Optical solitons to fractal nonlinear Schrödinger equation with non-Kerr law nonlinearity in magneto-optic waveguides

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This paper introduces the fractal model of the nonlinear Schrödinger equation with quadratic-cubic nonlinearity in magneto-optic waveguides, having plenty of applications in fiber optics. He's variational approach and Painlevé technique are used to obtain bright and kink soliton solutions of the governing system. The constraint conditions that ensure the existence of these solitons arise naturally from the model's solution structure. To quantify the behavior of different solutions, the effect of the fractal parameter is studied. These techniques may be very useful and efficient tools for solving nonlinear fractal differential equations that emerge in mathematical physics.

Keywords: Variational principle; Painlevé approach; nonlinear Schrödinger equation; solitons; quadratic-cubic nonlinearity.

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1. Introduction

Optical solitons are the basic component of fiber-optic telecommunication technology. Several models have been developed to investigate this mechanism, including the nonlinear Schrödinger's equation (NLSE). There are different forms of waveguides such as optical metamaterials, optical fibers and photonic crystal fibers, among others, that send a large amount of data across intercontinental distances [1, 2]. This paper considers a particular type of optical waveguides with an artificially generated magnetic field, known as magneto-optic waveguides. The benefit of such waveguides is that they reduce the soliton clutter effect ensuring smooth information propagation [3–5].

In the field of nonlinear science, the NLSE is a well-known model that can be used in a variety of physical instances, including nonlinear optics, nuclear physics, quantum mechanics, condensed matter physics, and plasma physics, etc. [6–12]. The fractal model is gaining significance in nonlinear evolution equations (NLEE) of physics and mathematics for its many attractive properties that traditional systems fail to provide. One form of fractal NLEE is coupled NLSE in nonlinear optics. This system can handle soliton solutions having applications in optical communications, logic gate devices, ultrafast soliton switches, and soliton lasers [13].

The optical soliton solutions of NLSE with various forms of nonlinearity possess a significant part in resolving real-world problems. In optics, a soliton is the wave that is unaltered during propagation due to a delicate balance between nonlinear and dispersive effects in the medium [14–16]. Solutions for various NLSE have been sought to investigate nonlinear phenomena with the solitons being either bright or dark depending on the details provided by the governing NLSE [17–20]. Researchers have been studying these solitons with quadratic-cubic nonlinearity since this form of nonlinearity was first suggested in 2011 [21]. The study of soliton dynamics in magneto-optic waveguides is crucial. Bright solitons can be formed from a state of attraction to a state of separation from each other by magneto-optic components. This allows us to manage the so-called soliton clutter. This article explores the soliton solutions of coupled NLSE with quadratic-cubic nonlinearity by implementing He's semi-inverse variational method and the Painlevé approach that may be conducive for engineers and physicist to physically comprehend this model.

The semi-inverse approach is an effective tool for finding different variational principles of physical problems [22, 23]. He suggested the semi-inverse variational theorem as an efficient and direct algebraic approach for computing soliton solutions [24]. Many authors went on to expand this approach and contributed to the analysis of fractal models in distinct fields of science [25–28]. Another method adopted here to obtain soliton solutions of the governing model is the Painlevé approach, which is the generalization of well-known algorithms: simplest equation method, tanh-function method, and the G'/G-expansion method [29]. This is a powerful and reliable scheme to find exact solutions of NLSE by avoiding the meromorphic solutions.

The article is organized as: Section 2 is devoted to the mathematical description. Section 3 covers the study of soliton solutions of the FLE along with their graphics. Discussion of the results is presented in Sec. 4 and 5 gives the conclusion.

