Optical soliton solutions of nonlinear Davey-Stewartson equation using an efficient method

H. Günerhan

Department of Mathematics, Faculty of Education, Kafkas University, Kars TR-36040, Turkey.
*e-mail: hatira.gunerhan@kafkas.edu.tr

Received 29 December 2020; accepted 30 June 2021

One of the most significant tools for expressing physical phenomena in the world around us is to express problems using differential equations with partial derivatives. The result of these considerations has been the invention and application of various analytical and numerical methods in solving this category of equations. In this work, we make use of a newly-developed technique called the generated exponential rational function method to compute the exact solution of the Davey-Stewartson equation. According to all the conducted research studies so far, results similar to those found in the present paper have not been published. The results attest to the efficiency of the proposed method. The method used in this paper has the ability to be implemented in other cases in solving equations with relative derivatives.

Keywords: Generalized exponential rational function method; exact soliton solutions; Davey-Stewartson equation; optical solutions; nonlinear partial differential equations.

DOI: https://doi.org/10.31349/RevMexFis.67.060702

1. Introduction

Many fields of physics benefit from the nonlinear partial differential equations (NPDEs) in a wide variety of applications to mechanics, electrostatics, quantum mechanics, and finance. In linear theory, solutions usually have representation formulas and conform to the superposition principle. Despite some equations governing nature were recognized to be linear since the 19th century, this advance was widely criticized in the 20th century. It is possible to observe NPDEs not only in non-Newtonian fluids, glaciology, rheology but also in stochastic game theory, nonlinear elasticity, flow through a porous medium, and image-processing. As a result, no available superposition can be found for nonlinear equations; therefore, there is a need for studying those equations. Over the last few years, a crucial number of new methods have been proposed to obtain exact solutions for NPDEs. The Davey-Stewartson equation (DSE) has been studied in many areas of research such as chemical engineering, nonlinear mechanics, biology, and physics. To define the evolution of a three-dimensional wave packet in finite depth water, Davey-Stewartson (1974) had presented the DSE in his research study about fluid dynamics [1].

In (2+1)-dimensions, the Davey-Stewartson equation is known as a solution equation that examines long and short wave resonances and other wave propagation patterns. For more details, we refer to the previous research works conducted in Refs. [2-3]. The Davey-Stewartson theory is an NPDE for a complex field (wave-amplitude) \( q \) and a real field (mean flow) \( \phi \) which is described by the following nonlinear coupled system

\[
iq_t + \frac{1}{2} (q_{xx} + \delta q_{yy}) + \lambda |q|^2 q - \phi_x q = 0, \quad (1)
\]

\[
\phi_{xx} - \delta^2 \phi_{yy} - 2\lambda (|q|^2)_x = 0. \quad (1.1)
\]

This important system of equations has attracted the attention of many researchers. For example, the line soliton [4], the semi-inverse variational principle method (SIVPM), the improved \( \tan(\phi/2) \)-expansion method (ITEM) along with the generalized \( G'/G \)-expansion method (GGM) [5], the \( G'/G \) method and the 1-soliton solution [6], the Galerkin methods [7], the extended tanh-function method [8], the generalized Kudryashov method [9], the traveling wave solutions [10], the first integral method [11], the solitary wave solution [12], the dynamical system method [13], the traveling waves solution and the exponential function method [14], the inverse scattering transform method and the soliton solutions [15], the self-similar solutions [16], the bilinear method [17, 18], the single soliton and multi-soliton solutions [19], the \( G'/G \) -expansion method [20], the generalized \( \tan(\phi/2) \) method and the He’s semi-inverse variational method [21], and the bifurcation method [22, 23]. Others popular techniques can be found in Ref. [24, 43].

In Ref. [44], Ghanbari and his collaborator developed an efficient methodology for obtaining exact solutions to NPDEs which is known as a generalized exponential rational function method (GERFM). The authors applied the technique to solve the resonant nonlinear Schrödinger equation (RNLS). It has been proven over time that the method enables us to be implemented in many different NPDEs arising in mathematics, physics, and engineering [45-54]. The proposed method reproduces many types of precise solutions, and it is very useful for finding the exact solutions of the equation with relative ease. Recently, a new version of the method for solving partial differential equations with local fractional derivatives has been considered in Refs. [55, 56].

In this paper, the GERFM is used to solve the Davey-Stewartson equation. This paper consists of the following parts: In Sec. 2, we introduce the methodology of the GERFM. In Sec. 3, the results of using the method in deter-
2. Methodology of the GERFM

The technique is a very efficient method in solving partial differential equations [44]. The basic steps of using this method are listed below.

1. NPDE will be accepted as follows:

\[ F(u(x,t), \frac{\partial u(x,t)}{\partial x}, \frac{\partial u(x,t)}{\partial t}, \frac{\partial^2 u(x,t)}{\partial x^2}, \ldots) = 0. \quad (2.1) \]

To abbreviate the NPDEs is the following ordinary differential equation (ODE), it will be used \( u(x,t) \) and \( \xi = kx - lt \).

\[ F(u, u', u'', \ldots) = 0, \quad (2.2) \]

2. The crucial part of the new methodology comes from the fact that Eq. (2.2) has the formal solution of

\[ u(\xi) = A_0 + \sum_{k=1}^{m} A_k \Xi(\xi)^k + \sum_{k=1}^{m} B_k \Xi(\xi)^{-k}, \quad (2.3) \]

where

\[ \Xi(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}}. \quad (2.4) \]

The real (or complex) unknown constants are \( A_0, A_k, B_k(1 \leq k \leq m) \), and \( p_k, q_k(1 \leq k \leq 4) \). These coefficients are determined in such a way that Eq. (2.3) satisfies the nonlinear ODE of Eq. (2.2).

Also, it is essential to determine the positive integer \( m \) by the principle of balancing.

3. By adding all terms and inserting Eq. (2.3) into Eq. (2.2), the left-hand side of Eq. (2.2) is converted into the polynomial equation \( P(Y_1, Y_2, Y_3, Y_4) = 0 \) in terms of \( Y_i = e^{q_i \xi} \) for \( i = 1, \ldots, 4 \). With the help of symbolic calculation in Maple, we obtain a set of simultaneous algebraic equations for \( p_n, q_n(1 \leq n \leq 4) \), and \( k, \omega, \lambda, A_0, A_1, B_1 \) by eliminating each coefficient of \( P \).

