New shape of the chirped bright, dark optical solitons and complex solutions for (2+1)-dimensional Ginzburg-Landau equation and modulation instability analysis

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The investigation of the Ginzburg-Landau equation (GLE) has been done to find out and investigate new chirped bright, dark periodic and singular function solutions. For this purpose, we have used the traveling wave hypothesis and the chirp component. From there it was pointed out the constraint relation to the different arbitrary parameters of the GLE. Thereafter, we have employed the improved sub-ODE method to handle the nonlinear ordinary differential equation (NODE). In the paper, the virtue of the used analytical method has been highlighted via new chirped solitary waves. Besides, to emphasize the confrontation between the nonlinearity and dispersion terms, we have investigated the steady state of the newly obtained results. It has been obtained the Modulation instability (MI) gain spectra under the effect of the power incident and the transverse wave number. In our knowledge, these results are new compared to Refs. [28–34], and are going to be helpful to explain physical phenomena.

Keywords: Chirped bright and dark; Complex solutions; (2+1)-Ginzburg-Landau equation; modulation instability.

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1. Introduction

The search for exact solutions of nonlinear systems has reached an unprecedented speed these days. The best known solitons solutions have found their applications in various fields just to name a few such as optical fibers, plasm, biology, quantum physics . Thus, solitons did not remain anonymous for a long time because of their direct implications in trans-continental and trans-oceanic data transport [1–24].
Without doubt, soliton is the one important wave which marveled in the field of data transport and securing it. It should also be noted that the most moving side of solitary waves comes from the fact that they are associated with chirped pulses. These chirped pulses, have been widely investigated in diverse shape in recent years by [27–29].
From this, many results in theoretical and experimentally have been followed with the mathematical tools to handle them [10, 35, 36]. These analytical methods facilitated the success of these results are among others, the Sine-Gordon expansion method, the modified $\exp(-\psi(\xi))$-expansion function method,(G'/G)-expansion scheme, the trial expansion method, the new mapping method, the auxiliary equation method, the rational function method, and the Riccati-Bernoulli sub-ODE method [25-30,35-45].
In this present work, an investigation will be carried out in order to formulate new shape of the chirped soliton solutions to the famous (2+1)-dimensional complex Ginzburg-Landau equation (GLE), of which skeletal structure is as follows [29–33]:

$$i\psi_t+\frac{1}{2}\psi_{xx}+\frac{1}{2}(\alpha-iG)\psi_{yy}+(1-i\lambda)|\psi|^2\psi+i\gamma\psi=0, \quad (1)$$

$\psi(x,t)$ represents complex wave profile on (2+1)-dimensional space time R^{2+1}, while t represents the temporal variable and x, y represent spatial variable. α, λ, γ and G are real. The set of the CGLE Eq. (1) was recently used to depict the beginning of stationary periodic solutions in nonlinear stability problems. It takes the name of real Ginzburg-Landau equation when ($\alpha = \lambda = 0$) [30]. To get right to the purpose, the work is organized as follows: Section 2 is devoted to the traveling-wave solution. Section 3 is used the linear stability technic to study the modulation instability gain spectrum. Section 4 will depicts the obtained analytical results with their physical explanation. The last part of the work will present the conclusions of this work.

1.1. Analytical investigation and traveling waves solution

The following envelope transformation is used to build soliton solution

$$\psi(x,y,t)=\phi(\zeta)\exp[i\,(\xi+A(\zeta))], \tag{2}$$

where $\phi(\zeta)$ is real function and the chirp component is $A(\zeta))$, with ξ and ζ given by $\xi = kx+ly+vt+\xi_0$ and $\zeta = nx+my+\omega t+\eta_0$, respectively. Inserting Eq. (2) into Eq. (1) splits imaginary and real parts as

$$\begin{aligned}(n^2+\alpha m^2)\phi A+2(n^2+\alpha m^2)\phi' A'+Gm^2{A'}^2\phi-Gm^2\phi''\\ +2(nk+\omega+\alpha l)\phi'+2Gml\phi A'+(Gl^2+2\gamma)\phi-2\lambda\phi^3=0,\end{aligned} \tag{3}$$

and

$$\begin{aligned}(n^2+\alpha m^2)\phi''-(n^2+\alpha m^2){A'}^2\phi+2Gm^2A'\phi'+Gm^2\phi A''\\ -2(nk+\omega\alpha ml)A'\phi+2Gml\phi'-(k^2+\alpha l^2+2v)\phi+2\phi^3=0,\end{aligned} \tag{4}$$

Suppose that the chirp is given by

$$A(\zeta)=A_0\ln\left(|\phi(\zeta)|\right), \tag{5}$$

where A_0 is an arbitrary constant to obtain later. Plugging Eq. (5) into Eq. (4) and Eq. (3), we get

$$\ell_1\phi\phi''+\ell_2{\phi'}^2+\ell_3\phi\phi'+\ell_4\phi^2-2\lambda\phi^4=0, \tag{6}$$

and

$$\Lambda_1\phi\phi''-\Lambda_2{\phi'}^2-\Lambda_3\phi\phi'-\Lambda_4\phi^2+2\phi^4=0, \tag{7}$$

where

$$\ell_1=A_0n^2+A_0\alpha m^2-Gm^2,\quad \ell_2=A_0\left(n^2+\alpha m^2+A_0Gm^2\right),\ \ell_3=2\left(nk+\omega+\alpha ml+GmlA_0\right),\ \ell_4=Gl^2+2\gamma.$$
$$\Lambda_1=n^2+\alpha m^2,\ \Lambda_2=A_0\left(A_0n^2+A_0\alpha m^2-Gm^2\right),\ \Lambda_3=2\left(A_0nk+A_0\omega+A_0\alpha ml-Gml\right),\ \Lambda_4=k^2+\alpha l^2+2v.$$

