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Quasi uniformly accelered motion of quasi normal magnetic biharmonic particles in Heisenberg space with cold plasma

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In this paper, we firstly review the quasi uniformly accelerated motion (QUAM) with Fermi derivative by cold plasma in Heisenberg space. We define new quasi uniformly accelerated potential electric energy of quasi normal magnetic biharmonic particles and some Lorentz fields. Moreover, we design the new relationship between the quasi uniformly accelerated motion and the Fermi parallel transportation in cold plasma. Also, we collect new physical geometric designs for a quasi uniformly accelerated motion of normal magnetic biharmonic particles in Heisenberg space. Finally, we construct quasi uniformly accelerated potential electric energy with respect to its electric field and some quasi curvatures.

Keywords: Biharmonic particle; quasi plasma; Heisenberg space; magnetic field; magnetic flux density; electrical energy.

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1. Introduction

Magnetic flux density tends to be pretty impressive. Fermi Walker systems are flexible connections of fluxes, solids, and streams, are full of relationship, model nonlinear equations, and are solidly influenced by magnetic variations. These quasi systems are more generous, enveloping biomaterials, liquids, polymers, gels, and foams, and have an extensive range of associated uniformly motions. Proceeding with these motions desires a hidden characterization of complexities of these profuse Magnetic flux systems [1].

A great hypersensitive magnetic torque calibrates for operation in fluctuated large electromagnetic fields has been importuned, exclusively for direction of formulation scientific electromagnetic densities of original magnetic particles. Thus, densities are of powerful influence from perspective of perception of peculiar magnetic attributes. Also, even with last advancement of crystal magnetic theory, it is not uncomplicated to expand magnetic crystals by magnetic moments.

Uniformly accelerated structures have been obtained in [2] and, more recently, in Refs. [3,4]. In this paper, authors suggest new methods for finding solutions for uniformly quasi accelerated motion in a general curved spacetime. This extends results of [5], where explicit solutions are computed for flat spacetime only. Other approaches rely on convenient use of Frenet equations [6,7].

Magnetic flux density combinations are extensively operated as a magnetic moment component considering numerous varieties of machines like potential generators, turbines and activators. Magnetic hybrids are frequently embittered with meanwhile application of these machines. Melting is a result of the destruction of electrical energy at the elastic magnetic torque. Accordingly, advancement in the capability of elastic magnetic torque is momentous to recover the electrical energy and to bypass to contaminate bordering machines. Hence, preserving electrical energy benefits at preserving the earth's ordinary reserves and climates. The inferior amorphous-forming capability of hybrids combines cripples with its operational capability at power generators and turbines [8].

Electromagnetic cover, just as an appropriate cover approach to establish a point, has explicit operation forecasts in various areas, being biomedicine, essential sensibility, and act cover [9]. The ultimate significant utilization areas are essentially nosy surgery [10]. The electromagnetic cover system may be managed like a split of the navigation structure to establish a certain-future situation for the pharmaceutical apparatus in the inmate's frame, implement extreme advantage to scientists [11]. Correlated with alternative navigation designs as optical new navigation, the electromagnetic cover has no radioactivity cripple or no constraint of radiation procedure. Also, an electromagnetic traverse system found on abstain energy of some magnetic flux density field characterized by some azimuth intersection is recommended [12].

The biharmonic particles also play an important role in geometry. Also, a large number of experts have analyzed geometric biharmonic particles and surface conditions, [13,14]. On the other hand, the energy concept has been obtained with some characterizations, [15-19].

Magnetic particles provide an impressive, reliable, and densely parallel synthesized attitude. By utilizing magnetic torque, principal magnetic matter experimenters have established advanced technologies in physics, optics, world sciences, and basic materials. Categorically, magnetic torque has been managed to construct particle interruptions, to assemble an influence fusion magnetic particles, and to apply and shape optical materials [20-25].

Aim of the research work, a general construction method is proposed for magnetic interpretation of new quasi uniformly accelerated potential electric energy and Fermi parallel transportation in cold plasma with illustrations of results. We consider the quasi uniformly accelerated motion with some Lorentz fields. The applications of these models are geometric and physical interpretations of quasi uniformly accelerated potential electric energy.

The construction of our article is as follows: Firstly, we construct the quasi uniformly accelerated motion (QUAM) with Fermi derivative by cold plasma in Heisenberg space. We define new quasi uniformly accelerated potential electric energy of quasi normal magnetic biharmonic particles and some Lorentz fields. Moreover, we design the new relationship between the quasi uniformly accelerated motion and the Fermi parallel transportation in cold plasma. Also, we collect new physical geometric designs for a quasi uniformly accelerated motion of normal magnetic biharmonic particles in Heisenberg space. Finally, we construct quasi uniformly accelerated potential electric energy with respect to its electric field and some quasi curvatures.

2. Backround on quasi frame and Heisenberg space

Heisenberg space have following metric [26]:

$$g = dx^2 + dy^2 + (dz - xdy)^2.$$

Heisenberg space's basis is given by

$$\mathbf{f}_1 = \frac{\partial}{\partial x}, \ \ \mathbf{f}_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}, \ \ \mathbf{f}_3 = \frac{\partial}{\partial z}.$$

Let α be a particle with arclength. Also, $\mathbf{t}$, $\mathbf{n}$, and $\mathbf{b}$ describe tangent, principal normal, and secondary normal fields, respectively. Then, Frenet equations are given by

$$\begin{bmatrix} \nabla_{\mathbf{t}}\mathbf{t} \\ \nabla_{\mathbf{t}}\mathbf{n} \\ \nabla_{\mathbf{t}}\mathbf{b} \end{bmatrix} = \begin{bmatrix} & \kappa & \\ -\kappa & & \tau \\ & -\tau & \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix},$$

where κ and τ are curvatures of particles.

A new quasi frame of a particle is presented by [27],

$$\mathbf{T_q} = \mathbf{t}, \mathbf{N_q} = \frac{\mathbf{t} \times \mathbf{k}}{\|\mathbf{t} \times \mathbf{k}\|}, \mathbf{B_q} = \mathbf{T_q} \wedge \mathbf{N_q},$$

where $k = (0, 0, 1)$ is the projection field.

Since the new quasi frame is provided by

$$\nabla_s \mathbf{T_q} = \varrho_1 \mathbf{N_q} + \varrho_2 \mathbf{B_q},$$
$$\nabla_s \mathbf{N_q} = -\varrho_1 \mathbf{T_q} + \varrho_3 \mathbf{B_q},$$
$$\nabla_s \mathbf{B_q} = -\varrho_2 \mathbf{T_q} - \varrho_3 \mathbf{N_q},$$

where the angle ψ is between the quasi field $\mathbf{n_q}$ and the vector $\mathbf{n}$ and

$$\varrho_1 = \kappa \cos\psi, \quad \varrho_2 = -\kappa \sin\psi, \quad \varrho_3 = \psi' + \tau.$$

3. Quasi uniformly accelerated motion of quasi normal magnetic biharmonic particles

In this section, we characterize the quasi uniformly accelerated motion of moving charged quasi normal magnetic biharmonic particles with unit speed normal magnetic biharmonic particles in Heisenberg space. We obtain necessary and sufficient conditions that have to be satisfied by the particle in terms of the Frenet scalars of the worldline of magnetic curves. We present the following definition of the quasi uniformly accelerated motion.

Let α be a biharmonic particle and $\mathcal{B}$ be a magnetic field in the Heisenberg space. We call the curve α as a quasi normal magnetic biharmonic particle if the quasi normal field of the biharmonic particle meets the following Lorentz force equation.

$$\nabla_s \mathbf{N_q} = \phi(\mathbf{N_q}) = \mathcal{B} \times \mathbf{N_q}.$$

Fermi Walker derivative of any field $\mathcal{R}$ is defined by

$$\nabla_s^{\boldsymbol{f}} \mathcal{R} = \nabla_s \mathcal{R} - h\left(\mathbf{T_q}, \mathcal{R}\right) \nabla_s \mathbf{T_q} + h(\nabla_s \mathbf{T_q}, \mathcal{R}) \mathbf{T_q},$$

where ∇ is Levi-Civita connection associated metric h [28].

