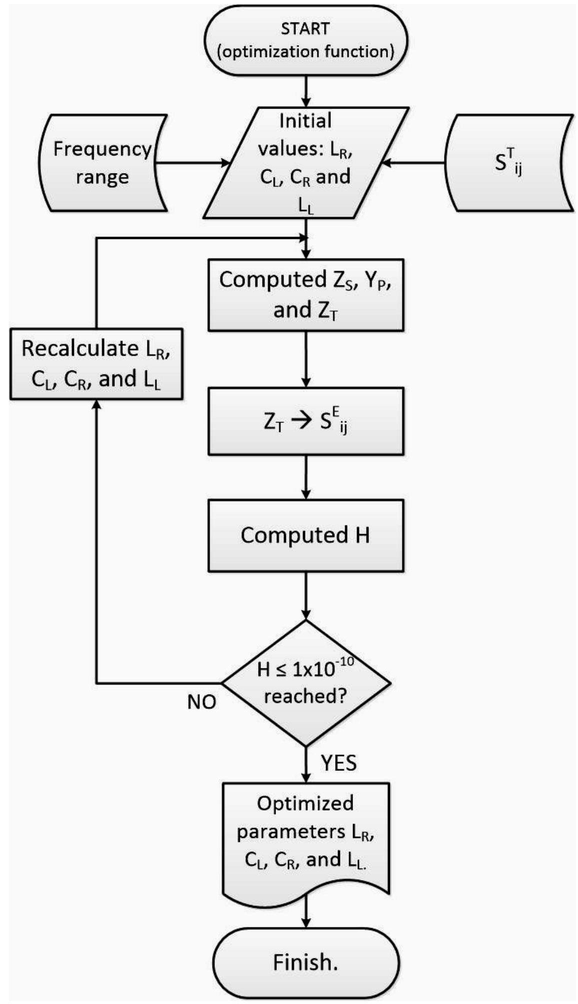

FIGURE 2. Optimization procedure.

where i, j=1,2, m is a number of simulated/measured frequency (f) points, S_{ij}^T are the S-parameters data, and S_{ij}^E are the estimated S-parameters using the equivalent circuit values, which were obtained with the proposed method. A termination tolerance of 1×10^{-10} was applied.

2.3.1. *Optimization procedure*

The optimization algorithm has been implemented in MATLAB™, as shown in Fig. 2, and is described as follows

(1) Initial parameters values are obtained from the proposed method using Eqs. (10), and (11).

(2) Z_S, Y_P, and Z_T are calculated from Eqs. (6), (7), and (1), respectively.

(3) Z_T is converted to S_{ij}^E-parameters.

(4) H is calculated from the previous knowledge of S_{ij}^T and S_{ij}^E as already indicated in Eq. (12).

(5) If the termination tolerance of H is reached 1×10^{-10}, the optimized parameters L_R, C_L, C_R, and L_L are achieved.

