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Anomalous photon emission from a solid

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The solution of the wave equation for electron in a solid can be formally singular on some line. The singularity is smeared out within the distance $\sim 10^{-11}\mathrm{cm}$ due to electron "vibrations" caused by its interaction with photons. Because of this localization, the large momentum uncertainty results in the local increase of the electron kinetic energy $\sim 1\ \mathrm{MeV}$. This energy enhancement is compensated by the local reduction of zero point energy of photons which can be considered as a potential well producing anomalous electron binding. The electron in this well gets to its bottom emitting photons of the total energy $\sim 1\ \mathrm{MeV}$ (anomalous emission). These effects can occur in a solid when its surface is bombarded by ions with the energy $\sim 1\ \mathrm{keV}$. Photons, produced inside the solid, escape from it and can be detected in addition to the usual Bremsstrahlung of incident ions.

Keywords: Ion irradiation; electron binding

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1. Introduction

An electron beam, irradiating a surface of a solid, produces Bremsstrahlung and the transition radiation. See, for example, Ref. 1. Irradiation of surfaces by ions may serve for various technical purposes including electron emission, chemical reactions, lithography, and ethichng [2, 3]. Bremsstrahlung is also expected when ions collide the surface. It happens, when charged particles bombard the surface of a solid an additional (anomalous) mechanism of photons emission exists. This mechanism is related to the electron-photon interaction in a solid and the incident flux of charged particles just initiates it.

The interaction of electrons with photons is weak. For example, the radiative correction to the ground state energy of electron in the Coulomb center Ze (the Lamb shift) is proportional to $Z^2(e^2/\hbar c)^3$ [4]. In this case the electron is on the distance $\hbar^2/(me^2Z)$ around the Coulomb center. Under increase of Z the electron wave function becomes more localized in space and radiative corrections are enhanced.

One cat put a general question about the strength of radiative effects when the electron is strongly localized in space due to some reasons. For example, Schrödinger equation has the formal solution with the peak, $\ln r$, on the z axis ($r^2 = x^2 + y^2$). This solution is not physical due to the singularity at $r \to 0$.

The simple question exists: what happens to that singularity if to account for the electron interaction with photons? Under this interaction the electron "vibrates". Its mean displacement $\langle\vec{u}\rangle$ is zero but the mean squared displacement $\langle u^2\rangle$ is finite. The finite $\langle u^2\rangle$ provides the Lamb shift since the electron probes various parts of the Coulomb potential.

It happens that the singularity gets smeared over the region with the size $\sqrt{\langle u^2\rangle} \sim 10^{-11}$ cm. Roughly speaking, $\langle\ln|\vec{r}+\vec{u}|\rangle$ becomes not singular as a function of r. That narrow but smooth distribution of the electron density results in a large momentum uncertainty. This corresponds to the local increase (~ 1 MeV) of the electron kinetic energy close to the line (thread) of the initial singularity.

The enhancement of the kinetic energy is compensated by the local reduction of zero point energy of photons. The latter can be considered as the certain potential well producing anomalous electron binding. The electron in this well gets to its bottom emitting photons of the total energy ~ 1 MeV. Being escaped from the solid, those photons constitute anomalous emission. Formation of the well, due to the reduction of photon zero point energy, reminds the van der Waals phenomenon when such a well is also created [5, 6]. The formation of anomalous electron binding is detailed step by step in Sec. 3.

There is an analogy with formation of hydrogen molecule. Two hydrogen atoms, in the ground state each, are acted by the attractive van der Waals force which brings them together (until activation of covalent forces) from a large distance. In this process the sum of the atoms kinetic energy and zero point energy of photons is conserved. Then the emission of the energy of 4.72 eV (H_2 binding energy) by photons transfers the system to the ground state. As a result, zero point photon energy is reduced by ~ 1 eV. Analogously, in anomalous electron binding it is reduced by ~ 1 MeV. In the both cases "energy from nothing" is emitted.

The electron state with anomalous binding is characterized, in contrast to the usual state, by the short range $\sim 10^{-11}$ cm. Therefore the perturbation, producing the anomalous state, should be of the short spatial range. Otherwise the proper matrix element is small. An incident charged particle with the de Broglie wave length of $\sim 10^{-11}$ cm, penetrated inside the solid, produces the charge density of that spatial range. This occurs due to interference of the incident and reflected (from a lattice center) waves of the particle.

For ions of light elements such de Broglie length corresponds to the energy of ~ 1 keV. For electron it would be of ~ 1 MeV. Using of ions is preferable. The ion of the en-

ergy ~ 1 keV, colliding the surface of a solid, can initiate the energy release (~ 1 MeV) in the form of emitted photons due to the reduction of zero point electromagnetic energy. The intensity and energy spectrum of this anomalous emission essentially differs from Bremsstrahlung. This is a pure electron-photon phenomenon.

2. Two Coulomb centers

We need a solution of the wave equation which is singular on some line. In the Schrödinger formalism there is the obvious solution of that type which is $\ln r$ at all z. In reality one should deal with a singular line of a finite length. For this reason, we consider, as the first step, the artificial situation when the electron is in the field of two positive charges fixed in the space.

2.1. Formalism

To study the electron in the Coulomb field of two positive point charges at the points $z = \pm\sigma$ it is convenient, instead of cylindrical coordinates $r = \sqrt{x^2+y^2}$, z, and φ, to use the elliptic ones ξ, η, and φ [7]

$$\xi, \eta = \frac{r_2 \pm r_1}{2\sigma}. \tag{1}$$

Here $r_{1,2} = \sqrt{(z \mp \sigma)^2 + r^2}$ are distances to the Coulomb centers shown in Fig. 1. The surface of a constant ξ is the ellipsoid

$$\frac{z^2}{\sigma^2\xi^2} + \frac{r^2}{\sigma^2(\xi^2-1)} = 1 \tag{2}$$

with the focuses at $z = \pm\sigma$. The surface of a constant η is the hyperboloid

$$\frac{z^2}{\sigma^2\eta^2} - \frac{r^2}{\sigma^2(1-\eta^2)} = 1 \tag{3}$$

with the focuses at the same points. The coordinate ξ takes values from 1 to ∞ and η from -1 to 1. Intersections of the surfaces (2) and (3) with the plane $y = 0$ are shown in Fig. 1.

