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Solitary wave solutions in two-core optical fibers with coupling-coefficient dispersion and Kerr nonlinearity

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This paper studies solitary wave solutions in two-core optical fibers with coupling-coefficient dispersion and intermodal dispersion. To construct bright, dark and W-shape bright solitons, the couple of nonlinear Schrödinger equations describing the pulses propagation along the two-core fiber have been reduced to one equivalent equation. By adopting the traveling-waves hypothesis, exact analytical solutions of the generalized nonlinear Schrödinger equation (GNSE) were obtained by using three relevant mathematical methods, namely, the auxiliary equation method, the modified auxiliary equation method and the sine-Gordon expansion approach. Lastly, the behavior of the soliton solutions was discussed and some contours of the plot evolution of the bright, W-shape bright and dark solitons are obtained.

Keywords: Two-core optical fiber; soliton solutions; nonlinear Schrödinger equation.

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1. Introduction

The single fiber that hold two parallel cores is knowing as two-core optical fiber. Nowadays, a lot off attention have been focussed to the devices based on two-core fibers. Thus, some authors have been demonstrated that the double-core single-mode can be tailored as a directional coupler, polarization splitters and power depend nonlinear couplers [1]. Furthermore, investigation of solitary waves in two-core optical fiber have been advanced beyond measure, such as soliton shape and mobility control in optical lattices [2], dark and bright solitons [3, 4] and so on. Moreover, these localized solutions in optical fibers usually take the forms of shift solitons, spatiotemporal soliton, temporal soliton and spatial soliton [5]. More recently, some works have been done to build soliton in a nonlinear coupler in a presence of a raman effect and solitary waves in asymmetric tin-core fibers [6, 7]. Today, analytical investigation of solitary waves become a big challenge, that is while some relevant methods have been used to construct exact solutions for partial differential equations (PDEs), such as unified method [8, 9], initial condition field distributions [10], the simplest equation approach [11], the split-Step method [12], the first integral method [13], new extended direct algebraic scheme [14, 15] and the generalized tanh method [16], just to name a few.

The following pair of nonlinear Schrödinger equations describes the pulse propagation through the two-core optical fibers [17]. The model has been studied by Samir *et al*. [18], and novel traveling-waves solutions have been retrieved by help of Jacobian elliptic functions.

$$
\begin{aligned}
&i\frac{\partial a_1}{\partial x}-\frac{\beta_2}{2}\frac{\partial^2 a_1}{\partial t^2}+\gamma|a_1|^2a_1-ca_1+ik_1\frac{\partial a_2}{\partial t}=0,\\
&i\frac{\partial a_2}{\partial x}-\frac{\beta_2}{2}\frac{\partial^2 a_2}{\partial t^2}+\gamma|a_2|^2a_2-ca_2+ik_1\frac{\partial a_1}{\partial t}=0, \qquad (1)
\end{aligned}
$$

where $a_1(x,t)$ and $a_2(x,t)$ represent the slowly envelopes of the electric field. However, x and t are, respectively, the propagation distance and time in a retarded frame. The parameter β_2 accounts for normal-or anomalous-GVD at the carrier frequency. $\gamma = 2\pi n_2/(\lambda A_{eff})$ controls the self phase modulation (SPM), where λ, n_2 and $A_{\rm eff}$ are, respectively, free-space optical wavelength, nonlinear refractive index of the fiber material and the effective area of each core, while c stands for the coupling coefficient and it is also proportional to the spatial encroachment between the mode fields in the two-core [18]. Here $k_1 = dc/d\omega$ accounts for the coupling-coefficient dispersion (CCD) at the carrier frequency and corresponds to the intermodal dispersion which arises from the group-delay difference between the only and uneven super-modes of the two-core fiber [18]. Recently, it has been obtained a photonic bandgap two-core fiber when the coupling coefficient is equal to zero [19]. More recently, some authors have investigated solitons in dual-core fiber by adopting the traveling wave method, and results for dark and bright solitons have been found [20].

We aim in this paper to construct solitary waves and discussed the behavior of the results obtained along with the constraint relation. To do so, we surmise $a_1(x,t) = a_2(x,t) = a(x,t)$, hence the set of couple of the generalized nonlinear Schrödinger equation which describing pulses propagating in two-core fibers reduces to one equivalent equation

$$i\frac{\partial a}{\partial x} - \frac{\beta_2}{2}\frac{\partial^2 a}{\partial t^2} + \gamma|a|^2 a - ca + ik_1\frac{\partial a}{\partial t} = 0. \quad (2)$$

To obtain the travelign wave solutions of Eq. (2), the following ansatz is adopted:

$$a(x,t) = \phi(\xi)\exp[if(\xi)], \quad \xi = x - vt, \quad (3)$$

where v is the velocity frame. Hence the phase $f(\xi)$ can be written in the following form

$$\delta\omega(x,t) = -\frac{\partial(f(\xi))}{\partial t}. \quad (4)$$

By inserting Eq. (3) into Eq. (2), we obtained the following set of equations

$$-\phi f' - \frac{\beta_2}{2}v^2\phi'' + \frac{\beta_2}{2}v^2\phi {f'}^2 + \gamma\phi^3 - c\phi + k_1 v\phi f' = 0, \quad (5)$$

and

$$(1-k_1 v)\phi' - \beta_2 v^2\phi' f' - \frac{\beta_2}{2}v^2\phi f'' = 0. \quad (6)$$

