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Analysis of nonrelativistic particles in noncommutative phase-space under new scalar and vector interaction terms

A. A. Safaei[a], H. Panahi[a,b,*], and H. Hassanabadi[c]
[a]*Department of Physics, University Campus 2,*
University of Guilan, Namjoo Avenue, P.O. Box 413351914, Rasht, Iran.
[*]*e-mail: t-panahi@guilan.ac.ir*
[b]*Department of Physics, Faculty of Science,*
University of Guilan, Namjoo Avenue, P.O. Box 413351914, Rasht, Iran.
[c]*Faculty of Physics, Shahrood University of Technology, Shahrood, Iran.*

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The Schrödinger equation in noncommutative phase space is considered with a combination of linear, quadratic, Coulomb, and inverse square terms. Using the quasi exact ansatz approach, we obtain the energy eigenvalues and the corresponding wave functions. We discuss the results for various values of θ and $\bar{\theta}$ in noncommutative phase space through various figures.

Keywords: Schrödinger equation; noncommutative phase space; Anstaz method.

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1. Introduction

In the last few years, there has been increasing interest in the study of noncommutative space (NCS) and noncommutative phase space (NCPS) due to their uses in several branches of physics including quantum field theories, string theories [1-5], relativistic and nonrelativistic quantum mechanics [6-17].

Hassanabadi *et al.* have studied the nonrelativistic [18] and relativistic quantum mechanics in NCS [19]. They also studied the $q-$deformed super statistics of the Schrödinger equation in commutative and noncommutative spaces with a magnetic field [20] and the two-dimensional Dirac equation for a fermion moving under Kratzer potential in the presence of an external magnetic field [21]. The two-dimensional harmonic oscillator in commutative and noncommutative spaces within the framework of minimal length quantum mechanics has been investigated by Ikot *et al.* in Ref. [22]. The noncommutative (2+1)-dimensional Duffin-Kemmer-Petiau oscillator under a magnetic field in minimal length formalism is studied in Ref. [23].

In Ref. [24], a general noncommutative quantum mechanical system in a central potential in two dimensions has been studied. The authors have shown that for large values of the anticommutative parameter, the system is equivalent to a commutative system described by the potential that is related to a two-dimensional harmonic oscillator and the $z-$component of the angular momentum. In Ref. [25], the description of nonrelativistic electron of mass m on a plane subject to a strong perpendicular magnetic field B in the lowest Landau level has been studied in noncommutative coordinates.

In other words, by imposing the modified commutator for momenta, it is seen that the Hamiltonian of the free particle becomes equivalent to that of the conventional Landau problem and the noncommutative parameters of momenta play the role of the magnetic field orthogonal to the plane. The authors have shown that in the $m/B \rightarrow 0$ limit, only the lowest Landau level is accessible and the coordinates on the plane appear as canonically conjugate dynamical variables [26-28]. A study on two-dimensional Hamiltonian in the noncommutative phase space has been done in Ref. [29], and the authors have introduced a rotation operator, which leaves the Hamiltonian invariant and have shown that the rotationally invariant Hermitian quadratic form of the coordinates and momenta generate a new abelian three-dimensional Lie algebra corresponding to sl(2,R) or su(2) algebra according to the critical point of noncommutative parameters $\theta\bar{\theta} < \hbar^2$ or $\theta\bar{\theta} > \hbar^2$. A similar study on the noncommutative quantum mechanics of a charged particle on plane and sphere which is subject to a magnetic field with a harmonic oscillator has been done on Ref. [30] and it is shown that there is an interesting interplay of the magnetic field B and the noncommutative parameter θ, with a critical point at $B\theta = 1$ where the density of states becomes infinite.