The Coulomb interaction potential is

$$U = -\frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} = -\frac{2Ze^2}{\sigma}\frac{\xi}{\xi^2-\eta^2}, \tag{4}$$

where Ze is the positive charge at each center. The Schrödinger equation for the electron

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial\vec{R}^2} + U\psi = E\psi, \tag{5}$$

where $R^2 = r^2 + z^2$, in elliptic coordinates takes the form

$$-\frac{\hbar^2}{2m\sigma^2(\xi^2-\eta^2)}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial\psi}{\partial\xi} + \frac{\partial}{\partial\eta}(1-\eta^2)\frac{\partial\psi}{\partial\eta}\right]$$
$$-\frac{2Ze^2}{\sigma}\frac{\xi}{\xi^2-\eta^2}\psi = E\psi. \tag{6}$$

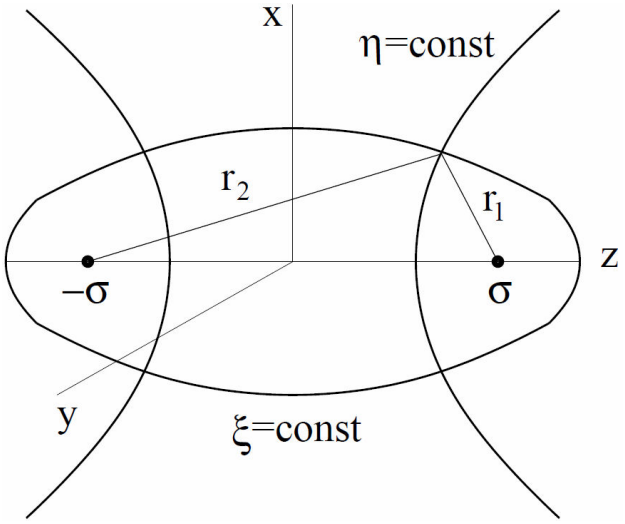

FIGURE 1. Intersections of surfaces of constant elliptic coordinates ξ and η with the plane $y = 0$ are shown. Two Coulomb centers are at the points $z = \pm\sigma$.

We consider an axially symmetric wave function. Since the variables are separated it has the form $\psi(\xi,\eta) = \psi_1(\xi)\psi_2(\eta)$. One can introduce dimensionless parameters $\nu = -2m\sigma^2E/\hbar^2$ and $p = 4Z\sigma/r_B$ where $r_B = \hbar^2/(me^2)$ is the Bohr radius.

After separation of variables the two Schrödinger equations are

$$-\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial\psi_1}{\partial\xi} = (p\xi + \beta - \nu\xi^2)\psi_1 \tag{7}$$

$$-\frac{\partial}{\partial\xi}(1-\eta^2)\frac{\partial\psi_2}{\partial\eta} = (\nu\eta^2 - \beta)\psi_2\,, \tag{8}$$

where β is some constant.

The variable η can be written as $\eta = \cos\chi$. Then Eq. (8) has the form

$$\frac{1}{\sin\chi}\frac{\partial}{\partial\chi}\left(\sin\chi\frac{\partial\psi_2}{\partial\chi}\right) = (\beta - \nu\cos^2\chi)\psi_2. \tag{9}$$

The solution, non-singular at $\chi = 0$, should be also non-singular after continuation to $\chi = \pi$. This is the condition to choose the parameter β for a given ν. When two Coulomb centers coincide ($\sigma = 0$ and therefore $\nu = 0$) that condition turns to $\beta = -l(l+1)$ as for Legendre polynomials [8]. In that case the variable χ coincides with the azimuthal angle θ.

2.2. Close Coulomb centers

Below two close Coulomb centers are considered under the condition $\sigma \ll r_B$. We study the state which is isotropic ($l = 0$) in the limit $\sigma = 0$. At a finite σ there is the small correction to the wave function which can be written as $\psi_2 = 1 + \delta\psi_2$. Using the relation

$$\sin\chi\frac{\partial\delta\psi_2}{\partial\chi} = \int d\chi(\beta - \nu\cos^2\chi)\sin\chi \tag{10}$$

one can obtain

$$\delta\psi_2 = \left(\frac{\nu}{3} - \beta\right) \ln\left(2\cos^2\frac{\chi}{2}\right) - \frac{\nu}{6}\sin^2\chi. \quad (11)$$

The solution (11) is finite at $\chi = 0$ ($\eta = 1$). In order to get it finite at $\chi = \pi$ ($\eta = -1$) it should be $\beta = \nu/3$. Note that $\nu \sim p^2 \sim \sigma^2/r_B^2$ are small.

At $\sigma \ll r_B$ there is a small region of the size σ around the centers in Fig. 1. This region corresponds to $\xi \sim \eta \sim 1$. The region of the order of the Bohr radius r_B is much larger and relates to large ξ. We consider first this region, $\sigma \ll R$.