4. Finally, exact solutions to Eq. (2.1) are derived through solving the algebraic nonlinear system of equations in step 3.

3. The results

The first step is the traveling wave transformation of Eq. (1.1) by utilizing the following new variables

\[ q = \Psi(\xi) e^{i\theta}, \quad \phi = \Phi(\xi) e^{i\theta}, \quad (3.1) \]

and

\[ \xi = i\mu (x + y - \eta t), \quad \theta = \alpha x + \beta y + \gamma t. \quad (3.2) \]

In addition, constants of \( \mu, \eta, \alpha, \) and \( \beta \) should be determined. Using the wave transformation of Eq. (3.1) and Eq. (1.1) together with \( \eta = \alpha \delta^2 + \beta \delta^4 \), the following system of nonlinear ODE is obtained [5]:

\[ \frac{1}{2} (2\gamma + \alpha^2 \delta^2 + \beta^2 \delta^4) \Psi'' - \frac{1}{2} \mu \delta^2 (1 + \delta^2) \Phi'' + \lambda \Psi^3 - i \mu \Phi \Psi' = 0, \quad (3.3) \]

\[ \mu (\delta^2 - 1) \Phi'' - 2i \lambda (\Psi^2) \Phi' = 0. \quad (3.4) \]

Integrating Eq. (3.4), we obtain

\[ \Psi' = \frac{2i \lambda}{\mu (\delta^2 - 1)} \int \Psi^2 d\xi. \quad (3.5) \]

Substituting Eq. (3.5) in Eq. (3.3) will turn into the following nonlinear differential equation:

\[ \frac{1}{2} \mu \delta^2 (\delta^4 - 1) \Psi'' + \frac{1}{2} (\delta^2 - 1) (2\gamma + \alpha^2 \delta^2 + \beta^2 \delta^4) \Psi - \lambda (1 + \delta^2) \Psi^3 = 0, \quad (3.6) \]
where primes denote the derivatives with respect to $\xi$. By balancing terms of $\mathcal{Y}''$ and $\mathcal{Y}^3$ in Eq. (3.6) using homogenous principle yields $3m = m + 2$, so $m = 1$. Accordingly, the solution of Eq. (1.1) is expressed as follows:

$$\mathcal{Y}(\xi) = A_0 + A_1 \Xi(\xi) + \frac{B_1}{\Xi(\xi)}.$$  \hspace{1cm} (3.7)

By following the described methodologies in section 2, we obtain several non-trivial solutions of (1.1).

**Family 1:** We attain the results for $p = [1, 1, -1, 1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Xi(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}.$$ \hspace{1cm} (3.8)

**Case 1:**

$$\alpha = \alpha, \quad \beta = \frac{\sqrt{\alpha^2 \delta^4 + (2 \, A_1^2 \lambda - \alpha^2 + 2 \, \gamma) \, \delta^2 + 2 \, A_1^2 \lambda - 2 \, \gamma}}{\delta^2 \sqrt{\delta^2 - 1}},$$

$$\mu = \frac{\sqrt{\lambda} \, A_1}{\delta \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0.$$ 

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

$$\mathcal{Y}_1(\xi) = \frac{A_1 \cosh(\xi)}{\sinh(\xi)},$$

$$\mathcal{V}_1(\xi) = \frac{2 \, i \, \lambda \, A_1^2 (\xi - \coth(\xi))}{\mu (\delta^2 - 1)}.$$ \hspace{1cm} (3.9)

Hence, the following exact solution was reached for Eq. (1.1):

$$q_1(x, y, t) = \left( \frac{A_1 \cosh(\xi)}{\sinh(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_1(x, y, t) = \left( \frac{2 \, i \, \lambda \, A_1^2 (\xi - \coth(\xi))}{\mu (\delta^2 - 1)} \right) e^{i(\alpha x + \beta y + \gamma t)}.$$ \hspace{1cm} (3.10)

**Case 2:**

$$\alpha = \alpha, \quad \beta = \frac{\sqrt{\alpha^2 \delta^4 + (8 \, A_1^2 \lambda - \alpha^2 + 6 \, \gamma) \, \delta^2 + 8 \, A_1^2 \lambda - 2 \, \gamma}}{\delta^2 \sqrt{\delta^2 - 1}},$$

$$\mu = \frac{\sqrt{\lambda} \, A_1}{\delta \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = A_1, \quad B_1 = A_1.$$ 

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

$$\mathcal{Y}_2(\xi) = \frac{2 \left( \cosh(\xi) \right)^2 - 1}{\cosh(\xi) \sinh(\xi)} A_1,$$

$$\mathcal{V}_2(\xi) = \frac{2 \, \lambda \, A_1^2 \left( 4 \, \cosh(\xi) \sinh(\xi) - 2 \left( \cosh(\xi) \right)^2 + 1 \right)}{\cosh(\xi) \sinh(\xi) \mu (\delta^2 - 1)}.$$ \hspace{1cm} (3.11)

Hence, the following exact solution has been reached for Eq. (1.1):

$$q_2(x, y, t) = \left( \frac{2 \left( \cosh(\xi) \right)^2 - 1}{\cosh(\xi) \sinh(\xi)} A_1 \right) e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_2(x, y, t) = \left( \frac{2 \, i \, \lambda \, A_1^2 \left( 4 \, \cosh(\xi) \sinh(\xi) - 2 \left( \cosh(\xi) \right)^2 + 1 \right)}{\cosh(\xi) \sinh(\xi) \mu (\delta^2 - 1)} \right) e^{i(\alpha x + \beta y + \gamma t)}.$$ \hspace{1cm} (3.12)
Figure 1. Dynamic behaviors modulus of solutions $q_3(x, y, t)$ (left) and $\phi_3(x, y, t)$ (right) for $\delta = 1.05, \gamma = 1.3, \alpha = \beta = 0.5, \lambda = 1.5$, and $t = 1$.