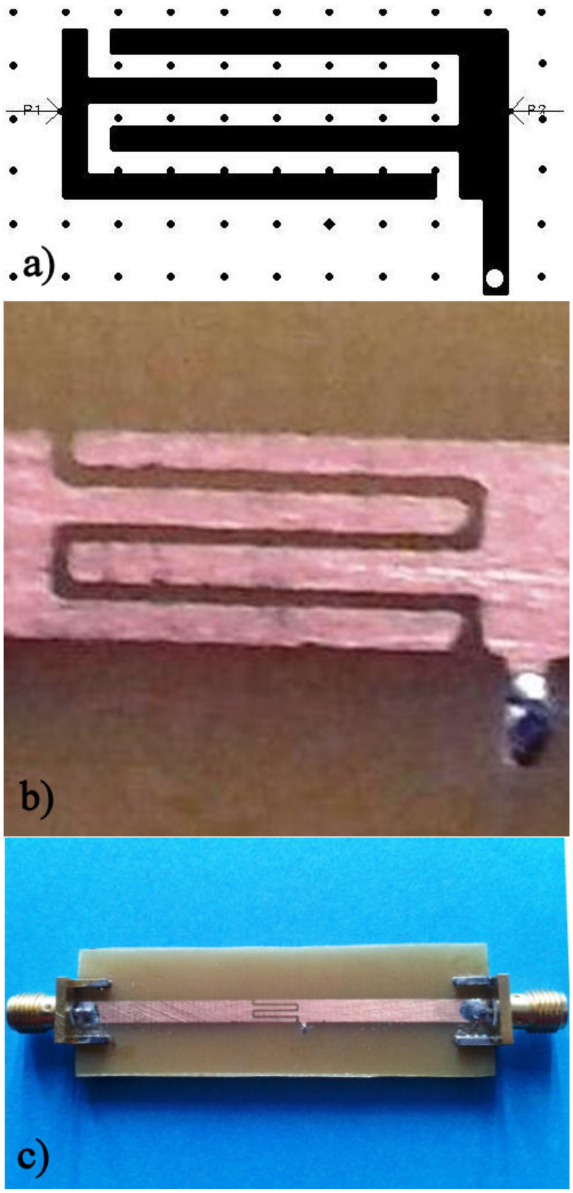

FIGURE 3. CRLH TL Unit Cell. (a) ADS Momentum Layout. (b) Fabricated unit cell. (c) Fabricated unit cell with coaxial-to-microstrip transition and SMA connectors.

(6) If not, the parameters L_R, C_L, C_R, and L_L are recalculated with MATLAB function *fminsearch*, and the procedure is repeated from step 2.

The MATLAB function *fminsearch* uses the Nelder-Mead simplex method to minimizes an objective function.

3. Experimental Results

To apply the proposed method, a CRLH-TL unit cell was designed with an interdigital capacitor value of 0.5 pF, and a stub inductor value of 1nH. The substrate FR4 with permittivity ϵ_r=4.3, and thickness h=1.6 mm has been used. Next,

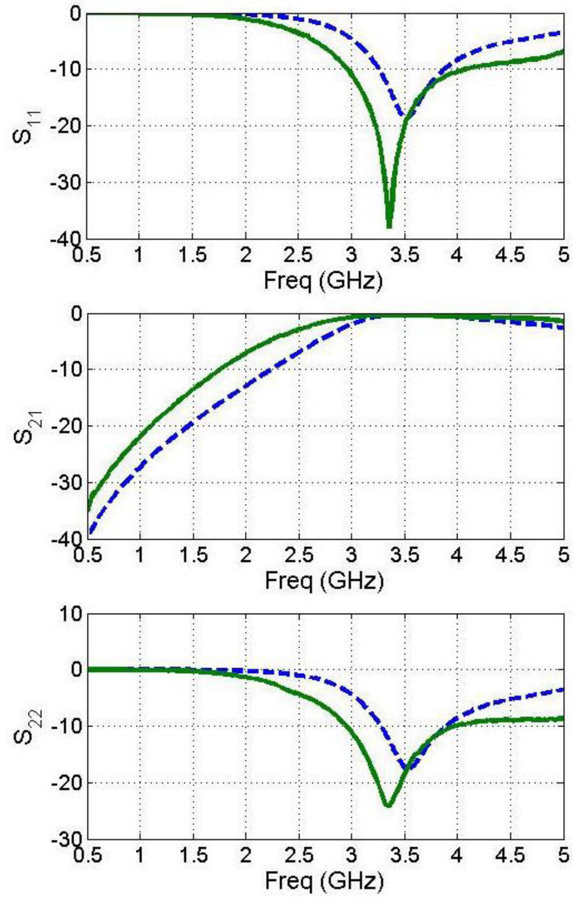

FIGURE 4. Simulation $(dashed)$ and measurement $(solid)$ S-Parameters S_{11}, S_{21} ($S_{12} = S_{21}$), and S_{22} of the structure shown in Fig. 3

we simulated the CRLH-TL unit cell using the Agilent's microwave circuit simulator ADS Momentum as shown in Fig. 3(a). Also, to verify the simulated characteristics of the CRLH-TL unit cell, it was fabricated (Fig. 3(b)) and measured with a vector network analyzer (VNA). To measure the unit cell with the VNA, a coaxial-to-microstrip transition was used as shown in Fig. 3(c). To remove the effect of the coaxial-to-microstrip transitions, the Thru-Reflect-Line (TRL) calibration technique [10] was used.