Multiplying Eq. (6) by ϕ and integrating once with the constant of integration having a value of zero, it is obtained:

$$f' = \frac{(1-k_1 v)}{\beta_2 v^2}, \quad (7)$$

and thus, Eq. (4) gives

$$\delta\omega = \frac{(1-k_1 v)}{\beta_2 v}. \quad (8)$$

The formula given for Eq. (7) depends on β_2, which measures the GVD at the carrier frequency ($\beta_2 < 0$ stands for anomalous dispersion and $\beta_2 > 0$ for normal dispersion), and the coupling coefficient dispersion (CCD) corresponds to the intermodal dispersion occuring from the group-delay difference between the even and odd supermodes of the two-core fiber. However, when the value of CCD is equal to zero, the phase corresponds to the specific operating condition for a conventional two-core fiber design [21]. To investigate analytical solutions which could propagate in two-core fiber optic, we substitute Eq. (7) into Eq. (5), which leads to the following ODE.

$$-\frac{\beta_2}{2}v^2\phi'' - \frac{1}{2}\frac{(2\,cv^2\beta_2 + v^2{k_1}^2 - 2\,vk_1 + 1)}{\beta_2 v^2}\phi + \gamma\phi^3 = 0. \quad (9)$$

So, the following section adopted three relevant integration techniques, namely, the auxiliary equation method, the modified auxiliary equation method and the sine-Gordon expansion approach to derive bright and dark soliton solutions and we will discuss the behavior of the results obtained.

2. Glimpse of the methods

2.1. The auxiliary equation method

The following steps describe the auxiliary equation method [26, 27]. Considering a given nonlinear partial differential equation (NPDE) with independent variables (x,t) and dependent variables $a(x,t)$

$$P(a, a_x, a_t, a_{xt}, a_{xx}, a_{tt}) = 0. \quad (10)$$

Step 1: The traveling-wave solution of Eq. (10) is used in the following form

$$a(x,t) = \phi(\xi), \quad \xi = x - vt, \quad (11)$$

where v is the traveling-wave speed. Then Eq. (10) was converted into a nonlinear ordinary differential equation as follows

$$N(\phi, \phi^{'}, \phi^{''}, \phi^{'''}, ...) = 0. \quad (12)$$

Step 2: Suppose that the exact solutions of Eq. (12) can be expressed [**?**]

$$\phi(\xi) = \sum_{i=0}^{n} A_i (g(\xi))^i, \quad (13)$$

and $g(\xi)$ satisfies the following auxiliary equation

$$g_\xi = \sqrt{2(C_0 + C_1 g + C_2 g^2 + C_3 g^3 + C_4 g^4)}, \quad (14)$$

$$g_{\xi\xi} = C_1 + 2C_2 g + 3C_3 g^2 + 4C_4 g^3, \quad (15)$$

with $g_\xi = (\partial g/\partial\xi)$, $C_i(i = (0,1,2,3,4))$, A_0, A_i $(i = 1,2,\ldots,n)$ are real constants to be determined later.

Step 3: Under the terms of the method, it is assumed that the solution of Eq.(12) can be written in the following form

$$\phi(\xi) = A_0 + A_1 g(\xi) + A_2 g(\xi)^2 + A_3 g(\xi)^3 + ...A_n g(\xi)^n, \quad (16)$$

where A_0, A_1, A_3, A_4, and A_n are real constants to be determined later. To calculate the value of n, we balance the highest-order nonlinear terms in Eq. (12), and then the value of n can be determined.

Step 4: Substituting Eqs. (16), (15) and (14) into Eq. (12) provides a polynomial of $g(\xi)$. Next, collecting all the coefficients $g(\xi)^i$, $(i = 0, 1, 2, ...n)$ forms a system of algebraic equations. Solving this system, we describe the variable coefficients of A_0, A_i, $i = (1, 2, ..., n)$, so the solution to Eq. (12) can be obtained in terms of $g(\xi)$.

Step 5: To obtain the exact solutions to Eq. (10), the following solutions of Eq. (14) or Eq. (15) are used.

Case 1: for $C_0 = C_1 = C_3 = 0,\ C_2 > 0,\ C_4 < 0$,

$$g(\xi) = \sqrt{\frac{-C_2}{C_4}}\text{sech}(\sqrt{2C_2}\xi), \tag{17}$$

Case 2: for $C_0 = \ (C_2^2/4C_4),\ C_1 = C_3 = 0,\ C_2 < 0,\ C_4 > 0$,

$$g(\xi) = \sqrt{\frac{-C_2}{2C_4}}\tanh(\sqrt{-C_2}\xi). \tag{18}$$

Case 3: for $C_0 = C_1 = 0\ C_2 > 0,\ C_4 > 0$,

$$g(\xi) = \frac{C_2\text{sech}^2(\sqrt{2C_2}\frac{\xi}{2})}{2\sqrt{C_2C_4}\tanh(\sqrt{2C_2}\frac{\xi}{2}) - C_3}. \tag{19}$$

