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Vector fields localization on brane worlds

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To confine vector bosons in the four-dimensional sector of a domain wall spacetime, we propose a mechanism in which the interaction among vectors is propagated via the self-interaction of the scalar wall. In the process, the vector acquires an asymptotic mass, defined by the bulk cosmological constant, and it ends up coupled to the wall by the tension of the brane. The mechanism is applied on the Randall Sundrum scenario and regular versions of it and singular domain walls. In any case, the electrostatic potential between two charged particles is defined by both the vector state attached to the wall and a continuous tower of massive vector states that propagate freely along the scenario's extra dimension.

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1. Introduction

The five-dimensional braneworld scenarios require standard model fields to be confined to the four-dimensional sector of the theory. In particular, when the electromagnetic field localization is considered, a non-normalizable gauge field is found [1]. This issue has been addressed in several opportunities, and some proposals have emerged in order to solve the problem. For instance, in [2], massive bulk vector fields are coupled to the Randall-Sundrum (RS) brane [3] through a quadratic interaction term, in such a way that the photon is the bounded state of a vector fields spectrum. On the other hand, in [4], where the brane is generated by a domain wall solution to the Einstein scalar field system, the coupling between the bulk gauge fields and a dilaton is required to find a normalizable gauge boson on the thick brane. In both mechanisms, the localized state generates the standard electromagnetic interaction. Outside of this framework, other proposals can be found in Refs. [5–10].

In this paper, we consider the vector field localization on self-gravitating domain walls via the coupling of the bulk vectors with the scalar field of the wall. We propose an interaction term defined by the scalar potential of the wall. As a result, the generalization of the Ghoroku-Nakamura mechanism [2] to thick walls is obtained. We also show that the four-dimensional degrees of freedom of the bulk vectors are determined by a supersymmetric quantum mechanics problem where the ground state yields the standard electrostatic potential on the wall.

We apply the mechanism on the RS scenario and regularized versions of it, and the so-called singular domain walls [11], where the scalar field interpolates between the lower values of the scalar potential. Whereas in the first case, standard electromagnetism on the wall is obtained, in the second one, the electrostatic interaction is the five-dimensional way.

Finally, we discuss coordinate covariance and gauge the symmetry of the model.

2. Vector field coupled to the wall

Consider the five-dimensional coupled Einstein-scalar field system (Latin and Greek indices correspond to five and four-dimensions, respectively)

$$\frac{\mathcal{L}_g}{\sqrt{g}} = \frac{1}{2}R - \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi). \qquad (1)$$

We are interested in domain wall geometries, *i.e.*, smooth scenarios where the scalar field ϕ interpolates between the minima of the self-interaction potential $V(\phi)$. Besides consider the coupling of the bulk vectors with the scalar field of the wall, namely

$$\frac{\mathcal{L}_A}{\sqrt{g}} = -\frac{1}{4}F_{ab}F^{ab} + \frac{2}{3}V(\phi)A_aA^a - \mathcal{Q}^2A_aJ^a, \qquad (2)$$

where $\mathcal{Q}$ is the five-dimensional coupling constant between two charged particles on the wall.

Before moving forward, it is necessary to point out two aspects regarding (2): i) we will assume that the vector fields do not modify the gravitation of the scenario and ii) the term $V(\phi)A_aA^a$ will not be justified. We want to show that it leads to standard electromagnetism in the four-dimensional sector of the wall.

In conformal coordinates,

$$ds^2 = e^{2a(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \qquad (3)$$

from (1), we have

$$(\partial_z\phi)^2 = 3\left[(\partial_z a)^2 + \partial_z^2 a\right] \tag{4}$$

and

$$V(\phi(z)) = -\frac{3}{2}\left[3(\partial_z a)^2 + \partial_z^2 a\right]e^{-2a}, \tag{5}$$