Figure 4 illustrates the simulated (Fig. 3(a)) and experimental (Fig. 3(b)) results of the designed unit cell, where we can observe a slight difference in the resonant frequency. This difference may be due to the gap and width of the interdigital capacitor fingers of the fabricated unit cell.

Then, the CRLH-TL unit cell S-parameters simulated are converted into Z-parameters. In Fig. 5(a) we show a plot of the imaginary part of the impedance Z_s multiplied by ω versus ω^2, at low frequencies it is observed the linear response from the simulated data according to Eq. 10. While Fig. 5(b) shows a plot of the imaginary part of the admittance Y_p multiplied by ω versus ω^2, where it is observed a nonlinear response at low frequencies, this nonlinear response is probably due to resistive effects introduced by the metallization of the stub inductor shorted to ground plane with a via hole. It

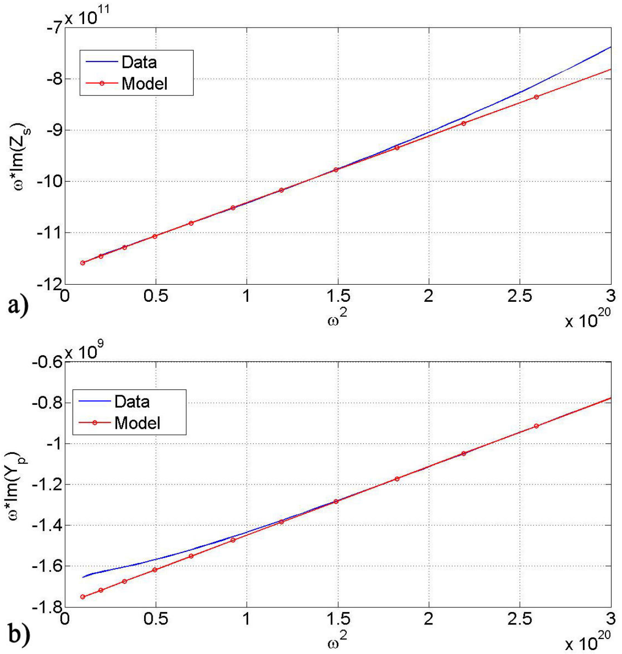

FIGURE 5. a) Plot of ω Im$\{Z_s\}$ vs. ω^2. b) Plot of ω Im$\{Y_p\}$ vs. ω^2.

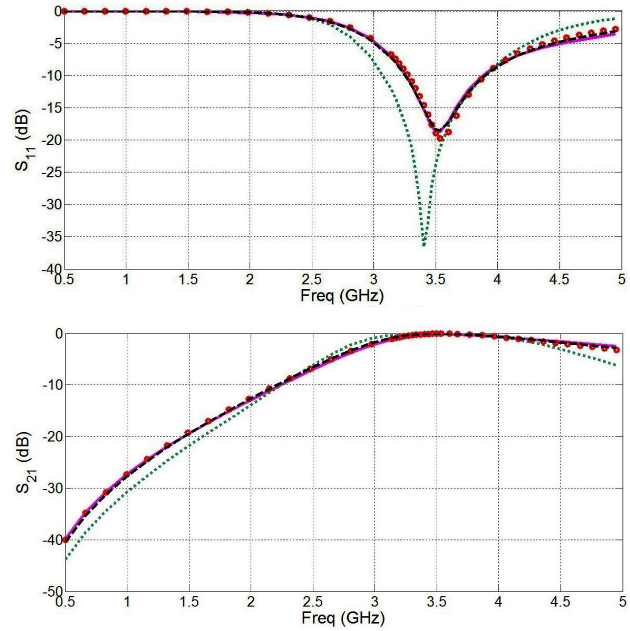

FIGURE 6. Comparison between simulation $(solid)$, proposed method $(circle)$, Caloz *et al.* method (dot), and optimize proposed method $(dash)$ of S-Parameters S_{11} and S_{12}.

TABLE I. Comparison between Caloz *et al.* Method [9] and the Proposed Method from the simulated unit cell.

Parameters	Caloz Method	Proposed Method	Optimization
C_R	3.61 pF	3.36 pF	2.97 pF
L_L	0.58 nH	0.56 nH	0.62 nH
C_L	0.54 pF	0.85 pF	0.76 pF
L_R	3.65 nH	1.30 nH	1.52 nH

TABLE II. Comparison between Caloz *et al.* Method [9], and the Proposed Method from the fabricated unit cell.