♣ *The particle $\mathbf{X}$ is quasi uniformly accelerated motion iff*

$$\nabla_s^{\boldsymbol{f}}(\nabla_s \mathbf{X}) = 0.$$

• *Lorentz force of $\mathbf{T_q}, \mathbf{N_q}, \mathbf{B_q}$ are presented*

$$\phi(\mathbf{T_q}) = (\varrho_1 \cos[\chi_1 s + \chi_2] + \omega \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\omega \cos\varphi \cos[\chi_1 s + \chi_2] - \varrho_1 \sin[\chi_1 s + \chi_2])\mathbf{f}_2 - \omega \sin\varphi \mathbf{f}_3,$$

$$\phi(\mathbf{N_q}) = (\varrho_3 \cos\varphi \sin[\chi_1 s + \chi_2] - \varrho_1 \sin\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3 \cos\varphi \cos[\chi_1 s + \chi_2] - \varrho_1 \sin\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 - (\varrho_3 \sin\varphi + \varrho_1 \cos\varphi)\mathbf{f}_3,$$

$$\phi(\mathbf{B_q}) = -(\varrho_3 \cos[\chi_1 s + \chi_2] + \omega \sin\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3 \sin[\chi_1 s + \chi_2] - \omega \sin\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 - \omega \cos\varphi \mathbf{f}_3,$$

where $\omega = \phi(\mathbf{T_q}) \cdot \mathbf{B_q}$.

• *Magnetic vector field* $\mathcal{B}$ *of normal magnetic biharmonic particle is given by*

$$\mathcal{B} = (\varrho_3 \sin\varphi \sin[\chi_1 s + \chi_2] - \omega \cos[\chi_1 s + \chi_2] + \varrho_1 \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3 \sin\varphi \cos[\chi_1 s + \chi_2] + \omega \sin[\chi_1 s + \chi_2] + \varrho_1 \cos\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 + (\varrho_3 \cos\varphi - \varrho_1 \sin\varphi)\mathbf{f}_3.$$

Lemma 1.

♣ *Fermi Walker derivative of* $\phi(\mathbf{T_q})$ *is*

$$\nabla_s^{\boldsymbol{f}} \phi(\mathbf{T_q}) = ((\omega' + \varrho_1\varrho_3)\cos\varphi \sin[\chi_1 s + \chi_2] + (\varrho_1' - \varrho_3\omega)\cos[\chi_1 s + \chi_2])\mathbf{f}_1 + ((\omega' + \varrho_1\varrho_3)\cos\varphi\cos[\chi_1 s + \chi_2] - (\varrho_1' - \varrho_3\omega)\sin[\chi_1 s + \chi_2])\mathbf{f}_2 - (\omega' + \varrho_1\varrho_3)\sin\varphi \mathbf{f}_3.$$

♣ *Fermi Walker derivative of* $\phi(\mathbf{N_q})$ *is*

$$\nabla_s^{\boldsymbol{f}} \phi(\mathbf{N_q}) = (\varrho_3' \cos\varphi \sin[\chi_1 s + \chi_2] - \varrho_1' \sin\varphi \sin[\chi_1 s + \chi_2] - \varrho_3^2 \cos[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3' \cos\varphi\cos[\chi_1 s + \chi_2] - \varrho_1' \sin\varphi \cos[\chi_1 s + \chi_2] + \varrho_3^2 \sin[\chi_1 s + \chi_2])\mathbf{f}_2 - (\varrho_3' \sin\varphi + \varrho_1' \cos\varphi)\mathbf{f}_3.$$

♣ *Fermi Walker derivative of* $\phi(\mathbf{B_q})$ *is*

$$\nabla_s^{\boldsymbol{f}} \phi(\mathbf{B_q}) = -(\omega' \sin\varphi \sin[\chi_1 s + \chi_2] + \varrho_3' \cos[\chi_1 s + \chi_2] + \varrho_3^2 \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3' \sin[\chi_1 s + \chi_2] - \varrho_3^2 \cos\varphi\cos[\chi_1 s + \chi_2] - \omega' \sin\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 + (\varrho_3^2 \sin\varphi - \omega' \cos\varphi)\mathbf{f}_3.$$

♣ *Fermi Walker derivative of* $\mathcal{B}$ *is*

$$\nabla_s^{\boldsymbol{f}} \mathcal{B} = (\varrho_3' \sin\varphi \sin[\chi_1 s + \chi_2] - (\omega' + \varrho_1\varrho_3)\cos[\chi_1 s + \chi_2] + (\varrho_1' - \omega\varrho_3) \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (\varrho_3' \sin\varphi\cos[\chi_1 s + \chi_2] + (\omega' + \varrho_1\varrho_3)\sin[\chi_1 s + \chi_2] + (\varrho_1' - \omega\varrho_3)\cos\varphi\cos[\chi_1 s + \chi_2])\mathbf{f}_2 + (\varrho_3' \cos\varphi - (\varrho_1' - \omega\varrho_3)\sin\varphi)\mathbf{f}_3.$$

Lemma 2.

♣ $\phi(\mathbf{T_q})$ *is Fermi Walker parallel iff*

$$((\omega' + \varrho_1\varrho_3)\cos\varphi\sin[\chi_1 s + \chi_2] + (\varrho_1' - \varrho_3\omega)\cos[\chi_1 s + \chi_2]) = 0$$
$$((\omega' + \varrho_1\varrho_3)\cos\varphi\cos[\chi_1 s + \chi_2] - (\varrho_1' - \varrho_3\omega)\sin[\chi_1 s + \chi_2]) = 0$$
$$(\omega' + \varrho_1\varrho_3)\sin\varphi = 0.$$

♣ $\phi(\mathbf{N_q})$ *is Fermi Walker parallel iff*

$$(\varpi' \cos\varphi \sin[\chi_1 s + \chi_2] - \varkappa_1' \sin\varphi\sin[\chi_1 s + \chi_2] - \varkappa_3\varpi\cos[\chi_1 s + \chi_2]) = 0,$$
$$(\varpi' \cos\varphi \cos[\chi_1 s + \chi_2] - \varkappa_1' \sin\varphi\cos[\chi_1 s + \chi_2] + \varkappa_3\varpi\sin[\chi_1 s + \chi_2]) = 0,$$
$$(\varpi' \sin\varphi + \varkappa_1' \cos\varphi) = 0.$$

♣ $\phi(\mathbf{B_q})$ *is Fermi Walker parallel iff*

$$(\omega' \sin\varphi\sin[\chi_1 s + \chi_2] + \varrho_3' \cos[\chi_1 s + \chi_2] + \varrho_3^2 \cos\varphi\sin[\chi_1 s + \chi_2]) = 0,$$
$$(\varrho_3' \sin[\chi_1 s + \chi_2] - \varrho_3^2 \cos\varphi\cos[\chi_1 s + \chi_2] - \omega' \sin\varphi\cos[\chi_1 s + \chi_2]) = 0,$$
$$(\varrho_3^2 \sin\varphi - \omega' \cos\varphi) = 0.$$

♣ *$\mathcal{B}$ is Fermi Walker parallel iff*

$$(\varrho_3'\sin\varphi\sin[\chi_1 s+\chi_2]-(\omega'+\varrho_1\varrho_3)\cos[\chi_1 s+\chi_2]+(\varrho_1'-\omega\varrho_3)\cos\varphi\sin[\chi_1 s+\chi_2])=0,$$
$$(\varrho_3'\sin\varphi\cos[\chi_1 s+\chi_2]+(\omega'+\varrho_1\varrho_3)\sin[\chi_1 s+\chi_2]+(\varrho_1'-\omega\varrho_3)\cos\varphi\cos[\chi_1 s+\chi_2])=0,$$
$$(\varrho_3'\cos\varphi-(\varrho_1'-\omega\varrho_3)\sin\varphi)=0.$$

Quasi uniformly accelerated motion

In this subsection, we will characterize the uniformly quasi accelerated motion (QUAM) in the Heisenberg space.