Now in this paper, we study a nonrelativistic particle in NCPS in the presence of an external magnetic field for a combination of linear and quadratic terms plus scalar and vector Kratzer potentials. This potential is a generalization of Cornell, Killingbeck, and Kratzer-type interactions and it is used to describe the atomic, molecular structure and thus plays an important role in quantum calculations [31-34]. It is also one of the rare potentials of quantum systems which is exactly solvable. We then compare the effect of an external magnetic field and noncommutative parameters on the spectrum of the system.

The paper is organized as follows: In Sec. 2, we briefly introduce the basic formulae of NC algebra in quantum mechanics. In Sec. 3, we study the Schrödinger equation in the presence of a uniform magnetic field for the mixed potentials in NCPS. Next, the corresponding energy spectra and the wave functions are derived. In Sec. 4, we study the effects of the NC parameter in the problems on the energy spectrum and discuss graphically. Finally, we present the results in our conclusion.

2. Noncommutative formalism

The momentum and position operators p^i and x^i satisfy the Heisenberg algebra in the commutative quantum mechanics as

$$[x^i, x^j] = [p^i, p^j] = 0, \quad [x^i, p^j] = i\delta^{ij}. \tag{1}$$

The NCPS algebra is [35]

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij}, \quad [\hat{p}^i, \hat{p}^j] = i\bar{\theta}^{ij}, \\ [\hat{x}^i, \hat{p}^j] = i\hbar_{\text{eff}}\delta^{ij}, \tag{2}$$

where $\hbar_{\text{eff}} = (1 + [\theta\bar{\theta}/4])$ and θ^{ij}, $\bar{\theta}^{ij}$ are the anti-symmetric constant tensors defined as

$$\theta^{ij} = \varepsilon^{ijk}\theta_k, \quad \theta_k = (0,0,0), \\ \bar{\theta}^{ij} = \varepsilon^{ijk}\bar{\theta}_k, \quad \bar{\theta}_k = (0,0,\bar{\theta}). \tag{3}$$

Introducing the following transformation, we can realize the deformed algebra

$$\hat{x}^i \to \lambda x^i - \frac{\theta^{ij}p_j}{2\lambda}, \\ \hat{p}^i \to \lambda p^i - \frac{\bar{\theta}^{ij}x_j}{2\lambda}, \tag{4}$$

where λ is a scaling constant factor. Also, the terms $\bar{\theta}^{ij}$, θ^{ij} and λ satisfy [36, 37]

$$\bar{\theta}^{ij}\theta^{ij} = \theta^{ij}\bar{\theta}^{ij} = \theta\bar{\theta}I = 4\lambda^4(\lambda^2 - 1)I, \tag{5}$$

where I is the unity matrix. By considering $\lambda = 1$, $\bar{\theta} = 0$, the noncommutative space is recovered from the noncommutative phase space. Then, the momentum and position operators in NCPS can be written in terms of the commutative space as

$$x^{(NC)} = x^{(C)} - \frac{1}{2}\theta p_y^{(C)}, \quad y^{(NC)} = y^{(C)} + \frac{1}{2}\theta p_x^{(C)}, \tag{6a}$$

$$p_x^{(NC)} = p_x^{(C)} + \frac{1}{2}\bar{\theta}y^{(C)}, \quad p_y^{(NC)} = p_y^{(C)} - \frac{1}{2}\bar{\theta}x^{(C)}. \tag{6b}$$

In the NCS, the Moyal-Weyl product can be replaced with the usual product as [38,39]

$$(f * g)(x) = f(x)e^{(i/2)\theta_{ij}\partial_i^x\partial_j^x + (i/2)\bar{\theta}_{ij}\partial_i^p\partial_j^p}g(x) \\ = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{i}{2}\theta_{ij}\partial x_i\partial x_j + \frac{i}{2}\bar{\theta}_{ij}\partial p_i\partial p_j\right)^n \\ \times f(x,p)g(x,p) = f(x,p)g(x,p) \\ + \frac{i}{2}\theta_{ij}\partial_i^x f(x,p)\partial_j^x g(x,p) \\ + \frac{i}{2}\bar{\theta}_{ij}\partial_i^p f(x,p)\partial_j^p g(x,p) \\ + O(\theta^2) + O(\bar{\theta}^2), \tag{7}$$