2.2.1. *Region $\sigma \ll R$*

In Eq. (7) one can omit β. In the limit $1 \ll \xi$ Eq. (7) takes the form

$$-\frac{\partial^2\psi_1}{\partial\xi^2} - \frac{2}{\xi}\frac{\partial\psi_1}{\partial\xi} - \frac{p}{\xi}\psi_1 = -\nu\psi_1, \quad (12)$$

which coincides with the radial Schrödinger equation with $l = 0$ in the Coulomb field of the point charge $2Ze$ [8]. The solution of (12), decaying on infinity and finite at small distances, corresponds to the eigenvalue $\nu = p^2/4$. This value relates to the ground state energy in the Coulomb field of the point charge $2Ze$.

For our purposes one needs a solution which also decays on infinity but is singular at $r = 0$ and $z^2 < \sigma^2$. To obtain that one should write the solution of (12) in the known form the confluent hypergeometric function [8]

$$\psi_1(\xi) = (2\xi\sqrt{\nu})^{p/(2\sqrt{\nu})-1}\exp(-\xi\sqrt{\nu}) \times G\left(1 - \frac{p}{2\sqrt{\nu}}, -\frac{p}{2\sqrt{\nu}}, -2\xi\sqrt{\nu}\right), \quad (13)$$

where

$$G(\alpha,\beta,v) = 1 + \frac{\alpha\beta}{1!v} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{2!v^2} + ... \quad (14)$$

At large distances $r_B \ll R$ ($r_B/\sigma \ll \xi$), as follows from (13) - (14) and the definition (1),

$$\psi(r,z) = \left(\frac{4ZR}{r_B}\sqrt{\frac{E}{E_0}}\right)^{\sqrt{E/E_0}-1} \exp\left(-\frac{2RZ}{r_B}\sqrt{\frac{E}{E_0}}\right), \quad (15)$$

where $E_0 = -m(2Ze^2)^2/(2\hbar^2)$.

To get the solution at shorter distances one should use the asymptotics of the function (13) at $\xi\sqrt{\nu} \ll 1$ [8]. Thus at small $[1 - p/(2\sqrt{\nu})]$ one can obtain from Eq. (13)

$$\psi_1(\xi) = 1 + \left(1 - \frac{p}{2\sqrt{\nu}}\right)\frac{1}{2\xi\sqrt{\nu}}, \quad 1 \ll \xi \ll r_B/\sigma. \quad (16)$$

2.2.2. *Region $R \ll r_B$*

On the other hand, at not large ξ the left-hand side of Eq. (7) is the principal one and

$$\psi_1(\xi) = 1 + \frac{1}{4\sqrt{\nu}}\left(1 - \frac{p}{2\sqrt{\nu}}\right)\ln\frac{\xi+1}{\xi-1}. \quad (17)$$

The wave function (17) is the solution of the homogeneous Eq. (7). ψ_1 contains also the small corrections

$$\delta\psi_1 = -\frac{p\xi}{2} - \left(1 - \frac{p}{2\sqrt{\nu}}\right) \times \left[\frac{\xi}{4}\ln\frac{\xi+1}{\xi-1} + \frac{1}{2}\ln(\xi^2-1)\right] \quad (18)$$

which is the solution of the inhomogeneous Eq. (7) where in the right-hand side one should keep $p\xi\psi_1$ only. Here instead of ψ_1 one has to substitute the expression (17). One can easily conclude that $\delta\psi_1$ can be dropped under the condition

$$1 < \xi \ll \frac{r_B}{\sigma}\sqrt{1 - \frac{p}{2\sqrt{\nu}}}, \quad (19)$$

where the last part is supposed to be large. Eq. (17) goes over into the form (16) when their applicability intervals, (16) and (19), overlap.

The wave function along the line, connecting two Coulomb centers in Fig. 1, now can be written at $|z^2 - \sigma^2|, r^2 \ll \sigma^2$ in the form

$$\psi(r,z) = 1 - \frac{r_B}{16Z\sigma}\left(1 - \frac{E}{E_0}\right) \times \ln\frac{8\sigma^2}{z^2 - \sigma^2 + \sqrt{(z^2-\sigma^2)^2 + 4\sigma^2r^2}}. \quad (20)$$

Equation (20) is also valid in the vicinity ($r^2 \ll (\sigma^2 - z^2)$) of the entire line ($z^2 < \sigma^2$) between the centers, where

$$\psi(r,z) = 1 - \frac{r_B}{8Z\sigma}\left(1 - \frac{E}{E_0}\right)\ln\frac{2\sqrt{\sigma^2 - z^2}}{r}. \quad (21)$$

2.2.3. *Entire region*

The wave function of the electron in the Coulomb field of two positive point charges Ze exponentially decays at large distances (15). But on the line, connecting two charges, ψ has the logarithmic singularity (21) if the energy does not coincide with the eigenvalue E_0. The absence of singularities is a usual condition to determine an eigenvalue. The eigenvalue E_0 coincides with one in the Coulomb field of one point charge $2Ze$. In the limit considered, $\sigma \ll r_B$, corrections to that eigenvalue are small.

When the energy differs from E_0 the singular wave function is not physical at the first sight. But the situation is more complicated as described in Sec. 3.

3. Anomalous electron binding

The electron wave function between two nuclei in a solid also can be of the type considered in Sec. 2. In a solid close to a lattice site the wave function is of the type (20). The same is valid for the lattice partner site. Between the partner sites the wave function is logarithmically singular, as (21), along the connecting line $-\sigma < z < \sigma$. In a real solid $\sigma \sim r_B$. In a close vicinity ($r \ll \sigma$) of the line, connecting two partner sites, the wave function, with the logarithmic accuracy, can be written in the form

$$\psi = \begin{cases} A(\sigma)\ln[\sqrt{(z-\sigma)^2+r^2}+(z-\sigma)], & |z-\sigma| \ll \sigma \\ 2A(z)\ln r\,, & \sqrt{\sigma^2-z^2} \sim \sigma \\ A(-\sigma)\ln[\sqrt{(z+\sigma)^2+r^2}-(z+\sigma), & |z+\sigma| \ll \sigma \end{cases} \quad (22)$$

Here $A(z)$ is the certain function accounting for exact forces on the electron. The exact form of this function is not crucial for our purposes. Properties of the singularity line (thread) between two nuclei are considered below.