Case 4: for $C_0 = C_1 = 0,\ C_2 > 0,\ {C_3}^2 - 4C_2C_4 > 0$,

$$g(\xi) = \frac{2C_2\text{sech}(\sqrt{2C_2}\xi)}{\sqrt{{C_3}^2 - 4C_2C_4} - C_3\text{sech}(\sqrt{2C_2}\xi)}. \tag{20}$$

Case 5: for $C_0 = C_1 = 0,\ C_2 > 0$,

$$g(\xi) = \frac{C_2C_3\text{sech}^2(\sqrt{2C_2}\frac{\xi}{2})}{C_2C_4(1 - \tanh(\sqrt{2C_2}\frac{\xi}{2}))^2 - C_3^2}, \tag{21}$$

where C_0, C_1, C_2, C_3 and C_4 are arbitrary constants. Therefore, using Eqs. (17-21) and (16), the exact solutions to Eq. (10) can be obtained.

2.2. The modified auxiliary equation method

Investigation of exact traveling wave solutions of certain nonlinear partial differential equations by using the modified auxiliary equation method has been carried out recently [24–26].

Consider a nonlinear evolution partial differential equation as in Eq. (10) where $a = a(x, t)$ is an unknown function of independent variables x and t.

The main steps of the method to obtain exact solutions of Eq. (10) can be given as follows.

Step 1: Suppose that the formal solution of the ODE in Eq. (12) can be expressed as

$$\phi(\xi) = A_0 + \sum_{i=1}^{n}\left(A_iK^{if(\xi)} + B_iK^{-if(\xi)}\right), \tag{22}$$

where A_i, B_i, K are arbitrary real constants and $f(\xi)$ satisfy the following auxiliary equation:

$$f^{'}(\xi) = \frac{\beta + \alpha K^{-f(\xi)} + \sigma K^{f(\xi)}}{\ln(K)}, \tag{23}$$

where α, β, σ are arbitrary constants and $K > 0, K \neq 1$. The solutions of Eq. (23) are given by

Case 1: for $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$,

$$K^{f(\xi)} = \frac{-\beta + \sqrt{-\beta^2 + 4\alpha\sigma}\tan(\frac{\sqrt{-\beta^2+4\alpha\sigma}\xi}{2})}{2\sigma}, \quad \text{or}$$

$$K^{f(\xi)} = -\frac{\beta + \sqrt{-\beta^2 + 4\alpha\sigma}\cot(\frac{\sqrt{-\beta^2+4\alpha\sigma}\xi}{2})}{2\sigma}, \tag{24}$$

Case 2: for $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$,

$$K^{f(\xi)} = -\frac{\beta + \sqrt{\beta^2 - 4\alpha\sigma}\tanh(\frac{\sqrt{\beta^2-4\alpha\sigma}\xi}{2})}{2\sigma}, \quad \text{or}$$

$$K^{f(\xi)} = -\frac{\beta + \sqrt{\beta^2 - 4\alpha\sigma}\coth(\frac{\sqrt{\beta^2-4\alpha\sigma}\xi}{2})}{2\sigma}, \tag{25}$$

Case 3: for $\beta^2 - 4\alpha\sigma = 0$ and $\sigma \neq 0$,

$$K^{f(\xi)} = -\frac{2 + \beta\xi}{2\sigma\xi}. \tag{26}$$

The explicit exact solutions of (10) can be obtained by inserting the values A_0, A_i, B_i $(j = 1, 2, 3, ..., n)$.

2.3. Sine-Gordon expansion approach

To integrate the ordinary differential equation (ODE) Eq. (12), we consider the sine-Gordon equation [27,28] given by

$$\omega^{'}(\xi) = \sin(\omega(\xi)). \tag{27}$$

The solutions of Eq. (27) are represented by

$$\sin(\omega(\xi)) = \text{sech}(\xi), \quad \text{or} \quad \cos(\omega(\xi)) = \tanh(\xi), \tag{28}$$

$$\sin(\omega(\xi)) = i\text{csch}(\xi), \quad \text{or} \quad \cos(\omega(\xi)) = \coth(\xi). \tag{29}$$

The series can be utilized to derive the solution of Eq. (12),

$$\phi(\xi) = \sum_{j=1}^{n}\cos(\omega)^{j-1}(B_j\sin(\omega) + A_j\cos(\omega)) + A_0. \tag{30}$$

The parameter n can be obtained using the balancing principle. Making all the necessary computations, the solutions of the NPDE under consideration can be obtained.

3. Application of the methods

3.1. The Auxiliary equation method

Now, to use the homogeneous balanced principle between ϕ'' and ϕ^3 in Eq. (9), it is obtained $n = 1$. Subsequently, it is introduced into Eq. (13) the value of n, and taking into account Eqs. (14-15), a system of equation in terms of $g(\xi)^i$ is obtained. Thus, we set all the coefficients of each individual term $g(\xi)^i$ to zero, which gives the results below.