♠ *The $\phi(\mathbf{T_q})$ obeys a quasi uniformly accelerated motion iff*

$$\begin{aligned}
&(((\omega'+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\\
&\quad\times\cos\varphi\sin[\chi_1 s+\chi_2]-((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)+\varrho_2(\omega'+\varrho_1\varrho_3)\\
&\quad-((\varrho_1'-\varrho_3\omega)\varrho_1+\varrho_2(\omega'+\varrho_1\varrho_3)))\sin\varphi\sin[\chi_1 s+\chi_2]+((\varrho_1'-\varrho_3\omega)'\\
&\quad-(\varrho_1^2+\varrho_2\omega)\varrho_1-\varrho_3(\omega'+\varrho_1\varrho_3)+(\varrho_1^2+\varrho_2\omega)\varrho_1)\cos[\chi_1 s+\chi_2])=0,\\
&(((\omega'+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\\
&\quad\times\cos\varphi\cos[\chi_1 s+\chi_2]-((\varrho_1'-\varrho_3\omega)'-(\varrho_1^2+\varrho_2\omega)\varrho_1-\varrho_3(\omega'+\varrho_1\varrho_3)\\
&\quad+(\varrho_1^2+\varrho_2\omega)\varrho_1)\sin[\chi_1 s+\chi_2]-((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)\\
&\quad+\varrho_2(\omega'+\varrho_1\varrho_3)-((\varrho_1'-\varrho_3\omega)\varrho_1+\varrho_2(\omega'+\varrho_1\varrho_3)))\sin\varphi\cos[\chi_1 s+\chi_2])=0,\\
&(((\omega'+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\sin\varphi\\
&\quad+((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)+\varrho_2(\omega'+\varrho_1\varrho_3)-((\varrho_1'-\varrho_3\omega)\varrho_1\\
&\quad+\varrho_2(\omega'+\varrho_1\varrho_3)))\cos\varphi)=0,
\end{aligned}$$

By Fermi derivative and following equations, we obtain above system.

$$\begin{aligned}
\nabla_s\phi(\mathbf{T_q})&=((\omega'+\varrho_1\varrho_3)\cos\varphi\sin[\chi_1 s+\chi_2]-(\varrho_1^2+\varrho_2\omega)\sin\varphi\sin[\chi_1 s+\chi_2]+(\varrho_1'-\varrho_3\omega)\\
&\quad\times\cos[\chi_1 s+\chi_2])\mathbf{f}_1+((\omega'+\varrho_1\varrho_3)\cos\varphi\cos[\chi_1 s+\chi_2]-(\varrho_1^2+\varrho_2\omega)\sin\varphi\cos[\chi_1 s+\chi_2]\\
&\quad-(\varrho_1'-\varrho_3\omega)\sin[\chi_1 s+\chi_2])\mathbf{f}_2-((\omega'+\varrho_1\varrho_3)\sin\varphi+(\varrho_1^2+\varrho_2\omega)\cos\varphi)\mathbf{f}_3.
\end{aligned}$$

It provides us the following equation

$$\begin{aligned}
\nabla_s^{\boldsymbol{f}}\nabla_s\phi(\mathbf{T_q})&=(((\omega'+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\\
&\quad\times\cos\varphi\sin[\chi_1 s+\chi_2]-((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)+\varrho_2(\omega'+\varrho_1\varrho_3)\\
&\quad-((\varrho_1'-\varrho_3\omega)\varrho_1+\varrho_2(\omega'+\varrho_1\varrho_3)))\sin\varphi\sin[\chi_1 s+\chi_2]+((\varrho_1'-\varrho_3\omega)'\\
&\quad-(\varrho_1^2+\varrho_2\omega)\varrho_1-\varrho_3(\omega'+\varrho_1\varrho_3)+(\varrho_1^2+\varrho_2\omega)\varrho_1)\cos[\chi_1 s+\chi_2])\mathbf{f}_1'\\
&\quad+(((\omega+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\cos\varphi\cos[\chi_1 s+\chi_2]\\
&\quad-((\varrho_1'-\varrho_3\omega)'-(\varrho_1^2+\varrho_2\omega)\varrho_1-\varrho_3(\omega'+\varrho_1\varrho_3)+(\varrho_1^2+\varrho_2\omega)\varrho_1)\sin[\chi_1 s+\chi_2]\\
&\quad-((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)+\varrho_2(\omega'+\varrho_1\varrho_3)-((\varrho_1'-\varrho_3\omega)\varrho_1\\
&\quad+\varrho_2(\omega'+\varrho_1\varrho_3)))\sin\varphi\cos[\chi_1 s+\chi_2])\mathbf{f}_2-(((\omega'+\varrho_1\varrho_3)'-(\varrho_1^2+\varrho_2\omega)\varrho_2\\
&\quad+\varrho_3(\varrho_1'-\varrho_3\omega)+(\varrho_1^2+\varrho_2\omega)\varrho_2)\sin\varphi+((((\varrho_1^2+\varrho_2\omega)'+\varrho_1(\varrho_1'-\varrho_3\omega)\\
&\quad+\varrho_2(\omega'+\varrho_1\varrho_3)-((\varrho_1'-\varrho_3\omega)\varrho_1+\varrho_2(\omega'+\varrho_1\varrho_3)))\cos\varphi)\mathbf{f}_3.
\end{aligned}$$

♠ *The* $\phi(\mathbf{N_q})$ *obeys a quasi uniformly accelerated motion iff*

$$\begin{aligned}
&(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&\quad-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\sin[\chi_1 s+\chi_2]-((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'\\
&\quad+\varrho_3(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\cos[\chi_1 s+\chi_2]+((\varrho_3'+\varrho_1\varrho_2)'\\
&\quad-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\sin[\chi_1 s+\chi_2])=0,\\
&(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&\quad-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\cos[\chi_1 s+\chi_2]+((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'\\
&\quad+\varrho_3(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\sin[\chi_1 s+\chi_2]+((\varrho_3'+\varrho_1\varrho_2)'\\
&\quad-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\cos[\chi_1 s+\chi_2])=0\\
&(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1^2+\varrho_3^2)\varrho_1))\cos\varphi\\
&\quad-((\varrho_3'+\varrho_1\varrho_2)'-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\sin\varphi)=0.
\end{aligned}$$

Also,

$$\begin{aligned}
\nabla_s\phi(\mathbf{N_q})&=((\varrho_3'+\varrho_1\varrho_2)\cos\varphi\sin[\chi_1 s+\chi_2]-(\varrho_1'+\varrho_3\varrho_2)\sin\varphi\sin[\chi_1 s+\chi_2]\\
&-(\varrho_1^2+\varrho_3^2)\cos[\chi_1 s+\chi_2])\mathbf{f}_1+((\varrho_3'+\varrho_1\varrho_2)\cos\varphi\cos[\chi_1 s+\chi_2]\\
&-(\varrho_1'+\varrho_3\varrho_2)\sin\varphi\cos[\chi_1 s+\chi_2]+(\varrho_1^2+\varrho_3^2)\sin[\chi_1 s+\chi_2])\mathbf{f}_2\\
&-((\varrho_3'+\varrho_1\varrho_2)\sin\varphi+(\varrho_1'+\varrho_3\varrho_2)\cos\varphi)\mathbf{f}_3.
\end{aligned}$$

By Fermi derivative and following equations, we easily obtain above system.

$$\begin{aligned}
\nabla_s^{\boldsymbol{f}}\nabla_s\phi(\mathbf{N_q})&=(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\sin[\chi_1 s+\chi_2]-((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'\\
&+\varrho_3(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\cos[\chi_1 s+\chi_2]+((\varrho_3'+\varrho_1\varrho_2)'\\
&-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\sin[\chi_1 s+\chi_2])\mathbf{f}_1\\
&+(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\cos[\chi_1 s+\chi_2]+((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'\\
&+\varrho_3(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\sin[\chi_1 s+\chi_2]+((\varrho_3'+\varrho_1\varrho_2)'\\
&-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\cos[\chi_1 s+\chi_2])\mathbf{f}_2\\
&+(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&-(\varrho_1^2+\varrho_3^2)\varrho_1))\cos\varphi-((\varrho_3'+\varrho_1\varrho_2)'-(\varrho_1^2+\varrho_3^2)\varrho_3\\
&-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\sin\varphi)\mathbf{f}_3.
\end{aligned}$$

♠ *The* $\phi(\mathbf{B_q})$ *obeys a quasi uniformly accelerated motion iff*

$$
\begin{aligned}
&((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 \\
&\quad + \omega\varrho_2)\varrho_2)) \sin\varphi \sin[\chi_1 s + \chi_2] + ((\varrho_3\varrho_1 - \omega')\varrho_1 - (\varrho_3' + \omega\varrho_1)' + \varrho_3(\varrho_3^2 \\
&\quad + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \cos[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 \\
&\quad + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \sin[\chi_1 s + \chi_2]) = 0, \\
&((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 \\
&\quad + \omega\varrho_2)\varrho_2)) \sin\varphi \cos[\chi_1 s + \chi_2] - ((\varrho_3\varrho_1 - \omega')\varrho_1 - (\varrho_3' + \omega\varrho_1)' + \varrho_3(\varrho_3^2 \\
&\quad + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \sin[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 + \omega\varrho_2)' \\
&\quad - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \cos[\chi_1 s + \chi_2]) = 0, \\
&((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 \\
&\quad + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \cos\varphi - (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 \\
&\quad + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \sin\varphi) = 0.
\end{aligned}
$$