Parameters	Caloz Method	Proposed Method	Optimization
C_R	2.08 pF	1.58 pF	2.23 pF
L_L	1.53 nH	1.41 nH	0.93 nH
C_L	0.39 pF	0.76 pF	0.97 pF
L_R	3.22 nH	2.70 nH	1.81 nH

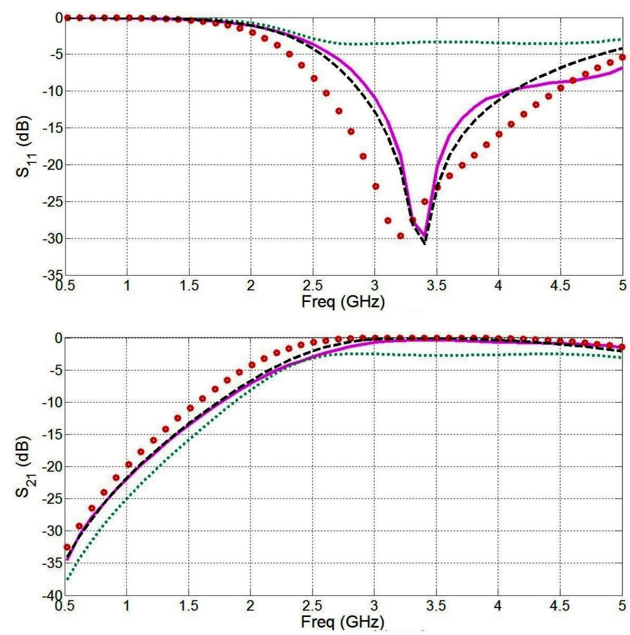

FIGURE 7. Comparison between measured data $(solid)$, proposed method $(circle)$, Caloz *et al.* method (dot), and optimize proposed method $(dash)$ of S-Parameters S_{11} and S_{12}.

should be noted that these effects are ignored in the parameters extraction method, because these effects are not significant in the equivalent circuit response, and besides, it is observed the linear response at higher frequencies according to Eq. (11).

Next, we followed the equivalent circuit parameter extraction method described in the previous section, which is applied to the unit cell. This procedure is also applied to the CRLH-TL unit cell measured S-parameters data.

The calculated values of the equivalent circuit elements using simulated and measured S-parameters data, are shown in Table I and Table II, respectively. Also we calculate the equivalent circuit elements of the unit cell by applying the method proposed by Caloz *et al.* in Ref. 9, and its results are also shown in Table I and Table II.

Once we have the parameters values of the equivalent circuit, we used them to generate the S-parameters of the unit cell, and they are compared with the simulated and measured results shown in Fig. 6 and Fig. 7, respectively. We observed that the proposed method provides more accurate frequency response curves than the Caloz *et al.* method.

We also apply an optimization algorithm to the results obtained with the proposed method, to get the best matching between model and simulation/experimental data. These results are also shown in Table I and Table II.

Then, we generate the S-parameters from the optimized model, and also compare them with the simulated and measured data. We observe that the optimization of the proposed method agrees very well with the predicted curves, both simulation and measured data.

4. Discussion

It has been illustrated by simulation and experimental results the proposed method. Fig. 6 shows the comparison between simulated data, the proposed method, Caloz *et al.* method, and optimized proposed method.

It has been found that our method shows a good agreement with the simulated data, and the optimized results. In the case of the measured results, shown in Fig. 7, we observed that our method predicts the results better than the existing one, and the optimized results agree with the measured data.

5. Conclusions

We have presented an improved method to extract the parameters of an equivalent circuit model of a CRLH-TL unit cell. This method uses the unit cell equivalent circuit admittance and impedance frequency dependence. It has the following advantages over the conventional one:

- It is based on linear equations instead of partial differential equations.
- It is easier to identify the frequency range at which the parameter extraction is performed, instead of finding a suitable region.

Moreover, a CRLH-TL unit cell was designed, simulated, and fabricated. Its equivalent circuit elements were extracted and also the S-parameters from the elements of the equivalent circuit of the unit cell were generated and compared. The results obtained both, in simulation and experimentation, exhibit a better agreement with the parameters extracted with the proposed method.

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