Case 3:

$$\alpha = \alpha, \quad \beta = \beta, \quad \mu = \frac{i/2\sqrt{2} \sqrt{\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma\sinh(\xi)}}{\delta \sqrt{\delta^2 + 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0,$$

$$A_1 = 0, \quad B_1 = \frac{-i\sqrt{2} \sqrt{\beta^2 - 1} \sqrt{\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma \sinh(\xi)}}{\sqrt{\lambda}(2 \delta^2 + 2)}.$$

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

$$\mathcal{U}_3(\xi) = \frac{-i/2\sqrt{2} \sqrt{\delta^4 - 1} \sqrt{\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma \sinh(\xi)}}{\sqrt{\lambda}(\delta^2 + 1) \cosh(\xi)},$$

$$\mathcal{Y}_3(\xi) = \frac{-i(\delta^4 - 1)(\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma)(\xi - \tanh(\xi))}{\mu(\delta^2 - 1)(\delta^2 + 1)^2}.$$  \hspace{1cm} (3.13)

Hence, the following exact solution has been reached for Eq. (1.1):

$$q_3(x, y, t) = \left(\frac{-i/2\sqrt{2} \sqrt{\delta^4 - 1} \sqrt{\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma \sinh(\xi)}}{\sqrt{\lambda}[\delta^2 + 1] \cosh(\xi)}\right) e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_3(x, y, t) = \left(\frac{-i[\delta^4 - 1][\beta^2 \delta^4 + \alpha^2 \delta^2 + 2 \gamma](\xi - \tanh(\xi))}{\mu[\delta^2 - 1][\delta^2 + 1]^2}\right) e^{i(\alpha x + \beta y + \gamma t)}. \hspace{1cm} (3.14)$$

Figure 1 shows the dynamic behavior of modulus of solutions $q_3(x, y, t)$ (left) and $\phi_3(x, y, t)$ for $\delta = 1.05, \gamma = 1.3, \alpha = \beta = 0.5, \lambda = 1.5$, and $t = 1$.

Family 2: We attain the results for $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$, and thus one gets

$$\Xi(\xi) = -\frac{\sin(\xi)}{\cos(\xi)}. \hspace{1cm} (3.15)$$

Case 1:

$$\alpha = \alpha, \quad \beta = \frac{\sqrt{\lambda} A_1}{\delta \sqrt{\delta^2 - 1}}, \quad \mu = \frac{i \sqrt{\alpha^2 \delta^4 + (\lambda - \alpha^2 + 2 \gamma) \delta^2 - 8 A_1^2 \lambda - 2 \gamma}}{\delta \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = A_1, \quad B_1 = -A_1.$$
We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ W_4(\xi) = \frac{2 \left[ \cos (\xi) \right]^2 - 1}{\sin (\xi) \cos (\xi)} A_1, \]
\[ Y_4(\xi) = \frac{2 i \lambda A_1^2 \left( -2 \left[ \cos (\xi) \right]^2 - 4 \xi \cos (\xi) \sin (\xi) + 1 \right)}{\mu \left( \delta^2 - 1 \right) \sin (\xi) \cos (\xi)}. \] (3.16)

Hence, the following exact solution has been reached for Eq. (1.1):

\[ q_4 (x, y, t) = \left( \frac{2 \left[ \cos (\xi) \right]^2 - 1}{\sin (\xi) \cos (\xi)} A_1 \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_4 (x, y, t) = \left( \frac{2 i \lambda A_1^2 \left( -2 \left[ \cos (\xi) \right]^2 - 4 \xi \cos (\xi) \sin (\xi) + 1 \right)}{\mu \left( \delta^2 - 1 \right) \sin (\xi) \cos (\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.17)

**Case 2:**

\[ \alpha = \alpha, \quad \beta = \frac{i \sqrt{\alpha^2 \delta^4 + (4 B_1^2 \lambda - \alpha^2 + 2 \gamma) \delta^2 + 4 B_1^2 \lambda - 2 \gamma}}{\delta \sqrt{\delta^2 - 1}}, \]
\[ \mu = \frac{\sqrt{\lambda B_1}}{\delta \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = B_1, \quad B_1 = B_1. \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ W_5(\xi) = -\frac{B_1}{\sin (\xi) \cos (\xi)}, \]
\[ Y_5(\xi) = \frac{2 i \lambda B_1^2 \left( -2 \left[ \cos (\xi) \right]^2 + 1 \right)}{\mu \left( \delta^2 - 1 \right) \sin (\xi) \cos (\xi)}. \] (3.18)

Hence, the following exact solution has been reached for Eq. (1.1):
\[ q_5(x, y, t) = \left( -\frac{B_1}{\sin(\xi) \cos(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_5(x, y, t) = \left( \frac{2i\lambda B_1^2 \left[ -2 \left\{ \cos(\xi) \right\}^2 + 1 \right]}{\mu \left[ \delta^2 - 1 \right] \sin(\xi) \cos(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.19)

Figure 2 shows the dynamic behavior of modulus of solutions \( q_5(x, y, t) \) (left) and \( \phi_5(x, y, t) \) for \( B_1 = 1, \delta = 8.5, \gamma = 2.01, \alpha = 4.25, \lambda = 0.92, \) and \( t = 1. \)

**Case 3:**

\[ \alpha = \alpha, \quad \beta = i\frac{\sqrt{\alpha^2 \delta^4 + (-2B_1^2 \lambda - \alpha^2 + 2\gamma) \delta^2 - 2B_1^2 \lambda - 2\gamma}}{\sqrt{\delta^2 - 1}}, \]
\[ \mu = \frac{\sqrt{\lambda B_1}}{\delta} \sqrt{\delta^2 - 1}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = 0, \quad B_1 = B_1. \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[ \mathcal{Z}_6(\xi) = -\frac{B_1 \cos(\xi)}{\sin(\xi)}, \]
\[ \mathcal{Y}_6(\xi) = \frac{2i\lambda B_1^2 \left( -\cot(\xi) - \xi \right)}{\mu \left( \delta^2 - 1 \right)}. \] (3.20)