Set 1:

$$A_0 = 0, \quad A_1 = A_1,$$
$$C_2 = -\frac{(2\,cv^2\beta_2 + v^2{k_1}^2 - 2\,vk_1 + 1)}{2v^4{\beta_2}^2},$$
$$C_4 = \frac{1}{2}\frac{\gamma\,{A_1}^2}{\beta_2 v^2}, \tag{31}$$

Case 1: For $C_0 = C_1 = C_3 = 0$, and $C_2 > 0,\ C_4 < 0$ using Eq. (31), the bright soliton to the governing model (2) is

$$a_{1,1}(x,t) = \left\{ A_1\sqrt{\frac{-C_2}{C_4}} \times \operatorname{sech}\left(\sqrt{2C_2}(x - vt)\right) \right\} e^{if(x-vt)}, \tag{32}$$

Case 2: For $C_0 = (C_2^2/4C_4)$, $C_1 = C_3 = 0$, and $C_2 < 0,\ C_4 > 0$, using Eq. (31), the dark soliton is found to be

$$a_{1,2}(x,t) = \left\{ A_1\sqrt{\frac{-C_2}{2C_4}} \times \tanh(\sqrt{-C_2}(x - vt)) \right\} e^{if(x-vt)}, \tag{33}$$

Set 2:

$$A_0 = \frac{1}{2}\frac{\sqrt{2}}{v}\sqrt{\frac{2\,cv^2\beta_2 + v^2{k_1}^2 - 2\,vk_1 + 1}{\gamma\,\beta_2}}, \quad A_1 = A_1,$$
$$C_2 = \frac{2\,cv^2\beta_2 + v^2{k_1}^2 - 2\,vk_1 + 1}{v^4{\beta_2}^2},$$
$$C_3 = \frac{\gamma\,\sqrt{2}A_1}{v^3\beta_2}\sqrt{\frac{2\,cv^2\beta_2 + v^2{k_1}^2 - 2\,vk_1 + 1}{\gamma\,\beta_2}},$$
$$C_4 = \frac{1}{2}\frac{\gamma\,{A_1}^2}{\beta_2 v^2}, \tag{34}$$

Case 3: For $C_0 = C_1 = 0$ $C_2 > 0$ $C_4 > 0$ using Eq. (34), one can find

$$a_{1,3}(x,t) = \left\{ A_0 + A_1 \times \frac{C_2 \operatorname{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{2\sqrt{C_2C_4}\tanh\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right) - C_3} \right\} e^{if(x-vt)}, \tag{35}$$

Case 4: For $C_0 = C_1 = 0$, and $C_2 > 0$, ${C_3}^2 - 4C_2C_4 > 0$, using Eq. (34), we obtain

$$a_{1,4}(x,t) = \left\{ A_0 + A_1 \times \frac{2C_2 \operatorname{sech}\left(\sqrt{2C_2}[x - vt]\right)}{\sqrt{{C_3}^2 - 4C_2C_4} - C_3 \operatorname{sech}\left(\sqrt{2C_2}[x - vt]\right)} \right\} e^{if(x-vt)}, \tag{36}$$

Case 5: For $C_0 = C_1 = 0$, and $C_2 > 0$, using Eq. (34),

$$a_{1,5}(x,t) = \left\{ A_0 + A_1 \times \frac{C_2C_3 \operatorname{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{C_2C_4\left(1 - \tanh\left[\sqrt{2C_2}\frac{(x-vt)}{2}\right]\right)^2 - C_3^2} \right\} e^{if(x-vt)}. \tag{37}$$

3.2. The modified auxiliary equation method

Now, the MAE method will be utilized to get the general solution of Eq.(2). In this perspective, we obtain a system of algebraic equations which solves to

Set 1:

$$A_0 = A_0, \quad A_1 = 0, \quad B_1 = \frac{2\alpha\,A_0}{\beta}, \quad v = v, \quad k_1 = k_1,$$
$$c = -\frac{1}{4}\frac{(4\,\alpha\,\sigma\,v^4{\beta_2}^2 - \beta^2v^4{\beta_2}^2 + 2\,v^2{k_1}^2 - 4\,vk_1 + 2)}{v^2\beta_2},$$
$$\beta_2 = \beta_2, \quad \gamma = \frac{1}{4}\frac{v^2\beta_2\beta^2}{{A_0}^2}. \tag{38}$$

Set 2:

$$A_0 = A_0, \quad A_1 = \frac{2\sigma\,A_0}{\beta}, \quad B_1 = 0, \quad v = v, \quad k_1 = k_1,$$
$$c = -\frac{1}{4}\frac{(4\,\alpha\,\sigma\,v^4{\beta_2}^2 - \beta^2v^4{\beta_2}^2 + 2\,v^2{k_1}^2 - 4\,vk_1 + 2)}{v^2\beta_2},$$
$$\beta_2 = \beta_2, \quad \gamma = \frac{1}{4}\frac{v^2\beta_2\beta^2}{{A_0}^2}. \tag{39}$$

Using the values of the parameters in Set 1 given by Eq. (38), we get the solitary wave solutions of Eq. (2) in the following formulas:

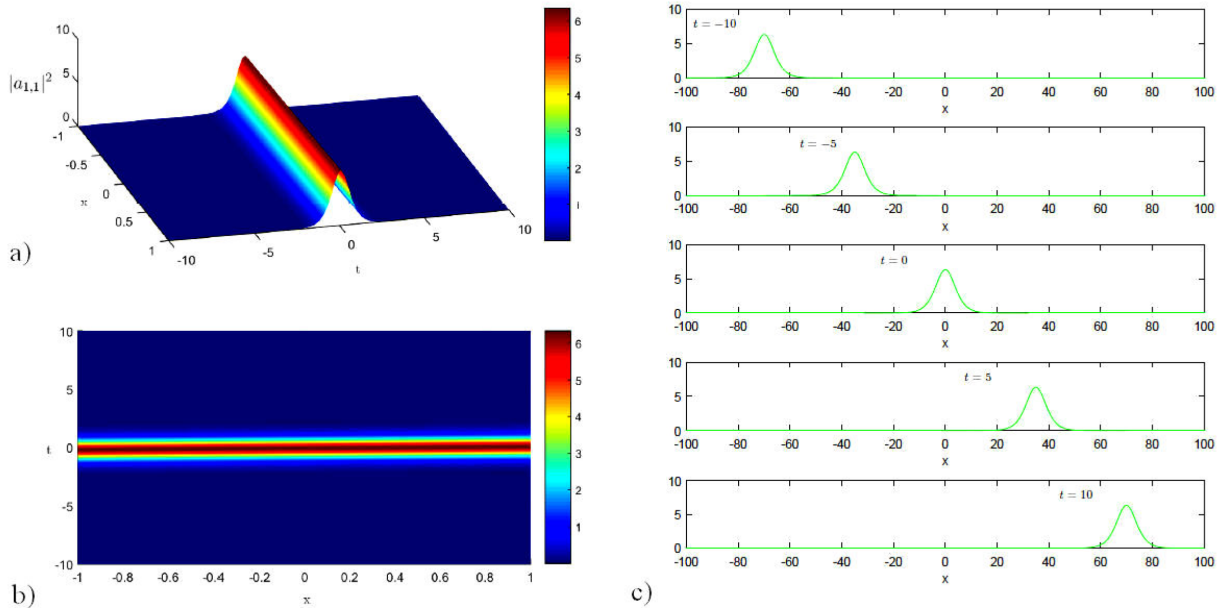

FIGURE 1. The plot of the bright soliton of $|a_{1,1}|^2$ of the solution Eq. (32) at $A_1 = 1$, $C_2 = 0.019$, $C_4 = -0.3$, $v = 7$.

When $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$, we obtain trigonometric function solutions:

$$a_{2,1}(x,t) = \left\{ A_0 + \frac{2B_1\sigma}{-\beta + \sqrt{4\,\alpha\,\sigma - \beta^2}\tan\left(\sqrt{4\,\alpha\,\sigma - \beta^2}\,\frac{(x-vt)}{2}\right)} \right\} e^{if(x-vt)}, \tag{40}$$

or

$$a_{2,2}(x,t) = \left\{ A_0 - \frac{2B_1\sigma}{\beta + \sqrt{4\,\alpha\,\sigma - \beta^2}\cot\left(\sqrt{4\,\alpha\,\sigma - \beta^2}\,\frac{(x-vt)}{2}\right)} \right\} e^{if(x-vt)}. \tag{41}$$

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ we obtain dark soliton solutions:

$$a_{2,3}(x,t) = \left\{ A_0 - \frac{2B_1\sigma}{\sqrt{-4\,\alpha\,\sigma + \beta^2}\tanh\left(\sqrt{-4\,\alpha\,\sigma + \beta^2}\,\frac{(x-vt)}{2}\right) + \beta} \right\} e^{if(x-vt)}, \tag{42}$$

or bright soliton solutions

$$a_{2,4}(x,t) = \left\{ A_0 - \frac{2B_1\sigma}{\sqrt{-4\,\alpha\,\sigma + \beta^2}\coth\left(\sqrt{-4\,\alpha\,\sigma + \beta^2}\,\frac{(x-vt)}{2}\right) + \beta} \right\} e^{if(x-vt)}. \tag{43}$$

When $\beta^2 - 4\alpha\sigma = 0$ and $\sigma \neq 0$ we obtain rational function solutions:

$$a_{2,5}(x,t) = \left\{ A_0 - \frac{2B_1\sigma\,(x-vt)}{\beta(x-vt)+2} \right\} e^{if(x-vt)}. \tag{44}$$

Using the values of the parameters in Set 2 given by Eq.(39), we get the solitary wave solutions of Eq.(2) in the following formulas: When $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$, we obtain trigonometric functions solutions:

$$a_{3,1}(x,t) = \left\{ A_0 + A_1 \left(\frac{-\beta + \sqrt{4\,\alpha\,\sigma - \beta^2}\tan\left(\sqrt{4\,\alpha\,\sigma - \beta^2}\,\frac{(x-vt)}{2}\right)}{2\sigma} \right) \right\} e^{if(x-vt)}, \tag{45}$$