Then, it is easy to see that

$$
\begin{aligned}
\nabla_s \phi(\mathbf{B_q}) = {} & ((\varrho_3\varrho_1 - \omega') \sin\varphi \sin[\chi_1 s + \chi_2] - (\varrho_3' + \omega\varrho_1) \cos[\chi_1 s + \chi_2] \\
& - (\varrho_3^2 + \omega\varrho_2) \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + ((\varrho_3\varrho_1 - \omega') \sin\varphi \cos[\chi_1 s + \chi_2] \\
& + (\varrho_3' + \omega\varrho_1) \sin[\chi_1 s + \chi_2] - (\varrho_3^2 + \omega\varrho_2) \cos\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 \\
& + ((\varrho_3\varrho_1 - \omega') \cos\varphi + (\varrho_3^2 + \omega\varrho_2) \sin\varphi)\mathbf{f}_3.
\end{aligned}
$$

In a similar way, we get

$$
\begin{aligned}
\nabla_s^{\boldsymbol{f}} \nabla_s \phi(\mathbf{B_q}) = {} & ((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 \\
& + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \sin\varphi \sin[\chi_1 s + \chi_2] + ((\varrho_3\varrho_1 - \omega')\varrho_1 - (\varrho_3' + \omega\varrho_1)' \\
& + \varrho_3(\varrho_3^2 + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \cos[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 + \omega\varrho_2)' \\
& - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \sin[\chi_1 s + \chi_2])\mathbf{f}_1 + ((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' \\
& + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \sin\varphi \cos[\chi_1 s + \chi_2] - ((\varrho_3\varrho_1 - \omega')\varrho_1 \\
& - (\varrho_3' + \omega\varrho_1)' + \varrho_3(\varrho_3^2 + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \sin[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') \\
& - (\varrho_3^2 + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \cos[\chi_1 s + \chi_2])\mathbf{f}_2 + ((\varrho_1(\varrho_3' + \omega\varrho_1) \\
& + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \cos\varphi \\
& - (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \sin\varphi)\mathbf{f}_3.
\end{aligned}
$$

4. Energy flux density in cold plasma

The force acting on an electron in the cold plasma is given by

$$\mathbf{F} = -q(\mathcal{E} + \mathbf{v} \times \mathcal{B}) - m\mathbf{v}, \quad \frac{m}{q} = \pi,$$

where $\mathbf{v}$ is a velocity of electrons, m is a mass, q is an electric charge, $\mathcal{B}$ is a magnetic field and $\mathcal{E}$ is an electric field [29-31].

From the force equation, the electric field is given by

$$\begin{aligned}\mathcal{E} = &((\omega - \pi\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2] - \pi\sin\varphi\sin[\chi_1 s + \chi_2] + (\varrho_1\\ &- \pi\varrho_1)\cos[\chi_1 s + \chi_2])\mathbf{f}_1 + ((\omega - \pi\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2]\\ &- \pi\sin\varphi\cos[\chi_1 s + \chi_2] - (\varrho_1 - \pi\varrho_1)\sin[\chi_1 s + \chi_2])\mathbf{f}_2\\ &- ((\omega - \pi\varrho_2)\sin\varphi + \pi\cos\varphi)\mathbf{f}_3.\end{aligned}$$

♠ *The $\mathcal{E}$ obeys a quasi uniformly accelerated motion iff*

$$\begin{aligned}&((((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))' - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\varrho_2\\ &\quad + \varrho_3((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)))\cos\varphi\sin[\chi_1 s + \chi_2] - ((\pi + \varrho_1(\varrho_1 - \pi\varrho_1)\\ &\quad + \varrho_2(\omega - \pi\varrho_2))' + \varrho_1((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)) + \varrho_2((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1\\ &\quad - \pi\varrho_1)))\sin\varphi\sin[\chi_1 s + \chi_2] + (((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2))' - (\pi + \varrho_1(\varrho_1\\ &\quad - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\varrho_1 - \varrho_3((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1)))\cos[\chi_1 s + \chi_2]) = 0,\\ &((((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))' - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\varrho_2\\ &\quad + \varrho_3((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)))\cos\varphi\cos[\chi_1 s + \chi_2] - ((\pi + \varrho_1(\varrho_1 - \pi\varrho_1)\\ &\quad + \varrho_2(\omega - \pi\varrho_2))' + \varrho_1((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)) + \varrho_2((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1\\ &\quad - \pi\varrho_1)))\sin\varphi\cos[\chi_1 s + \chi_2] - (((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2))' - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1)\\ &\quad + \varrho_2(\omega - \pi\varrho_2))\varrho_1 - \varrho_3((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1)))\sin[\chi_1 s + \chi_2]) = 0,\\ &((((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))' - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\varrho_2 + \varrho_3((\varrho_1\\ &\quad - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)))\sin\varphi + ((\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))'\\ &\quad + \varrho_1((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2)) + \varrho_2((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1)))\cos\varphi) = 0.\end{aligned}$$

We instantly calculate

$$\begin{aligned}\nabla_s\mathcal{E} = &((((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))\cos\varphi\sin[\chi_1 s + \chi_2] + ((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2))\\ &\times\cos[\chi_1 s + \chi_2] - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\sin\varphi\sin[\chi_1 s + \chi_2])\mathbf{f}_1\\ &+ (((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))\cos\varphi\cos[\chi_1 s + \chi_2] - (\pi + \varrho_1(\varrho_1 - \pi\varrho_1)\\ &+ \varrho_2(\omega - \pi\varrho_2))\sin\varphi\cos[\chi_1 s + \chi_2] - ((\varrho_1 - \pi\varrho_1)' - \varrho_3(\omega - \pi\varrho_2))\sin[\chi_1 s + \chi_2])\mathbf{f}_2\\ &- (((\omega - \pi\varrho_2)' + \varrho_3(\varrho_1 - \pi\varrho_1))\sin\varphi + (\pi + \varrho_1(\varrho_1 - \pi\varrho_1) + \varrho_2(\omega - \pi\varrho_2))\cos\varphi)\mathbf{f}_3.\end{aligned}$$

So, we obtain

$$
\begin{aligned}
\nabla_s^{f}\nabla_s\mathcal{E} = {} & ((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_2\\
& +\varrho_3((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\cos\varphi\sin[\chi_1 s+\chi_2]-((\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))'\\
& +\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))+\varrho_2((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\sin\varphi\sin[\chi_1 s+\chi_2]\\
& +(((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_1-\varrho_3((\omega-\pi\varrho_2)'\\
& +\varrho_3(\varrho_1-\pi\varrho_1)))\cos[\chi_1 s+\chi_2])\mathbf{f}_1+((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))'-(\pi+\varrho_1(\varrho_1\\
& -\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_2+\varrho_3((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\cos\varphi\cos[\chi_1 s+\chi_2]-((\pi\\
& +\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))'+\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))+\varrho_2((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1\\
& -\pi\varrho_1)))\sin\varphi\cos[\chi_1 s+\chi_2]-(((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega\\
& -\pi\varrho_2))\varrho_1-\varrho_3((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\sin[\chi_1 s+\chi_2])\mathbf{f}_2-((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1\\
& -\pi\varrho_1))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_2+\varrho_3((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\sin\varphi\\
& +((\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))'+\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))\\
& +\varrho_2((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\cos\varphi)\mathbf{f}_3.
\end{aligned}
$$

♠ *The $\mathcal{B}$ obeys a uniformly quasi accelerated motion iff*

$$
\begin{aligned}
& (((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)'+\varrho_1\omega'-\varrho_2(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)-\omega'\varrho_1+\varrho_2(\varrho_1'+\varrho_3\varrho_2\\
& \quad -\omega\varrho_3))\sin\varphi\sin[\chi_1 s+\chi_2]+((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_1-\varrho_3(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)-\omega''\\
& \quad -(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_1)\cos[\chi_1 s+\chi_2]+(\varrho_2(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)+(\varrho_1'+\varrho_3\varrho_2\\
& \quad -\omega\varrho_3)'-\varrho_3\omega'-(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_2)\cos\varphi\sin[\chi_1 s+\chi_2])=0,\\
& (((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)'+\varrho_1\omega'-\varrho_2(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)-\omega'\varrho_1+\varrho_2(\varrho_1'\\
& \quad +\varrho_3\varrho_2-\omega\varrho_3))\sin\varphi\cos[\chi_1 s+\chi_2]-((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_1-\varrho_3(\varrho_1'+\varrho_3\varrho_2\\
& \quad -\omega\varrho_3)-\omega''-(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_1)\sin[\chi_1 s+\chi_2]+(\varrho_2(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\\
& \quad +(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)'-\varrho_3\omega'-(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_2)\cos\varphi\cos[\chi_1 s+\chi_2])=0,\\
& (((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)'+\varrho_1\omega'-\varrho_2(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)-\omega'\varrho_1+\varrho_2(\varrho_1'\\
& \quad +\varrho_3\varrho_2-\omega\varrho_3))\cos\varphi-(\varrho_2(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)+(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)'\\
& \quad -\varrho_3\omega'-(\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\varrho_2)\sin\varphi)=0.
\end{aligned}
$$