To deal with an analytical solutions to Eq. (7) and Eq. (6), we suppose that $\Lambda_3=\ell_3=0$. Consequently it is recovered

$$A_0=\frac{Gml}{nk+\omega+\alpha ml}. \tag{8}$$

Then Eq. (7) and Eq. (6) become

$$\ell_1\phi\phi''+\ell_2{\phi'}^2+\ell_4\phi^2-2\lambda\phi^4=0, \tag{9}$$

$$\Lambda_1\phi\phi''-\Lambda_2{\phi'}^2-\Lambda_4\phi^2+2\phi^4=0, \tag{10}$$

It has become easy to investigate soliton-like solutions now. For this purpose, we considered the following expression as solution [45]

$$\phi=\mu F^n(\zeta),\quad \mu>0, \tag{11}$$

and n an arbitrary constant, while $F(\xi)$ is taken like solutions of the following ordinary differential equation [45]

$${F'}^2(\zeta)=AF^{2-2p}(\zeta)+BF^{2-p}(\zeta)+CF^2(\zeta)+DF^{2+p}(\zeta)+EF^{2+2p}(\zeta).\quad p>0. \tag{12}$$

With the homogeneous balance principle between $\phi\phi''$ and ϕ^4, it is obtained $n+n+2p=4n\Rightarrow p=n$. From which Eq. (11) turns to

$$\phi=\mu F^p(\zeta). \tag{13}$$

Using the set of equations given by Eq. (13) together with Eq. (12) and make use of into Eq. (10) or Eq. (9) gives the set of system of equation in terms of $F^{jp}(\zeta)(j = 2,3,4,5,6,8)$

$$-2\,\lambda\,\mu^4 F^{8p}(\zeta) + \left(6\ell_1\mu^2p^2E + 4\ell_2\mu^2p^2E\right)F^{6p}(\zeta) + \left(4\ell_2\mu^2p^2D + 5\ell_1\mu^2p^2D\right)F^{5p}(\zeta)$$
$$+ \left(4\ell_2\mu^2p^2C + 4\ell_1\mu^2p^2C + \ell_4\mu^2\right)F^{4p}(\zeta) + \left(4\ell_2\mu^2p^2B + 3\ell_1\mu^2p^2B\right)F^{3p}(\zeta)$$
$$+ \left(2\ell_1\mu^2p^2A + 4\ell_2\mu^2p^2A\right)F^{2p}(\zeta) = 0. \tag{14}$$

Thereafter solving the obtained set of system of Eq. (14) by using the mathematical software `Maple 18`, it is revealed.

$$A = 0,\ B = 0,\ C = -\frac{1}{4}\frac{\ell_4}{p^2(\ell_2+\ell_1)},\ D = D,\ E = E. \tag{15}$$

We can thus unroll the types of solutions of Eq. (1), as well as the corresponding chirped solutions
Case 1: If $A = 0,\ B = 0, D = 0$, it is recovered bright soliton of Eq. (1):

$$\psi_{1,1}(x,y,t) = e^{(i\xi)}\mu\left[\varepsilon\sqrt{-\frac{C}{E}}\,\mathrm{sech}\left(p\sqrt{C}\zeta\right)\right]^{\frac{1}{p}}\left[\left|\mu\left(\varepsilon\sqrt{-\frac{C}{E}}\,\mathrm{sech}\left(p\sqrt{C}\zeta\right)\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},$$
$$C>0,\quad E<0,\quad \varepsilon\pm1, \tag{16}$$

and the bright chirp is

$$A_{1,1}(x,y,t) = A_0\ln\left|\mu\left[\varepsilon\sqrt{-\frac{C}{E}}\,\mathrm{sech}\left(p\sqrt{C}\zeta\right)\right]^{\frac{1}{p}}\right|,\quad C>0,\quad E<0,\quad \varepsilon\pm1, \tag{17}$$

a periodic function solutions

$$\psi_{1,2}(x,y,t) = e^{(i\xi)}\mu\left[\varepsilon\sqrt{-\frac{C}{E}}\sec\left(p\sqrt{-C}\zeta\right)\right]^{\frac{1}{p}}\left[\left|\mu\left(\varepsilon\sqrt{-\frac{C}{E}}\sec\left(p\sqrt{-C}\zeta\right)\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},$$
$$C<0,\quad E>0,\quad \varepsilon\pm1, \tag{18}$$

and the periodic chirp

$$A_{1,2}(x,y,t) = A_0\ln\left|\left[\varepsilon\sqrt{-\frac{C}{E}}\sec\left(p\sqrt{-C}\zeta\right)\right]^{\frac{1}{p}}\right|,\quad C<0,\quad E>0,\quad \varepsilon\pm1, \tag{19}$$

then a rational solution

$$\psi_{1,3}(x,y,t) = e^{(i\xi)}\mu\left[\frac{\varepsilon}{p\sqrt{E}\zeta}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{\varepsilon}{p\sqrt{E}\zeta}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},\quad C=0,\quad E>0,\quad \varepsilon\pm1. \tag{20}$$

and the rational chirp

$$A_{1,3}(x,y,t) = A_0\ln\left|\mu\left[\frac{\varepsilon}{p\sqrt{E}\zeta}\right]^{\frac{1}{p}}\right|,\quad C=0,\quad E>0,\quad \varepsilon\pm1. \tag{21}$$

Case 2: By setting the variables $A = 0,\ B = 0$, we deduce three forms of solutions of Eq. (5):

$$\psi_{2,1}(x,y,t) = e^{(i\xi)}\mu\left[\frac{1}{\cosh(p\sqrt{C}\zeta)-\frac{D}{2C}}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{1}{\cosh(p\sqrt{C}\zeta)-\frac{D}{2C}}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},$$
$$C>0,\ D<2C,\quad E=\frac{D^2}{4C}-C, \tag{22}$$

and the chirp

$$A_{2,1}(x,y,t) = A_0 \ln \left| \mu \left[\frac{1}{\cosh(p\sqrt{C}\zeta) - \frac{D}{2C}} \right]^{\frac{1}{p}} \right|, \quad C > 0,\ D < 2C, \quad E = \frac{D^2}{4C} - C, \tag{23}$$

