Conversion of zero point energy into high-energy photons

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An unusual phenomenon, observed in experiments, is studied. X-ray laser bursts of keV energy are emitted from a metal where long-living states, resulting in population inversion, are totally unexpected. Anomalous electron-photon states are revealed to be formed inside the metal. These states are associated with narrow, 10^{-11} cm, potential well created by the local reduction of zero point electromagnetic energy. In contrast to analogous van der Waals potential well, leading to attraction of two hydrogen atoms, the depth of the anomalous well is on the order of 1 MeV . The states in that well are long-living which results in population inversion and subsequent laser generation observed. The X-ray emission, occurring in transitions to lower levels, is due to the conversion of zero point electromagnetic energy.

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1. Zero point electromagnetic energy

The concept of zero point energy was developed by A. Einstein and O. Stern in 1913. This energy depends on mutual positions of atoms or macroscopic objects in space resulting in interaction forces among them. The first calculation of this force between two atoms (van der Waals force) has been done by F. London in 1928 [1]. As shown by H. Casimir in 1948, van der Waals forces exist also between macroscopic bodies [2, 3]. These results were expanded by E. Lifshitz in 1955 [4, 5].

A significant problem is extraction of zero point energy and conversion of it into a macroscopic form. It is possible in reality and there is an example of this. Two hydrogen atoms, in the ground state each, are acted by the attractive van der Waals force which brings them together (until activation of covalent forces) from a large distance. In this process the sum of the atoms kinetic energy and zero point energy of photons is conserved. Then the emission of the energy of 4.72 eV (H_2 binding energy) by photons transfers the system to the ground state. As a result, zero point photon energy is reduced by $\sim 1\,\mathrm{eV}$. In this case the vacuum energy is emitted ("energy from nothing").

The macroscopic version of the above process was proposed in Ref. 6 and discussed in Ref. 7. See also Refs. 8 to 10.

Apart from that, the mechanism of conversion of zero point electromagnetic energy into high-energy photons (up to a few MeV) has been revealed in Ref. 11. This mechanism is based on anomalous electron-photon states.

2. Glow discharge and X-ray bursts

In papers [12, 13] and references therein unusual results were reported on high-current glow discharge in various gases. At discharge voltage of (1-2) kV X-ray emission (up to 10 keV) from the metal cathode was registered. Both diffuse and collimated X-ray emissions were observed. Collimated X-ray bursts of the duration of 20 μs were generated approximately every 50 μs during 0.1 s after stopping the discharge.

Moreover, some collimated X-ray bursts have been seen up to 20 hours after switching off the discharge voltage. An emission of separate photons by radioactive isotopes from the cathode material is easy understandable. But here one deals with strongly collimated X-ray laser bursts. So it was the laser emission from "dead" sample, namely, which was acted by nothing during 20 hours.

The essential point is that above experiments were repeatedly performed for years and could be reproduced any time on demand. Indeed, the array of macroscopic laser bursts unlikely is an artifact.

It is hard, using a combination of known effects, to explain phenomena observed. First, it is unclear how energy inside the isolated and equilibrium solid is suddenly collected to get converted into the macroscopic laser burst. Second, even if this happens, a mechanism of creation of population inversion is unclear since electronic lifetimes in the keV regime are very short [12].

Misinterpretation of experiments [12, 13] is possible by attributing the energy source to nuclear reactions. These reactions are impossible here since energies of phonons (0.01 eV) and electrons (1 eV) inside a solid are too low compared to MeV . It is not real to expect phonons in a solid to suddenly get collected into the MeV energy.

Below the conversion mechanism of zero point energy into high-energy photons [11] is shown to be likely the base of phenomena observed in Refs. 12 and 13.

3. Anomalous electron-photon states

In this section we study some aspects of the electron-photon interaction.

3.1. Electron-photon system

Electron-photon interaction in quantum electrodynamics is described by the part H_{e-ph} [5]. One can apply multi-dimensional quantum mechanics to the electron-photons system since photons are the infinite set of harmonic oscillators. This method was proposed in Ref. 14 and developed in Ref. 15 and further publications.