and from (2), we get

$$\left[\eta^{\gamma\alpha}\left(\partial_z + \partial_z a\right)\partial_z + \frac{4V(\phi)e^{2a}}{3}\eta^{\gamma\alpha}\right]A_\alpha + \eta^{\beta[\sigma}\eta^{\alpha]\gamma}\partial^2_{\beta\sigma}A_\alpha - \eta^{\gamma\alpha}\left(\partial_z + \partial_z a\right)\partial_\alpha A_z = \mathcal{Q}^2 e^{4a}J^\gamma \tag{6}$$

and

$$\left(\eta^{\beta\sigma}\partial^2_{\beta\sigma} + \frac{4V(\phi)e^{2a}}{3}\right)A_z - \eta^{\beta\sigma}\partial_z\partial_\beta A_\sigma = \mathcal{Q}^2 e^{4a}J^z. \tag{7}$$

We want to study the electromagnetic interaction in the four-dimensional sector of the five-dimensional spacetime. To do this we will calculate the electrostatic potential, which can be determined from the following generating functional

$$W[J] = W[0]\exp\left[-\frac{i}{2}\int d^4x dz\sqrt{g(z)}\int d^4x' dz'\sqrt{g(z')} \times J_1^a(x,z)G_{ab}(x-x',z,z')J_2^b(x',z')\right]. \tag{8}$$

with G_{ab} in the bulk propagator. In the momentum space, (8) can be written as

$$W[J] = W[0]\exp\left[-\frac{i}{2}\int dt\int dz\sqrt{g(z)}\int dz'\sqrt{g(z')} \times \int\frac{d^3p}{(2\pi)^3}\tilde{J}^a(-\vec{p},z)\tilde{G}_{ab}(\vec{p},z,z')\tilde{J}^b(\vec{p},z')\right], \tag{9}$$

where, tilde indicates four-dimensional Fourier transform. So, if we assume that the electrostatic sources are at $z = z' = 0$, namely $\tilde{J}^a(\vec{p},z) = \delta(z)\delta^a_\mu\tilde{j}^\mu(\vec{p})$, we find that the potential is given by

$$U[j] = \int\frac{d^3p}{(2\pi)^3}\tilde{j}_1^\mu(-\vec{p})\tilde{G}_{\mu\nu}(\vec{p},0,0)\tilde{j}_2^\nu(\vec{p}). \tag{10}$$

where $G_{\mu\nu}$ are the four-dimensional components of propagator G_{ab}.

By choosing

$$A_a = \int d^4x' dz'\sqrt{g(z')}\,G_{ab}(x-x';z,z')J^b(x',z'), \tag{11}$$

from (6) and (7), in the momentum space, we find a coupled system of equations for the components of $\tilde{G}_{ab}$ given by

$$\eta^{\sigma\alpha}\left[e^{-a}\partial_z\left(e^a\partial_z\right) - \bar{p}^2 + \frac{4}{3}V(\phi)e^{2a}\right]\tilde{G}_{\alpha b}(p,z,z') + \bar{p}^\sigma\bar{p}^\alpha\tilde{G}_{\alpha b}(p,z,z') + i\bar{p}^\sigma\left(\partial_z + \partial_z a\right)\tilde{G}_{zb}(p,z,z') = \frac{\mathcal{Q}^2}{(2\pi)^2}e^{-a}\delta^\sigma_b\delta(z-z') \tag{12}$$

and

$$\left(-\bar{p}^2 + \frac{4}{3}V(\phi)e^{2a}\right)\tilde{G}_{zb}(p,z,z') + i\bar{p}^\alpha\partial_z\tilde{G}_{\alpha b}(p,z,z') = \frac{\mathcal{Q}^2}{(2\pi)^2}e^{-a}\delta^z_b\delta(z-z'), \tag{13}$$

where $\bar{p}^\alpha \equiv \eta^{\alpha\beta}p_\beta$. Now, in (12, 13), the transversal and longitudinal decomposition of the four-dimensional components of the propagator is considered, *i.e*.