1.1. Governing system

The coupled model of NLSE with quadratic-cubic nonlinearity in magneto-optic waveguides is given as:

$$iu_t + l_1 u_{xx} + (m_1|u| + n_1|u|^2 + p_1|v| + s_1|v|^2)u = R_1 v + i(\beta_1 u_x + \mu_1(|u|u)_x + \upsilon_1(|u|)_x u + \eta_1|u|u_x) \quad (1)$$

$$iv_t + l_2 v_{xx} + (m_2|v| + n_2|v|^2 + p_2|u| + s_2|u|^2)v = R_2 u + i(\beta_2 v_x + \mu_2(|v|v)_x + \upsilon_2(|v|)_x v + \eta_2|v|v_x) \quad (2)$$

where $l_i, m_i, n_i, p_i, s_i, R_i, \beta_i, \mu_i, \upsilon_i$ and η_i for $i = 1, 2$ are constants, while $i = \sqrt{-1}$. In Eqs. (1) and (2), t and x are independent and represent the temporal and spatial variables, respectively, while the dependent variables are $u(x,t)$ and $v(x,t)$ which show the complex valued soliton profiles. The constants l_i denote chromatic dispersion, whereas m_i and n_i are the self-phase modulation coefficients. The cross-phase modulation is expressed by the parameters p_i and s_i. On the right hand side of Eqs. (1) and (2), inter-modal dispersion and the magneto-optic parameter are denoted by the coefficients β_i and R_i, respectively. μ_i stands for self-steepening term and the coefficients of nonlinear dispersion are symbolized by υ_i and η_i.

2. Mathematical analysis

To continue, the initial assumptions are as follows:

$$u(x,t) = F_1(\xi)e^{i\chi(x,t)}, \qquad v(x,t) = F_2(\xi)e^{i\chi(x,t)}, \quad (3)$$

where

$$\xi = x - at, \qquad \chi(x,t) = -hx + \nu t + \eta_0. \quad (4)$$

Here a, h, ν and η_0 are speed, frequency, wave number, and phase constant of the wave, respectively. $F_i(x,t)$ for $i = 1, 2$ denote the amplitude of the pulses, whereas $\chi(x,t)$ represents the phase component of the pulses.

Substituting Eqs. (3) and (4) into Eqs. (1) and (2). So, the real parts become

$$l_1 F_1'' - (\nu + l_1 h^2 + h\beta_1)F_1 - R_1 F_2 + (m_1 - h\mu_1 - h\eta_1)F_1^2 + n_1 F_1^3 + s_1 F_2^2 F_1 + p_1 F_2 F_1 = 0, \quad (5)$$

$$l_2 F_2'' - (\nu + l_2 h^2 + h\beta_2)F_2 - R_2 F_1 + (m_2 - h\mu_2 - h\eta_2)F_2^2 + n_2 F_2^3 + s_2 F_1^2 F_2 + p_2 F_1 F_2 = 0, \quad (6)$$

while the imaginary parts are given as:

$$(a + 2l_1 h + \beta_1)F_1' + (2\mu_1 + \upsilon_1 + \eta_1)F_1 F_1' = 0, \quad (7)$$

$$(a + 2l_2 h + \beta_2)F_2' + (2\mu_2 + \upsilon_2 + \eta_2)F_2 F_2' = 0. \quad (8)$$

Integrating Eqs. (7) and (8) and setting the integration constants to zero yields

$$(a + 2l_1 h + \beta_1)F_1 + \frac{1}{2}(2\mu_1 + \upsilon_1 + \eta_1)F_1^2 = 0, \quad (9)$$

$$(a + 2l_2 h + \beta_2)F_2 + \frac{1}{2}(2\mu_2 + \upsilon_2 + \eta_2)F_2^2 = 0. \quad (10)$$

Equating the coefficients of linearly independent functions to zero in Eqs. (9) and (10), provides the constraints:

$$-(2l_1 h + \beta_1) = a, \quad (11)$$

$$2\mu_1 + \upsilon_1 + \eta_1 = 0, \quad (12)$$

and

$$-(2l_2 h + \beta_2) = a, \quad (13)$$

$$2\mu_2 + \upsilon_2 + \eta_2 = 0. \quad (14)$$

It can be deduced from Eqs. (11) and (13) that the soliton frequency is

$$h = \frac{\beta_2 - \beta_1}{2(l_1 - l_2)}, \quad (15)$$

provided $l_1 \neq l_2$ and $\beta_1 \neq \beta_2$. Furthermore, we set

$$F_1(\xi) = \varepsilon F_2(\xi), \quad (16)$$

where $\varepsilon \neq 0, 1$. As a consequence, Eqs. (5) and (6) become

$$l_1 F_1'' - [\nu + l_1 h^2 + h\beta_1 + R_1\varepsilon]F_1 + [m_1 - h(\mu_1 + \eta_1) + \varepsilon p_1]F_1^2 + (n_1 + s_1\varepsilon^2)F_1^3 = 0, \quad (17)$$