it is gained for $C > 0,\ E > 0, D = -2\sqrt{CE}, \quad \varepsilon = \pm 1,$

$$\psi_{2,2}(x,y,t) = e^{(i\xi)} \mu \left[\frac{1}{2}\sqrt{\frac{C}{E}} \left(1 + \varepsilon \tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right) \right]^{\frac{1}{p}} \left[\left| \mu \left(\frac{1}{2}\sqrt{\frac{C}{E}} \left(1 + \varepsilon \tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right) \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \tag{24}$$

the corresponding chirp gives

$$A_{2,2}(x,y,t) = A_0 \ln \left| \mu \left[\frac{1}{2}\sqrt{\frac{C}{E}} \left(1 + \varepsilon \tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right) \right]^{\frac{1}{p}} \right|, \quad C > 0, \quad E > 0, \quad D = -2\sqrt{CE}, \quad \varepsilon = \pm 1 \tag{25}$$

and

$$\psi_{2,3}(x,y,t) = e^{(i\xi)} \mu \left[\frac{4D}{(pD\zeta)^2 - 4E} \right]^{\frac{1}{p}} \left[\left| \mu \left(\frac{4D}{(pD\zeta)^2 - 4E} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \qquad C = 0,\ E < 0. \tag{26}$$

with the chirp

$$A_{2,3}(x,y,t) = A_0 \left| \mu \left[\frac{4D}{(pD\zeta)^2 - 4E} \right]^{\frac{1}{p}} \right|, \quad C = 0,\ E < 0. \tag{27}$$

Case 3: Considering $A = B = 0,\ C > 0$, we have gained combined bright soliton and hyperbolic functions solutions of Eq. (5):

$$\psi_{3,1}(x,y,t) = e^{(i\xi)} \mu \left[\frac{2C \operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} + D)\operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)} \right]^{\frac{1}{p}}$$

$$\times \left[\left| \mu \left(\frac{2C \operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} + D)\operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)} \quad D^2 - 4CE > 0, \tag{28}$$

the corresponding chirp

$$A_{3,1}(x,y,t) = A_0 \ln \left| \left[\frac{2C \operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} + D)\operatorname{sech}^2(\frac{p}{2}\sqrt{C}\zeta)} \right]^{\frac{1}{p}} \right|, \quad D^2 - 4CE > 0, \tag{29}$$

$$\psi_{3,2}(x,y,t) = e^{(i\xi)} \mu \left[\frac{2C \operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} - D)\operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)} \right]^{\frac{1}{p}}$$

$$\times \left[\left| \mu \left(\frac{2C \operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} - D)\operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \quad D^2 - 4CE > 0, \tag{30}$$

with the corresponding chirp

$$A_{3,2}(x,y,t) = A_0 \ln \left| \mu \left[\frac{2C \operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} - D)\operatorname{csch}^2(\frac{p}{2}\sqrt{C}\zeta)} \right]^{\frac{1}{p}} \right|, \quad D^2 - 4CE > 0, \tag{31}$$

for $D^2 - 4CE > 0$, $\varepsilon = \pm 1$

$$\psi_{3,3}(x,y,t) = e^{(i\xi)} \mu \left[\frac{2C}{\varepsilon\sqrt{D^2 - 4CE}\cosh(p\sqrt{C}\xi) - D} \right]^{\frac{1}{p}} \left[\left| \mu \left(\frac{2C}{\varepsilon\sqrt{D^2 - 4CE}\cosh(p\sqrt{C}\zeta) - D} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \tag{32}$$

the chirp gives

$$A_{3,3}(x,y,t) = A_0 \ln \left| \mu \left[\frac{2C}{\varepsilon\sqrt{D^2-4CE}\cosh(p\sqrt{C}\zeta) - D} \right]^{\frac{1}{p}} \right|, \quad D^2-4CE>0, \quad \varepsilon=\pm 1 \tag{33}$$

for $D^2-4CE<0$, $\varepsilon=\pm 1$

$$\psi_{3,4}(x,y,t) = e^{(i\xi)}\mu \left[\frac{2C}{\varepsilon\sqrt{-(D^2-4CE)}\sinh(p\sqrt{C}\zeta) - D} \right]^{\frac{1}{p}} \times \left[\left| \mu \left(\frac{2C}{\varepsilon\sqrt{-(D^2-4CE)}\sinh(p\sqrt{C}\zeta) - D} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \tag{34}$$

the chirp is

$$A_{3,4}(x,y,t) = A_0 \ln \left| \mu \left[\frac{2C}{\varepsilon\sqrt{-(D^2-4CE)}\sinh(p\sqrt{C}\zeta) - D} \right]^{\frac{1}{p}} \right|, \quad D^2-4CE<0, \quad \varepsilon=\pm 1 \tag{35}$$

$$\psi_{3,5}(x,y,t) = e^{(i\xi)}\mu \left[-\frac{C}{D}\left(1+\varepsilon\tanh(\frac{p}{2}\sqrt{C}\zeta)\right) \right]^{\frac{1}{p}} \left[\left| \mu \left(-\frac{C}{D}\left(1+\varepsilon\tanh(\frac{p}{2}\sqrt{C}\zeta)\right) \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}$$
$$D^2-4CE=0, \quad \varepsilon=\pm 1 \tag{36}$$

the chirp is given by

$$A_{3,5}(x,y,t) = A_0 \ln \left| \mu \left[-\frac{C}{D}\left(1+\varepsilon\tanh\left[\frac{p}{2}\sqrt{C}\zeta\right]\right) \right]^{\frac{1}{p}} \right|, \quad D^2-4CE=0, \quad \varepsilon=\pm 1 \tag{37}$$

for $D^2-4CE=0$, $\varepsilon=\pm 1$

$$\psi_{3,6}(x,y,t) = e^{(i\xi)}\mu \left[-\frac{C}{D}\left(1+\varepsilon\coth(\frac{p}{2}\sqrt{C}\zeta)\right) \right]^{\frac{1}{p}} \left[\left| \mu \left(-\frac{C}{D}\left(1+\varepsilon\coth\left[\frac{p}{2}\sqrt{C}\zeta\right]\right) \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \tag{38}$$