$$\tilde{G}_{\alpha\beta} = \left(\eta_{\alpha\beta} - \frac{p_\alpha p_\beta}{\bar{p}^2}\right)G_1 + \frac{p_\alpha p_\beta}{\bar{p}^2}G_2, \tag{14}$$

where G_1 and G_2 satisfy

$$e^{-a}\partial_z\left(e^a\partial_z G_1\right) + \left(-\bar{p}^2 + \frac{4}{3}V(\phi)e^{2a}\right)G_1 = \frac{\mathcal{Q}^2}{(2\pi)^2}e^{-a}\delta(z-z') \tag{15}$$

and

$$e^{-a}\bar{p}^2\partial_z\left[e^a\left(-\bar{p}^2 + \frac{4}{3}V(\phi)e^{2a}\right)^{-1}\partial_z G_2\right] + \frac{4}{3}V(\phi)e^{2a}G_2 + e^{-a}\partial_z\left(e^a\partial_z G_2\right) = \frac{\mathcal{Q}^2}{(2\pi)^2}e^{-a}\delta(z-z'). \tag{16}$$

On the other hand, for $b = z$, we find

$$\tilde{G}_{z\beta} = -\frac{ip_\beta}{-\bar{p}^2 + 4V(\phi)e^{2a(z)}/3}\partial_z G_2, \tag{17}$$

$$\tilde{G}_{\beta z} = \frac{ip_\beta}{-\bar{p}^2 + 4V(\phi)e^{2a(z')}/3}\partial_{z'}G_2, \tag{18}$$

$$\tilde{G}_{zz} = -\frac{ip_\beta}{-\bar{p}^2 + 4V(\phi)e^{2a(z)}/3}\partial_z\tilde{G}_{\beta z} + \frac{\mathcal{Q}^2}{(2\pi)^2}\frac{e^{-a(z)}}{-\bar{p}^2 + 4V(\phi)e^{2a(z)}/3}\delta(z-z'). \tag{19}$$

Due to that $p_\alpha\tilde{j}^\alpha = 0$, only the traversal sector of (14) contributes with the electrostatic potential (10). Therefore, we will focus on finding a solution to (15).

Let us consider a continuous set of states $\psi_m(z)$ satisfying the Schrödinger like the equation

$$\left(-\partial_z^2 + V_{QM}\right)\psi_m = m^2\psi_m, \tag{20}$$

$$V_{QM} = \frac{1}{2}\partial_z^2 a + \frac{1}{4}\left(\partial_z a\right)^2 - \frac{4}{3}V(\phi)e^{2a} \tag{21}$$

and the following orthonormality and closure relationships

$$\int_{-\infty}^{\infty} \psi^*_{m'}(z)\psi_m(z)dz = \delta(m-m'), \tag{22}$$

$$\int \psi^*_{m}(z')\psi_m(z)dm = \delta(z'-z). \tag{23}$$

Notice that, for $V(\phi)$ given by (5), the eigenvalues equation (20, 21) can be written as a supersymmetric quantum mechanics problem,

$$\left(\partial_z + \frac{5}{2}\partial_z a\right)\left(-\partial_z + \frac{5}{2}\partial_z a\right)\psi_m = m^2\psi_m, \tag{24}$$

and hence, in the eigenfunctions spectrum, there are no states with negative mass [12,13]. On the other hand, the orthonormality condition (22) is divergent for all $m = m'$, which is a technical problem that can be solved by introducing two regularity branes at $\pm z_r$ [14], in such a way that in limit $z_r \to \infty$ the initial scenario is recovered. As a consequence of the regulatory branes, the basis is discretized, and the orthonormality relationships satisfied in the sense of Kronecker

$$\frac{1}{z_r}\int_{-z_r}^{z_r} \psi^*_{m'}(z)\psi_m(z)dz = \frac{1}{z_r}\delta_{m'm}. \tag{25}$$