$$l_2 \varepsilon F_1'' - [\varepsilon(\nu + l_2 h^2 + h\beta_2) + R_2]F_1 + [\varepsilon^2(m_2 - h(\mu_2 + \eta_2)) + \varepsilon p_2]F_1^2 + (\varepsilon^3 n_2 + s_2\varepsilon)F_1^3 = 0. \quad (18)$$

Equations (17) and (18) are equivalent by taking the constraint conditions:

$$l_1 = \varepsilon l_2, \quad (19)$$

$$n_1 + \varepsilon^2 s_1 = \varepsilon^3 n_2 + \varepsilon s_2, \quad (20)$$

$$\nu + l_1 h^2 + h\beta_1 + R_1\varepsilon = \varepsilon(\nu + l_2 h^2 + h\beta_2) + R_2, \quad (21)$$

$$m_1 - h(\mu_1 + \eta_1) + \varepsilon p_1 = \varepsilon^2(m_2 - h(\mu_2 + \eta_2)) + \varepsilon p_2. \quad (22)$$

From the constraint Eq. (21), the wave number ν appears to be

$$\nu = \frac{h^2(\varepsilon l_2 - l_1) + h(\varepsilon\beta_2 - \beta_1) + (R_2 - \varepsilon R_2)}{1 - \varepsilon}. \quad (23)$$

Next, Eq. (17) can be rewritten as

$$F_1'' + \delta_1 F_1 + \delta_2 F_1^2 + \delta_3 F_1^3 = 0, \quad (24)$$

where

$$\delta_1 = -\frac{\nu + l_1 h^2 + h\beta_1 + R_1\varepsilon}{l_1}, \quad \delta_2 = \frac{m_1 - h(\mu_1 + \eta_1) + \varepsilon p_1}{l_1}, \quad \delta_3 = \frac{n_1 + \varepsilon^2 s_1}{l_1}, \quad (25)$$

provided $l_1 \neq 0$.

In the view of [30], the fractal form of Eq. (1) and Eq. (2) can be written as:

$$\frac{d}{d\xi^\alpha}\left(\frac{dF_1}{d\xi^\alpha}\right) + \delta_1 F_1 + \delta_2 F_1^2 + \delta_3 F_1^3 = 0, \qquad (26)$$

where α is the fractal dimension value and $dF_1/d\xi^\alpha$ is the fractal derivative represented as follows:

$$\frac{dF_1}{d\xi^\alpha} = \Gamma(1+\alpha) \lim_{\xi-\xi_0 \to \Delta\xi, \Delta\xi \neq 0} \frac{F_1(\xi) - F_1(\xi_0)}{(\xi - \xi_0)^\alpha}. \qquad (27)$$

3. Extraction of solitons by proposed methods

3.1. Semi-inverse method

By He's variational principle [22] we can derive the following variational formulation for Eq. (26) as:

$$J = \int L d\xi = \int (K - E) d\xi$$

$$= \int_0^\infty \left(\frac{1}{2}\left(\frac{dF_1}{d\xi^\alpha}\right)^2 - \delta_1 \frac{F_1^2}{2} - \delta_2 \frac{F_1^3}{3} - \delta_3 \frac{F_1^4}{4} \right) d\xi^\alpha, \qquad (28)$$

where

$$L = \frac{1}{2}\left(\frac{dF_1}{d\xi^\alpha}\right) - \delta_1 \frac{F_1^2}{2} - \delta_2 \frac{F_1^3}{3} - \delta_3 \frac{F_1^4}{4}$$

be the Lagrangian, $K = 1/2\,(dF_1/d\xi^\alpha)$ is the kinetic energy and

$$E = \delta_1 \frac{F_1^2}{2} + \delta_2 \frac{F_1^3}{3} + \delta_3 \frac{F_1^4}{4}$$

is the potential energy.