Equation (22) corresponds to the Schrödinger formalism which is applicable at $r_c < r$ where $r_c = \hbar/(mc) \simeq 3.86 \times 10^{-11}\mathrm{cm}$ is the electron Compton length [4]. At $r < r_c$ one should use Dirac equations (Appendix A). In this case, besides the term $\ln r$, the wave function also contains the term r_c/r which is small at $r_c \ll r$ (A.6). So the expression (22) corresponds to distances r and $|\sigma^2 - z^2|^{1/2}$ larger than r_c.

3.1. Usual radiative effects

Usually the wave function, as a solution of quantum mechanical equation, is smooth and does not have singularities. Since electron-photon interaction H_{e-ph} is weak radiative effects are also weak.

Due to the interaction with electromagnetic zero point oscillations the electron "vibrates" with the amplitude $\vec{u}$ and becomes spatially smeared [4]. The mean displacement $\langle\vec{u}\rangle$ is zero and the mean squared displacement $r_T^2 = \langle u^2\rangle$ is finite. The evaluation of r_T is done in Appendix B.

Radiative corrections to electron energy levels in atom are small. The electron, due to the uncertainty in its position, probes various parts of the electrostatic potential $V(\vec{R})$ and therefore electron energy levels becomes shifted (Lamb shift [4]). Besides the rigorous calculations of the Lamb shift one can use the approximate method [9]. In this approach the electron moves in the effective potential $\langle V(\vec{R}+\vec{u})\rangle$ averaged on fast electron motions $\vec{u}$. Expansion up to the second order in $\vec{u}$ results in

$$\langle V(\vec{R}+\vec{u})\rangle \simeq V(\vec{R}) + \frac{\langle u^2\rangle}{6}\nabla^2 V(\vec{R}). \quad (23)$$

The second term in (23) produces the Lamb shift of the electron energy level.

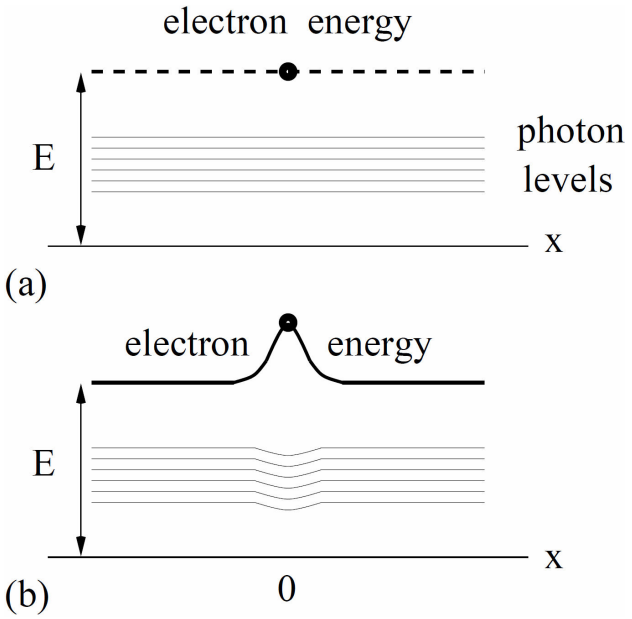

FIGURE 2. (a) The standard situation of a smooth electron wave function. The total energy is the sum of electron one E and the infinite photon contribution $\sum \hbar\omega/2$. (b) The thread state of the same energy. The enhancement of the electron energy inside the thread is compensated by the reduction of the zero point electromagnetic energy at the same region.

3.2. No electron-photon interaction

One can apply the quantum mechanical description to the electron-photons system since photons are an infinite set of harmonic oscillators. This method was proposed in Ref. 10 and developed in Ref. 11 and further publications.

Let us formally suppose the electron-photon interaction to be zero, $H_{e-ph} = 0$. Then the total energy is the sum of the electron energy E (5) and the zero point photon energy $\sum \hbar\omega/2$. This is shown schematically in Fig. 2(a). The stationary state of the system with that total energy is described by the wave function

$$\psi_0 = \begin{pmatrix} \Phi \\ \Theta \end{pmatrix} \psi_{ph}\,, \quad (24)$$

where the electron bispinor is multiplied by the multidimensional photon function. With spinors of Appendix A this is a formally correct solution of the problem at all r excepting $r = 0$ which corresponds to the singularity line (thread).

3.3. Far from the thread

The finite H_{e-ph} turns the wave function (24) into exact one, ψ. In this case ψ (as ψ_0) also corresponds to a stationary state of the total Hamiltonian with the certain total energy. The role of H_{e-ph} is minor far from the thread. In that region the difference between ψ and ψ_0 is not significant.