the corresponding chirp

$$A_{3,6}(x,y,t) = A_0 \ln \left| \mu \left[-\frac{C}{D}\left(1+\varepsilon\coth\left[\frac{p}{2}\sqrt{C}\zeta\right]\right) \right]^{\frac{1}{p}} \right|, \quad D^2-4CE=0, \quad \varepsilon=\pm 1 \tag{39}$$

for $E>0, \quad \varepsilon=\pm 1$

$$\psi_{3,7}(x,y,t)=e^{(i\xi)}\mu \left[-\frac{C\,\mathrm{sech}^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)} \right]^{\frac{1}{p}} \left[\left| \mu \left(-\frac{C sech^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)} \right)^{\frac{1}{p}} \right| \right]^{(iA_0)}, \tag{40}$$

the chirp is revealed as

$$A_{3,7}(x,y,t) = A_0 \ln \left| \mu \left[-\frac{C sech^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)} \right]^{\frac{1}{p}} \right|, \quad E>0, \quad \varepsilon=\pm 1 \tag{41}$$

for $E>0, \quad \varepsilon=\pm 1$

$$\psi_{3,8}(x,y,t) = e^{(i\xi)}\mu \left[\frac{C\,\mathrm{csch}^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)} \right]^{\frac{1}{p}} \left[\left| \mu \left(\frac{C\,\mathrm{csch}^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)^{\frac{1}{2}}} \right) \right| \right]^{(iA_0)}, \tag{42}$$

and the corresponding chirp

$$A_{3,8}(x,y,t) = A_0 \ln\left|\mu\left[\frac{Ccsch^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D+2\varepsilon\sqrt{CE}\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)}\right]^{\frac{1}{p}}\right|, \quad E>0, \quad \varepsilon=\pm 1 \tag{43}$$

$$\psi_{3,9}(x,y,t) = e^{(i\xi)}\mu\left[\frac{-CDsech^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right]^{\frac{1}{2}} \times\left[\left|\mu\left(\frac{-CDsech^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right)^{\frac{1}{P}}\right|\right]^{(iA_0)}, \tag{44}$$

the chirp gives

$$A_{3,9}(x,y,t) = A_0 \ln\left|\mu\left[\frac{-CDsech^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\tanh\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right]^{\frac{1}{P}}\right|, \tag{45}$$

$$\psi_{3,10}(x,y,t) = e^{(i\xi)}\mu\left[\frac{CDcsch^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right]^{\frac{1}{P}} \times\left[\left|\mu\left(\frac{CDcsch^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}. \tag{46}$$

and the corresponding chirp as

$$A_{3,11}(x,y,t) = A_0 \ln\left|\mu\left[\frac{CDcsch^2\left(\frac{p}{2}\sqrt{C}\zeta\right)}{D^2-CE\left(1+\varepsilon\coth\left(\frac{p}{2}\sqrt{C}\zeta\right)\right)^2}\right]^{\frac{1}{p}}\right|. \tag{47}$$

Case 4: Considering $A=B=0,\ C<0$, we have gained combined bright soliton and hyperbolic functions as solutions For $D^2-4CE>0$

$$\psi_{4,1}(x,y,t) = e^{(i\xi)}\left[\frac{-2C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}-D)\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}} \times\left[\left|\mu\left(\frac{-2C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}-D)\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \tag{48}$$

the chirp gives

$$A_{4,2}(x,y,t) = A_0 \ln\left|\mu\left[\frac{-2C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}-D)\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad D^2-4CE>0, \tag{49}$$

for $D^2-4CE>0$,

$$\psi_{4,3}(x,y,t)=e^{(i\xi)}\mu\left[\frac{2C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}+D)\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}} \times\left[\left|\mu\left(\frac{2C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}+D)\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \quad (50)$$

the chirp gives

$$A_{4,4}(x,y,t)=A_0\ln\left|\mu\left[\frac{2C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{2\sqrt{D^2-4CE}-(\sqrt{D^2-4CE}+D)\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad D^2-4CE>0, \quad (51)$$

for $D^2-4CE>0,\ \varepsilon=\pm1$,

$$\psi_{4,5}(x,y,t)=e^{(i\xi)}\mu\left[\frac{2C\sec(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\sec(p\sqrt{-C}\zeta)}\right]^{\frac{1}{p}} \times\left[\left|\mu\left(\frac{2C\sec(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\sec(p\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \quad (52)$$

the chirp reads as follows

$$A_{4,5}(x,y,t)=A_0\ln\left|\mu\left[\frac{2C\sec(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\sec(p\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad D^2-4CE>0, \quad \varepsilon=\pm1, \quad (53)$$

for $D^2-4CE>0$, $\varepsilon=\pm1$

$$\psi_{4,6}(x,y,t)=e^{(i\xi)}\mu\left[\frac{2C\csc(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\csc(p\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{2C\csc(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\csc(p\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},$$

the chirp is given by

$$A_{4,6}(x,y,t)=A_0\ln\left|\mu\left[\frac{2C\csc(p\sqrt{-C}\zeta)}{\varepsilon\sqrt{D^2-4CE}-D\csc(p\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad D^2-4CE>0, \quad \varepsilon=\pm1, \quad (54)$$

for $E>0,\ \varepsilon=\pm1$,

$$\psi_{4,7}(x,y,t)=e^{(i\xi)}\mu\left[-\frac{C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\tan(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\left[\left|\mu\left(-\frac{C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\tan(\frac{p}{2}\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \quad (55)$$

the chirp is obtained

$$A_{4,7}(x,y,t)=A_0\ln\left|\mu\left[-\frac{C\sec^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\tan(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad E>0, \quad \varepsilon=\pm1, \quad (56)$$

for $D^2-4CE>0$, $E>0$, $\varepsilon=\pm1$

$$\psi_{4,8}(x,y,t)=e^{(i\xi)}\mu\left[-\frac{C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\cot(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\left[\left|\mu\left(-\frac{C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\cot(\frac{p}{2}\sqrt{-C}\zeta)}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \quad (57)$$

the chirp is

$$A_{4,8}(x,y,t)=A_0\ln\left|\mu\left[-\frac{C\csc^2(\frac{p}{2}\sqrt{-C}\zeta)}{D+2\varepsilon\sqrt{-CE}\cot(\frac{p}{2}\sqrt{-C}\zeta)}\right]^{\frac{1}{p}}\right|, \quad D^2-4CE>0, E>0, \quad \varepsilon=\pm1. \quad (58)$$