Hence, the following exact solution has been reached for equation (1.1):
\[ q_6(x, y, t) = \left( -\frac{B_1 \cos(\xi)}{\sin(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_6(x, y, t) = \left( \frac{2i\lambda B_1^2 \left[ -\cot(\xi) - \xi \right]}{\mu \left[ \delta^2 - 1 \right]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.21)

**Family 3:** We attain \( p = [1, 1, -1, 1] \) and \( q = [2, 0, 2, 0] \), which gives
\[ \Xi(\xi) = \frac{e^{2\xi} + 1}{e^{2\xi} - 1}. \] (3.22)

**Case 1:**

\[ \alpha = \alpha, \quad \beta = i\frac{\sqrt{\alpha^2 \delta^4 + (8A_1^2 \lambda - \alpha^2 + 2\gamma) \delta^2 + 8A_1^2 \lambda - 2\gamma}}{\sqrt{\delta^2 - 1}}, \]
\[ \mu = \frac{\sqrt{\lambda A_1}}{\delta} \sqrt{\delta^2 - 1}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = A_1, \quad B_1 = B_1. \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[ \mathcal{Z}_7(\xi) = \frac{2A_1 \left( e^{4\xi} + 1 \right)}{e^{4\xi} - 1}, \]
\[ \mathcal{Y}_7(\xi) = \frac{2i\lambda \left( -4 + 4\xi \left( e^{4\xi} - 1 \right) \right) A_1^2}{\mu \left( \delta^2 - 1 \right) \left( e^{4\xi} - 1 \right)}. \] (3.23)

Hence, the following exact solution has been reached for Eq. (1.1):
\[ q_7(x, y, t) = \left( \frac{2A_1 \left[ e^{4\xi} + 1 \right]}{e^{4\xi} - 1} \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_7(x, y, t) = \left( \frac{2i\lambda \left[ -4 + 4\xi \left( e^{4\xi} - 1 \right) \right] A_1^2}{\mu \left[ \delta^2 - 1 \right] \left( e^{4\xi} - 1 \right)} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.24)
Case 2:

\[ \begin{align*}
\alpha &= \alpha, \\
\beta &= \frac{i \sqrt{\alpha^2 \delta^4 + \left(-4 A_1^2 \lambda - \alpha^2 + 2 \gamma\right) \delta^2 - 4 A_1^2 \lambda - 2 \gamma}}{\sqrt{\delta^2 - 1}}, \\
\mu &= \frac{\sqrt{\lambda A_1}}{\sqrt{\delta^2 - 1}}, \\
\delta &= \delta, \\
\gamma &= \gamma, \\
A_0 &= 0, \\
A_1 &= A_1, \\
B_1 &= -A_1.
\end{align*} \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ \begin{align*}
\mathcal{W}_s(\xi) &= \frac{4 e^{2 \xi} A_1}{e^4 \xi - 1}, \\
\mathcal{Y}_s(\xi) &= \frac{-8 i \lambda A_1^2}{\mu \left(\delta^2 - 1\right) \left(e^4 \xi - 1\right)}.
\end{align*} \] (3.25)

Hence, the following exact solution has been reached for Eq. (1.1):

\[ \begin{align*}
q_s(x, y, t) &= \left(\frac{4 e^{2 \xi} A_1}{e^4 \xi - 1}\right) e^{i(\alpha x + \beta y + \gamma t)}, \\
\phi_s(x, y, t) &= \left(\frac{-8 i \lambda A_1^2}{\mu \left[\delta^2 - 1\right] \left[e^4 \xi - 1\right]}\right) e^{i(\alpha x + \beta y + \gamma t)}.
\end{align*} \] (3.26)

Figure 3 shows the dynamic behavior of modulus of solutions \(q_s(x, y, t)\) (left) and \(\phi_s(x, y, t)\) for \(A_1 = 1, \delta = 1.01, \gamma = 2.3, \alpha = 0.2, \lambda = 0.5, \text{ and } t = 1\).

**Family 4:** We attain the results for \(p = [-1, 3, 1, -1]\) and \(q = [2, 0, 2, 0]\). So, it reads

\[ \Xi(\xi) = \frac{-e^{2 \xi} + 3}{e^{2 \xi} - 1}. \] (3.27)

Case 1:

\[ \begin{align*}
\alpha &= \alpha, \\
\beta &= \frac{i \sqrt{\alpha^2 \delta^4 + \left(2 A_1^2 \lambda - \alpha^2 + 2 \gamma\right) \delta^2 + 2 A_1^2 \lambda - 2 \gamma}}{\sqrt{\delta^2 - 1}}, \\
\mu &= \frac{\sqrt{\lambda A_1}}{\sqrt{\delta^2 - 1}}, \\
\delta &= \delta, \\
\gamma &= \gamma, \\
A_0 &= 2 A_1, \\
A_1 &= A_1, \\
B_1 &= 0.
\end{align*} \]

**Rev. Mex. Fis. 67 060702**
We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ \mathcal{U}_9(\xi) = \frac{A_1 (e^{2\xi} + 1)}{e^{2\xi} - 1}, \]
\[ \mathcal{V}_9(\xi) = \frac{2i\lambda \left(-4 + 2\xi \left[e^{2\xi} - 1\right]\right) A_1^2}{\mu \left(\delta^2 - 1\right)(2 e^{2\xi} - 2)}. \]  (3.28)

Hence, the following exact solution has been reached for equation (1.1):

\[ q_9(x, y, t) = \left(A_1 \left[e^{2\xi} + 1\right] \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_9(x, y, t) = \left(\frac{2i\lambda \left[-4 + 2\xi \left[e^{2\xi} - 1\right]\right]}{\mu \left[\delta^2 - 1\right]\left[2 e^{2\xi} - 2\right]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \]  (3.29)