When formally $H_{e-ph} = 0$ the total energy is the sum of the electron energy E and the zero point photon energy $\sum \hbar\omega/2$. The stationary state of the system with that total energy is described by the wave function

$$\psi_0 = \psi_e(\vec{r}, z)\psi_{ph}\,, \tag{1}$$

where ψ_{ph} is the multi-dimensional photon function. We suppose the part ψ_e in (1) to be the usual electron wave function of the ground state with the energy E in the potential $U_0(r)$. This is shown in Fig. 1 where $\rho_0 = |\psi_e|^2$ is the electron density. As the first step, we consider the general problem in a smooth axially symmetric potential and do not specify a particular physical situation.

The finite H_{e-ph} turns the wave function (1) into exact one, ψ. In this case ψ (as ψ_0) also corresponds to a stationary state of the total Hamiltonian with the certain total energy E_{tot}. One can present E_{tot} as

$$E_{\text{tot}} = E(\vec{r}, z) + \sum \frac{\hbar\omega}{2} - \left(\sum \frac{\hbar\omega}{2}\right)_0, \tag{2}$$

where the first term relates to the electron part which also includes H_{e-ph}. The last term is zero point energy of photons in absence of the electron. According to [14, 15], the electron density

$$\rho = \langle |\psi|^2 \rangle_{ph} \tag{3}$$

corresponds to the average on photon coordinates. At $H_{e-ph} = 0$ the density is $\rho = |\psi_e|^2$.

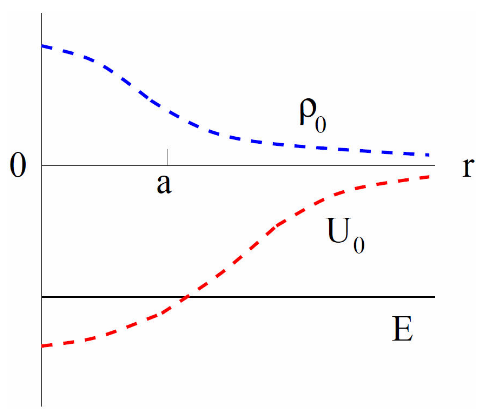

FIGURE 1. $H_{e-ph} = 0$. Ground state energy E of electron in the potential $U_0(r)$. The electron density is $\rho_0 = |\psi_e|^2$. a is the spatial scale of the potential.

3.2. Lamb shift

When $H_{e-ph} \neq 0$, for the potential $U_0(r)$ in Fig. 1 the modified ground state energy E_{tot} hardly differs from bare E by the Lamb shift [5]. Analogously the inclusion of the electron-photon interactions hardly influences the electron density in Fig. 1.

This is because of smallness of radiative corrections [5]. Due to the interaction with zero point electromagnetic oscillations the electron "vibrates" [16, 17]. This idea was proposed by W. Nernst in 1916. The mean electron displacement $\langle \vec{u} \rangle$ is zero but the mean squared displacement $r_T^2 = \langle u^2 \rangle$ is finite. One can estimate for the electron in hydrogen atom [11, 16]

$$r_T \simeq r_c \sqrt{\frac{4e^2}{\pi\hbar c} \ln \frac{\hbar c}{e^2}} \simeq 0.81 \times 10^{-11} cm, \tag{4}$$

where $r_c = \hbar/(mc) \simeq 3.86 \times 10^{-11} cm$ is the Compton length. The electron, due to the uncertainty r_T in its positions, probes various parts of the electrostatic potential and therefore electron energy levels become slightly shifted (Lamb shift [5]).