Now, by expanding G_1 in the discrete basis

$$G_1 = -\mathcal{Q}^2\sum_m \frac{1}{p^2+m^2}\psi^*_m(z')\psi_m(z)e^{[(a(z')+a(z)]/2}, \tag{26}$$

we find that (26) satisfies automatically the Eq. (15) and that the electrostatic potential, in the coordinates space, between two charged particles q_1 and q_2, *i.e.*, $j^\mu_i(\vec{x}) = q_i\delta(\vec{x}-\vec{x_i})\delta^\mu_0$, is determined by

$$U(r) = \frac{\mathcal{Q}^2}{2\pi}\frac{q_1q_2}{r}\left(|\psi_0(0)|^2 + \sum_{m>0}^{\infty}|\psi_m(0)|^2 e^{-mr}\right), \tag{27}$$

where $r = |\vec{x_2}-\vec{x_1}|$. Notice that, in the presence of regulatory branes, the system resembles a well of the infinite potential of width $2z_r$, so it is estimated that m is quantized in units of $1/z_r$. Thus, for $z_r \to \infty$, the potential (27) can be written as

$$U(r) = \frac{\mathcal{Q}^2}{2\pi}\frac{q_1q_2}{r}\Bigg(|\psi_0(0)|^2 + \kappa \lim_{z_r\to\infty}\int_0^\infty |\psi_m(0)|^2 e^{-mr} z_r dm\Bigg). \tag{28}$$

where κ is a proportionality constant defined by boundary conditions at $\pm z_r$.

For a domain wall spacetime, we expect a zero-mode localized around $z = 0$, $\psi_0 \sim \exp(5a/2)$ in correspondence with standard electromagnetism, and a tower of unbounded KK modes generating corrections to the Coulomb law.

Next, we will determine the electrostatic potential (28) on three scenarios: RS brane, regular domain wall, and singular domain wall.

3. Vector fields on RS brane world

The RS scenario [3] in the conformal coordinates (3) is determined by

$$a(z) = -\ln(1+\alpha|z|), \tag{29}$$

and

$$V(\phi(z)) = -\frac{3}{4}\left(\frac{4}{3}|\Lambda| - \frac{2}{3}\tau\delta(z)\right), \tag{30}$$

with $\tau = 6\alpha$ the brane's tension and $|\Lambda| = 6\alpha^2$ the bulk cosmological constant.

In this scenario, the localization mechanism defined by action (2) takes the form

$$\frac{\mathcal{L}_A}{\sqrt{g}} = -\frac{1}{4}F_{ab}F^{ab} - \frac{1}{2} \times \left(\frac{4}{3}|\Lambda| - \frac{2}{3}\tau\delta(z)\right)A_aA^a - \mathcal{Q}^2A_aJ^a, \tag{31}$$

which is similar to the proposed in Ref. [2]: a five-dimensional Proca theory with generic massive terms in five and four dimensions. However, in (31), the bulk mass of the vector fields and the four-dimensional coupling parameter are determined by the cosmological constant and the brane's tension, respectively.

To evaluate the electrostatic potential (28), from (21) and (29) we obtain

$$V_{QM} = \frac{35}{4}\frac{\alpha^2}{(1+\alpha|z|)^2} - 5\alpha\delta(z); \tag{32}$$

such that the eigenstates of the problem (20, 32) are given by a normalizable zero mode

$$\psi_0 = \sqrt{2\alpha}\,(1+\alpha|z|)^{-5/2}, \tag{33}$$

and a discrete spectrum of unbounded eigenstates

$$\psi_{m+} = N_m\sqrt{\alpha^{-1}+z}\Big(J_3\left[m\{\alpha^{-1}+z\}\right] + B_+Y_3\left[m\{\alpha^{-1}+z\}\right]\Big),\ z>0, \tag{34}$$

$$\psi_{m-} = N_m\sqrt{\alpha^{-1}-z}\Big(A_-J_3\left[m\{\alpha^{-1}-z\}\right] + B_-Y_3\left[m\{\alpha^{-1}-z\}\right]\Big),\ z<0, \tag{35}$$

with J and Y, the Bessel functions of the first and second kind and A, B, and N_m the integration and normalization constants.