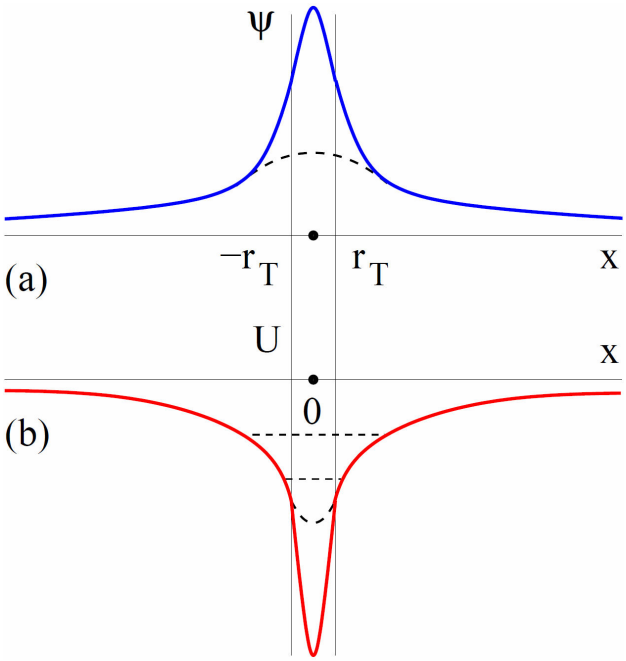

FIGURE 3. Features of the thread state at $y = 0$ and constant η. Dashed curves correspond to the absence of thread. Thin vertical lines restrict the thread region where the wave function and the potential should not be interpreted literally. (a) Thread wave function. (b) Potential well. Energy levels in the potential well in the absence of thread are shown by dashed horizontal lines.

3.4. Close to the thread

One can follow the exact wave function ψ approaching the line $r = 0$. Due to the average on photon coordinates [10,11], the wave function, as a function of r, becomes smeared around the singularity line. This phenomenon is generic with the Lamb shift. In the both cases the electron "vibrates" because of the interaction with photons.

3.5. Density peak at the thread

The smeared singularity relates to a narrow but smooth peak, shown in Fig. 3(a), of the electron density around the thread line where the density would be singular at $H_{e-ph} = 0$. This corresponds to the local enhancement of the electron kinetic energy $\sqrt{(mc^2)^2 + (\hbar c/r_T)^2} - mc^2 \simeq 1.97$ MeV in the thread region. The estimate of r_T in Appendix B is used.

3.6. Thread well

Since the state is stationary (with the certain total energy) the total electron-photon energy far from the thread is approximately the sum of electron one and zero point energy of photons as in Fig. 2(a). Close to the thread the same energy is redistributed between the locally enhanced kinetic energy of the electron and the local reduction of zero point energy of photons Fig. 2(b). The latter can be considered as the certain potential well sketched in Fig. 3(b). This well, as well as the electron wave function in Fig. 3(a), should not be interpreted literally in the thread region. In that region single particle quantum mechanics is not applicable even approximately.

3.6.1. *Analogy with the van der Waals phenomenon*

Analogous reduction of zero point photon energy occurs in formation of van der Waals forces [5, 6]. The essence of the van der Waals phenomenon is a spatial dependence of zero point energy analogously to Fig. 2(b). It is mostly clear from the simplest example of two parallel plates when the photon spectrum becomes discrete $\omega = c(k^2 + \pi^2 n^2/R^2)^{1/2}$. Here R is the distance between two plates and $\vec{k}$ is the two-dimensional wave vector parallel to them. The zero point energy $\sum \hbar\omega/2$ is decreased compared to the case of $R \to \infty$.

So the spatial restriction locally reduces the zero point electromagnetic energy. In this manner one can also explain the reduction of zero point energy of photons near the thread. Within r_T smearing of the singularity is a strong electron-photon effect when the perturbation theory is not applicable. It is clear from the simple argument that an expansion of $\psi(\vec{r} + \vec{u})$ on $\vec{u}$ is impossible in the singular case.

Due to that, there is a geometrical restriction for photons. This is roughly a tube of the radius r_T. Inside this tube the zero point energy is reduced. This reduction, which should locally compensate the kinetic energy 1.97 MeV, is roughly estimated as $-\hbar c/r_T \sim -2.43$ Mev. This value can be treated as a depth of the potential well in Fig. 3(b).

3.7. Anomalous electron binding

Generally, a discrete energy spectrum in a well is a consequence of the absence of a singularity in a solution of wave equations at small distances. In our case such condition is not imposed. According to Eqs. (20) and (21), the singular solution of the wave equation exists at any non-zero prelogarithmic factor $(1 - E/E_0)$. Here E_0 is the discrete energy eigenvalue. For this reason the energy spectrum of the thread is continuous. This is not surprising since a discrete spectrum of a particle turns into a continuous one under interaction with a bath.

Qualitatively Fig. 3 reminds a wave function (a) of a particle localized in the potential well (b). Usually the electron energy E has a typical value of (1-10) eV in solids. This corresponds to the top of the well. The electron emits photons to get to the bottom of the well. The emitted energy is on the order of 2 MeV and the electron binding energy

$$E_B \simeq \frac{\hbar c}{r_T} \tag{25}$$

is on the same order.

Besides that general estimate, the coordinate dependence of E_B is significant. The mean squared displacement of the electron $\langle u^2 \rangle$ is determined by the logarithmic cut off Ω at low frequencies (B.9). The meaning of Ω is clear from Eq. (B.1). This frequency increases with increasing of the external potential. In other words, "vibration" amplitude of

the electron depends on the local strength of the Coulomb potential at each point of the space. Therefore one can approximately ascribe the coordinate dependence to Ω. This is qualitatively confirmed by the exact results for the Lamb shift [4] if to extract $\langle u^2\rangle$ using Eq. (23). The low frequency logarithmic cut off becomes smaller for higher atomic levels, that is for larger distances.

Then, according to (25) and (B.11), the potential, probed by the electron, is

$$-E_B(R) \simeq -mc^2\sqrt{\frac{\pi\hbar c}{2e^2}}\left[\ln\frac{mc^2}{\hbar\Omega(R)}\right]^{-1/2}. \qquad (26)$$

The rough approach (26) provides just the tendency of the spatial dependence of E_B.

The binding energy E_B increases upon approaching each of two nuclei in Fig. 1. Therefore the energy emitted in the form of photons is expected to be more than 2 MeV following from the simple estimate. As a result, the ground state corresponds to the electron localized in the vicinity of a nucleus. The radius of such anomalous atom is no larger than $r_T \simeq 10^{-11}\mathrm{cm}$.