Case 2:

\[ \alpha = \alpha, \quad \beta = \frac{i\sqrt{2} \sqrt{2\alpha^2 \delta^4 + (A_0^2 \lambda - 2 \alpha^2 + 4 \gamma) \delta^2 + A_0^2 \lambda - 4 \gamma}}{2\sqrt{\delta^2 - 1}}, \]
\[ \mu = \frac{A_0 \sqrt{\lambda}}{2\delta \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = 3/2 A_0. \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ \mathcal{U}_{10}(\xi) = -\frac{A_0 \left(e^{2\xi} + 3\right)}{2 e^{2\xi} - 6}, \]
\[ \mathcal{V}_{10}(\xi) = \frac{2i\lambda A_0^2 \left(-12 + 2\xi \left[e^{2\xi} - 3\right]\right)}{\mu \left[\delta^2 - 1\right]\left[8 e^{2\xi} - 24\right]} \]  (3.30)

Hence, the following exact solution has been reached for equation (1.1):

\[ q_{10}(x, y, t) = \left(-A_0 \left[e^{2\xi} + 3\right] \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[ \phi_{10}(x, y, t) = \left(\frac{2i\lambda A_0^2 \left[12 + 2\xi \left[e^{2\xi} - 3\right]\right]}{\mu \left[\delta^2 - 1\right]\left[8 e^{2\xi} - 24\right]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \]  (3.31)

Family 5: We attain the results for \( p = [\alpha, 1, 1, 1] \) and \( q = [1, -1, 1, -1] \), which gives

\[ \Xi(\xi) = -\frac{\sinh(\xi)}{\cosh(\xi)}. \]  (3.32)

Case 1:

\[ \alpha = \alpha, \quad \beta = \frac{i\sqrt{\alpha^2 \delta^4 + \left(-4 A_1^2 \lambda - \alpha^2 + 2 \gamma\right) \delta^2 - 4 A_1^2 \lambda - 2 \gamma}}{\sqrt{\delta^2 - 1}}, \]
\[ \mu = \frac{\sqrt{\lambda A_1}}{\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = 0, \quad A_1 = 1, \quad B_1 = -A_1. \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ \mathcal{U}_{11}(\xi) = \frac{A_1}{\cosh(\xi) \sinh(\xi)}, \]
\[ \mathcal{V}_{11}(\xi) = \frac{2i\lambda A_1^2 \left(-2 \left[\cosh(\xi)\right]^2 + 1\right)}{\mu \left[\delta^2 - 1\right] \cosh(\xi) \sinh(\xi)}. \]  (3.33)
Figure 4. Dynamic behaviours modulus of solutions $q_{11}(x,y,t)$ (left) and $\phi_{11}(x,y,t)$ (right) for $A_1 = 1, \delta = 1.01, \gamma = 2.3, \alpha = 0.2, \lambda = 0.5$, and $t = 1$.

Hence, the following exact solution has been reached for Eq. (1.1):

\[
q_{11}(x,y,t) = \left( \frac{A_1}{\cosh(\xi)\sinh(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)},
\]

\[
\phi_{11}(x,y,t) = \left( \frac{2 i \lambda A_1^2 [-2 \{ \cosh(\xi) \}^2 + 1]}{\mu [\delta^2 - 1] \cosh(\xi) \sinh(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}. \tag{3.34}
\]

Figure 4 shows the dynamic behavior of modulus of solutions $q_{11}(x,y,t)$ (left) and $\phi_{11}(x,y,t)$ for $A_1 = 1, \delta = 1.01, \gamma = 2.3, \alpha = 0.2, \lambda = 0.5$, and $t = 1$.

**Family 6:** We attain the results for $p = [-1-i, 1-i, -1, 1]$ and $q = [i, -i, i, -i]$, and one obtains

\[
\Xi(\xi) = \frac{\cos(\xi) + 2 \sin(\xi)}{\sin(\xi)}. \tag{3.35}
\]

**Case 1:**

\[
\alpha = \alpha, \quad \beta = i \sqrt{\frac{\alpha^2 \delta^4 + (-2 A_0^2 \lambda - \alpha^2 + 2 \beta) \delta^2 - 2 A_0^2 \lambda - 2 \gamma}{\delta^2 - 1}},
\]

\[
\mu = \frac{A_0 \sqrt{\lambda}}{\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = -2 A_0.
\]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[
\mathcal{W}_{12}(\xi) = \frac{A_0 \left[ \sin(\xi) + \cos(\xi) \right]}{\cos(\xi) + \sin(\xi)},
\]

\[
\mathcal{Y}_{12}(\xi) = \frac{-2 i \lambda A_0^2 \left( \xi \tan(\xi) + \xi + 2 \right)}{\mu (\delta^2 - 1) (\tan(\xi) + 1)}. \tag{3.36}
\]

Hence, the following exact solution has been reached for Eq. (1.1):

\[
q_{12}(x,y,t) = \left( \frac{A_0 \left[ \sin(\xi) + \cos(\xi) \right]}{\cos(\xi) + \sin(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)},
\]

\[
\phi_{12}(x,y,t) = \left( \frac{-2 i \lambda A_0^2 \left[ \xi \tan(\xi) + \xi + 2 \right]}{\mu [\delta^2 - 1] (\tan(\xi) + 1)} \right) e^{i(\alpha x + \beta y + \gamma t)}. \tag{3.37}
\]

**Family 7:** We obtain $p = [-2-i, 2-i, -1, 1]$ and $q = [i, -i, i, -i]$, and thus one attains

\[
\Xi(\xi) = \frac{\cos(\xi) + 2 \sin(\xi)}{\sin(\xi)}. \tag{3.38}
\]
Case 1:
\[\alpha = \alpha, \quad \beta = \frac{i/2\sqrt{2}\sqrt{2\alpha^2\delta^4 + (-A_0^2\lambda - 2\alpha^2 + 4\gamma)\delta^2 - A_0^2\lambda - 4\gamma}}{\sqrt{\delta^2 - 1}}\]

\[\mu = 1/2 \frac{A_0\sqrt{\lambda}}{\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = -5/2 A_0.\]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[\Upsilon_{13}(\xi) = \frac{A_0 (- \sin (\xi) + 2 \cos (\xi))}{2 \cos (\xi) + 4 \sin (\xi)}, \]
\[\Upsilon_{13}(\xi) = \frac{-2 i \lambda A_0 \left[ 4 \xi \tan (\xi) + 2 \xi + 5 \right]}{\mu (\delta^2 - 1) \left[ 8 + 16 \tan (\xi) \right]}. \] (3.39)

