Prediction of rms charge radius of proton using proton-proton elastic scattering data at $\sqrt{s} = 2.76$ TeV

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Received 3 October 2020; accepted 18 January 2021

Using proton-proton elastic scattering data at $\sqrt{s} = 2.76$ TeV and squared four-momentum transfer $0.36 < -t < 0.76$ (GeV/c)^2 for 13 σ_{Beam} distance and $0.07 < -t < 0.46$ (GeV/c)^2 for 4.3 σ_{Beam} distance, the electromagnetic form factor of proton is predicted. The simplest version of Chou-Yang model is employed to extract the form factor by fitting experimental data of differential cross section from TOTEM experiment (for 13 σ_{Beam} and 4.3 σ_{Beam} distance) to a single Gaussian. Root mean square charge radius of proton is calculated using this form factor and is found to be equal to 0.91 fm and 0.90 fm, respectively. This result is in good agreement with experimental data and theoretically predicted values.

Keywords: Chou-Yang model; p-p scattering; electromagnetic form factor of proton.

DOI: https://doi.org/10.31349/RevMexFisE.67.491

1. Introduction

The structure of particles can be probed with the help of scattering experiments. The high energy scattering processes are now approachable present us with an opportunity to examine the hadronic structure at higher energies [1-9]. Among hadrons, the proton structure has remained a topic of interest between researchers since its discovery. Proton's radius is a prime problem in the study of its structure. The root-mean-square (rms) radius of a proton can be experimentally measured by two methods; electron proton scattering [10] and atomic spectroscopy technique. In hydrogen spectroscopy, two methods are adopted: one by using atomic hydrogen [11] and a second by using muonic hydrogen [12]; both of these methods give contradictory results, giving rise to the so-called "proton radius puzzle". There are many theoretical approaches to find out the rms radius of proton, including MIT Bag model [13], self-consistent model [14], by using Lattice QCD [15-17], etc.

The form factor also plays a dynamic role in the study of hadronic structure. It is related to the distribution of matter inside a hadron. Theories claiming to explain the structure of hadrons must be able to calculate their form factors from first principles. Continuous efforts of decades led the researchers to obtain the form factors of proton from different calculation schemes, as discussed in [18-22]. Experimentally, the magnitude of the form factor is determined by the ratio of the measured cross-section to the Mott cross-section: $(d\sigma/d\Omega)_{\text{exp}} = (d\sigma/d\Omega)^*_{\text{Mott}} \cdot |F(q^2)|^2$. One therefore measures the cross-section for a fixed beam energy at various angles (and thus different values of $|q|$) and divides by the calculated Mott cross-section.

In this work, proton-proton elastic scattering data from TOTEM experiment [9] is used to calculate the proton form factor employing the simplest version of Chou-Yang model. The Chou-Yang model [23,24] is a geometrical model. In this model two hadrons are considered to be scattering elastically and are supposed to be translucent objects passing through each other without attenuation. The differential cross section ($d\sigma/dt$) and total cross-section (σ) are the two measuring quantities involved in such processes. Let us consider two hadrons A and B, scattering elastically ($A+B \to A+B$). Let $a_{AB}(t)$ represent the asymptotic scattering amplitude. Here we are interested in a case, where Gaussian (in t) could be used to approximate the differential cross section. In this situation the differential cross section is written as

$$\frac{d\sigma}{dt} = \alpha e^{\beta t}. \tag{1}$$

Hadron's radius is associated to the form factor by the relation $F_A(t) = 1 - (1/6\hbar^2)t\langle r^2\rangle$. The product of both form factors ($F_A(t)$ and $F_B(t)$) of two scattering hadrons A and B is given as:

$$F_A(t)F_B(t) = \text{constant} \times \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{\alpha}{\pi}\right)^{n/2}\left(\frac{1}{\beta}\right)^{n}\frac{\beta}{n}e^{\beta t/2n}, \tag{2}$$

(for details see Ref. [25]). This relation is very useful for finding out the form factor of scattering hadrons. Many form factors of proton were suggested by researchers at lower values of $\sqrt{s}$ [26-28].