where $f(x,p)$, $g(x,p)$ are two arbitrary functions. In NCS, the product can be replaced by a Bopp's shift. By considering the following functions

$$f(x,p) = x, \quad g(x,p) = \psi(x), \tag{8a}$$

we obtain

$$(x * \psi) = x\psi + \frac{i}{2}\theta_{xy}\partial_x x\partial_y\psi = \left(x - \frac{1}{2}\theta_{xy}p_y\right)\psi. \tag{8b}$$

Also by choosing

$$f(x,p) = y, \quad g(x,p) = \psi(x), \tag{9a}$$

we get

$$(y * \psi) = y\psi + \frac{i}{2}\theta_{yx}\partial_y y\partial_x\psi \\ = \left(y + \frac{1}{2}\theta_{xy}p_x\right)\psi. \tag{9b}$$

It is evident from Eqs. (8b) and (9b) that the Bopp's shift is an exact equivalent to the star product. Therefore, instead of solving problems in NC space by using the star product procedure, we replace the star-product with the usual product by making a Bopp's shift.

3. Schrödinger equation in the presence of a magnetic field in noncommutative phase-space

Let us first recall that to analyze a charged particle in a magnetic field, the momentum is transformed as $\vec{p} \to (\vec{p} - [e/c]\vec{A})$. Here, we consider a uniform magnetic field of the form $\vec{B} = (0,0,B)$.

The Schrödinger equation in the presence of a magnetic field in noncommutative phase space is given as

$$\left(\frac{1}{2M}\left[\left\{p_x^{(NC)} - \frac{e}{c}A_x^{(NC)}\right\}\hat{i} + \left\{p_y^{(NC)} - \frac{e}{c}A_y^{(NC)}\right\}\hat{j}\right]^2 \\ + V^{(NC)}(\vec{r})\right)\psi(\vec{r}) = E\psi(\vec{r}), \tag{10}$$

where A_x and A_y are the vector potential components and $V(\vec{r})$ is the scalar potential.

We introduce the symmetric gauge

$$\vec{A}^{(NC)} = \left(-\frac{1}{2}B_0 y^{(NC)}, \frac{1}{2}B_0 x^{(NC)}, 0\right),$$

where B_0 is the intensity of the field and the direction of the magnetic field is considered along the $z-$axis ($\vec{B} = B_0\hat{k}$). Now, Eq. (10) appears as

$$\frac{1}{2M}\left(\left[p_x^{(NC)}\right]^2 + \left[p_y^{(NC)}\right]^2 + \frac{e^2B_0^2}{4c^2} \times \left[\left\{x^{(NC)}\right\}^2 + \left\{y^{(NC)}\right\}^2\right] - \frac{eB_0}{c} \times \left[x^{(NC)}p_y^{(NC)} - y^{(NC)}p_x^{(NC)}\right]\right)\psi(\vec{r}) = \left(E - V^{(NC)}(\vec{r})\right)\psi(\vec{r}). \quad (11)$$

In NCPS, by substitution of Eq. (6) into Eq. (11), we obtain

$$\left(\left[1 + \frac{e^2B_0^2\theta^2}{16c^2} + \frac{eB_0\theta}{2c}\right]\left[\left\{p_x^{(C)}\right\}^2 + \left\{p_y^{(C)}\right\}^2\right] + \left[\frac{\bar{\theta}^2}{4} + \frac{e^2B_0^2}{4c^2} + \frac{eB_0\bar{\theta}}{2c}\right]\left[\left\{x^{(C)}\right\}^2 + \left\{y^{(C)}\right\}^2\right] - \left[\bar{\theta} + \frac{e^2B_0^2\theta}{4c^2} + \frac{eB_0}{c} + \frac{eB_0\theta\bar{\theta}}{4c}\right] \times \left[x^{(C)}p_y^{(C)} - y^{(C)}p_x^{(C)}\right] + 2M\left(V^{(NC)}(r) - E\right)\right)\psi(\vec{r}) = 0. \quad (12)$$

We consider the scalar potential $V(r)$ as

$$V = ar^2 + br + \frac{c}{r} + \frac{d}{r^2}, \quad (13)$$

which is a generalization of Cornell, Killingbeck, and Kratzer-type interactions.