Hence, the following exact solution has been reached for equation (1.1):
\[q_{13}(x, y, t) = \left( \frac{A_0 \left[ - \sin (\xi) + 2 \cos (\xi) \right]}{2 \cos (\xi) + 4 \sin (\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[\phi_{13}(x, y, t) = \left( \frac{-2 i \lambda A_0 \left[ 4 \xi \tan (\xi) + 2 \xi + 5 \right]}{\mu (\delta^2 - 1) \left[ 8 + 16 \tan (\xi) \right]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.40)

Family 8: We attain the results for \( p = [1 - i, -1 - i, -1, 1] \) and \( q = [i, -i, i, -i] \), and thus we obtain
\[\Xi(\xi) = \frac{- \sin (\xi) + \cos (\xi)}{\sin (\xi)}. \] (3.41)

Case 1:
\[\alpha = \alpha, \quad \beta = \frac{i\sqrt{2\alpha^2\delta^4 + (A_0^2\lambda - 2\alpha^2 + 4\gamma)\delta^2 + A_0^2\lambda - 4\gamma}}{2\sqrt{\delta^2 - 1}}, \]
\[\mu = \frac{A_0\sqrt{\lambda}}{2\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = 2 A_0.\]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[\Upsilon_{14}(\xi) = \frac{2 A_0 \sin (\xi) \cos (\xi) + A_0}{2 \left( \cos (\xi) \right)^2 - 1}, \]
\[\Upsilon_{14}(\xi) = \frac{-2 i \lambda A_0 \left[ \xi \tan (\xi) - \xi + 2 \right]}{\mu (\delta^2 - 1) \left[ \tan (\xi) - 1 \right]}. \] (3.42)

Hence, the following exact solution has been reached for Eq. (1.1):
\[q_{14}(x, y, t) = \left( \frac{2 A_0 \sin (\xi) \cos (\xi) + A_0}{2 \left( \cos (\xi) \right)^2 - 1} \right) e^{i(\alpha x + \beta y + \gamma t)}, \]
\[\phi_{14}(x, y, t) = \left( \frac{-2 i \lambda A_0 \left[ \xi \tan (\xi) - \xi + 2 \right]}{\mu (\delta^2 - 1) \left[ \tan (\xi) - 1 \right]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.43)

Family 9: We attain the results for \( p = [-3, -1, 1, 1] \) and \( q = [1, -1, 1, -1] \), and thus we have
\[\Xi(\xi) = \frac{- \sinh (\xi) - 2 \cosh (\xi)}{\cosh (\xi)}. \] (3.44)

Case 1:
\[\alpha = \alpha, \quad \beta = \frac{i2\sqrt{2\alpha^2\delta^4 + (A_0^2\lambda - 2\alpha^2 + 4\gamma)\delta^2 + A_0^2\lambda - 4\gamma}}{2\sqrt{\delta^2 - 1}}, \]
\[\mu = \frac{A_0\sqrt{\lambda}}{2\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = 3/2 A_0.\]
We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ W_{15}(\xi) = \frac{A_0 (2 \sinh(\xi) + \cosh(\xi))}{4 \cosh(\xi) + 2 \sinh(\xi)}, \]

\[ Y_{15}(\xi) = \frac{2i\lambda \left(4 \cosh(\xi) \ln (\cosh(\xi) - 1 + \sinh(\xi)) + 2 \sinh(\xi) \ln (\cosh(\xi) - 1 + \sinh(\xi))\right) A_0^2}{\mu \left(\delta^2 - 1\right) (16 \cosh(\xi) + 8 \sinh(\xi))} + \frac{2i\lambda \left(-4 \cosh(\xi) \ln (\cosh(\xi) - 1 - \sinh(\xi)) - 2 \sinh(\xi) \ln (\cosh(\xi) - 1 - \sinh(\xi)) - 3 \sinh(\xi)\right) A_0^2}{\mu \left(\delta^2 - 1\right) (16 \cosh(\xi) + 8 \sinh(\xi))}. \] (3.45)

Hence, the following exact solution has been reached for Eq. (1.1):

\[ q_{15}(x, y, t) = \left(\frac{A_0 [2 \sinh(\xi) + \cosh(\xi)]}{4 \cosh(\xi) + 2 \sinh(\xi)}\right) e^{i(\alpha x + \beta y + \gamma t)}, \]

\[ \phi_{15}(x, y, t) = Y_{15}(\xi) e^{i(\alpha x + \beta y + \gamma t)}. \] (3.46)

**Family 10:** We attain the results for

\[ p = [-2 - i, -2 + i, 1, 1] \] and \[ q = [i, -i, i, -i], \] and thus one has

\[ \Xi(\xi) = \frac{\sin(\xi) - 2 \cos(\xi)}{\cos(\xi)}. \] (3.47)

**Case 1:**

\[ \begin{align*}
\alpha &= \alpha, \\
\beta &= \frac{i\sqrt{2} \sqrt{2 \alpha^2 \delta^4 + (-A_0^2 \lambda - 2 \alpha^2 + 4 \gamma) \delta^2 - A_0^2 \lambda - 4 \gamma}}{2 \sqrt{\delta^2 - 1}}, \\
\mu &= \frac{A_0 \sqrt{\lambda}}{2 \sqrt{\delta^2 - 1}}, \\
\delta &= \delta, \\
\gamma &= \gamma, \\
A_0 &= A_0, \\
A_1 &= 0, \\
B_1 &= 5/2 A_0.
\end{align*} \]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[ W_{16}(\xi) = \frac{A_0 (\cos(\xi) + 2 \sin(\xi))}{2 \sin(\xi) - 4 \cos(\xi)}, \]

\[ Y_{16}(\xi) = \frac{-2 i\lambda A_0^2 (\xi \tan(\xi) - 2 \xi + 5)}{\mu \left(\delta^2 - 1\right) (4 \tan(\xi) - 8)}. \] (3.48)

Hence, the following exact solution has been reached for Eq. (1.1):