Then

$$
\begin{aligned}
\nabla_s\mathcal{B} = {} & ((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\sin\varphi\sin[\chi_1 s+\chi_2]-\omega'\cos[\chi_1 s+\chi_2]+(\varrho_1'+\varrho_3\varrho_2\\
& -\omega\varrho_3)\cos\varphi\sin[\chi_1 s+\chi_2])\mathbf{f}_1+((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\sin\varphi\cos[\chi_1 s+\chi_2]\\
& +\omega'\sin[\chi_1 s+\chi_2]+(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)\cos\varphi\cos[\chi_1 s+\chi_2])\mathbf{f}_2\\
& +((\varrho_3'+\omega\varrho_1-\varrho_1\varrho_2)\cos\varphi-(\varrho_1'+\varrho_3\varrho_2-\omega\varrho_3)\sin\varphi)\mathbf{f}_3.
\end{aligned}
$$

By calculating Fermi Walker derivative

$$\begin{aligned}\nabla_s^{\boldsymbol{f}}\nabla_s\mathcal{B} &= (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' - \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 \\ &+ \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3))\sin\varphi\sin[\chi_1 s + \chi_2] + ((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1 \\ &- \varrho_3(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1)\cos[\chi_1 s + \chi_2] \\ &+ (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3)' - \varrho_3\omega' - (\varrho_3' + \omega\varrho_1 \\ &- \varrho_1\varrho_2)\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2])\mathbf{f}_1 + (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' \\ &- \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 + \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3))\sin\varphi\cos[\chi_1 s + \chi_2] \\ &- ((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1 - \varrho_3(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'' - (\varrho_3' + \omega\varrho_1 \\ &- \varrho_1\varrho_2)\varrho_1)\sin[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' + \varrho_3\varrho_2 \\ &- \omega\varrho_3)' - \varrho_3\omega' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2])\mathbf{f}_2 \\ &+ (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' - \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 + \varrho_2(\varrho_1' \\ &+ \varrho_3\varrho_2 - \omega\varrho_3))\cos\varphi - (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' + \varrho_3\varrho_2 \\ &- \omega\varrho_3)' - \varrho_3\omega' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_2)\sin\varphi)\mathbf{f}_3.\end{aligned}$$

5. Application to quasi uniformly circular potential electric energy

In this section, we obtain the polar plot for the time variation of potential electrical energy with respect to its electric field and energy flux density in the radial direction.

The quasi uniformly circular potential electric energy of field $\mathbf{X}$ in the electric field $\mathcal{E}$ is defined by

$$\mathcal{L}^{\boldsymbol{f}}\mathbf{X} = \nabla_s^{\boldsymbol{f}}(\nabla_s\mathbf{X}) \cdot \mathcal{E}.$$

In previous work, we investigate the special type of magnetic trajectories such that it corresponds to a moving charged particle in an associated magnetic field in Heisenberg space. This study differs from the former studies in the literature since it is considered in the Heisenberg space. We considered uniformly accelerated motion (UAM), unchanged direction motion (UDM), and uniform circular motion (UCM) of normal magnetic biharmonic particles in Heisenberg space, [32].

We further improve an alternative method to find uniformly quasi circular potential electric energy of biharmonic normal magnetic particles in the Heisenberg space. We also give the relationships between physical and geometrical characterizations of uniformly quasi circular potential electric energy. Finally, we illustrate important figures for quasi uniformly circular potential electric energy with respect to its electric field in the radial direction.

• *Quasi uniformly circular potential electric energy of* $\phi(\mathbf{T_q})$ *in the electric field* $\mathcal{E}$

$$\begin{aligned}\mathcal{L}^{\boldsymbol{f}}\phi(\mathbf{T_q}) &= (((\omega' + \varrho_1\varrho_3)' - (\varrho_1^2 + \varrho_2\omega)\varrho_2 + \varrho_3(\varrho_1' - \varrho_3\omega) + (\varrho_1^2 \\ &+ \varrho_2\omega)\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2] - (((\varrho_1^2 + \varrho_2\omega)' + \varrho_1(\varrho_1' - \varrho_3\omega) + \varrho_2(\omega' \\ &+ \varrho_1\varrho_3) - ((\varrho_1' - \varrho_3\omega)\varrho_1 + \varrho_2(\omega' + \varrho_1\varrho_3)))\sin\varphi\sin[\chi_1 s + \chi_2] + ((\varrho_1' \\ &- \varrho_3\omega)' - (\varrho_1^2 + \varrho_2\omega)\varrho_1 - \varrho_3(\omega' + \varrho_1\varrho_3) + (\varrho_1^2 + \varrho_2\omega)\varrho_1)\cos[\chi_1 s + \chi_2])((\omega \\ &- \pi\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2] - \pi\sin\varphi\sin[\chi_1 s + \chi_2] + (\varrho_1 - \pi\varrho_1)\cos[\chi_1 s + \chi_2]) + (((\omega' \\ &+ \varrho_1\varrho_3)' - (\varrho_1^2 + \varrho_2\omega)\varrho_2 + \varrho_3(\varrho_1' - \varrho_3\omega) + (\varrho_1^2 + \varrho_2\omega)\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2] \\ &- ((\varrho_1' - \varrho_3\omega)' - (\varrho_1^2 + \varrho_2\omega)\varrho_1 - \varrho_3(\omega' + \varrho_1\varrho_3) + (\varrho_1^2 + \varrho_2\omega)\varrho_1)\sin[\chi_1 s + \chi_2] \\ &- (((\varrho_1^2 + \varrho_2\omega)' + \varrho_1(\varrho_1' - \varrho_3\omega) + \varrho_2(\omega' + \varrho_1\varrho_3) - ((\varrho_1' - \varrho_3\omega)\varrho_1 + \varrho_2(\omega' \\ &+ \varrho_1\varrho_3)))\sin\varphi\cos[\chi_1 s + \chi_2])((\omega - \pi\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2] - \pi\sin\varphi\cos[\chi_1 s + \chi_2] \\ &- (\varrho_1 - \pi\varrho_1)\sin[\chi_1 s + \chi_2]) + (((\omega' + \varrho_1\varrho_3)' - (\varrho_1^2 + \varrho_2\omega)\varrho_2 + \varrho_3(\varrho_1' - \varrho_3\omega) \\ &+ (\varrho_1^2 + \varrho_2\omega)\varrho_2)\sin\varphi + (((\varrho_1^2 + \varrho_2\omega)' + \varrho_1(\varrho_1' - \varrho_3\omega) + \varrho_2(\omega' + \varrho_1\varrho_3) - ((\varrho_1' \\ &- \varrho_3\omega)\varrho_1 + \varrho_2(\omega' + \varrho_1\varrho_3)))\cos\varphi)((\omega - \pi\varrho_2)\sin\varphi + \pi\cos\varphi).\end{aligned}$$

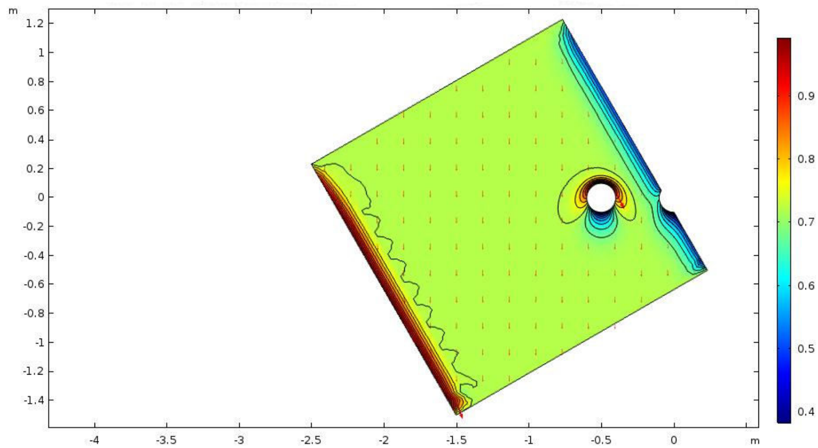

FIGURE 1. The shape of uniformly quasi circular potential density for $\phi(\mathbf{T_q})$.