2. Calculations

In this work recent data of elastic proton-proton scattering at $\sqrt{s} = 2.76$ TeV from TOTEM experiment [9] is used. Al-

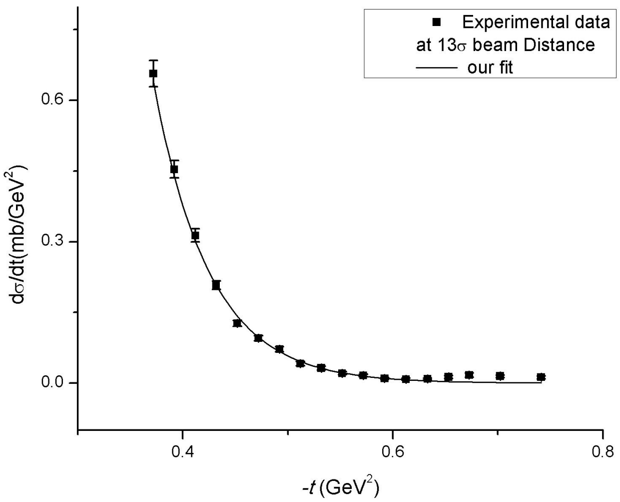

FIGURE 1. Fitting of differential cross section data of proton-proton elastic scattering at $\sqrt{s}=2.76$ TeV (for 13 σ_{Beam} distance) to a single Gaussian.

though the same procedure has been adopted in [18] for finding out form factor and rms radius of proton by using the data from TOTEM at $\sqrt{s} = 8$ TeV. Here elastic proton-proton data of two different kinematical regions is analyzed at $\sqrt{s} = 2.76$ TeV which would be highly beneficial for getting precise results. The experimental setup of TOTEM experiment is explained in Ref. [9], where one of the data sets of differential cross section has been obtained by placing Roman Pot detectors at 13 times the transverse beam size (σ_{Beam}). This setup allowed to measure elastic differential cross section at $t = 0.36$ GeV^2 to 0.74 GeV^2. The second data set was obtained by inserting Roman Pot detectors at 4.3 times the transverse beam size and measured elastic differential cross section at $t = 0.07$ GeV^2 to 0.45 GeV^2. The differential cross section data plotted against $-t$ for 13 σ_{Beam} and 4.3 σ_{Beam} distance, and fitted to a single Gaussian, shown in Fig. 1 and 2, respectively.

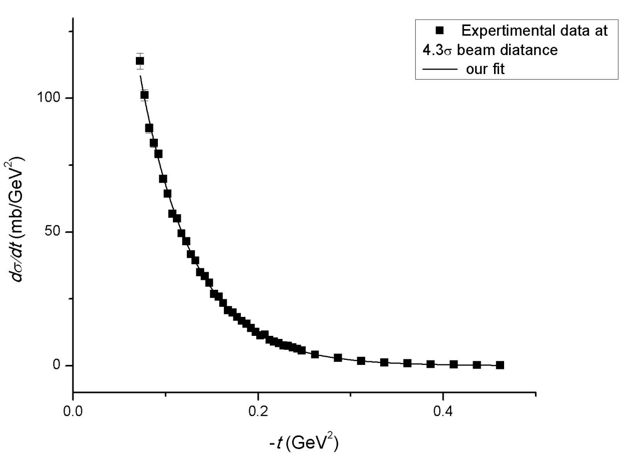

FIGURE 2. Fitting of differential cross section data of proton-proton elastic scattering at $\sqrt{s}=2.76$ TeV (for 4.3 σ_{Beam} distance) to a single Gaussian.

TABLE I. Fitted parameters for two set of data, *i.e.*, for 13 σ_{Beam} and 4.3 σ_{Beam} distance

j	α_j (mb^{-1}/GeV / c)	β_j (GeV)$^{-2}$	Adj. R-squared for fit
1	683.06 ± 228.29	-18.75 ± 0.76886	0.96907
2	375.62 ± 4.09935	-17.15 ± 0.06806	0.99914

TABLE II. Computed values of a_{ji} and b_{ji}.

	For 13 σ_{Beam} distance $j=1$		For 4.3 σ_{Beam} distance $j=2$	
i	a_{ji}	b_{ji}	a_{ji}	b_{ji}
1	-2.898290	-18.75452	-1.742644	-17.1527
2	14.74531	-9.37726	10.93454	10.93454

TABLE III. Computed values of δ_j.