In agreement with [15], each eigenvalue of (20) is associated with a pair of eigenfunctions, ψ_m^c and ψ_m^d; which, for the particular case (32), where $V_{QM}(z) = V_{QM}(-z)$, corresponds to odd and even eigenstates of the problem (20, 32). Hence, on the brane, they satisfy

$$\psi_m^c(0^-) = \psi_m^c(0^+) = 0, \tag{36}$$

$$\frac{d}{dz}\psi_m^c(0^-) - \frac{d}{dz}\psi_m^c(0^+) = 0, \tag{37}$$

and

$$\psi_m^d(0^-) = \psi_m^d(0^+), \tag{38}$$

$$\frac{d}{dz}\psi_m^d(0^-) - \frac{d}{dz}\psi_m^d(0^+) = 5\alpha\psi_m^d(0); \tag{39}$$

while, on the regular branes

$$\psi_{m\pm}^d(\pm z_r) \simeq 0, \qquad \frac{d}{dz}\psi_{m\pm}^d(\pm z_r) \simeq 0. \tag{40}$$

Therefore, the constants of integration set for ψ_m^c and ψ_m^d can be determined by (36, 37) and (25) in the first case; and by (38, 39) and (25) in the second one. On the other hand, the boundary condition (40) induces the discretization of the mass, *i.e.*, $m \sim n\pi/z_r$ with $n \in Z^+$.

Thus, for the integration constants, we have

$$A_-^c = 1, \qquad B_-^c = -B_+^c, \qquad B_+^c = -\frac{J_3(m/\alpha)}{Y_3(m/\alpha)}, \tag{41}$$

and

$$A_-^d = \left[1 + \frac{Y_3(m/\alpha)}{J_3(m/\alpha)}\right]^{-1}, \qquad B_-^d = -B_+^d, \tag{42}$$

$$B_+^d = -\frac{1}{2}\left(\frac{Y_3(m/\alpha)}{J_3(m/\alpha)} + \frac{J_2(m/\alpha)}{Y_2(m/\alpha)}\right) \times \left(1 + \frac{Y_3^2(m/\alpha)}{J_3^2(m/\alpha)}\right)^{-1} - \frac{1}{2}\frac{J_2(m/\alpha)}{Y_2(m/\alpha)}. \tag{43}$$

Finally, on the brane and for $m \ll \alpha$, we find

$$z_r\left|\psi_m^d(0)\right|^2 \simeq \frac{\pi}{32}\left(\frac{m}{\alpha}\right)^3 + \frac{\pi}{48}\left(\frac{m}{\alpha}\right)^5 + \frac{\pi}{86}\left(\frac{m}{\alpha}\right)^7, \tag{44}$$

such that the electrostatic interaction

$$U(r) \sim \frac{1}{r}\left(1 + \frac{3}{32}\frac{1}{(\alpha r)^4} + \frac{5}{4}\frac{1}{(\alpha r)^6} + \frac{1260}{43}\frac{1}{(\alpha r)^8}\right), \tag{45}$$

is in correspondence with the standard four-dimensional potential for $r \gg \alpha^{-1}$.

4. Regular Domain Walls

Let $a(z)$ be the metric factor of a domain wall corresponding to a regularized version of the RS brane [16–21]. Following [22], it is possible to estimate at least the order of corrections as follows. Asymptotically, (where the effects of wall thickness are negligible) the metric factor of a regularized wall resembles the RS solution: $a(z) \sim -\ln(1 + \alpha|z|)$. Hence, for $z \gg \alpha^{-1}$ the quantum mechanics potential (21)

$$V_{QM} \longrightarrow \frac{5}{2}\left(\frac{5}{2} + 1\right)\frac{1}{z^2}. \tag{46}$$

In Ref. [22], it is shown that for a V_{QM} with asymptotic behavior similar to (46), the density of the state on the wall is determined by $\psi_m(0) \sim (m/\alpha)^{(5/2)-1}$ in such a way that, the corrections to the electrostatic potential (28) go as r^{-5} and they are negligible compared to the Coulomb term from a critical radius determined by the parameters of the regular scenario.

5. Singular Domain Walls

The domain walls are understood as solutions to the coupled Einstein-scalar field system where ϕ interpolates between the minimums of the scalar potential. However, there is another family of walls where $V(\phi)$ does not have minimums, but ϕ interpolates among the lower values of the scalar potential. These scenarios are called singular domain walls, and like standard walls, they can locate gravity [11].