3.8. How to create anomalous photon emission

The typical range of the electron state in solids $10^{-8}\mathrm{cm}$ is comparable with the Bohr radius r_B. The range of the anomalous bound state is on the order of $r_T \sim 10^{-11}\mathrm{cm}$. This estimate also relates to the diameter of the thread state of the electron. This thread state, before photon emission, corresponds to the usual electron energy in solids. Therefore the perturbation, producing the thread from the usual state, should be varied on the distance of r_T.

The charge density, varying in space on the typical distance r_T, can be created by an incident charged particle which is reflected by lattice sites of the solid. The resulting density, related to such particle, is due to interference of its incident and reflected waves. Omitting details, this charge density is approximately proportional to $\cos(2R\sqrt{2ME_p}/\hbar)$ where M is the particle mass and E_p is its energy. For example, if to use deuterons as light ions, $M \simeq 3.346\times10^{-24}\mathrm{g}$, one can estimate

$$\text{charge density} \sim \cos\left[1.96\frac{R}{r_T}\sqrt{E_p(\mathrm{keV})}\right], \qquad (27)$$

where r_T is taken to be $10^{-11}\mathrm{cm}$. If to use electrons, the same wave length corresponds to high electron energy on the order of 1 MeV.

We see that one can bombard the surface of the solid by ions with the energy of approximately 1 keV to produce anomalous electron binding and photon emission within the depth of ions penetration.

The charge density of the anomalous atom (Sec. 3.7), varying on the distance of $10^{-11}\mathrm{cm}$, can also serve as a perturbation for formation of anomalous state of other electrons. One can consider a possibility when anomalous atoms remain the only source for the photon emission after stopping surface irradiation by incident ions.

4. Description of anomalous binding in quantum electrodynamics

The link from phenomena considered to the standard formalism of quantum electrodynamics is in the form of the exact electron propagator [4]

$$G(\varepsilon,\vec{R},\vec{R}') = \sum_\nu \frac{\langle 0|\psi(\vec{R})|\nu\rangle\langle\nu|\bar\psi(\vec{R}')|0\rangle}{\varepsilon-\varepsilon_\nu+i0} = G_0+G_B. \qquad (28)$$

In this equation ε_ν is the set of exact (with all radiative corrections) electron energy levels in the external field. In (28) the analogous positron part and bispinor indices are dropped. The propagator (28) can be a basis, for example, for calculations of small radiative corrections to discrete energy levels.

Besides the part G_0, related to the usual energy levels, the sum (28) contains also the part G_B due to the energy spectrum in the deep well in Fig. 3(b). So the electron density, corresponding to the propagator (28), has the peak of the type shown in Fig. 3(a).

Unlike the Lamb shift, it is impossible to apply the perturbation theory to study smearing of the singularity. This phenomenon, with an essential electron-photon influence, corresponds to the non-perturbative part G_B in the electron propagator. So the thread state does not follow from the usual one, of the same energy, through perturbation theory with respect to the electron-photon interaction.

If to increase H_{e-ph}, formally starting with the usual state of free electron, the state of this type continuously exists at small H_{e-ph}. This means that the usual state is separated

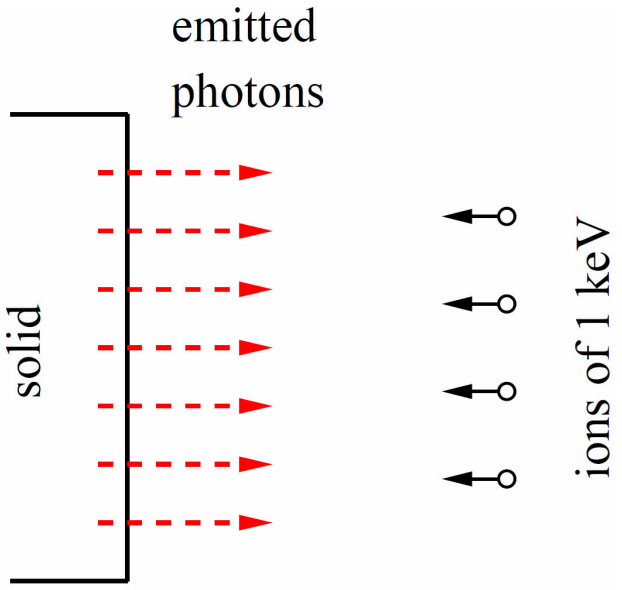

FIGURE 4. Scheme of the experimental setup for observation of anomalous photon emission.

by a barrier from the thread one whose fraction is exponentially small. Analogously the exact thread state contains the exponentially small part of usual one.

In the absence of an external field the propagator $G(\varepsilon, \vec{R}, \vec{R})$ does not depend on $\vec{R}$ and the electron density is homogeneous. This corresponds to the spectral representation of the exact electron propagator (with no external fields) in quantum electrodynamics. Without an external field ($\Omega = 0$, Appendix B) the mean squared displacement (B.9) becomes infinite. In this approach, according to the general theory, the state is homogeneous since the singular solution of the wave equation is smeared over the entire space.

5. Discussions

Irradiation of the surface of a solid by ions results first in Bremsstrahlung due to ion braking by the surface. In addition to this, there is a more delicate phenomenon of anomalous photon emission. The underlying mechanism is of electron-photon nature. The point is that the electron density can be formally singular on some line if to ignore the electron-photon interaction. This interaction leads to smearing of the singularity within the narrow cylinder (thread). The resulting state is physical.