Case 5: For $A = B = 0, \quad C > 0, \quad \varepsilon = \pm 1,$

$$\psi_{5,1}(x,y,t) = e^{(i\xi)}\mu\left[\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\zeta)}}{\left(e^{\varepsilon p\sqrt{C}\zeta} - Dp^2\right)^2 - 4CEp^4}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\zeta)}}{\left(e^{\varepsilon p\sqrt{C}\zeta} - Dp^2\right)^2 - 4CEp^4}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \tag{59}$$

the chirp component is

$$A_{5,1}(x,y,t) = A_0 \ln\left|\mu\left[\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\xi)}}{\left(e^{\varepsilon p\sqrt{C}\zeta} - Dp^2\right)^2 - 4CEp^4}\right]^{\frac{1}{p}}\right|, \quad C > 0, \quad \varepsilon = \pm 1, \tag{60}$$

$$\psi_{5,2}(x,y,t) = e^{(i\xi)}\mu\left[\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\zeta)}}{-1 + 4CEp^4 e^{2\varepsilon p\sqrt{C}\zeta}}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\zeta)}}{-1 + 4CEp^4 e^{2\varepsilon p\sqrt{C}\zeta}}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)},$$
$$C > 0, \ D = 0, \quad \varepsilon = \pm 1, \tag{61}$$

the chirp gives

$$A_{5,2}(x,y,t) = A_0 \ln\left|\mu\left[\frac{4Cp^2 e^{(p\varepsilon\sqrt{C}\zeta)}}{-1 + 4CEp^4 e^{2\varepsilon p\sqrt{C}\zeta}}\right]^{\frac{1}{p}}\right|, \quad C > 0, \ D = 0, \quad \varepsilon = \pm 1, \tag{62}$$

$$\psi_{5,3}(x,y,t) = e^{(i\xi)}\mu\left[\frac{\varepsilon}{p\sqrt{E}\zeta}\right]^{\frac{1}{p}}\left[\left|\mu\left(\frac{\varepsilon}{p\sqrt{E}\zeta}\right)^{\frac{1}{p}}\right|\right]^{(iA_0)}, \quad E > 0, \ C = D = 0, \quad \varepsilon = \pm 1. \tag{63}$$

The last chirp gives

$$A_{5,3}(x,y,t) = A_0 \ln\left|\mu\left[\frac{\varepsilon}{p\sqrt{E}\zeta}\right]^{\frac{1}{p}}\right|, \quad E > 0, \ C = D = 0, \quad \varepsilon = \pm 1. \tag{64}$$

1.2. Modulation analysis

This section will be using linear analysis technique to take out the modulation instability (MI) gain spectrum. Assuming the steady state solution of Eq. (1) in the form of:

$$\psi(x,y,t) = [\sqrt{P_0} + B(x,y,t)]e^{i\phi_{NL}}, \quad \phi_{NL} = \delta P_0 x \tag{65}$$

where P_0 is the incident power. $B(x,y,t)$ is the small perturbation component and $B^*(x,y,t)$ is the complex conjugate. Inserting Eq. (66) into Eq. (1) lead to

$$\psi(x,y,t) = iB_t + \frac{1}{2}B_{xx} + \frac{1}{2}(1-\alpha G)B_{yy} + (1-i\lambda)(P_0(2B+B^*) + i\gamma B = 0. \tag{66}$$

Suppose the solution of Eq. (67) is in the following expression

$$B(x,y,t) = a_1 e^{i[Kx+\Gamma y-\Omega t]} + a_2 e^{-i[Kx+\Gamma y-\Omega t]}. \tag{67}$$

where $a_j (j = 1,2)$ are reals, and K and Ω are wave numbers and the modulation frequency, respectively. The quantity Γ is the transverse wave number of the perturbation. Inserting Eq. (68) into Eq. (67) gives the set of linear of coupled equations for a_1 and a_2

$$\left(\Omega - 2i\lambda P_0 - \frac{1}{2}K^2 + 2P_0 - \frac{1}{2}\alpha\Gamma^2 + \lambda\gamma + \frac{1}{2}iG\Gamma^2 + i\gamma\right)a_1 + (P_0 - i\lambda P_0)\,a_2 = 0,$$
$$(P_0 - i\lambda P_0)\,a_1 + \left(-\frac{1}{2}K^2 + 2P_0 - \Omega - 2i\lambda P_0 + \frac{1}{2}iG\Gamma^2 + i\gamma - \frac{1}{2}\alpha\Gamma^2 + \lambda\gamma\right)a_2 = 0. \tag{68}$$

This set of coupled of equation has a nontrivial solution when the determinant of the matrix below vanishes

$$\begin{bmatrix} \Omega-2\,i\lambda\,P_0-\frac{1}{2}K^2+2\,P_0-\frac{1}{2}\alpha\Gamma^2+\lambda\,\gamma+\frac{1}{2}iG\Gamma^2+i\gamma & P_0-i\lambda P_0 \\ P_0-i\lambda\,P_0 & -\frac{1}{2}K^2+2P_0-\Omega-2i\lambda\,P_0+\frac{1}{2}iG\Gamma^2+i\gamma-\frac{1}{2}\alpha\Gamma^2+\lambda\gamma \end{bmatrix} \tag{69}$$