Next, let us explore the four-dimensional effective behavior of the vector field on the singular scenario reported in [11] wherein in conformal coordinate (3), the warp factor, the scalar field, and potential are given by

$$a(z) = -\ln\cosh(\alpha z), \quad \alpha > 0. \tag{47}$$

$$\phi = \sqrt{\frac{3}{2}}\,\alpha z\,, \tag{48}$$

and

$$V(\phi) = 3\,\alpha^2\,\left[1 - \frac{3}{4}\,\cosh^2\left(\sqrt{2/3}\phi\right)\right]. \tag{49}$$

Notice that the scalar field interpolates between $\pm\infty$, and the scalar potential has a maximum in $\phi = 0$. On the other hand, the scalar curvature for this geometry is determined by

$$R = 14\alpha^2\left[1 - \frac{3}{2}\cosh(2\alpha z)\right] \tag{50}$$

which diverges for $z \to \pm\infty$. Thus, the solution represents a wall embedded in a five-dimensional spacetime that interpolates between two subspace with naked singularities in the horizon.

To calculate the electrostatic potential (28), the density of states associated to (20) with

$$V_{QM} = m_{\bar{0}}^2 - \frac{7}{5}m_{\bar{0}}^2\cosh^{-2}(\alpha z)\,, \quad m_{\bar{0}} \equiv \frac{5}{2}\alpha\,, \tag{51}$$

is required. In this case, we can see that the zero mode is separated from continuous modes by a mass gap defined by $m_{\bar{0}}$.

By considering the change of variable $\xi = \alpha z$, the equation (20, 51) takes the form of a Schrödinger equation with a potential of Pöschl-Teller [23],

$$-\frac{1}{2}\frac{d^2}{d\xi^2}\psi(\xi) - \frac{35}{8}\cosh^{-2}(\xi)\psi(\xi) = E\psi(\xi)\ , \qquad (52)$$

where

$$E = \frac{25}{8}\left(\frac{m^2}{m_{\bar{0}}^2} - 1\right), \qquad (53)$$

with bounded states determined by

$$E_0 = -\frac{25}{8}\ , \quad \psi_0 = N_0 \cosh^{-5/2}(\xi)\ , \qquad (54)$$

and

$$E_1 = -\frac{9}{8}\ , \quad \psi_1 = N_1 \cosh^{-5/2}(\xi)\ \sinh(\xi), \qquad (55)$$

such that

$$m_0^2 = 0, \qquad m_1^2 = \frac{16}{25}\,m_{\bar{0}}^2, \qquad N_0^2 = \frac{16}{15\pi}m_{\bar{0}}. \qquad (56)$$

Regarding the free states, under the change of variable

$$u = \tanh(\xi), \qquad (57)$$

the differential equation (52) takes the form of a Legendre equation

$$\frac{d}{du}\left[\left(1-u^2\right)\frac{d}{du}\psi\right] + \left[\frac{35}{4} + \frac{2E^2}{(1-u^2)}\right]\psi = 0\ , \qquad (58)$$

whose solutions are the associated Legendre functions of the first and second kind, of $5/2$ grade and order $\mu = \pm\sqrt{-2E}$, given by

$$P^{\mu}_{5/2}(u) = \frac{1}{\Gamma(1-u)}\left(\frac{1+u}{1-u}\right)^{\mu/2} \times {}_2F_1\left[-\frac{5}{2},\ \frac{7}{2};\ 1-\mu;\ \frac{1-u}{2}\right], \qquad (59)$$

$$Q^{\mu}_{5/2}(u) = \frac{\sqrt{\pi}\Gamma(7/2+\mu)}{6}\frac{e^{i\pi\mu}(u^2-1)^{\mu/2}}{u^{7/2+\mu}} \times {}_2F_1\left[\frac{7}{4}+\frac{\mu}{2}, \frac{9}{4}+\frac{\mu}{2}; 4; \frac{1}{u^2}\right]. \qquad (60)$$