3.3. Singular electron density

Let us consider the potential $U(r)$ which coincides with $U_0(r)$ at $a < r$, where a is the scale of the potential in the space. At $r < a$ the potential $U(r)$ exceeds $U_0(r)$ as shown in Fig. 2. The energy E, the same as in Fig. 1, becomes below the ground state energy in the modified potential. The corresponding ψ_e (coinciding with one in Fig. 1 at $a < r$) is no more the eigenfunction since it has the singularity at $r = 0$ sketched in Fig. 2. This is a formally correct solution of the problem at all r excepting $r = 0$ which is the singularity line

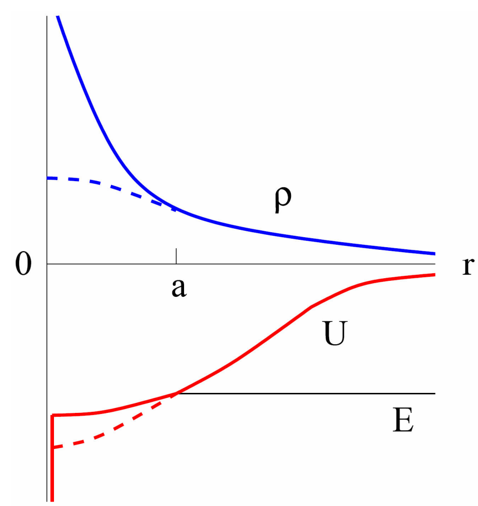

FIGURE 2. $H_{e-ph} = 0$. Compared to Fig. 1, the potential $U(r)$ is larger at $r < a$. Now E (the same as in Fig. 1) is below the ground state energy. The electron density ρ is singular at $r = 0$ since the related ψ_e is not the electron eigenfunction for $U(r)$. Dashed curves correspond to Fig. 1 and at $a < r$ $U = U_0$ and $\rho = \rho_0$.

(thread) of ψ_e. The electron density ρ in Fig. 2 coincides with ρ_0 in Fig. 1 at $a < r$. To formally support the singular wave function (density) in Fig. 2 the potential should be singular $U(r) \sim -\delta(r)$. This is shown in Fig. 2.

Let us look at Fig. 1. Inclusion of the electron-photon interaction just slightly modifies the ground state energy in the potential $U_0(r)$, which becomes $E_{\rm tot}$ (Sec. 3.3). The related electron density is also close to ρ_0. The associated total wave function ψ, which produces that electron density according to (3), follows from the milti-dimensional Schrödiger equation (Sec. 3.1). The solution ψ, defined on the total set of coordinates with a given large r, is an initial condition for continuation to smaller r according to the formalism of differential equations. During the continuation from large r down to the certain fixed r the wave function ψ "does not know" what happens at values of this variable smaller than that fixed r.

Let us look now at Fig. 2. At $H_{e-ph} \neq 0$, the total wave function ψ (and therefore ρ), being continued from large r, is the same at $a < r$ for the both potentials, $U_0(r)$ and $U(r)$, if to substitute E by $E_{\rm tot}$. In the potential $U_0(r)$ the latter is the exact ground state energy which obviously has zero imaginary part. Therefore there is the stationary state with this exact real energy $E_{\rm tot}$ also for the potential $U(r)$. We track this state from large r. But what happens to this state at $r < a$?

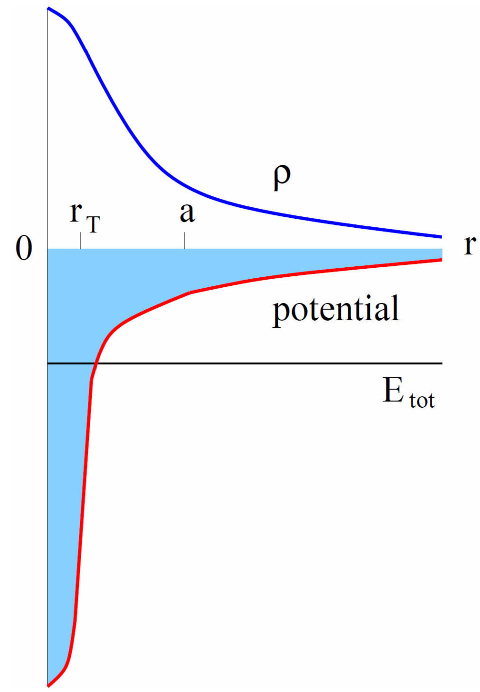

FIGURE 3. Features of anomalous electron-photon state. This is the case of Fig. 2 but with the electron-photon interaction. The bare energy E in Fig. 2 goes over into $E_{\rm tot}$ at $H_{e-ph} \neq 0$. The depth of the potential well is in the range of MeV. The energy spectrum in the well is continuous.