Figure 1 shows the shape of the uniformly quasi circular potential density for $\phi(\mathbf{T_q})$ with appropriate values of the $\varrho_1,\ \varrho_2,$ and ϱ_3.

- *Quasi uniformly circular potential electric energy of* $\phi(\mathbf{N_q})$ *in the electric field* $\mathcal{E}$

$$
\begin{aligned}
\mathcal{L}^{\boldsymbol{f}}\phi(\mathbf{N_q}) &= (((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)\\
&-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\sin[\chi_1 s+\chi_2]-((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'+\varrho_3(\varrho_3'+\varrho_1\varrho_2)\\
&-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\cos[\chi_1 s+\chi_2]+((\varrho_3'+\varrho_1\varrho_2)'-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2\\
&+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\sin[\chi_1 s+\chi_2])((\omega-\pi\varrho_2)\cos\varphi\sin[\chi_1 s+\chi_2]\\
&-\pi\sin\varphi\sin[\chi_1 s+\chi_2]+(\varrho_1-\pi\varrho_1)\cos[\chi_1 s+\chi_2])+(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'\\
&+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1^2+\varrho_3^2)\varrho_1))\sin\varphi\cos[\chi_1 s+\chi_2]\\
&+((\varrho_1'+\varrho_3\varrho_2)\varrho_1+(\varrho_1^2+\varrho_3^2)'+\varrho_3(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)\varrho_1)\sin[\chi_1 s+\chi_2]\\
&+((\varrho_3'+\varrho_1\varrho_2)'-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\cos\varphi\\
&\cos[\chi_1 s+\chi_2])((\omega-\pi\varrho_2)\cos\varphi\cos[\chi_1 s+\chi_2]-\pi\sin\varphi\cos[\chi_1 s+\chi_2]-(\varrho_1\\
&-\pi\varrho_1)\sin[\chi_1 s+\chi_2])-(((\varrho_1^2+\varrho_3^2)\varrho_1-\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1'+\varrho_3\varrho_2)'\\
&+(\varrho_2(\varrho_3'+\varrho_1\varrho_2)-(\varrho_1^2+\varrho_3^2)\varrho_1))\cos\varphi-((\varrho_3'+\varrho_1\varrho_2)'-(\varrho_1^2+\varrho_3^2)\varrho_3-(\varrho_1'\\
&+\varrho_3\varrho_2)\varrho_2+\varrho_2(\varrho_1'+\varrho_3\varrho_2))\sin\varphi)((\omega-\pi\varrho_2)\sin\varphi+\pi\cos\varphi).
\end{aligned}
$$

Figure 2 shows the shape of the uniformly quasi circular potential density for $\phi(\mathbf{N_q})$ with appropriate values of the $\varrho_1,\ \varrho_2,$ and ϱ_3.

- *Quasi uniformly circular potential electric energy of* $\phi(\mathbf{B_q})$ *in the electric field* $\mathcal{E}$

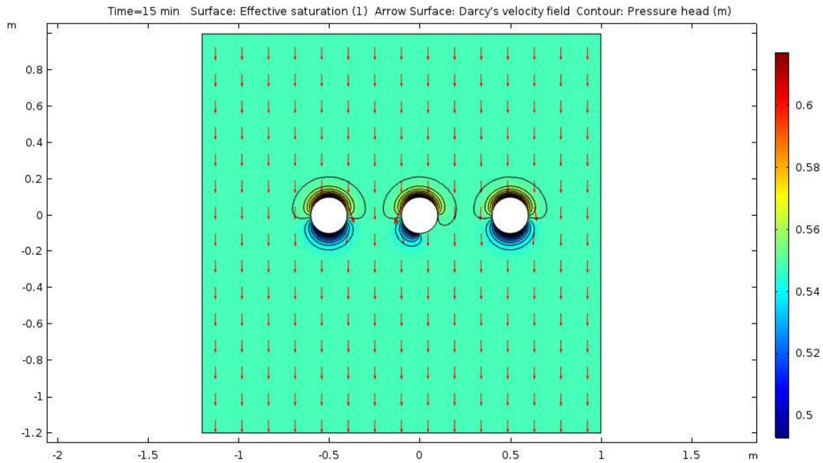

FIGURE 2. The shape of uniformly quasi circular potential density for $\phi(\mathbf{N_q})$.

$$\begin{aligned}
\mathcal{L}^{\boldsymbol{f}}\phi(\mathbf{B_q}) = {} & ((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 \\
& + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \sin\varphi \sin[\chi_1 s + \chi_2] + ((\varrho_3\varrho_1 - \omega')\varrho_1 - (\varrho_3' + \omega\varrho_1)' \\
& + \varrho_3(\varrho_3^2 + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \cos[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 \\
& + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \sin[\chi_1 s + \chi_2])((\omega \\
& - \pi\varrho_2) \cos\varphi \sin[\chi_1 s + \chi_2] - \pi \sin\varphi \sin[\chi_1 s + \chi_2] + (\varrho_1 - \pi\varrho_1) \cos[\chi_1 s + \chi_2]) \\
& + ((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 \\
& + \omega\varrho_2)\varrho_2)) \sin\varphi \cos[\chi_1 s + \chi_2] - ((\varrho_3\varrho_1 - \omega')\varrho_1 - (\varrho_3' + \omega\varrho_1)' + \varrho_3(\varrho_3^2 \\
& + \omega\varrho_2) - (\varrho_3\varrho_1 - \omega')\varrho_1) \sin[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3\varrho_1 - \omega') - (\varrho_3^2 + \omega\varrho_2)' - \varrho_3(\varrho_3' \\
& + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \cos\varphi \cos[\chi_1 s + \chi_2])((\omega - \pi\varrho_2) \cos\varphi \cos[\chi_1 s + \chi_2] \\
& - \pi \sin\varphi \cos[\chi_1 s + \chi_2] - (\varrho_1 - \pi\varrho_1) \sin[\chi_1 s + \chi_2]) - ((\varrho_1(\varrho_3' + \omega\varrho_1) + (\varrho_3\varrho_1 \\
& - \omega')' + \varrho_2(\varrho_3^2 + \omega\varrho_2) - ((\varrho_3' + \omega\varrho_1)\varrho_1 + (\varrho_3^2 + \omega\varrho_2)\varrho_2)) \cos\varphi - (\varrho_2(\varrho_3\varrho_1 - \omega') \\
& - (\varrho_3^2 + \omega\varrho_2)' - \varrho_3(\varrho_3' + \omega\varrho_1) - (\varrho_3\varrho_1 - \omega')\varrho_2) \sin\varphi)((\omega - \pi\varrho_2) \sin\varphi + \pi \cos\varphi).
\end{aligned}$$

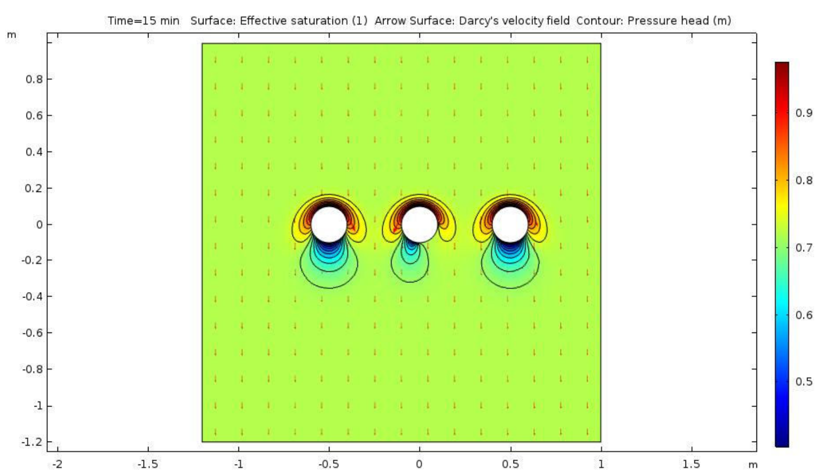

FIGURE 3. The shape of uniformly quasi circular potential density for $\phi(\mathbf{B_q})$.

Figure 3 shows the shape of the uniformly quasi circular potential density for $\phi(\mathbf{B_q})$ with appropriate values of the ϱ_1, ϱ_2, and ϱ_3.