The functions $P^{\mu}_{5/2}$ and $Q^{\mu}_{5/2}$ are orthogonal for $|1-u| < 2$ and $|u| > 1$, respectively; in particular, the change (57) satisfies $|1-u| < 2$. Hence

$$\psi_m(u) = \frac{N_m}{2}\left[A_m\left(\Gamma^- P^{\mu}_{5/2}(u) + \Gamma^+ P^{-\mu}_{5/2}(u)\right) - i\left(\Gamma^- P^{\mu}_{5/2}(u) - \Gamma^+ P^{-\mu}_{5/2}(u)\right)\right] \qquad (61)$$

where $\Gamma^- = \Gamma(1-\mu)$ and $\Gamma^+ = \Gamma(1+\mu)$.

The solution is doubly degenerate [15] and satisfies the following boundary conditions

$$\psi^c_m(0^-) = \psi^c_m(0^+) = 0\ , \quad \psi'^c_m(0^-) = \psi'^c_m(0^+)\ , \qquad (62)$$

$$\psi^d_m(0^-) = \psi^d_m(0^+)\ , \quad \psi'^d_m(0^-) = \psi'^d_m(0^+). \qquad (63)$$

Therefore, the density of the state in $z = 0$ is determined by

$$z_r\left|\psi^d_m(0)\right|^2 = \Gamma^-\Gamma^+ P^{\mu}_{5/2}(0) P^{-\mu}_{5/2}(0)\ . \qquad (64)$$

Now, the integral in the electrostatic potential saturates for $m \gg m_{\bar{0}}$, and in this case, the states (64) reduces to

$$z_r\left|\psi^d_m(0)\right|^2 \simeq 1 - \frac{49}{16}\,\frac{m_{\bar{0}}^2}{m^2}\ . \qquad (65)$$

By substituting the modes (54), (55) y (65) in (28), we obtain

$$U(r) \sim \frac{1}{r}\left(-\frac{33}{16} - \frac{49}{16}m_{\bar{0}}r\ln m_{\bar{0}}r + \frac{1-m_{\bar{0}}r}{m_{\bar{0}}r}\right) \qquad (66)$$

where the dominant term for $r \ll \alpha^{-1}$ is the order of r^{-2} such that the electrostatic interaction on the four-dimensional sector of the scenario is defined for a five-dimensional potential.

6. Discussion

In this paper, the mechanics of Ghoroku-Nakamura [2] for confining vector fields on the RS brane was extended to self-gravitating domain walls in a five-dimensional bulk.

This was achieved by considering the coupling of the vector boson with the scalar field of the wall. We found that the four-dimensional degrees of freedom of the vector field can be expanded in terms of a basis of eigenstates of the Schrödinger equation, free of tachyonic states. In the electrostatic potential, the ground state defines the Coulomb interaction on the wall, while the massive state density generates corrections to the potential.

When mechanism (2) is applied on the RS brane, the bulk cosmological constant plays the role of mass for the vector field while the brane tension defines the parameter coupling with the wall. The corrections generated by the massive states in the electrostatic potential (28) are exponentially suppressed, and the standard electromagnetism can be recovered on the brane from a critical radius similar to the gravitational case.

In the case of thick brane, the mechanism was applied on regularized versions of the RS scenario and singular walls. In the first case, in agreement with the Csaki theorem [22], we find that deviations to the Coulomb law are the order of r^{-5}. In the second case, the deviations are not negligible, and the electrostatic interaction goes to r^{-2} on the four-dimensional sector of the wall.