	δ_j
For 13 σ_{Beam} Distance	0.2905
For 4.3 σ_{Beam} Distance	0.3298

The most appropriate values of α_1, β_1 and α_2, β_2 are found. Where α_1, β_1 are the fitted parameters for differential cross section data at 13 σ_{Beam} distance and are for 4.3 σ_{Beam} distance. These fitted parameters given in Table I.

The measure of goodness of fit is determined by R-square, given in Table I. Which shows that our fit is 100% successful in both set of data. These values of α and β are used in Eq. (2) directly, and a computer program is used to solve this equation for finite values of n. The square of form factors of proton is found to be equal to

$$(F_{jp}(t))^2 = \delta_j \sum_{i=1}^{2} a_{ji} e^{b_{ji} t}. \tag{3}$$

Here $j = 1$ and 2 for differential cross section data at 13 σ_{Beam} and 4.3 σ_{Beam} distance respectively. δ_j is the normalization constant. Computed values are given in Table II and Table III.

Using electromagnetic form factor from Eq. (3) we can easily compute rms charge radius of proton by using following relation $\langle r^2 \rangle = 6\hbar^2 (dF(t)/dt)|_{t=0}$. We have therefore computed $\langle r_p \rangle = 0.91$ fm and $\langle r_p \rangle = 0.90$ fm for 13 σ_{Beam} and 4.3 σ_{Beam} distance respectively which is in good agreement with the experiment $\langle r_p \rangle = 0.84 \pm 0.00039$ fm [29].

3. Discussion

The Chou Yang model is successful in its predictions for elastic scattering process at higher as well as the lower values of $\sqrt{s}$. The proton electromagnetic form factor is obtained at low squared momemtm transfer, *i.e.*, $0.36 < -t < 0.76$ (GeV/c)^2

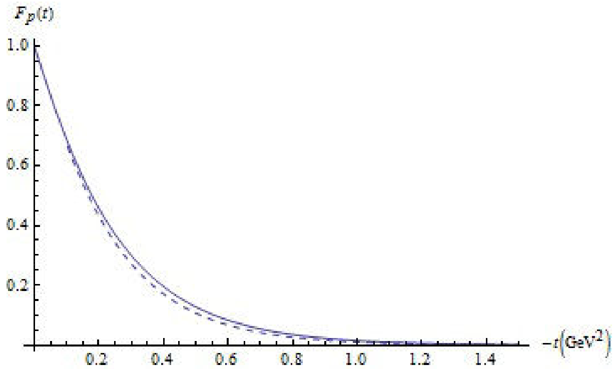

FIGURE 3. Form factor of proton predicted (for 13 σ_{Beam} distance and 4.3 σ_{Beam} distance).

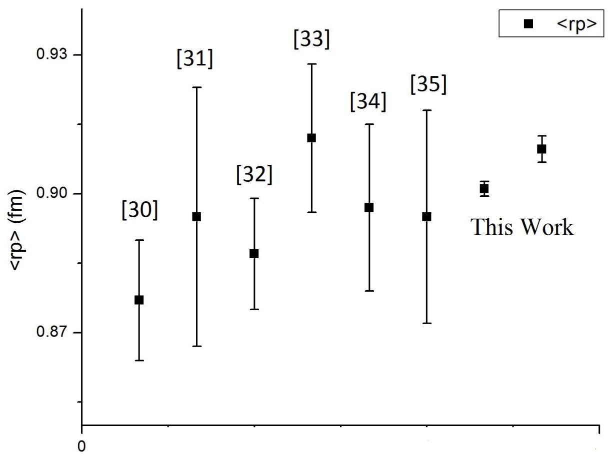

FIGURE 4. Comparison of calculated rms radii.

and $0.07 < -t < 0.46$ (GeV/c)^2, but high centre-of-mass energy *i.e* $\sqrt{s} = 2.76$ TeV. Figures 1 and 2 show the points of the experimental data from TOTEM [9] represented by squares, whereas our fit is drawn with a solid line. We have obtained the most suitable fit. Figure 3 shows the comparison plot of our predicted form factor for both sets of data, where the dotted line shows the form factor for 13 σ_{Beam} distance data and solid line shows the form factor predicted for 4.3 σ_{Beam} distance. The novel aspect of this work is that the simplest method is employed and our calculated rms charge radius of proton that agree well with experiment and theory. A comparison of calculated values is given in Fig. 4.

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