The potential in NCPS is written as [40]

$$V^{(NC)}(r) = V(r) + \frac{1}{2}(\vec{\theta}\times\vec{p})\cdot\nabla V(r) + O(\theta^2). \quad (14)$$

Substituting Eq. (13) into Eq. (14) and by doing some calculations, we obtain the noncommutative potential up to the first order in θ as

$$V^{(NC)}(r) = a(r^2 - \theta L_z) + b\left(r - \frac{1}{2r}\theta L_z\right) + c\left(\frac{1}{r} + \frac{\theta L_z}{2r^3}\right) + d\left(\frac{1}{r^2} + \frac{\theta L_z}{r^4}\right), \quad (15)$$

which, upon substitution in Eq. (12), yields

$$\left(\left[1 + \frac{e^2B_0^2\theta^2}{16c^2} + \frac{eB_0\theta}{2c}\right]\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{1}{r^2}\frac{d^2}{d\varphi^2}\right] + \left[\bar{\theta} + \frac{e^2B_0^2\theta}{4c^2} + \frac{eB_0}{c} + \frac{eB_0\theta\bar{\theta}}{4c} + 2Ma\theta + \frac{Mb\theta}{r} - \frac{Mc\theta}{r^3} - \frac{-2Md\theta}{r^4}\right]L_z - \left[\frac{\bar{\theta}^2}{4} + \frac{e^2B_0^2}{4c^2} + \frac{eB_0\bar{\theta}}{2c} + 2Ma\right]r^2 - 2Mbr - \frac{2Mc}{r} - \frac{2Md}{r^2} + 2ME\right)\psi(\vec{r}) = 0. \quad (16)$$

Now we choose the wave function as

$$\psi(\vec{r}) = u(r)r^{-1/2}e^{il\varphi}, \quad (17)$$

where l is an integer parameter. Hence, Eq. (16) appears as

$$\frac{d^2u(r)}{dr^2} + \left(-A_1r^2 + A_2r - A_3 + \frac{A_4}{r} + \frac{A_5}{r^2} - \frac{A_6}{r^3} - \frac{A_7}{r^4}\right)u(r) = 0, \quad (18)$$

where

$$A_1 = \frac{1}{\eta}\left(\frac{\bar{\theta}^2}{4} + \frac{e^2B_0^2}{4c^2} + \frac{eB_0\bar{\theta}}{2c} + 2Ma\right),$$
$$A_2 = -\frac{2Mb}{\eta},$$
$$A_3 = -\frac{1}{\eta}\left(\left[\bar{\theta} + \frac{e^2B_0^2\theta}{4c^2} + \frac{eB_0}{c} + \frac{eB_0\theta\bar{\theta}}{4c} + 2Ma\theta\right]l + 2ME\right),$$
$$A_4 = -\frac{2Mc}{\eta} + \frac{Mb\theta l}{\eta},$$
$$A_5 = -\frac{2Md}{\eta} + \frac{1}{4} - l^2,$$
$$A_6 = \frac{Mcl\theta}{\eta}$$
$$A_7 = \frac{2Mdl\theta}{\eta}, \quad (19)$$

with

$$\eta = \left(1 + \frac{e^2B_0^2\theta^2}{16c^2} + \frac{eB_0\theta}{2c}\right). \quad (20)$$