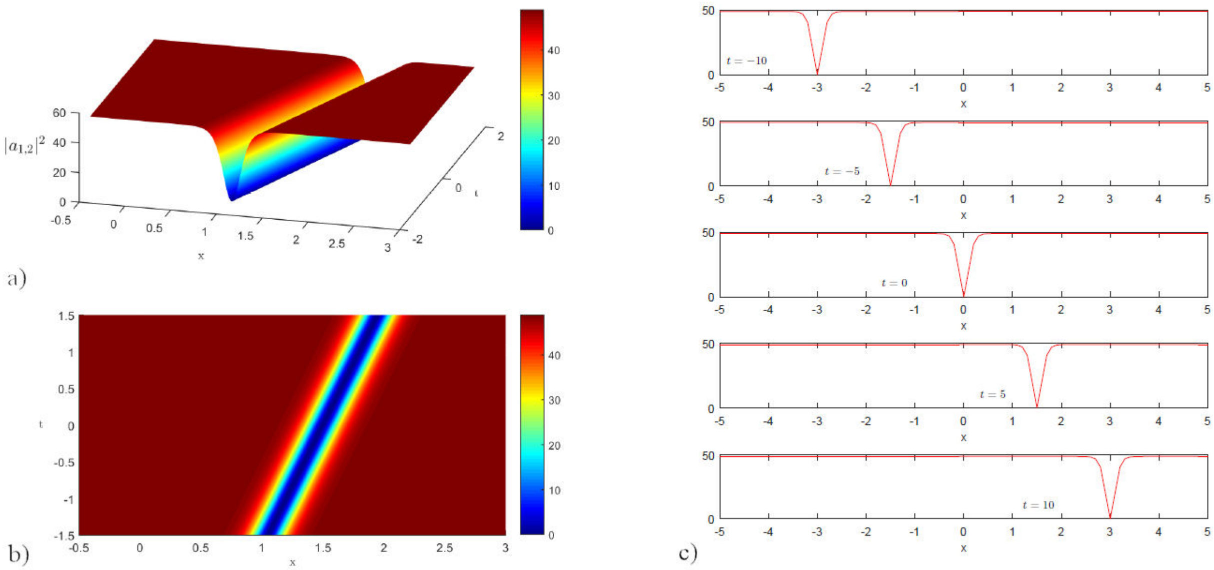

FIGURE 2. The plot of dark soliton of $|a_{1,2}|^2$ of the solution Eq. (33) at $A_1 = 1$, $C_0 = 520.62$, $C_2 = -58.91$, $C_4 = 1.66$, $v = 0.3$.

or

$$a_{3,2}(x,t) = \left\{ A_0 + A_1 \left(-\frac{\sqrt{4\,\alpha\,\sigma - \beta^2}\cot\left(\sqrt{4\,\alpha\,\sigma - \beta^2}\,\frac{(x-vt)}{2}\right) + \beta}{2\sigma} \right) \right\} e^{if(x-vt)}. \tag{46}$$

When $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$ we obtain dark soliton solutions:

$$a_{3,3}(x,t) = \left\{ A_0 + A_1 \left(-\frac{\sqrt{-4\,\alpha\,\sigma + \beta^2}\tanh\left(\sqrt{-4\,\alpha\,\sigma + \beta^2}\,\frac{(x-vt)}{2}\right) + \beta}{\sigma} \right) \right\} e^{if(x-vt)}, \tag{47}$$

or bright soliton solutions

$$a_{3,4}(x,t) = \left\{ A_0 + A_1 \left(-\frac{\sqrt{-4\,\alpha\,\sigma + \beta^2}\coth\left(\sqrt{-4\,\alpha\,\sigma + \beta^2}\,\frac{(x-vt)}{2}\right) + \beta}{2\sigma} \right) \right\} e^{if(x-vt)}. \tag{48}$$

When $\beta^2 - 4\alpha\sigma = 0$ and $\sigma \neq 0$ we obtain rational function solutions:

$$a_{3,5}(x,t) = \left\{ A_0 + A_1 \left(-\frac{\beta(x-vt) + 2}{2\sigma\,(x-vt)} \right) \right\} e^{if(x-vt)}. \tag{49}$$

Figures 1 and 2 show (a) the spatiotemporal evolution in 3D, (b) contour plot, and (c) the evolution at a different time in 2D of the bright and dark soliton solutions in optical fibers for $|a_{1,1}|^2$ and $|a_{1,2}|^2$, respectively. It is observed that the evolution plot of bright and dark solitons (c) for $-100 \leq x \leq 100$, at $t = 0$, $t = 10$, $t = 15$, $t = 20$ shift from left to right caused by the group velocity dispersion (GVD) term (β_2), which is fully annulled by the nonlinear phase shift modulation caused by (SPM), coming from a pulse that spreads undisturbed down the fiber.

3.3. The Sine-Gordon expansion approach

Plugging the predicted solution Eq. (27) and its necessary derivatives into Eq. (9), we get an over-determined system containing the combination of $\cos(\omega)$ and $\sin(\omega)$. Setting the same powers of $\sin^i(\omega)\cos^j(\omega)$ equal to zero, a system of algebraic equations is deduced. Solving this system, we report the following result:

Set 1:

$$A_0 = B_1 = 0, \quad A_1 = \sqrt{\frac{\left(c + \sqrt{2\,v^2{k_1}^2 + c^2 - 4\,vk_1 + 2}\right)}{2\gamma}}, \quad \beta_2 = \frac{c + \sqrt{2\,v^2{k_1}^2 + c^2 - 4\,vk_1 + 2}}{2v^2} \tag{50}$$