3.4. Why the singularity is cut off

One can look at the exact density ρ moving from the region $a < r$ in Fig. 3 to the region $r < a$. When we are not close to $r = 0$, ρ slightly differs from one in Fig. 2 since the small uncertainty in electron positions, due to its "vibrations", produces a small effect on the electron motion. This statement in terms of quantum electrodynamics [5] is equivalent to weakness of the electron-photon interaction. As known, some aspects of quantum electrodynamics can be formulated in the approximate manner using "vibrational" representation [17]. For example, the Lamb shift of atomic levels can be almost exactly calculated within that approximation [11, 16].

Electron "vibrations" become significant close to the line $r = 0$ when $r \lesssim r_T$. There is a question will the density ρ, when $H_{e-ph} \neq 0$, remain singular on that line. As plotted in Fig. 3, the singularity is cut off by the electron-photon interaction. This is clear from below.

In quantum electrodynamics the electron propagator may be calculated at a given (in space-time) macroscopic electromagnetic field with the subsequent averaging on those fields with the certain weight [5]. Before this average the electron wave function can be presented as a path integral on classical trajectories [18]. For simplicity we consider the Schrödinger formalism.

The electron wave function, given at the initial moment of time t_1, is "transferred" by classical trajectories to the next moment t_2. In the absence of the electromagnetic field the wave function, singular in space when $t = t_1$, goes over at t_2 into one with the same singularity at the same position in space. With the macroscopic electromagnetic field classical trajectories are deformed and bring the singularity to a different position in space at the moment t_2. For example, in the harmonic potential the argument of the wave function is shifted by the time-dependent coordinate given by the classical trajectory in the macroscopic non-stationary field [19]. Since the singularity shift depends on photon coordinates, averaging on them results in cutting off the singularity on the distance r_T.

Thus the resulting stationary state with the energy $E_{\rm tot}$ in the potential $U(r)$ becomes non-singular and therefore physical.

The exact real energy $E_{\rm tot}$ in the potential $U(r)$ can be varied by the variation of the bare ground state energy E in the potential $U_0(r)$. It occurs if to vary $U_0(r)$ at $r < a$ keeping the same $U(r)$. For each E the related $E_{\rm tot}$ has zero imaginary part as the ground state energy in the potential $U_0(r)$. This means that the energy spectrum of stationary states in the well in Fig. 3 is continuous and non-decaying. The above $E_{\rm tot}$ in Sec. 3.3 is just one of the total set of levels. There is no contradiction since this is not one particle quantum mechanics.

3.5. Potential well

The above scheme of formation of anomalous electron-photon state is also applicable when the electron moves in the potential of lattice sites in a solid [11]. In this case the thread is not infinite but is restricted by two lattice sites.

The smeared singularity is referred to the narrow (on the order of r_T) but smooth peak of ρ, shown in Fig. 3. This corresponds to the local enhancement of the electron kinetic energy $\sqrt{(mc^2)^2 + (\hbar c/r_T)^2} - mc^2 \simeq 1.9$ MeV in the thread region $r \lesssim r_T$.

One has to make a remark on the estimate (4). Generally it contains $\ln\sqrt{mc^2/U}$ where U is the scale of the potential energy [16]. For the hydrogen atom U coincides with the Rydberg energy resulting $\ln \hbar c/e^2$ in Eq. (4). In our case mc^2 is involved into the effective potential. So one can take the estimate $r_T \sim r_c\sqrt{e^2/\hbar c}$. The subatomic r_T is universal that is independent of atomic potentials which hardly contribute to r_T.