Thereafter, the MI gain spectrum is revealed as

$$\begin{aligned} 2Im(K) = \Big(& -4iK^2\gamma - 4G\Gamma^2\gamma - 24i\lambda P_0^2 - 8P_0\alpha\Gamma^2 + 16iP_0\gamma + 8i\lambda\gamma^2 - 4K^2\lambda\gamma + 4\lambda^2\gamma^2 - G^2\Gamma^4 - 12\lambda^2P_0^2 \\ & - 8K^2P_0 + 12P_0^2 + \alpha^2\Gamma^4 + 8i\lambda P_0\alpha\Gamma^2 + 4i\lambda\gamma G\Gamma^2 + 2K^2\alpha\Gamma^2 + 32P_0\lambda\,\gamma + 8\lambda P_0 G\Gamma^2 + 8i\lambda P_0K^2 \\ & - 2K^2iG\Gamma^2 + 8P_0iG\Gamma^2 - 2i\alpha\,\Gamma^4G - 16i\lambda^2P_0\gamma - 4i\alpha\Gamma^2\gamma - 4\alpha\Gamma^2\lambda\gamma - 4\gamma^2 + K^4\Big)^{\frac{1}{2}}, \end{aligned} \tag{70}$$

Now to investigate the behavior of Eq. (71), we first point out the condition of the obtaining the steady state. So, it is important to highlight the fact that the perturbation grow exponentially, when the wave number value contains the imaginary part, in this condition the steady state solution is unstable. However, in case of small perturbation and having the wave number with real value, the steady state of the solution is stable.

Here two cases are going to be discussed:

- Case 1: In this case, the steady state is stable again and a small perturbution is seen if K is real

$$\begin{aligned} \Big(& -4iK^2\gamma - 4G\Gamma^2\gamma - 24i\lambda P_0^2 - 8P_0\alpha\Gamma^2 + 16iP_0\gamma + 8i\lambda\gamma^2 - 4K^2\lambda\gamma + 4\lambda^2\gamma^2 - G^2\Gamma^4 - 12\lambda^2P_0^2 - 8K^2P_0 \\ & + 12P_0^2 + \alpha^2\Gamma^4 + 8i\lambda P_0\alpha\Gamma^2 + 4i\lambda\gamma G\Gamma^2 + 2K^2\alpha\Gamma^2 + 32P_0\lambda\,\gamma + 8\lambda P_0G\Gamma^2 + 8i\lambda P_0K^2 - 2K^2iG\Gamma^2 + 8P_0iG\Gamma^2 \\ & - 2i\alpha\,\Gamma^4G - 16i\lambda^2P_0\gamma - 4i\alpha\Gamma^2\gamma - 4\alpha\Gamma^2\lambda\gamma - 4\gamma^2 + K^4\Big)^{\frac{1}{2}} > 0, \end{aligned} \tag{71}$$

but if λ, γ and G are non zero value, the inequality Eq. (72) is invalid. Consequently the steady is still unstable.

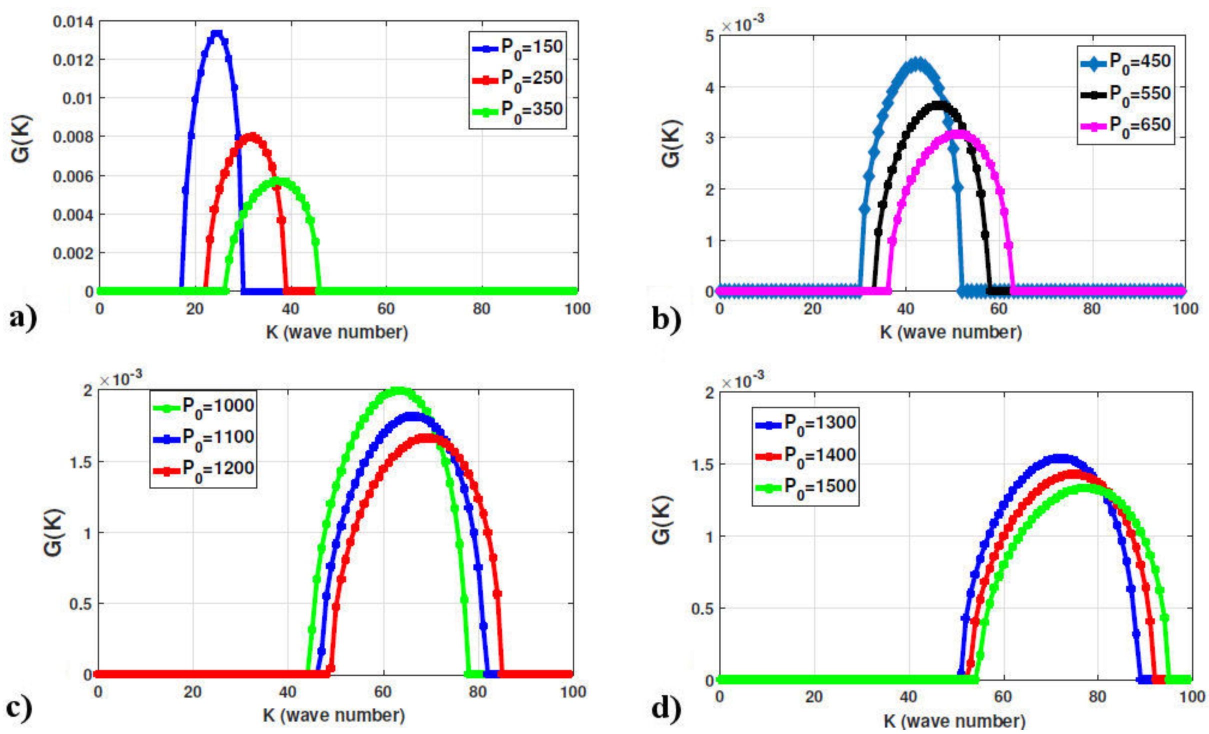

FIGURE 1. (Color online) Variation of MI gain spectrum versus wave number with the effect of the incident power a) $[P_0 = 150,\ P_0 = 250,\ P_0 = 350]$, b) $[P_0 = 450,\ P_0 = 550,\ P_0 = 650]$, c) $[P_0 = 1000,\ P_0 = 1100,\ P_0 = 1200]$, d) $[P_0 = 1300,\ P_0 = 1400,\ P_0 = 1500]$ at $\alpha = 0.1, \Gamma = 0.5$.