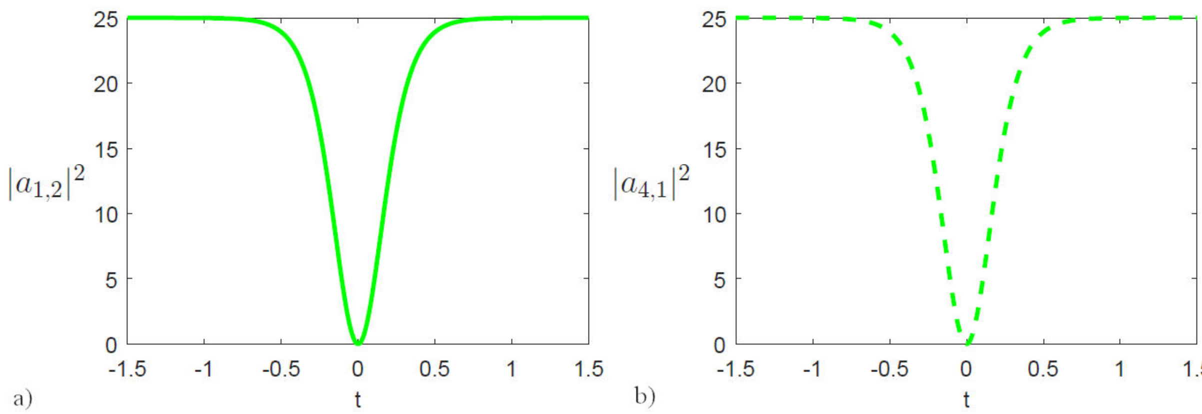

FIGURE 3. The plot of dark soliton $|a_{1,2}|^2$ of the solution Eq. (33) at $A_1 = 1.5$, $C_0 = 0.5$, $C_4 = 0.5$, $v = 4.5$, a) $[C_2 = -0.25, C_2 = -0.5, C_2 = -0.75]$; b) $[C_2 = -1.25, C_2 = -1.50, C_2 = -1.75]$ respectively.

Then, the solutions of Eq. (2) corresponding to Eq. (50) are

$$a_{4,1}(x,t) = \{A_1 \tanh(x - vt)\}\, e^{if(x-vt)}, \tag{51}$$

$$a_{4,2}(x,t) = \{A_1 \coth(x - vt)\}\, e^{if(x-vt)}, \tag{52}$$

where Eqs. (51) and (52) represent dark optical and singular soliton solutions, respectively.

Furthermore, Eq. (33) is read like Eq. (51) under the constraint conditions $C_2 = -1$, and $C_4 = 1/2$ (Fig. 3). On the other side when $C_2 \neq -1$, Eq. (33) and Eq. (51) (dark-solitons) have the same form, but differ in width and amplitude during long distance communication taking advantage of its stability under the influence of the material losses (see Fig. 4). So, it is important to conserve the obtained two results.

Set 2:

$$A_0 = A_1 = 0,$$

$$B_1 = \frac{\sqrt{\gamma\left(\sqrt{c^2 - (vk_1 - 1)^2} + c\right)}}{\gamma},$$

$$\beta_2 = -\frac{\sqrt{c^2 - (vk_1 - 1)^2} + c}{v^2}. \tag{53}$$

Then, the solutions of Eq. (2) corresponding to Eq. (53) are

$$a_{5,1}(x,t) = \{B_1 \mathrm{sech}(x - vt)\}\, e^{if(x-vt)}, \tag{54}$$

$$a_{5,2}(x,t) = \{iB_1 \mathrm{csch}(x - vt)\}\, e^{if(x-vt)}, \tag{55}$$

where Eq. (54) and Eq. (55) represent bright optical and singular soliton solutions, respectively.

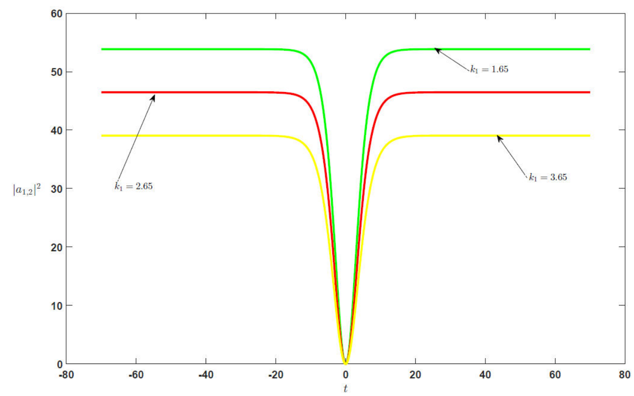

FIGURE 4. The plot of dark soliton $|a_{1,2}|^2$ of the solution Eq. (33) at $[A_1 = 1.5, C_0 = 51.62, C_2 = -2.31, C_4 = 0.26, c = 1.2, v = 0.15, \gamma = 0.35, \beta_2 = 1, k_1 = 1.65]$, $[A_1 = 1.5, C_0 = 38.43, C_2 = -2.002, C_4 = 0.026, c = 1.2, v = 0.15, \gamma = 0.35, \beta_2 = 1, k_2 = 2.65]$, $[A_1 = 1.5, C_0 = 27.18, C_2 = -1.68, C_4 = 0.026, c = 1.2, v = 0.15, \gamma = 0.35, \beta_2 = 1, k_1 = 3.65]$ respectively.