The narrow but smooth peak of the electron density on the thread is associated with the narrow ($\sim 10^{-11}\mathrm{cm}$) and deep ($\sim 1\mathrm{MeV}$) potential well in the thread region. That well is formed by the spatial redistribution of zero point electromagnetic energy. The electron state of a usual energy inside the solid corresponds to upper states in the well. The electron can emit energy in the form of photons getting to the bottom of the well. The total energy emitted is in the range of MeV. This is anomalous photon emission from the solid which can be detected outside it.

The source of high-energy photons is of the electron-photon origin

$$mc^2\sqrt{\frac{\hbar c}{e^2}} \simeq 6 \quad \mathrm{MeV}. \tag{29}$$

The energy generation occurs due to the reduction of zero point photon energy (from the photon vacuum of the infinite energy $\sum \hbar\omega/2$).

The direct way to form the thread state from the usual one is to use an external perturbation rapidly varying in space (on the distance of $10^{-11}\mathrm{cm}$). In this case its matrix element between those states is not small, Sec. 3.7. The role of such perturbation can be played by the incident flux of ions directed to the surface of a solid as shown in Fig. 4.

The spatially oscillating charge density is produced inside the solid by a superposition of the incident and reflected (by a lattice site) waves of the incident ion. The matrix element of that perturbation provides thread formation with a not small probability. The process in Fig. 4 is characterized by the generation of photons of the energy up to a few MeV. These photons are emitted from the surface of the solid and can be registered. The intensity and the energy spectrum of emitted photons depends on processes inside the solid including energy losses. The energy of incident ions around 1 keV is optimal for the creation of electron binding. At much lower and much higher energies the effect is reduced.

After the photon emission the electron gets to the bottom of the deep well and remains on the distance of $10^{-11}\mathrm{cm}$ from the nucleus. The charge density of such anomalous atom (Sec. 3.7), varying on the distance of $10^{-11}\mathrm{cm}$, can also serve as a perturbation for formation of anomalous state of other electrons. One can consider a possibility when anomalous atoms remain the only source for the photon emission after stopping surface irradiation by incident ions.

High energy photons, generated inside the solid, can cause nuclear transmutations of lattice nuclei. Misinterpretation of such experiments is possible by ascribing the energy source to nuclear reactions. These reactions are impossible since the energy of incident particles ($\sim 1\mathrm{keV}$) is too low.

6. Conclusions

The ion of the energy $\sim$ 1 keV, colliding the surface of a solid, can initiate the energy release ($\sim$ 1 MeV) in the form of emitted photons (anomalous emission) due to the reduction of zero point electromagnetic energy. The intensity and the energy spectrum of anomalous emission essentially differs from Bremsstrahlung. This is a pure electron-photon phenomenon.

Appendix

A. Singular solution of Dirac equations

The static Schrödinger equation formally has the solution $\ln r$ in cylindrical coordinates. Below we establish the continuation of this singular solution to the region $r < r_c$ where one should use the Dirac formalism. In this case the wave function is bispinor consisting of two spinors φ and χ [4]. Since we are interested by the singular wave functions (large kinetic energy part) one can ignore, as the first step, the potential energy and consider free electron Dirac equations

$$(\varepsilon + i\hbar c\hat{\vec{\sigma}}\nabla)\varphi = mc^2\chi, \quad (\varepsilon - i\hbar c\hat{\vec{\sigma}}\nabla)\chi = mc^2\varphi. \tag{A.1}$$

Here ε is the total relativistic energy and $\hat{\vec{\sigma}}$ are the Pauli matrices [4]. We consider for simplicity two dimensional case $\vec{r} = \{x, y\}$ when z derivatives are zero. The solution of equations (A.1) is

$$\varphi(\vec{r}) = (\varepsilon + mc^2 - i\hbar c\hat{\vec{\sigma}}\nabla)F(\vec{r})$$
$$\chi(\vec{r}) = (\varepsilon + mc^2 + i\hbar c\hat{\vec{\sigma}}\nabla)F(\vec{r}), \tag{A.2}$$

where one accounts for the relation $(\hat{\vec{\sigma}}\nabla)(\hat{\vec{\sigma}}\nabla) = \nabla^2$ and the spinor function $F(\vec{r})$ satisfies the equation

$$(\varepsilon^2 + \hbar^2c^2\nabla^2)F = m^2c^4F. \tag{A.3}$$

The solution of (A.3) is the Neumann function [12]

$$F(\vec{r}) = \frac{\pi\varphi_0}{4} N_0\left(\frac{r}{\hbar c}\sqrt{\varepsilon^2 - m^2c^4}\right) \qquad (A.4)$$

with the asymptotics $N_0(z) \simeq (2/\pi)\ln z$ at small argument. φ_0 is some constant spinor. Accordingly, at short distances two spinors are

$$\varphi(\vec{r}) = \left(\frac{\varepsilon + mc^2}{2}\ln r - \frac{i\hbar c}{2r^2}\hat{\vec{\sigma}}\vec{r}\right)\varphi_0,$$

$$\chi(\vec{r}) = \left(\frac{\varepsilon + mc^2}{2}\ln r + \frac{i\hbar c}{2r^2}\hat{\vec{\sigma}}\vec{r}\right)\varphi_0. \qquad (A.5)$$

In the standard representation $\Phi = \varphi + \chi$ and $\Theta = \varphi - \chi$

$$\Phi(\vec{r}) = (\varepsilon + mc^2)\varphi_0 \ln r\,, \qquad \Theta(\vec{r}) = -\frac{i\hbar c}{r^2}\hat{\vec{\sigma}}\vec{r}\varphi_0. \quad (A.6)$$

At distances $r_c < r$ (non-relativistic limit) Θ is small compared to Φ and the wave function is the usual spinor Φ.

If the electron is not free but is acted by some potential $V(\vec{r})$, one should substitute ε by $\varepsilon - V(\vec{r})$ in Eqs. (A.1). In this case the parts at short distances remains of the same type as (A.5) and (A.6) with the substitution $\varepsilon \to \varepsilon - V(0)$.