At distances $r_c < r$ nonrelativistic quantum mechanics is applicable. At $r_T < r < r_c$ one should use the Dirac formalism. At shorter distances the electron-photon interaction is substantial and the perturbation theory on $e^2/\hbar c$ does not work. This is an unusual case of enhancement of the electron-photon interaction within the narrow region. There is the known phenomenon of enhancement of the electron-photon interaction. It happens when the constant $e^2/\hbar c$ is multiplied by the large logarithm [5, 20, 21].

Since the state is stationary, its total energy (2) far from the thread is approximately the sum of electron one and zero point energy of photons. Close to the thread E_{tot} is redistributed between the above locally enhanced kinetic energy of the electron (the first term in (2)) and the local reduction of zero point energy of photons (the last two terms in (2)). The latter can be interpreted as the certain potential well sketched in Fig. 3. At $r_T < r$ this potential well is close to U in Fig. 2. That well can also be treated as smeared δ-function shown in Fig. 2. Analogous reduction of zero point photon energy occurs in formation of van der Waals (Casimir) forces [2–5].

As in van der Waals forces, the depth of the well in Fig. 3 can be estimated as $\hbar c/r_T$. In our case it is 2.4 MeV. This rough estimate of the related energy reduction, -2.4 MeV, corresponds to the local enhancement of the electron energy, 1.9 MeV, to compensate it.

4. Properties of anomalous states

Usually discrete electron levels in a given potential well become of finite width under the interaction with photons. The transition of the electron to the ground state is accompanied by photon emission. One can formally consider the exact state (with continuous energy) of the total electron-photon system as the electron in the ground state and a photon propagating to the infinity. This exact state is delocalized.

In our case the well is self consistently formed due to the electron-photon interaction. At $H_{e-ph} \neq 0$ smearing of the singular wave function is accompanied by broadening of the δ function part of the potential (Fig. 2) which goes over into the well in Fig. 3. The exact electron-photon states are localized (no photons propagating to the infinity). These states are of zero imaginary part of energy if to consider the electron-photon interaction only (Sec. 3.3, 3.4). Under these conditions emission of photons is impossible. States in the well in Fig. 3 can be classified as anomalous ones.

There is the qualitative explanation why anomalous state is not decaying. The region $r < r_T$ plays a role of the point where the electron is tightly connected to electromagnetic coordinates and is dragged by them. One can treat the electron to be localized at $r < r_T$. Under photon emission the region $r < r_T$ would oscillate increasing the electron kinetic energy. This prevents the electron to lose its total energy going to a lower level.

Anomalous electron-photon states are non-decaying (infinite lifetime) if to account for solely electron-photon interaction. But besides the substantial interaction with photons at $r < r_T$ there are weak interactions (electron-electron for example) leading to a small finite width of those states. As a result, the states in the well in Fig. 3 become not exact but long-living.

Anomalous states are of continuous energy spectrum (Sec. 3.3, 3.4) since the usual condition of absence of singularity, leading to levels quantization in quantum mechanics, is not imposed.

The scheme considered above is based on the clear subsequent steps which do not require detailed calculations. The state, which is singular in quantum mechanics, becomes cut off under the interaction with photons. It is established that

- there are long-living states in the narrow (10^{-11} cm) and deep (1 MeV) potential well,
- these long-living states are of continuous energy spectrum.

Electron transitions in the system of long-living states are accompanied by emission of X-ray or even MeV quanta since the well depth is on this order of magnitude. The above properties may provide a possibility for population inversion and therefore X-ray laser emission.

4.1. Extraction of zero point energy

One should make a note about the energy balance when zero point energy $\sum \hbar\omega/2$ is involved. Since $\omega = ck$ this sum is divergent because it is reduced to the integration $d^3k/(2\pi)^3$. The infinite energy of zero point oscillations corresponds to the rules of quantum electrodynamics [5].