Remark: By integrating from Eq. (4) with along to Eq. (7), we have

$$f(\xi) = \int \frac{(1 - k_1 v)}{\beta_2 v^2} d\xi + \xi_0, \tag{56}$$

where ξ_0 is an integration constant. However when $k_1 = 0$, which corresponds to the conventional two-core fiber design [21], the chirp obtained depends only with the GVD and the speed of the soliton.

4. Conclusion

In this paper, we have investigated solitary waves for two-core optical fiber with coupling-coefficient dispersion and Kerr nonlinearity. It have been constructed an exact analytical soliton-like solutions by utilizing the traveling-wave hypothesis and hence the constraint relation fall out by adopting three integration techniques. By adopting the modified auxiliary equation and the sine-Gordon expansion approch, we obtain

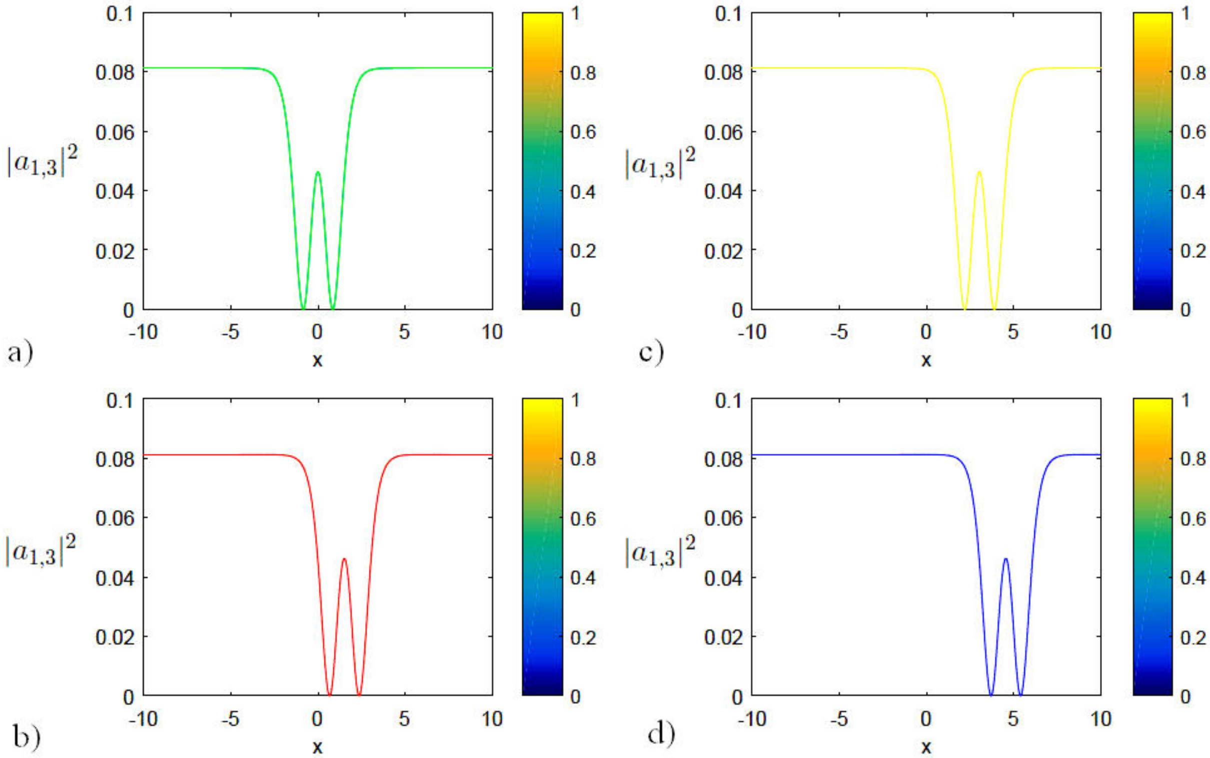

FIGURE 5. The plot of W-shape bright soliton $|a_{1,3}|^2$ of the solution Eq. (35) at a) $[k_1 = 0.975; c = 0.0461; v = 1.981; \gamma = 1.98005; A_1 = 0.71; \beta_2 = .90; \quad t = 0]$, b) $[k_1 = 0.975; c = 0.0461; v = 1.981; \gamma = 1.98005; A_1 = 0.71; \beta_2 = .90; \quad t = 3]$, c) $[k_1 = 0.975; c = 0.0461; v = 1.981; \gamma = 1.98005; A_1 = 0.71; \beta_2 = .90; \quad t = 6]$, d) $[k_1 = 0.975; c = 0.0461; v = 1.981; \gamma = 1.98005; A_1 = 0.71; \beta_2 = .90; \quad t = 9]$ respectively.

solitary waves; additionally, trigonometric function solutions and rational function solutions also emerge. Moreover, some new solitons solutions have been obtained, among which Eqs. (35)-(37), (40), (41), (43), (44), (49), and (55) are not reported in the standard integration method results summarized in Table Iof Ref. [30]. The righteousness of the auxiliary equation method in this work is that, without a lot convoluted calculations, we obtain bright, dark and W-shape bright solitons. Thus, it is also important to note that the auxiliary equation method is independent of the integrability of the nonlinear differential equation Eq. (9). The obtained results will certainly have an important effect in nonlinear optical fibers in the field of solitary waves and can be helpful in describing communication systems ans ultra fast phenomena.

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