The solution of Eq. (18) is considered as

$$u(r) = f_n(r)u_0(r), \quad (21)$$

where $f_n(r)$ is defined as

$$f_n(r) = \begin{cases} 1 & n = 0 \\ \Pi_{i=1}^{n}(r - \alpha_i^n) & n \geq 1 \end{cases}. \quad (22)$$

We use the ansatz technique as [41]

$$u_0(r) = \exp\left(\alpha r^2 + \beta r + \gamma Lnr + \frac{\lambda}{r}\right). \quad (23)$$

Equation (18) takes the form

$$4\alpha^2 r^2 + 4\alpha\beta r + (2\alpha + 4\alpha\gamma + \beta^2) + \frac{(2\beta\gamma - 4\alpha\lambda)}{r} + \frac{(\gamma^2 - \gamma - 2\beta\lambda)}{r^2} + \frac{(2\lambda - 2\gamma\lambda)}{r^3} + \frac{\lambda^2}{r^4} + \Big(-A_1 r^2 + A_2 r - A_3 + \frac{A_4}{r} + \frac{A_5}{r^2} - \frac{A_6}{r^3} - \frac{A_7}{r^4}\Big) = 0. \quad (24)$$

We can obtain the following condition from Eq. (24)

$$-A_1 + 4\alpha^2 = 0 \Rightarrow \alpha = -\frac{\sqrt{A_1}}{2},$$
$$A_2 + 4\alpha\beta = 0 \Rightarrow \beta = \frac{A_2}{2\sqrt{A_1}},$$
$$-A_3 + 2\alpha + \beta^2 + 4\alpha\gamma = 0,$$
$$A_4 + 2\beta\gamma - 4\alpha\lambda = 0,$$
$$A_5 + (\gamma^2 - \gamma - 2\beta\lambda) = 0,$$
$$-A_7 + \lambda^2 = 0 \Rightarrow \lambda = -\sqrt{A_7},$$
$$-A_6 + (2\lambda - 2\gamma\lambda) = 0 \Rightarrow \gamma = \frac{A_6}{2\sqrt{A_7}} + 1, \quad (25)$$

which gives the relation of energy as

$$-A_3 - \sqrt{A_1} + \frac{A_2^2}{4A_1} - 2\sqrt{A_1}\left(\frac{A_6}{2\sqrt{A_7}} + 1\right) = 0. \quad (26)$$

Using Eq. (19), we can obtain the energy in the compact form

$$E = -\frac{1}{2M}\left(\bar{\theta} + \frac{e^2 B_0^2 \theta}{4c^2} + \frac{eB_0}{c} + \frac{eB_0\theta\bar{\theta}}{4c} + 2Ma\theta\right)l + \frac{\sqrt{\eta\gamma}}{2M} - \frac{Mb^2}{2\gamma} + \frac{\sqrt{\eta\gamma}}{M}\left(\sqrt{\frac{Mc^2 l\theta}{8\eta d}} + 1\right), \quad (27)$$

where

$$\gamma = \left(\frac{\bar{\theta}^2}{4} + \frac{e^2 B_0^2}{4c^2} + \frac{eB_0\bar{\theta}}{2c} + 2Ma\right). \quad (28)$$

The wave function of the system is also derived as

$$u(r) = \exp\left(-\frac{\sqrt{A_1}}{2}r^2 + \frac{A_2}{2\sqrt{A_1}}r + \left[\frac{A_6}{2\sqrt{A_7}} + 1\right]Lnr - \frac{\sqrt{A_7}}{r}\right), \quad (29)$$

which, using Eq. (19) appears as

$$u(r) = \exp\left(-\frac{1}{2}\sqrt{\frac{\gamma}{\eta}}r^2 - \frac{Mb}{\sqrt{\eta\gamma}}r + \left[\sqrt{\frac{Mc^2 l\theta}{8\eta d}} + 1\right] \times Lnr - \sqrt{\frac{2Mdl\theta}{\eta}}\frac{1}{r}\right). \quad (30)$$