When the electromagnetic field is not free (interaction with electrons, atoms or macroscopic bodies) the above k-integration should be with the additional factor ρ_k which is the density of states. At large k $\rho_k \to 1$. For free electromagnetic field $\rho_k = 1$. The last two terms in Eq. (2) correspond to the k-integration with the weight $(\rho_k - 1)$. The resulting integral is not strongly divergent.

Since ρ_k depends on coordinates, the last two terms in (2) are equivalent to the certain potential energy (Fig. 3) caused by the spatially dependent reduction of zero point energy. In the case of two atoms the analogous reduction corresponds to van der Waals (Casimir) potential well [2–5]. Therefore the energy of emitted quanta, under electron transitions in the well, comes from zero point electromagnetic energy (from vacuum).

4.2. Creation of anomalous electron-photon states

The typical size of the usual electron state in solids is 10^{-8} cm. The size of the anomalous state is on the order of $r_T \sim 10^{-11}$ cm. Therefore the perturbation, producing anomalous state from the usual one, should be varied on the distance of r_T. Otherwise the corresponding matrix element is small and the anomalous fraction in the total wave function is negligible [11].

The charge density, varying in space on the typical distance r_T, can be created by an incident charged particle which is reflected by lattice sites of the solid. The resulting density, related to such particle, is due to interference of incident and reflected waves. This charge density is approximately proportional to $\cos(2r\sqrt{2ME_p}/\hbar)$ where M is the particle mass and E_p is its energy [11]. For example, if to use deuterons, $M \simeq 3.346 \times 10^{-24}$ g, one can estimate

$$\text{charge density} \sim \cos\left[1.96\frac{r}{r_T}\sqrt{E_p(\text{keV})}\right]. \qquad (5)$$

If to use electrons, the same wave length corresponds to high electron energy on the order of 1 MeV .

We see that one can bombard the surface of the solid by ions with the energy of approximately 1 keV to produce anomalous electron states within the depth of ions penetration. This correlates with conditions of the glow discharge experiments [12].

5. Post-irradiation emission of photons

In this section non-rigorous, rather heuristic, arguments are considered which help to understand the post-irradiation X-ray laser bursts in experiments [12, 13].

Anomalous electron-photon state in a solid is not infinite in space. It is the thread of the diameter 10^{-11} cm between two lattice sites [11]. An electron in the anomalous state is distributed along that thread. The related charge density varies on the distance of 10^{-11} cm. This serves as a perturbation for formation either of other anomalous state by conductivity electrons or by addition of a new electron to the existing anomalous state. When irradiation of the surface by ions is switched off, remaining anomalous states are the only source for formation of new anomalous ones.

Anomalous state with one electron can accept the second one to the same thread. Since the thread length is on the order of 10^{-8} cm, the second electron cannot qualitatively change the anomalous state which already exists. This is due to smallness of the electron-electron interaction. In other words, the second electron cannot convert the localized electron-photon state into delocalized one (Sec. 4.) that is with a photon propagating to the infinity.

So (two electrons)-photon state is with zero width. Subsequent filling out the thread by other electrons do not delocalize the state until the number of electrons is not large. In this case every new electron can be accounted for by the perturbation theory with respect to the electron-electron interaction.

When the number of electrons in the thread becomes large, the mutual influence of electrons is not a small perturbation. Indeed, in this case all collected electrons can oscillate along the thread providing electromagnetic radiation. This means a conversion of the localized state into delocalized one that is with a finite width of levels. The total state becomes decaying when the number of collected electrons, increasing in time, exceeds the certain critical value. After that the states, which were initially non-decaying, emit an avalanche of photons with continuous spectrum.

The common phase in the laser radiation is similar to the order parameter in phase transitions. Phases, direction, and polarization of all individual photons in the emitted avalanche are not random. This gives rise to the collimated beams observed in experiments [12, 13] (post irradiation emission).