- Case 2: In this case, the MI occurs when K is imaginary due to the fact that the perturbation increases exponentially and in the same time

$$\Big(-4iK^2\gamma - 4G\Gamma^2\gamma - 24i\lambda P_0^2 - 8P_0\alpha\Gamma^2 + 16iP_0\gamma + 8i\lambda\gamma^2 - 4K^2\lambda\gamma + 4\lambda^2\gamma^2 - G^2\Gamma^4 - 12\lambda^2P_0^2 - 8K^2P_0 + 12P_0^2 + \alpha^2\Gamma^4 + 8i\lambda P_0\alpha\Gamma^2 + 4i\lambda\gamma G\Gamma^2 + 2K^2\alpha\Gamma^2 + 32P_0\lambda\gamma + 8\lambda P_0G\Gamma^2 + 8i\lambda P_0K^2 - 2K^2iG\Gamma^2 + 8P_0iG\Gamma^2 - 2i\alpha\Gamma^4G - 16i\lambda^2P_0\gamma - 4i\alpha\Gamma^2\gamma - 4\alpha\Gamma^2\lambda\gamma - 4\gamma^2 + K^4\Big)^{\frac{1}{2}} < 0, \tag{72}$$

in the same way if (λ, γ , $G \neq 0$) it remains invalid. To analyze the MI gain spectrum, we suppose that $\lambda = \gamma = G = 0$. Consequently the increase rate of MI gain spectrum $G(K) = 2Im(K)$ is given by

$$G(K) = \sqrt{-8\alpha P_0\Gamma^2 - 8K^2P_0 + 12P_0^2 + \alpha^2\Gamma^4 + 2\alpha K^2\Gamma^2 + K^4}, \tag{73}$$

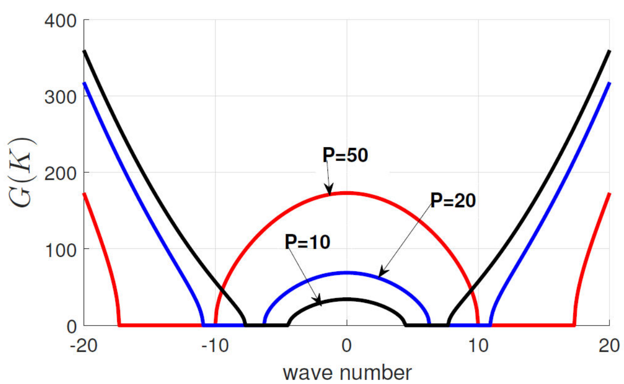

FIGURE 2. Plot of the MI gain spectrum versus wave number with the effect of the incident power (black line [$P_0 = 10$], blue line [$P = 50$] and red line [$P = 20$]) at $\alpha = 0.75, \Gamma = 0.75$.

1.3. Physical explanation and Modulation instability analysis

Figure 1 is the illustration of the MI gain spectrum versus wave number with the effect of incident power. It is observed that when the incident power increase the unstable band also increase. So, at the maximum incident power, it remains stable (see Fig. 1d). Meanwhile, Fig. 2 is illustration of the MI gain spectrum with small value of the incident power. The unstable plage increase when the incident power value is to small (see black line). Futhermore, Fig. 3 is the illustration of the MI gain spectra versus wave number under the effect of the transverse wave number. For $\Gamma = 70.50$, one side lobe is obtained and $38 \leq K \leq 100$. Figures 4a) and b) have depicted the chirped bright 3D and 2D for values of n=0.5, m=0.14,

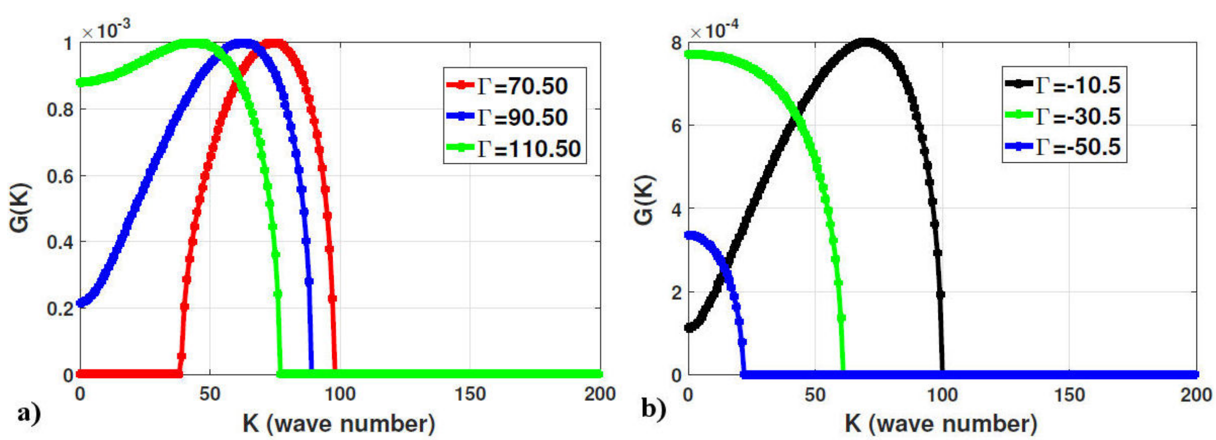

FIGURE 3. (Color online) variation of the MI gain spectra versus wave number with the effect of the transverse wave number a) [$\Gamma = 70.50,\ \Gamma = 90.50,\ \Gamma = 110.50$] and b) [$\Gamma = -10.50,\ \Gamma = -30.50,\ \Gamma = -50.50$] at $\alpha = 0.05,\ P_0 = 2500$ respectively.