4. Results and discussion

We solved the Schrödinger equation in NCPS under an external magnetic field. After some rather cumbersome algebra, we found the energy eigenvalues and eigen functions of the system. According to Eq. (27), the ground-state energy is related to the noncommutative parameter and the strange of the magnetic field. In Fig. 1, we have plotted the energy of the system in terms of the magnetic field for three different values of the parameter $\bar{\theta}$. It is shown that the total energy monotonically decreases at a low magnetic field and then monotonically increases at higher magnetic field values for each value of $\bar{\theta}$. By increasing B_0, the effect of noncommutativity increases.

In Figs. 3-5, we have investigated the energy variation versus different potential parameters. In Fig. 3, it can be seen

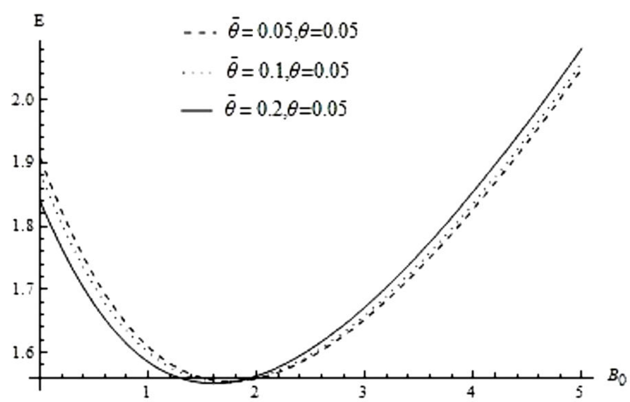

FIGURE 1. Total energy vs. B_0 for different values of $\theta = 0.05, (\bar{\theta} = 0.05, 0.1, 0.2)$.

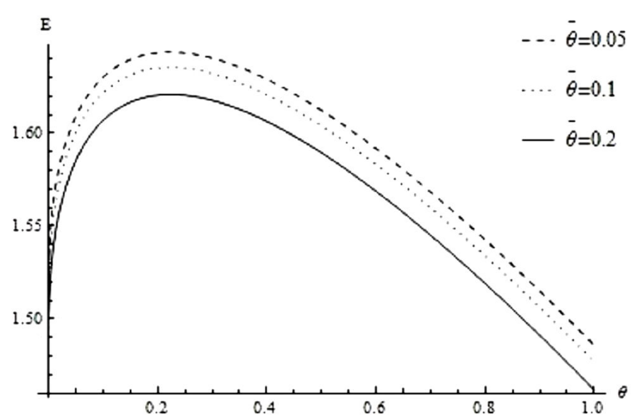

FIGURE 2. Total energy vs. θ for different $\bar{\theta}$ values.

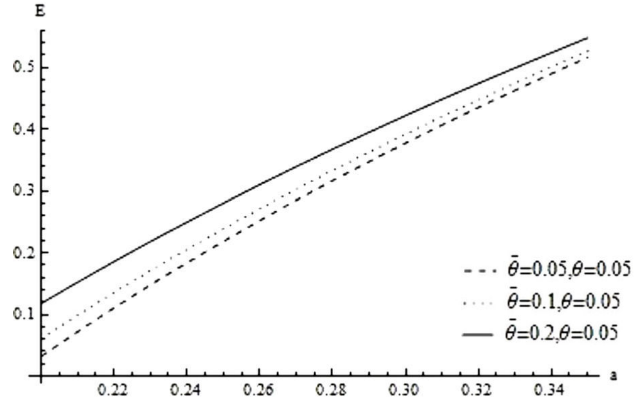

FIGURE 3. Total energy vs. a for different $\bar{\theta}$ values.