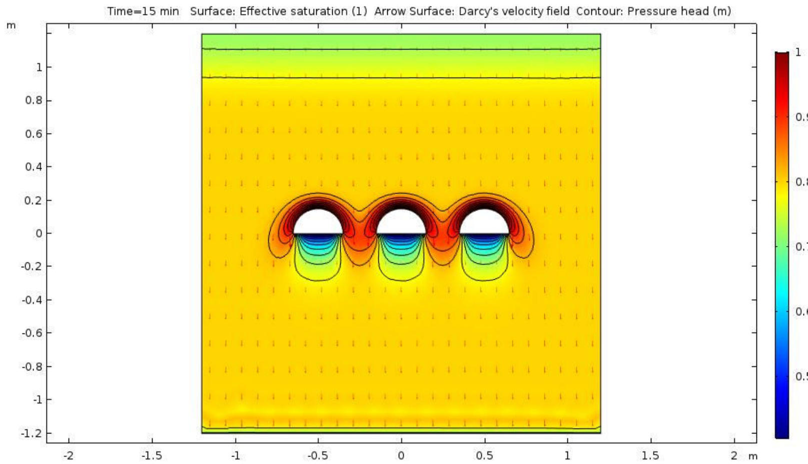

FIGURE 4. The shape of uniformly quasi circular potential density for $\mathcal{E}$.

• *Quasi uniformly circular potential electric energy of $\mathcal{E}$ in the electric field $\mathcal{E}$*

$$\begin{aligned}
\mathcal{L}^{f}(\mathcal{E}) = {} & ((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_2+\varrho_3((\varrho_1 \\
& -\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\cos\varphi\sin[\chi_1 s+\chi_2]-((\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))' \\
& +\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))+\varrho_2((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\sin\varphi\sin[\chi_1 s+\chi_2] \\
& +(((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_1-\varrho_3((\omega-\pi\varrho_2)' \\
& +\varrho_3(\varrho_1-\pi\varrho_1)))\cos[\chi_1 s+\chi_2])((\omega-\pi\varrho_2)\cos\varphi\sin[\chi_1 s+\chi_2]-\pi\sin\varphi\sin[\chi_1 s+\chi_2]+(\varrho_1 \\
& -\pi\varrho_1)\cos[\chi_1 s+\chi_2])+((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega \\
& -\pi\varrho_2))\varrho_2+\varrho_3((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\cos\varphi\cos[\chi_1 s+\chi_2]-((\pi+\varrho_1(\varrho_1-\pi\varrho_1) \\
& +\varrho_2(\omega-\pi\varrho_2))'+\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))+\varrho_2((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))) \\
& \times\sin\varphi\cos[\chi_1 s+\chi_2]-(((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))'-(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega \\
& -\pi\varrho_2))\varrho_1-\varrho_3((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\sin[\chi_1 s+\chi_2])((\omega-\pi\varrho_2)\cos\varphi\cos[\chi_1 s+\chi_2] \\
& -\pi\sin\varphi\cos[\chi_1 s+\chi_2]-(\varrho_1-\pi\varrho_1)\sin[\chi_1 s+\chi_2])+((((\omega-\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1))' \\
& -(\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))\varrho_2+\varrho_3((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2)))\sin\varphi \\
& +((\pi+\varrho_1(\varrho_1-\pi\varrho_1)+\varrho_2(\omega-\pi\varrho_2))'+\varrho_1((\varrho_1-\pi\varrho_1)'-\varrho_3(\omega-\pi\varrho_2))+\varrho_2((\omega \\
& -\pi\varrho_2)'+\varrho_3(\varrho_1-\pi\varrho_1)))\cos\varphi)((\omega-\pi\varrho_2)\sin\varphi+\pi\cos\varphi).
\end{aligned}$$

Figure 4 shows the shape of the uniformly quasi circular potential density for $\mathcal{E}$ with appropriate values of the $\varrho_1,\ \varrho_2$, and ϱ_3.

• *Quasi uniformly circular potential electric energy of $\mathcal{B}$ in the electric field $\mathcal{E}$*

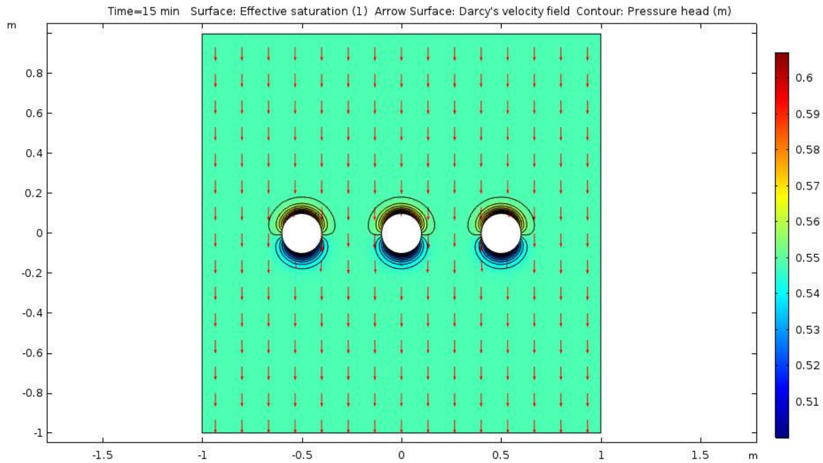

FIGURE 5. The shape of uniformly quasi circular potential density for $\mathcal{E}$.

$$\begin{aligned}\mathcal{L}^{\mathbf{f}}(\mathcal{B}) = {} & (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' - \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 + \varrho_2(\varrho_1' + \varrho_3\varrho_2 \\ & - \omega\varrho_3))\sin\varphi\sin[\chi_1 s + \chi_2] + ((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1 - \varrho_3(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'' - (\varrho_3' \\ & + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1)\cos[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3)' - \varrho_3\omega' \\ & - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2])((\omega - \pi\varrho_2)\cos\varphi\sin[\chi_1 s + \chi_2] - \pi\sin\varphi\sin[\chi_1 s + \chi_2] \\ & + (\varrho_1 - \pi\varrho_1)\cos[\chi_1 s + \chi_2]) + (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' - \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 \\ & + \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3))\sin\varphi\cos[\chi_1 s + \chi_2] - ((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1 - \varrho_3(\varrho_1' + \varrho_3\varrho_2 \\ & - \omega\varrho_3) - \omega'' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_1)\sin[\chi_1 s + \chi_2] + (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' + \varrho_3\varrho_2 \\ & - \omega\varrho_3)' - \varrho_3\omega' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2])((\omega - \pi\varrho_2)\cos\varphi\cos[\chi_1 s + \chi_2] \\ & - \pi\sin\varphi\cos[\chi_1 s + \chi_2] - (\varrho_1 - \pi\varrho_1)\sin[\chi_1 s + \chi_2]) - (((\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)' + \varrho_1\omega' - \varrho_2(\varrho_1' \\ & + \varrho_3\varrho_2 - \omega\varrho_3) - \omega'\varrho_1 + \varrho_2(\varrho_1' + \varrho_3\varrho_2 - \omega\varrho_3))\cos\varphi - (\varrho_2(\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2) + (\varrho_1' \\ & + \varrho_3\varrho_2 - \omega\varrho_3)' - \varrho_3\omega' - (\varrho_3' + \omega\varrho_1 - \varrho_1\varrho_2)\varrho_2)\sin\varphi)((\omega - \pi\varrho_2)\sin\varphi + \pi\cos\varphi).\end{aligned}$$

Figure 5 shows the shape of the uniformly quasi circular potential density for $\mathcal{E}$ with appropriate values of the $\varrho_1,\ \varrho_2$, and ϱ_3.

6. Conclusion

The uniform motion of the physical system, in particular, a mechanism defined along with the particle in a given appropriate spacetime structure, can be described by minimizing the action functional and can further computation be obtained by the principle of the least action [33-35]. The investigation of the uniform motion of the particle is very efficient for the exact comprehension of many physical processes such as vortex filaments, integrable systems, dynamics of Heisenberg chain, soliton equation theory, relativity, sigma models, water wave theory, field theories, fluid dynamics, linear and nonlinear optics, and so on.

We have examined the quasi uniformly accelerated motion (QUAM) with cold plasma in Heisenberg space. Thus we investigate the shape of the uniformly quasi circular potential density for a quasi uniformly accelerated motion of normal magnetic biharmonic particles in Heisenberg space. Finally, we construct quasi uniformly accelerated potential electric energy with respect to its electric field and some quasi curvatures by Figs. 1-5.