The collimated beam exists during the time when electrons in anomalous states emit photons and undergo to lower levels. After that a new anomalous states start to be formed. This continues periodically as observed in [12, 13]. Remaining distortions of the lattice sites after photon emission look like the certain impurities in the crystal lattice.

As noted in Sec. 1., zero point energy is converted into one of emitted photon in formation of H_2 molecule. Suppose that there is a number N of pairs of hydrogen atoms. Each pair has the tendency to get converted into H_2 and there is the subsequent formation of hydrogen molecules with photon emission. If $N \to \infty$ we have the steady process of conversion of infinite zero point energy into emitted photons ("energy from nothing").

This is similar to the process considered in this paper since there is also subsequent formation of potential wells (Fig. 3) at various points of the solid. At present it is unclear how long this process of the vacuum energy extraction will continue. In experiments [12, 13] the time of 20 hours was established.

6. Discussions

Well reproducible experiments [12, 13] look paradoxical because of the mysterious phenomenon of X-ray laser bursts emitted from the solid. These macroscopic bursts are suddenly emitted by a conventional metal disconnected long time ago from any external energy source. An explanation in terms of combination of known effects does not work. Necessary condition for laser emission, long-living states, seemed impossible. Indeed, an excitation of nuclear degrees of freedom by keV ions is not effective. Also the lifetime of keV electrons is short.

In this paper the revealed mechanism is described which is likely the base of the phenomena observed in Refs. 12

and 13. This mechanism is based on anomalous electron-photon states in quantum electrodynamics. In these states the strong electron-photon influence occurs since even small “vibrations” of the electron are able to cut off the singularity. Anomalous states are connected with the reduction of zero point electromagnetic energy resulting in the deep (on the order of MeV) potential well for the electron. Formation of anomalous state allows to explain generation of high-energy photons which can be even more energetic than exciting ions [12, 13]. There is no violation of the total energy balance since conversion of zero point electromagnetic energy enters the game. It was shown that the energy of bombarding ions of ~ 1 keV is optimal for creation of anomalous states. This correlates with results of Refs. 12 an 13.

The anomalous states in the well are long-living. This explains the paradoxical phenomenon of keV-laser radiation when long-living states of this energy were unexpected. The radiation occurs due to the conversion of vacuum energy.

The energy of the MeV range is a typical nuclear scale. Nevertheless it appears in the electron-photon process. High energy photons, generated inside the solid, can cause nuclear transmutations of lattice nuclei. Misinterpretation of such experiments is possible by attributing the energy source to nuclear reactions. These reactions are impossible here since the energies of incident particles (1 keV), phonons (0.01 eV), and electrons (1 eV) inside a solid are too low compared to MeV. It is not real to expect phonons in a solid to suddenly get collected into the MeV energy.

One can suppose that after switching off the irradiation by ions, remaining anomalous states, having the short range variation of the charge density, serve as a source for further creation of anomalous states. This mechanism would generally explain X-ray laser radiation from a “dead” sample, that is from a piece of metal which is out of external influences.

In glow discharge experiments one can measure a difference between the thermal energy produced and the supplied electric energy. There is no theoretical ban for positive excess energy. Positive excess energy coming from vacuum may have practical applications.

There is a question on creation of anomalous states in a biological matter, for example, in DNA. In that case one can expect an X-ray generation by a living system and transmutation of elements inside it.

7. Conclusions

An unusual phenomenon, observed in experiments, is studied. X-ray laser bursts of keV energy are emitted from a metal where long-living states, resulting in population inversion, are totally unexpected. Anomalous electron-photon states are revealed to be formed inside the metal. These states are associated with narrow, 10^{-11} cm, potential well created by the local reduction of zero point electromagnetic energy. In contrast to analogous van der Waals potential well, leading to attraction of two hydrogen atoms, the depth of the anomalous well is on the order of 1 MeV. The states in that well are long-living which results in population inversion and subsequent laser generation observed. The X-ray emission, occurring in transitions to lower levels, is due to the conversion of zero point electromagnetic energy.

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