Using the two scale transformation $b = \xi^\alpha$, Eq. (28) takes the form

$$J = \int_0^\infty \left(\frac{1}{2}\left(\frac{dF_1}{db}\right)^2 - \delta_1 \frac{F_1^2}{2} - \delta_2 \frac{F_1^3}{3} - \delta_3 \frac{F_1^4}{4} \right) db. \qquad (29)$$

Using the Ritz's approach, consider the solitary wave solution as follows

$$F_1 = X \operatorname{sech}(Yb), \qquad (30)$$

where unknown constants X and Y are to be computed further. Substituting Eq. (30) into Eq. (29) gives

$$J = \frac{1}{6}X^2 Y - \frac{\delta_1 X^2}{2Y} - \frac{\delta_2 \pi X^3}{12Y} - \frac{\delta_3 X^4}{6Y}. \qquad (31)$$

Taking the corresponding derivatives of J with respect to X and Y gives

$$\frac{\partial J}{\partial X} = \frac{1}{3}XY - \frac{\delta_1 X}{Y} - \frac{\delta_2 \pi X^2}{4Y} - \frac{2}{3}\frac{\delta_3 X^3}{Y} = 0, \qquad (32)$$

$$\frac{\partial J}{\partial Y} = \frac{1}{6}X^2 + \frac{1}{2}\frac{\delta_1 X^2}{Y^2} + \frac{\delta_2 \pi X^3}{12Y^2} + \frac{1}{6}\frac{\delta_3 X^4}{Y^2} = 0. \qquad (33)$$

From Eq. (32) and Eq. (33) we have

$$X = \frac{-5\pi\delta_2 \pm \sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{24\delta_3}, \qquad (34)$$

$$Y = \pm\frac{1}{2}\sqrt{\frac{5\pi^2\delta_2^2 - \pi\delta_2\sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{72\delta_3} - 4\delta_1}\,. \qquad (35)$$

Equation (30) becomes

$$F_1 = \frac{-5\pi\delta_2 \pm \sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{24\delta_3} \operatorname{sech}\left(\pm\frac{1}{2}\sqrt{\frac{5\pi^2\delta_2^2 - \pi\delta_2\sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{72\delta_3} - 4\delta_1}\; b \right). \qquad (36)$$

The solitary wave solution for Eq. (26) is

$$u(x,t) = \frac{-5\pi\delta_2 \pm \sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{24\delta_3}\, e^{\iota(hx+\nu t+\eta_0)} \operatorname{sech}\left[\pm\frac{1}{2}\sqrt{\frac{5\pi^2\delta_2^2 - \pi\delta_2\sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{72\delta_3} - 4\delta_1}\; (x-at)^\alpha \right], \qquad (37)$$

$$v(x,t) = \varepsilon\left(\frac{-5\pi\delta_2 \pm \sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{24\delta_3} \right) e^{\iota(hx+\nu t+\eta_0)}$$

$$\times \operatorname{sech}\left[\pm\frac{1}{2}\sqrt{\frac{5\pi^2\delta_2^2 - \pi\delta_2\sqrt{25\pi^2\delta_2^2 - 1152\delta_1\delta_3}}{72\delta_3} - 4\delta_1}(x-at)^\alpha \right], \qquad (38)$$

provided $\varepsilon \neq 0, 1$.

Now, consider another possible soliton solution, this time of the form

$$F_1 = W \operatorname{sech}^4(Zb), \qquad (39)$$

where unknown constants W and Z are to be calculated later. Plugging Eq. (39) into Eq. (29) yields

$$J = \frac{128}{315} W^2 Z - \frac{8}{35}\frac{\delta_1 W^2}{Z} - \frac{256}{2079}\frac{\delta_2 W^3}{Z} - \frac{512}{6435}\frac{\delta_3 W^4}{Z}. \tag{40}$$