Notice that theory (2) is covariant but not gauge-invariant due to the presence of the quadratic term in the action. This can also be seen in the four-dimensional effective theory (see Appendix),

$$\mathcal{L}_A^{(4)}=-\frac{1}{4}f_{\alpha\beta}^2+\sum_n\left[-\frac{1}{4}\left(f_{\alpha\beta}^n\right)^2-\frac{1}{2}\left(m_n a_\mu^n+\partial_\mu a_5^n\right)^2\right]$$
$$+4\sum_{p,q}\left[\lim_{z_r\to\infty}\int_{-z_r}^{z_r}dz\partial_z a\ \psi_q\varphi_p\right]\partial^\mu a_5^p\ a_\mu^q$$
$$+\frac{2}{3}\sum_{p,q}\frac{m_p}{m_q}\left[\lim_{z_r\to\infty}\int_{-z_r}^{z_r}dz\ e^{2a}V(\phi(z))\psi_p\psi_q\right]a_5^p a_5^q \tag{67}$$

with

$$f_{\alpha\beta}=\partial_\alpha a_\beta-\partial_\beta a_\alpha,\qquad f_{\alpha\beta}^n=\partial_\alpha a_\beta^n-\partial_\beta a_\alpha^n, \tag{68}$$

where we have considered $A_b\to e^{-a/2}A_b$ and

$$A_\mu(x,z)=a_\mu(x)\ \psi_0(z)+\sum_{n\neq 0}a_\mu^n(x)\ \psi_n(z), \tag{69}$$

$$A_5(x,z)=\sum_{p\neq 0}a_5^p(x)\ \varphi_p(z), \tag{70}$$

such that $\varphi_n=(-\partial_z+5/2\ \partial_z a)\psi_n/m_n$ is the supersymmetric partner of ψ_n for $m_n\neq 0$ [12]. In (67), the first two terms correspond to the Maxwell action for a_μ and the Stueckelberg action for the massive vector a_μ^n. The absence of gauge symmetry is due to the final two terms; the second to last is a term of interaction between a_μ^p and a_5^n via the dynamics of the scalar modes $\partial^\mu a_5^n$, and the last one corresponds to massive terms for $(a_5^n)^2$ when $n=p$ and to interaction terms between a_5^n and a_5^p when $n\neq p$.

In a forthcoming paper, we look forward to reporting a gauge-invariant generalization of (2) and justify the term of interaction between vector fields.

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Appendix

A. The eigenvalues problem

To find the four-dimensional effective theory (67), we have considered

$$\int_{-\infty}^{\infty}dz\int d^4x\ \mathcal{L}_A\ \longrightarrow\ \lim_{z_r\to\infty}\int_{-z_r}^{z_r}dz\int d^4x\ \mathcal{L}_A, \tag{A.1}$$

where the spacetime is bounded by two regulatory branes, each with negative tension and located at $\pm z_r$. Under this approach, it is possible to expand the components of A_b in terms of a discrete base of functions with support along the extra dimension.

In this sense, consider the operators

$$\boldsymbol{Q}=\partial_z+\frac{5}{2}a'\qquad\text{and}\qquad\boldsymbol{Q}^+=-\partial_z+\frac{5}{2}a'. \tag{A.2}$$

If ψ_n is the set of states of the eigenvalues problem

$$\boldsymbol{Q}\boldsymbol{Q}^+\psi_n=m_n^2\psi_n,$$
$$\boldsymbol{Q}^+\psi_n\Big|_{\pm z_r}=0,\qquad n=0,1,2,3,\ldots \tag{A.3}$$

with $m_n^2\geq 0$ (because the differential operator is factorizable) then, φ_n is the set of states of the problem

$$\boldsymbol{Q}^+\boldsymbol{Q}\varphi_n=m_n^2\varphi_n,\quad n=1,2,3,\ldots \tag{A.4}$$

where $\varphi_n=\boldsymbol{Q}^+\psi_n/m_n$ for all $m_n\neq 0$. Hence, ψ_n y φ_n always come in pairs, except for the zero modes of ψ_n (see Ref. [12] for details). On the other hand, the orthonormality relationship for each discrete set of functions is determined by

$$\int_{-z_r}^{z_r}dz\ \psi_n\psi_p=\delta_{np}\quad\text{and}\quad\int_{-z_r}^{z_r}dz\ \varphi_n\varphi_p=\delta_{np}. \tag{A.5}$$

For a five-dimensional vector field A_b, the components expand as indicated in (69) and (70) where $\psi_0(z)\sim e^{5a(z)/2}$.

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