**Figure 5.** Dynamic behaviours modulus of solutions \(q_{16}(x, y, t)\) (left) and \(\phi_{16}(x, y, t)\) (right) for \(A_0 = 1, \delta = 1.1, \gamma = 0.8, \alpha = 0.9, \lambda = 0.2\), and \(t = 1\).
\[
q_{16}(x, y, t) = \left( \frac{A_0 \cos(\xi) + 2 \sin(\xi)}{2 \sin(\xi) - 4 \cos(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}, \\
\phi_{16}(x, y, t) = \left( \frac{-2i\lambda A_0^2 [\xi \tan(\xi) - 2\xi + 5]}{\mu [\delta^2 - 1] [4 \tan(\xi) - 8]} \right) e^{i(\alpha x + \beta y + \gamma t)}.
\]
(3.49)

Figure 5 shows the dynamic behavior of modulus of solutions \(q_{16}(x, y, t)\) (left) and \(\phi_{16}(x, y, t)\) for \(A_0 = 1, \delta = 1.1, \gamma = 0.8, \alpha = 0.9, \lambda = 0.2, \) and \(t = 1.\)

**Family 11:** We attain the results for \(p = [1 - i, 1 + i, 1, 1]\) and \(q = [i, -i, i, -i]\), and then one results
\[
\Xi(\xi) = \frac{\cos(\xi) + \sin(\xi)}{\cos(\xi)}.
\]
(3.50)

Case 1:
\[
\alpha = \alpha, \quad \beta = \frac{i \sqrt{\alpha^2 \delta^4 + (2 A_1^2 \lambda - \alpha^2 + 2 \gamma) \delta^2 - 2 A_1^2 \lambda - 2 \gamma}}{\sqrt{\delta^2 - 1}}, \quad \mu = \frac{\sqrt{\lambda A_1}}{\sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = -A_1, \quad A_1 = A_1, \quad B_1 = 0,
\]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[
\mathcal{W}_{17}(\xi) = \frac{A_1 \sin(\xi)}{\cos(\xi)}, \\
\mathcal{W}_{17}(\xi) = \frac{2i\lambda A_1^2 [\tan(\xi) - \xi]}{\mu [\delta^2 - 1]}.
\]
(3.51)

Hence, the following exact solution was reached for Eq. (1.1):
\[
q_{17}(x, y, t) = \left( \frac{A_1 \sin(\xi)}{\cos(\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}, \\
\phi_{17}(x, y, t) = \left( \frac{2i\lambda A_1^2 [\tan(\xi) - \xi]}{\mu [\delta^2 - 1]} \right) e^{i(\alpha x + \beta y + \gamma t)}.
\]
(3.52)

Figure 6 shows the dynamic behavior of modulus of solutions \(q_{17}(x, y, t)\) (left) and \(\phi_{17}(x, y, t)\) for \(A_1 = 1, \delta = 1.5, \gamma = 0.5, \alpha = 0.1, \lambda = 0.2, \) and \(t = 1.\)
Figure 7. Dynamic behaviours modulus of solutions $q_{18}$ $(x, y, t)$ (left) and $\phi_{18}$ $(x, y, t)$ (right) for $A_0 = 1, \delta = 5, \gamma = 0.1, \alpha = 1.5, \lambda = 0.3$, and $t = 1$.

**Family 12:**
We attain the results for $p = [-3, -2, 1, 1]$ and $q = [0, 1, 0, 1]$, and we get

$$\Xi (\xi) = \frac{-3 - 2 e^\xi}{1 + e^\xi}.$$  \hspace{1cm} (3.53)

**Case 1:**

$$\alpha = \alpha, \quad \beta = \frac{i/5 \sqrt{25 \alpha^2 \delta^4 + (2 A_0^2 \lambda - 25 \alpha^2 + 50 \gamma) \delta^2 + 2 A_0^2 \lambda - 50 \gamma}}{\sqrt{\delta^2 - 1}},$$

$$\mu = \frac{2 A_0 \sqrt{\lambda}}{5 \sqrt{\delta^2 - 1}}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = \frac{12 A_0}{5},$$

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

$$\mathcal{W}_{18} (\xi) = -\frac{A_0 (2 e^\xi - 3)}{15 + 10 e^\xi},$$

$$\mathcal{Y}_{18} (\xi) = \frac{2 i \lambda A_0^2 (2 \xi e^\xi + 3 \xi + 12)}{\mu (\delta^2 - 1) (75 + 50 e^\xi)}.$$  \hspace{1cm} (3.54)

Hence, the following exact solution has been reached for equation (1.1):

$$q_{18} (x, y, t) = \left( -\frac{A_0 \left[ 2 e^\xi - 3 \right]}{15 + 10 e^\xi} \right) e^{i(\alpha x + \beta y + \gamma t)},$$

$$\phi_{18} (x, y, t) = \left( \frac{2 i \lambda A_0^2 \left[ 2 \xi e^\xi + 3 \xi + 12 \right]}{\mu \left[ \delta^2 - 1 \right] (75 + 50 e^\xi)} \right) e^{i(\alpha x + \beta y + \gamma t)}.$$  \hspace{1cm} (3.55)

Figure 7 shows the dynamic behavior of modulus of solutions $q_{18} (x, y, t)$ (left) and $\phi_{18} (x, y, t)$ for $A_0 = 1, \delta = 5, \gamma = 0.1, \alpha = 1.5, \lambda = 0.3$, and $t = 1$.