In future studies, we will investigate the physical implications of the uniformly quasi circular motion (UQCM) by obtaining different trajectories in de Sitter spacetime, Anti de Sitter spacetime, etc.

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1. J.L. Synge, *Relativity: The General Theory*. North Holland, Amsterdam (1960).
2. B. Mashhoon, U. Muench, Length measurement in accelerated systems. *Annalen der Physik* **11** (2002) 532, https://doi.org/10.1002/1521-3889(200208)11:7⟨532::AID-ANDP532⟩3.0.CO;2-3.
3. D. de la Fuente, A. Romero, Uniformly accelerated motion in General Relativity: completeness of inextensible trajectories. *Gen. Relativ. Gravit.* **47** (2015) 33, https://doi.org/10.1007/s10714-015-1879-3.
4. D. de la Fuente, A. Romero, and P. Torres, Unchanged direction motion in General Relativity: the problems of prescribing acceleration and the extensibility of trajectories. *J. Math. Phys.* **56** (2015) 112501, https://doi.org/10.1063/1.4935854.
5. Y. Friedman, and T. Scarr, Uniform acceleration in general relativity. *Gen. Relativ. and Gravit.* **47** (2015) 121, DOI: https://doi.org/10.1007/s10714-015-1966-5.
6. Y. Friedman, T. Scarr, Making the relativistic dynamics equation covariant: explicit solutions for motion under a constant force, *Phys. Scr.* **86** (2012) 065008, https://doi.org/10.1088/0031-8949/86/06/065008.
7. B. O'Neill, *Semi-Riemannian Geometry*. Academic Press, Massachusetts (1983).
8. K. Takenaka, A.D. Setyawan, P. Sharma, N. Nishiyama, A. Makino, Industrialization of nanocrystalline Fe–Si–B–P–Cu alloys for high magnetic flux density cores, *J. Magn. Magn. Mater*, **401** (2016) 479, https://doi.org/10.1016/j.jmmm.2015.10.091.
9. M. Li, T. Bien, G. Rose, Construction of a conductive distortion reduced electromagnetic tracking system for computer assisted image-guided interventions, *Med. Eng. Phys.* **36** (2014) 1496, DOI: https://doi.org/10.1016/j.medengphy.2014.07.018.
10. A.M. Franz, T. Haidegger, W. Birkfellner, K. Cleary, T.M. Peters, L. Maier-Hein, Electromagnetic tracking in medicine - a review of technology, validation and applications, *IEEE Trans. Med. Imag.* **33** (2014) 1702, https://doi.org/10.1109/TMI.2014.2321777.
11. A. De Lambert, S. Esneault, A. Lucas, P. Haigron, P. Cinquin, J.L. Magne, Electromagnetic tracking for registration and navigation in endovascular aneurysm repair: a phantom study, *Eur. J. Vasc. Endovasc. Surg.* **43** (2012) 684, https://doi.org/10.1016/j.ejvs.2012.03.007.
12. M. Sha, Y. Wang, N. Ding, X. Wu, Z. Fang, An electromagnetic tracking method based on fast determination of the maximum magnetic flux density vector represented by two azimuth angles, *Measurement*, **109** (2017) 160, https://doi.org/10.1016/j.measurement.2017.04.027.
13. E. Turhan, T. Körpınar, On Characterization Canal Surfaces around Timelike Horizontal Biharmonic Curves in Lorentzian Heisenberg Group Heis^3, *Z. Naturforsch.* **66a** (2011) 441, https://doi.org/10.5560/ZNA.2011.66a0441.
14. E. Turhan, T. Körpınar, On Characterization of Time-Like Horizontal Biharmonic Curves in the Lorentzian Heisenberg Group Heis^3, *Z. Naturforsch.* **65a** (2010) 641, https://doi.org/10.1515/zna-2010-8-904.
15. A. Altin, On the energy and Pseduoangle of Frenet Vector Fields in R^n_v. *Ukranian Mathematical J.* **63** (2011), 969, https://doi.org/10.1007/s11253-011-0556-2.
16. V. Asil, Velocities of Dual Homothetic Exponential Motions in D^3, *Iranian Journal of Science & Tecnology Transaction A, Science* **31** (2007) 265, https://doi.org/10.22099/IJSTS.2008.2301.
17. T. Körpınar, R.C. Demirkol, A new approach on the curvature dependent energy for elastic curves in a Lie Group. *Honam Mathematical J.* **39** (2017), 637, https://doi.org/10.5831/HMJ.2017.39.4.637.
18. T. Körpınar, R.C. Demirkol, A New characterization on the energy of elastica with the energy of Bishop vector fields in Minkowski space. *Journal of Advanced Physics.* **6** (2017) 562, https://doi.org/10.1166/jap.2017.1375.
19. T. Körpınar, R.C. Demirkol, Energy on a timelike particle in dynamical and electrodynamical force fields in De-Sitter space. *Rev. Mex. Fis.* **63** (2017) 560-568.
20. T. Adachi, Kahler magnetic on a complex projective space. Proc. Japan Acad. Ser. A: Math Sci. **70** (1994) 12, https://doi.org/10.3792/pjaa.70.12.
21. M. Barros, A. Romero, J.L. Cabrerizo, M. Fernandez, The Gauss–Landau–Hall problem on Riemannian surfaces, *J. Math. Phys.* **46** (2005) 112905, https://doi.org/10.1063/1.2136215.
22. J.L. Cabrerizo, M. Fernandez, J.S. Gómez, The contact magnetic flow in 3D Sasakian manifolds, *J. Phys. A* **42** (2009) 195201, https://doi.org/10.1088/1751-8113/42/19/195201.
23. J.L. Cabrerizo, Magnetic fields in 2D and 3D sphere. *Journal of Nonlinear Mathematical Physics* **20** (2013), 440, https://doi.org/10.1080/14029251.2013.855052.
24. T. Körpınar, R. C. Demirkol, Frictional magnetic curves in 3D Riemannian manifolds, *International Journal of Geometric Methods in Modern Physics*, **15** (2018) 1850020, https://doi.org/10.1142/S0219887818500202.
25. T. Körpınar, On T-Magnetic Biharmonic Particles with Energy and Angle in the Three Dimensional Heisenberg Group H, *Adv. Appl. Clifford Algebras*, **28** (2018) 1, https://doi.org/10.1007/s00006-018-0834-2.
26. S. Rahmani, Metriqus de Lorentz sur les groupes de Lie unimodulaires, de dimension trois, *Journal of Geometry and Physics* **9** (1992), 295, https://doi.org/10.1016/0393-0440(92)90033-W.
27. M. Dede, C. Ekici, H. Tozak, Directional Tubular Surfaces, *International Journal of Algebra*, **9** (2015), 527, http://dx.doi.org/10.12988/ija.2015.51274.
28. E. Pina, Lorentz transformation and the motion of a charge in a constant electromagnetic field. *Rev. Mex. Fis.* **16** (1967) 233.
29. H. Ringermacher, Intrinsic geometry of curves and the Minkowski force, *Phys. Lett. A* **74** (1979) 381, https://doi.org/10.1016/0375-9601(79)90229-9.

30. B. M. Barbashov and V. Nesterenko, *Introduction to the relativistic string theory* (World Scientific, 1990).

31. T. Körpınar, A new characterization of quasi velocity magnetic biharmonic particles in the Heisenberg space with cold plasma, *Rev. Mex. Fis.*. Accepted.

32. T. Körpınar, Z. Körpınar, Y-M. Chu, M A Akinlar and M. Inc, New Uniform Motion and Fermi–Walker Derivative of Normal Magnetic Biharmonic Particles in Heisenberg Space, *Symmetry*, **12** (2020) 1017, https://doi.org/10.3390/sym12061017.

33. T. Körpınar, R. C. Demirkol and Z. Körpınar, Soliton propagation of electromagnetic field vectors of polarized light ray traveling in a coiled optical fiber in the ordinary space, *International Journal of Geometric Methods in Modern Physics* **16** (8)(2019) 1950117, https://doi.org/10.1142/S0219887819501172.

34. T. Körpınar R. C. Demirkol, On the uniform motion of a relativistic charged particle in a homogeneous electromagnetic field in Minkowski space E_2^4, *Math Meth Appl Sci.* **42** (2019) 3069, https://doi.org/10.1002/mma.5567.

35. T. Körpınar, Optical directional binormal magnetic flows with geometric phase: Heisenberg ferromagnetic model, *Optik - International Journal for Light and Electron Optics* **219** (2020) 165134, https://doi.org/10.1016/j.ijleo.2020.165134.