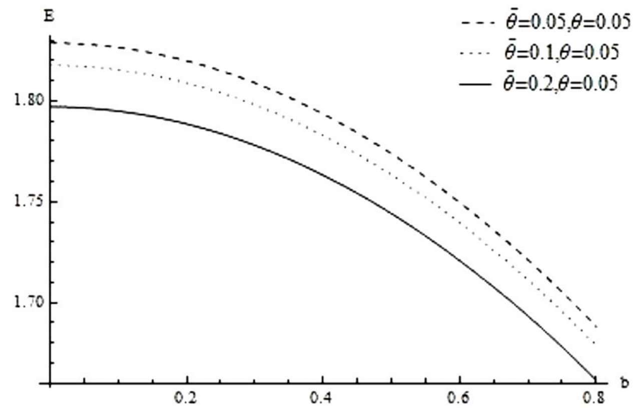

FIGURE 4. Total energy vs. b for different $\bar{\theta}$ values.

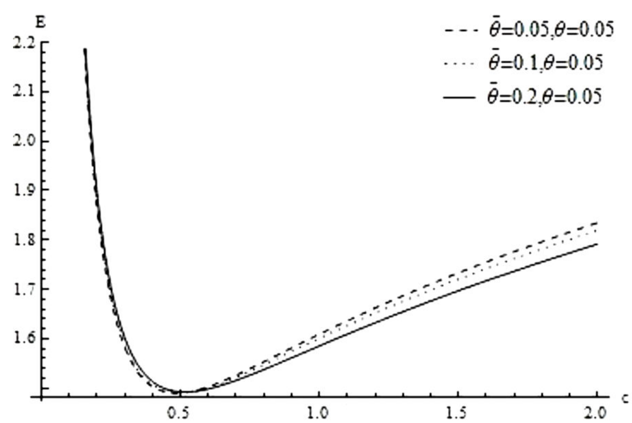

FIGURE 5. Total energy vs. c for different $\bar{\theta}$ values.

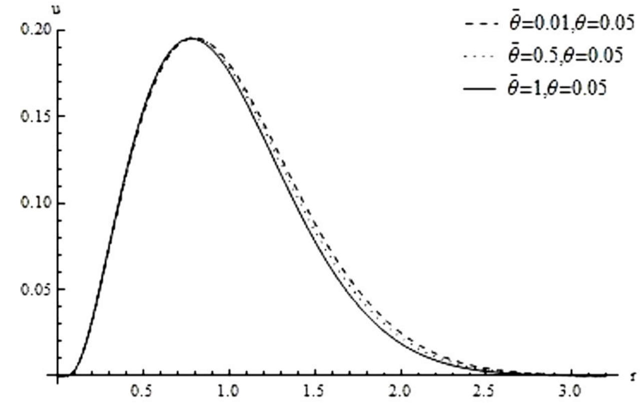

FIGURE 6. Wavefunction vs. r for different values ($\bar{\theta} = 0.001, 0.5, 1$).

that by increasing the parameter a, energy increases, whereas the energy decreases for increasing values of the parameter b as shown in Fig. 4. Also, it is observed that in Fig. 5 by increasing the parameter c, the total energy first decreases and then rises.

In Fig. 6, we have plotted the ground state wave function and it is seen that the particle has been localized at the region $0 \leq r \leq 3$. It is also seen that the amplitude of wave function remains the same when decreasing $\bar{\theta}$ and the width of the wave function becomes wider.

5. Conclusion

In this article, after introducing noncommutative phase space quantum mechanics, we derived the Schrödinger equation in this space for a particle under a potential containing linear, quadratic terms plus scalar and vector Kratzer potentials, *i.e*. a generalization of Cornell, Killingbeck, and Kratzer-type interactions, in the presence of an external magnetic field. We used the phase space coordinates based on Bopp's shift definition and solved the problem in the usual product. We obtained the spectrum and wave function of the system and plotted the energy of the system for different values of noncommutative parameters θ and $\bar{\theta}$. The procedure can be applied for other types of potentials or in relativistic wave equations such as Klein-Gordon or Dirac equations.

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