**Family 13:**
We attain the results for $p = [-1, -2, 1, 1]$ and $q = [1, 0, 1, 0]$, and one finds

$$\Xi (\xi) = \frac{-e^\xi - 2}{e^\xi + 1}.$$  \hspace{1cm} (3.56)
Case 1:
\[
\alpha = \alpha, \quad \beta = \frac{i \sqrt{9 \alpha^2 \delta^4 + (2 A_0^2 \lambda - 9 \alpha^2 + 18 \gamma) \delta^2 + 2 A_0^2 \lambda - 18 \gamma}}{3 \sqrt{\delta^2 - 1} \delta^2},
\]
\[
\mu = \frac{2 A_0 \sqrt{\lambda}}{3 \sqrt{\delta^2 - 1} \delta}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = 4/3 A_0.
\]
We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[
\mathcal{W}_19(\xi) = -\frac{A_0 (e^\xi - 2)}{3 e^\xi + 6},
\]
\[
\mathcal{Y}'_19(\xi) = \frac{2 i \lambda A_0^2 (\xi e^\xi + 2 \xi + 8)}{\mu (\delta^2 - 1) (9 e^\xi + 18)}. \tag{3.57}
\]
Hence, the following exact solution has been reached for equation (1.1):
\[
q_{19}(x, y, t) = \left( -\frac{A_0 [e^\xi - 2]}{3 e^\xi + 6} \right) e^{i(\alpha x + \beta y + \gamma t)},
\]
\[
\phi_{19}(x, y, t) = \left( \frac{2 i \lambda A_0^2 [\xi e^\xi + 2 \xi + 8]}{\mu [\delta^2 - 1] [9 e^\xi + 18]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \tag{3.58}
\]
Family 14: We attain the results for \( p = [2, 1, 1, 1] \) and \( q = [1, 0, 1, 0] \), and then we obtain
\[
\Xi(\xi) = \frac{2 e^\xi + 1}{e^\xi + 1}. \tag{3.59}
\]
Case 1:
\[
\alpha = \alpha, \quad \beta = \frac{i \sqrt{9 \alpha^2 \delta^4 + (2 A_0^2 \lambda - 9 \alpha^2 + 18 \gamma) \delta^2 + 2 A_0^2 \lambda - 18 \gamma}}{3 \sqrt{\delta^2 - 1} \delta^2},
\]
\[
\mu = \frac{2 A_0 \sqrt{\lambda}}{3 \sqrt{\delta^2 - 1} \delta}, \quad \delta = \delta, \quad \gamma = \gamma, \quad A_0 = A_0, \quad A_1 = 0, \quad B_1 = -4/3 A_0.
\]
We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)
\[
\mathcal{W}_{20}(\xi) = \frac{A_0 (2 e^\xi - 1)}{6 e^\xi + 3},
\]
\[
\mathcal{Y}'_{20}(\xi) = \frac{2 i \lambda A_0^2 (2 \xi e^\xi + \xi + 4)}{\mu [\delta^2 - 1] (18 e^\xi + 9)}. \tag{3.60}
\]
Hence, the following exact solution has been reached for equation (1.1):
\[
q_{20}(x, y, t) = \left( \frac{A_0 [2 e^\xi - 1]}{6 e^\xi + 3} \right) e^{i(\alpha x + \beta y + \gamma t)},
\]
\[
\phi_{20}(x, y, t) = \left( \frac{2 i \lambda A_0^2 [2 \xi e^\xi + \xi + 4]}{\mu [\delta^2 - 1] [18 e^\xi + 9]} \right) e^{i(\alpha x + \beta y + \gamma t)}. \tag{3.61}
\]
Family 15: We attain the results for \( p = [-1, 0, 1, 1] \) and \( q = [0, 0, 1, 0] \), and then we find
\[
\Xi(\xi) = \frac{1}{1 + e^\xi}. \tag{3.62}
\]
Case 1:

\[
\alpha = \frac{i \sqrt{\beta^2 \delta^6 - \beta^2 \delta^4 + (2 A_0^2 \lambda + 2 \gamma) \delta^2 + 2 A_0^2 \lambda - 2 \gamma}}{\sqrt{\delta^2 - 1}} , \quad \beta = \beta , \\
\mu = \frac{2 A_0 \sqrt{\lambda}}{\delta \sqrt{\delta^2 - 1}} , \quad \delta = \delta , \quad \gamma = \gamma , \quad A_0 = A_0 , \quad A_1 = 2 A_0 , \quad B_1 = 0 .
\]

We have replaced the above values with Eqs. (3.7) and (3.8) together with (3.5)

\[
\mathcal{Z}_{21}(\xi) = \frac{A_0 (e^\xi - 1)}{1 + e^\xi} , \\
\mathcal{V}_{21}(\xi) = \frac{2 i \lambda A_0^2 (\xi e^\xi + \xi + 4)}{\mu (\delta^2 - 1) (1 + e^\xi)} .
\]

Hence, the following exact solution has been reached for equation (1.1):

\[
q_{21}(x, y, t) = \left( \frac{A_0 [e^\xi - 1]}{1 + e^\xi} \right) e^{i(\alpha x + \beta y + \gamma t)} , \\
\phi_{21}(x, y, t) = \left( \frac{2 i \lambda A_0^2 [\xi e^\xi + \xi + 4]}{\mu [\delta^2 - 1] [1 + e^\xi]} \right) e^{i(\alpha x + \beta y + \gamma t)} .
\]

Figure 8 shows the dynamic behavior of modulus of solutions \(q_{21}(x, y, t)\) (left) and \(\phi_{21}(x, y, t)\) (right) for \(A_0 = 1, \delta = 2, \gamma = 0.9, \beta = 0.5, \lambda = 0.7,\) and \(t = 1.\)

Remark 1 In each of the above cases, we take \(\xi = i \mu \left( x + y - (\alpha \delta^2 + \beta \delta^4) t \right) .\)

4. Conclusion

Partial differential equations have many applications in modeling practical problems in our lives. This importance has created additional motivation for researchers to develop new and efficient methods. Some of these techniques enable us to achieve exact solutions to such problems. However, determining such solutions is impossible or very difficult for some categories of equations. The method used in this paper, called the GERFM, is a powerful technique to determine the exact solutions to different types of PDEs. In this survey, the method has been utilized to solve the Davey-Stewartson equation. It was shown that the method is a suitable technique to solve the Davey-Stewartson equation with this study. The results are quite reliable for solving this problem. Further, we believe that the presented methods and results in this paper are valuable to all researchers in the field of mathematical physics. Therefore, GERFM offers an excellent opportunity for future research studies on related topics of the research. This emphasizes the power of the method used in providing exact solutions to various real-world applied models.


Rev. Mex. Fis. 67 060702