B. Estimate of the thread radius

Unlike the Lamb shift calculations, it is impossible to apply the perturbation theory to study smearing of a singular wave function of the electron. For this reason, we use below the approximate method just to estimate the electron mean squared displacement [9]. The method is successfully applied for study of the Lamb shift. In this method the electron motion under the action of zero point oscillations can be described by the equation

$$m\frac{d^2\vec{u}}{dt^2} + m\Omega^2\vec{u} = -e\vec{\mathcal{E}}, \qquad (B.1)$$

where m is the electron mass and $\Omega \sim me^4/\hbar^3$ is the electron rotation frequency in the atom.

One can use the Fourier expansion

$$\vec{u}(\vec{R},t) = \sum_k \vec{u}_k \exp(i\vec{k}\vec{R} - i\omega_k t) \qquad (B.2)$$

and analogous one for the fluctuating electric field $\vec{\mathcal{E}}(\vec{R},t)$. Since $\vec{u}(\vec{R},t)$ is real it should be $\vec{u}_k^* = \vec{u}_{-k}$ and $\omega_{-k} = -\omega_k$ in the expansion (B.2). The condition $uk \ll 1$ has to be held in this method. The solution of Eq. (B.1) is of the form

$$\vec{u}_k = \frac{e\vec{\mathcal{E}}_k}{2m|\omega_k|}\left(\frac{1}{|\omega_k| + \Omega} + \frac{1}{|\omega_k| - \Omega}\right). \qquad (B.3)$$

According to the quantum mechanical approach, Eq. (B.3) should be modified as

$$\vec{u}_k = \frac{e\vec{\mathcal{E}}_k}{2m|\omega_k|}\left(\frac{\sqrt{1+n_k}}{|\omega_k| + \Omega} + \frac{\sqrt{n_k}}{|\omega_k| - \Omega}\right), \qquad (B.4)$$

where n_k is the number of quanta, the first term relates to the quanta emission, and the second one to the absorption.

The mean squared displacement is

$$\langle u^2\rangle = \int \frac{d^3R}{V}\langle u^2\rangle = \sum_k \langle|\vec{u}_k|^2\rangle, \qquad (B.5)$$

where V is the volume of the system. Since in our case $n_k = 0$, the mean squared displacement has the form

$$\langle u^2\rangle = \frac{e^2}{4m^2}\sum_k \frac{\langle|\vec{\mathcal{E}}_k|^2\rangle}{\omega_k^2(|\omega_k| + \Omega)^2}\,. \qquad (B.6)$$

The energy of zero point oscillations is

$$\int \frac{d^3R}{4\pi}\langle\mathcal{E}^2\rangle = \frac{V}{4\pi}\sum_k \langle|\vec{\mathcal{E}}_k|^2\rangle = \sum_k \frac{\hbar|\omega_k|}{2}. \qquad (B.7)$$

It follows from here that $\langle|\vec{\mathcal{E}}_k|^2\rangle = 2\pi\hbar|\omega_k|/V$. Using the summation rule

$$\sum_k = 2\int \frac{4\pi k^2 dk V}{(2\pi)^3} \qquad (B.8)$$

(the coefficient 2 accounts for two photon polarizations) and the relation $\omega_k = ck$, one can obtain from Eq. (B.6) [9]

$$\langle u^2\rangle = \frac{r_B^2}{2\pi}\left(\frac{e^2}{\hbar c}\right)^3 \int_0^{\omega_{\max}} \frac{\omega d\omega}{(\omega + \Omega)^2}$$

$$= \frac{r_c^2}{2\pi}\,\frac{e^2}{\hbar c}\ln\frac{mc^2}{\hbar\Omega}\,, \qquad (B.9)$$

where $r_c = \hbar/(mc) \simeq 3.86 \times 10^{-11}$cm is the electron Compton length. The upper limit $\omega_{\max}$ is determined by the condition of non-relativistic motion, that is $\omega_{\max} \simeq mc^2/\hbar$. In the relativistic region $\vec{u}_k$ decreases due to the enhancement of the relativistic mass. The exact cut off Ω is not crucial for the estimate. The applicability condition of the approach used $\langle u^2\rangle k_{\max}^2 \sim \langle u^2\rangle/r_c^2 \ll 1$ is valid since $\langle u^2\rangle/r_c^2 \sim e^2/(\hbar c)$.

The mean squared electron displacement can be also evaluated from the known expression for the Lamb shift δE_L [4]. As follows from Eq. (23),

$$\delta E_L = \frac{\langle u^2\rangle}{6}\int \psi^*(\vec{R})\nabla^2 V(\vec{R})\psi(\vec{R})d^3R. \qquad (B.10)$$

Since $\nabla^2 V(\vec{R}) = 4\pi e^2\delta(\vec{R})$ the result depends on the wave function at $\vec{R} = 0$. For the ground state of the hydrogen atom $|\psi(0)|^2 = 1/(\pi r_B^3)$. Using (B.10) and the exact value of the Lamb shift [4], one can obtain

$$r_T = \sqrt{\langle u^2\rangle} = r_c\sqrt{\frac{2e^2}{\pi\hbar c}\ln\frac{mc^2}{\hbar\Omega}}$$

$$= r_c\sqrt{\frac{4e^2}{\pi\hbar c}\ln\frac{\hbar c}{e^2}}\,. \qquad (B.11)$$

The expression (B.9) is four times less than the more exact estimate following from (B.11). We use (B.11) for the mean squared electron displacement. As follows from Eq. (B.11), $r_T \simeq 0.21\, r_c \simeq 0.81 \times 10^{-11}$ cm.

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