Taking the corresponding derivatives of J with respect to W and Z leads to

$$\frac{\partial J}{\partial W} = \frac{256}{315} WZ - \frac{16}{35}\frac{\delta_1 W}{Z} - \frac{256}{693}\frac{\delta_2 W^2}{Z} - \frac{2048}{6435}\frac{\delta_3 W^3}{Z} = 0\ , \tag{41}$$

$$\frac{\partial J}{\partial Z} = \frac{128}{315} W^2 + \frac{8}{35}\frac{\delta_1 W^2}{Z^2} + \frac{256}{2079}\frac{\delta_2 W^3}{Z^2} + \frac{512}{6435}\frac{\delta_3 W^4}{Z^2} = 0\ . \tag{42}$$

From Eqs. (41) and (42) we have

$$W = \frac{-325\delta_2 \pm \sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}}{504\delta_3}\ , \tag{43}$$

$$Z = \pm\frac{1}{12}\sqrt{\frac{10\left(325\delta_2^2 - \delta_2\sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}\right)}{693\delta_3} - 27\delta_1}\ , \tag{44}$$

with the help of which Eq. (39) takes the form

$$F_1 = \frac{-325\delta_2 \pm \sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}}{504\delta_3}\,\mathrm{sech}\left(\pm\frac{1}{12}\sqrt{\frac{10\left(325\delta_2^2 - \delta_2\sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}\right)}{693\delta_3} - 27\delta_1}\ b\right). \tag{45}$$

The solitary wave solution for Eq. (26) is given as:

$$u(x,t) = \frac{-325\delta_2 \pm \sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}}{504\delta_3}\, e^{\iota(hx+\nu t+\eta_0)} \times \mathrm{sech}^4\left[\pm\frac{1}{12}\sqrt{\frac{10\left(325\delta_2^2 - \delta_2\sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}\right)}{693\delta_3} - 27\delta_1}(x-at)^\alpha\right], \tag{46}$$

$$v(x,t) = \varepsilon\left(\frac{-325\delta_2 \pm \sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}}{504\delta_3}\right) e^{\iota(hx+\nu t+\eta_0)} \times \mathrm{sech}^4\left[\pm\frac{1}{12}\sqrt{\frac{10\left(325\delta_2^2 - \delta_2\sqrt{105625\delta_2^2 - 486486\delta_1\delta_3}\right)}{693\delta_3} - 27\delta_1}(x-at)^\alpha\right], \tag{47}$$

provided $\varepsilon \neq 0, 1$.

3.2. Painlevé Approach

According to Paul Painlevé, the exact solution of Eq. (26) has the form:

$$F_1(\xi) = e_0 + f(U)e^{-c\xi}, \qquad U = g(\xi) = e_1 - \frac{e^{-c\xi}}{c}, \tag{48}$$

and $f(U)$ in Eq. (48) satisfies $f_U - AU^2 = 0$, which is a Riccati-equation.
The solution to this equation is given as

$$f(U) = \frac{1}{AU + U_0}. \tag{49}$$

Differentiating Eq. (48) with respect to ξ and using Riccati equation give:

$$F_{1\xi} = -ce^{-c\xi}f + Ae^{-2c\xi}f^2,$$
$$F_{1\xi\xi} = c^2e^{-c\xi}f - 3Ace^{-2c\xi}f^2 + 2A^2e^{-3c\xi}f^3,$$
$$F_{1\xi\xi\xi} = -c^3e^{-c\xi}f + 7Ac^2e^{-2c\xi}f^2 - 12A^2ce^{-3c\xi}f^3 + 6A^3e^{-4c\xi}f^4.$$

Substituting F_1, $F_{1\xi}$ and $F_{1\xi\xi}$ in Eq. (26) and comparing the coefficients of like powers of $e^{-c\xi}f(U)$ equal to zero, we obtain the system of equations as:

$$2A^2 + \delta_3 = 0,$$
$$-3Ac + \delta_2 = 0,$$
$$c^2 + \delta_1 = 0, \tag{50}$$

which implies the following four cases:

(i) If $A = \sqrt{-\delta_3/2}$ and $c = \sqrt{-\delta_1}$ then the solution is

$$F_1(\xi) = \frac{e^{\sqrt{-\delta_1}\xi}}{\sqrt{\frac{-\delta_3}{2}U + U_0}}. \tag{51}$$

(ii) If $A = \sqrt{-\delta_3/2}$ and $c = -\sqrt{-\delta_1}$ then the solution is

$$F_1(\xi) = \frac{e^{-\sqrt{-\delta_1}\xi}}{\sqrt{\frac{-\delta_3}{2}U + U_0}}. \tag{52}$$

(iii) If $A = -\sqrt{-\delta_3/2}$ and $c = \sqrt{-\delta_1}$ then the solution is

$$F_1(\xi) = \frac{e^{\sqrt{-\delta_1}\xi}}{-\sqrt{\frac{-\delta_3}{2}U + U_0}}. \tag{53}$$

(iv) If $A = -\sqrt{-\delta_3/2}$ and $c = -\sqrt{-\delta_1}$ then the solution is

$$F_1(\xi) = \frac{e^{-\sqrt{-\delta_1}\xi}}{-\sqrt{\frac{-\delta_3}{2}U + U_0}}. \tag{54}$$

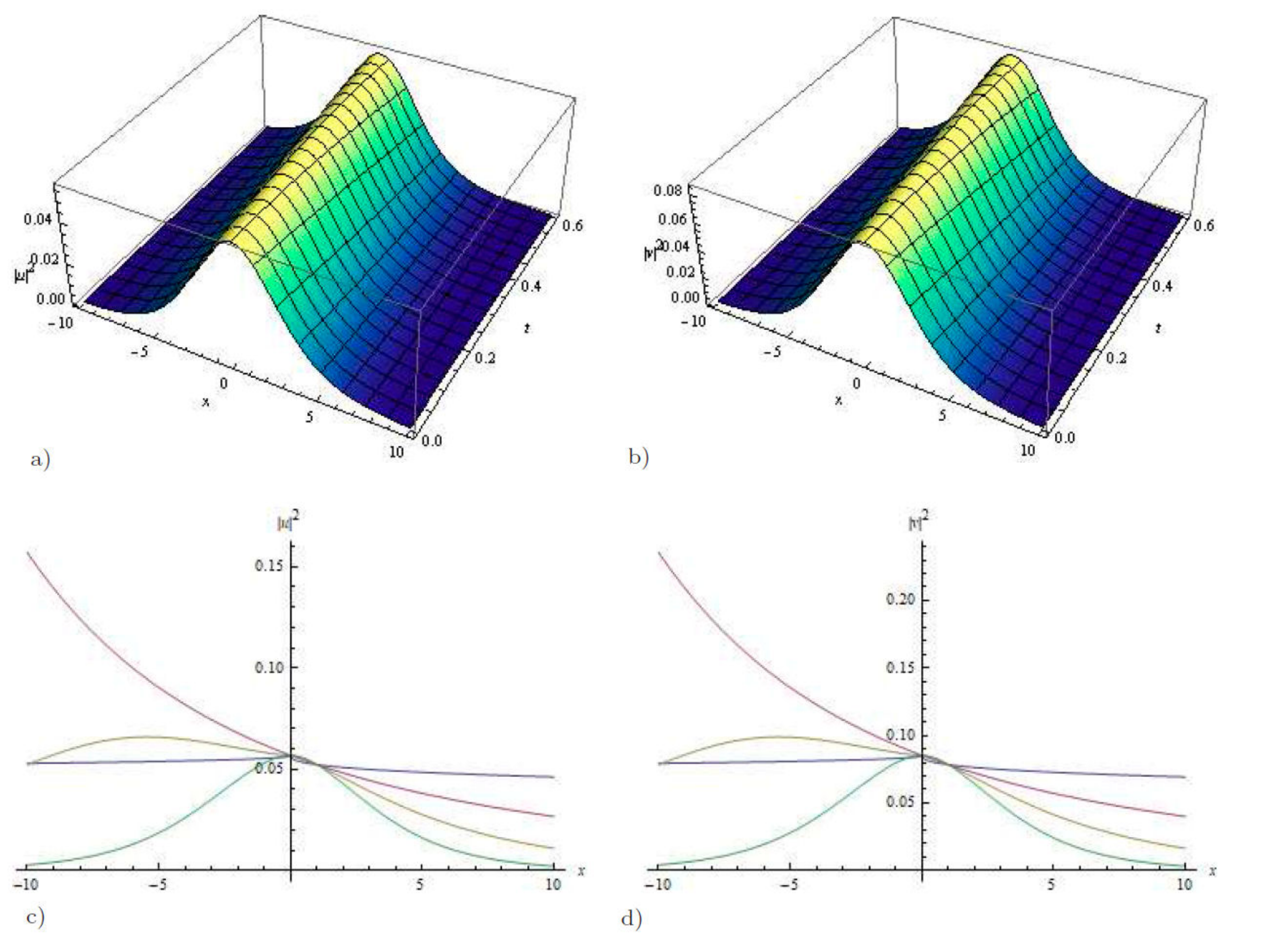

FIGURE 1. The 3D profile of a) Eq. (37) for $|u|^2$ and b) Eq. (38) for $|v|^2$ for the parameters: $\delta_1 = -0.12$, $\delta_2 = 0.55$, $\delta_3 = 1.2$, $\pi = 22/7$, $a = -3$, $\alpha = 1$, $\epsilon = 1.5$ 2D plots of c) $|u|^2$ and d) $|v|^2$ against x at $t = 0$ for fractal dimension value $\alpha = 0.2, 0.5, 0.7, 0.9$.