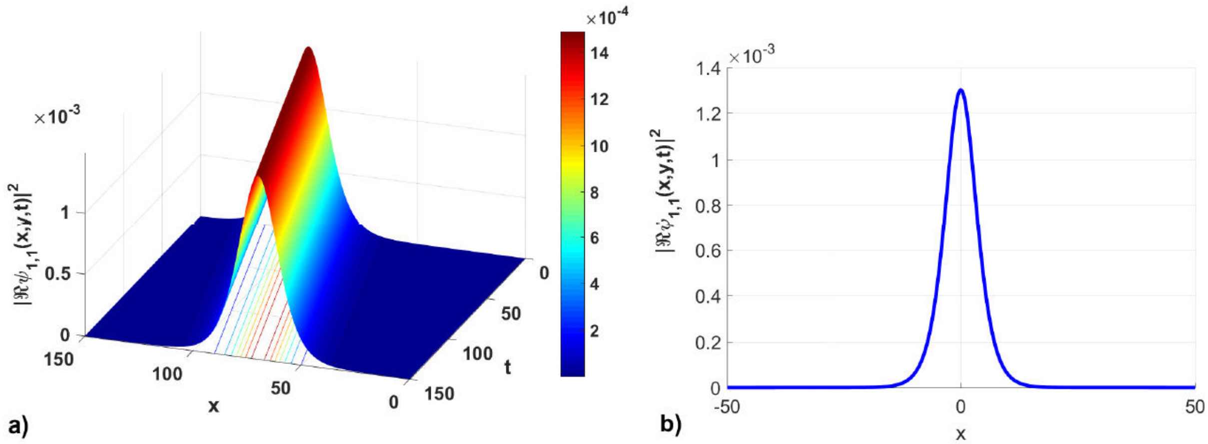

FIGURE 4. Spatiotemporal plot evolution a) 3-D (left panel) and b) 2-D (right panel) of the chirp bright of $|\Re\psi_{1,1}(x,y,t)|^2$ of Eq. (16) at $n = 0.5, m = 0.14, \alpha = 0.02, G = 2.7, l = -0.3, \gamma = 0.12, y = 0.5, E = -0.5, C = 0.1106, \omega = 0.15$.

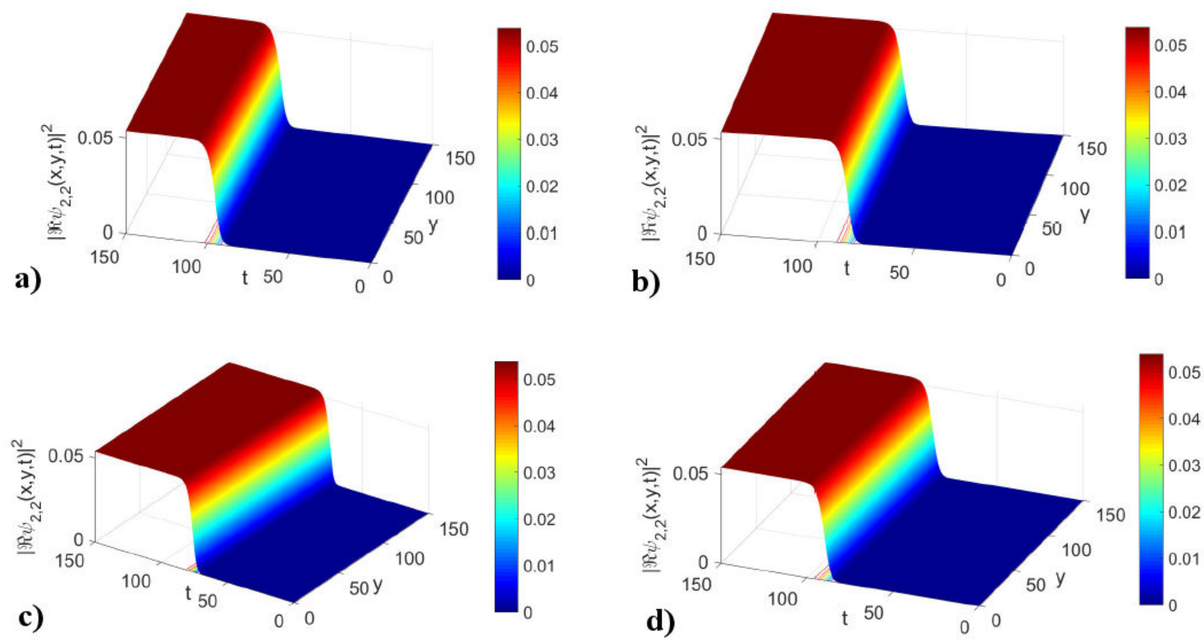

FIGURE 5. Spatiotemporal plot evolution 3-D of the chirp kink-like soliton solutions of $|\Re\psi_{2,2}(x,y,t)|^2$ of Eq. (24) at a) $\omega = 2.05$, b) $\omega = 2.005$, c) $\omega = 2.0005$, d) $\omega = 2.00005$, for $A_0 = -7.5$, $n = 0.5$, $m = 0.14$, $\alpha = 0.2$, $G = 1.7$, $l = -1.3$, $\gamma = 0.12$, $x = 0$, $E = 1.5$, $C = 0.3820$.

$\alpha = 0.02$, $G = 2.7$, $l = -0.3$, $\gamma = 0.12$, $y = 0.5$, $E = -0.5$, $C = 0.1106$, $\omega = 0.15$. Moreover, Figs. 5a), b), c) and d) are chirp kink-like soliton obtained for the values of $A_0 = -7.5, n = 0.5, m = 0.14, \alpha = 0.2, G = 1.7, l = -1.3, \gamma = 0.12, x = 0, E = 1.5, C = 0.3820$.

2. Conclusion

This work addresses new shape of the chirped bright and dark soliton solutions through the CGLE by using the new sub-ODE equations. By using a special ansatz of the traveling wave transformation, we obtain a new shape of the chirp comparatively to the previous works reported in literature [26,31–34,36,45]. In addition, new singular soliton solutions, trigonometric function solutions and complex traveling waves have been obtained. The obtained results are new in the field of solitons. The authors hope that these results will be very useful to explain physical phenomenons in diverse field of science and engineering. In addition, the different parameters of the CGLE have play an important role during graphical representation of the analytical results. Finally, the linear analysis technique has been applied to the investigation the steady state of the MI gain spectrum and also to point out the different regime of instability.

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