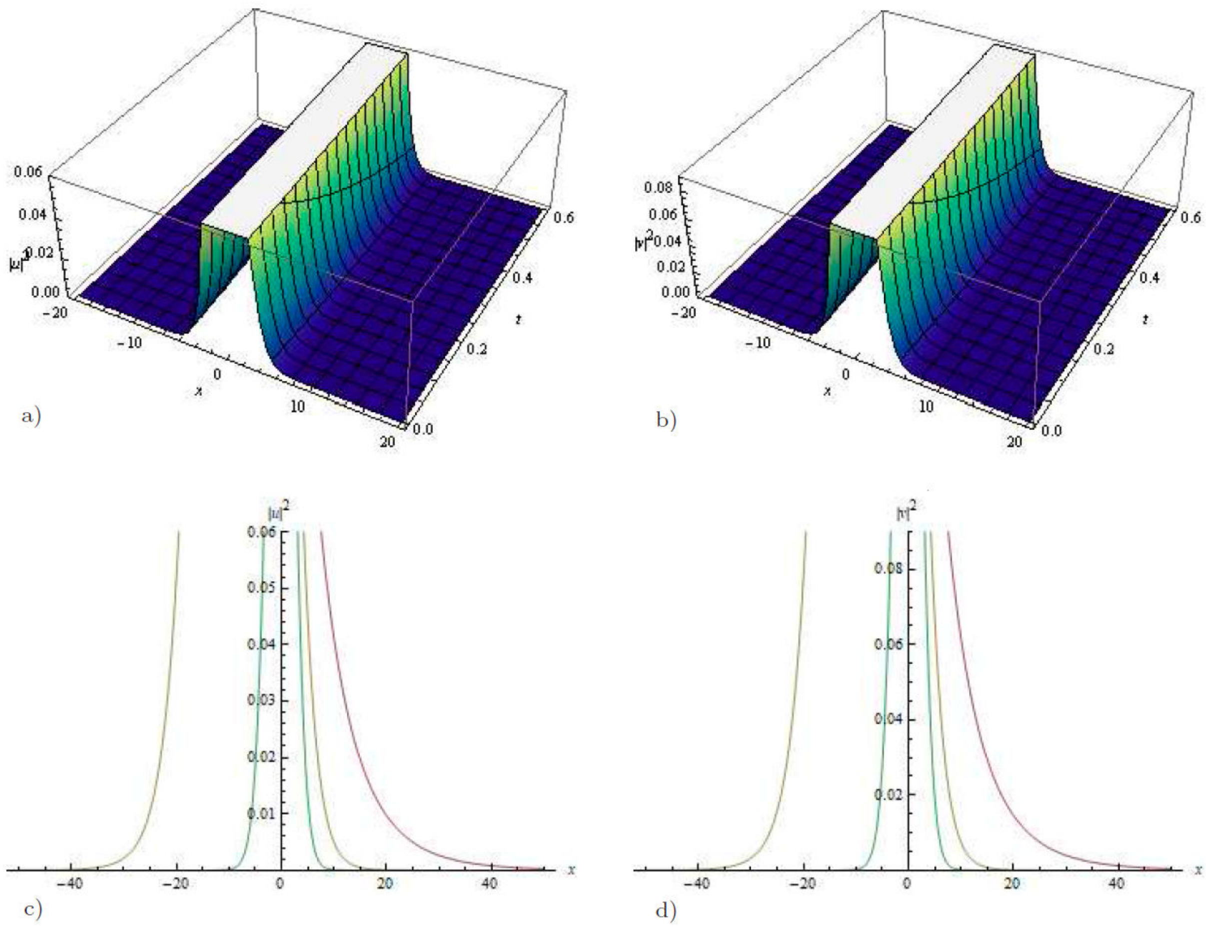

FIGURE 2. The 3D profile of a) Eq. (46) for $|u|^2$ and b) Eq. (47) for $|v|^2$ the parameters: $\delta_1 = -0.3$, $\delta_2 = 0.55$, $\delta_3 = 1.2$, $a = -3$, $\alpha = 1$, $\epsilon = 1.5$, 2D plots of c) $|u|^2$ and d) $|v|^2$ against x at $t = 0$ for fractal dimension value $\alpha = 0.2, 0.5, 0.7, 0.9$.

TABLE I. Comparison of the results following the Painlevé approach, ϕ^6 expansion, and semi-inverse methods.

Methods	NLSE	Fractal NLSE
Painlevé		$F_1(\xi) = \frac{e^{\pm\sqrt{-\delta_1}\xi}}{\pm\sqrt{\frac{-\delta_3}{2}}U+U_0}$
ϕ^6 expansion	$P_1(\varsigma) = \left[\sqrt{\frac{(2n+1)(2n^2\mu_1-h_2)}{3n^2\mu_3}}\left(1+\frac{(n^2\mu_1+h_2)U^2(\varsigma)}{3h_0(fU^2(\varsigma)+g)}\right)\right]^{\frac{1}{2n}}$	
Semi-inverse	$q(x,t) = \frac{A}{D+\cosh[B(x-vt)]}e^{i(-kx+\omega t+\theta_0)}$	$u(x,t) = X\,\mathrm{sech}[Y(x-at)^{\alpha}]e^{\iota(hx+\nu t+\eta_0)}$

4. Results and discussion

The graphical interpretation of the obtained results and the effect of fractal parameter on them are discussed in this section. The semi-inverse variational method yields the bright soliton solutions given by Eqs. (37), (38), (46), and (47). The physical appearance of these solitons is shown in terms of $|u|^2$ and $|v|^2$ by assigning different parameteric values. In Figs.1 and 2, the 2D profiles are provided for fractal dimension values $\alpha = 0.2, 0.5, 0.7, 0.9$ while 3D plots are the standard solitary waves of Eqs. (37), (38), (46) and (47). Kink soliton solutions, *i.e.*, Eq. (51-54) of a given model are obtained following the Painlevé approach. In Figs. 3 and 4, the 3D plots of Eq. (51) and Eq. (52) are shown for distinct fractal dimension values $\alpha = 0.2, 0.5, 0.7, 1$. In Fig. 3, the oscillation spikes on the surface are due to the fractal effect. In Fig. 4, the fractal effect on the solution is shown by the irregularity in the surface. Equations (53) and (54) display the same graphical behavior with just reflection as in Figs. 3 and 4, respectively.

Remark The obtained results are compared to those existing in the literature [3, 23] and found to be novel. Kink solitons of the governing system are obtained following the Painlevé approach, while for the semi-inverse variational method we considered the fractal model of NLSE.

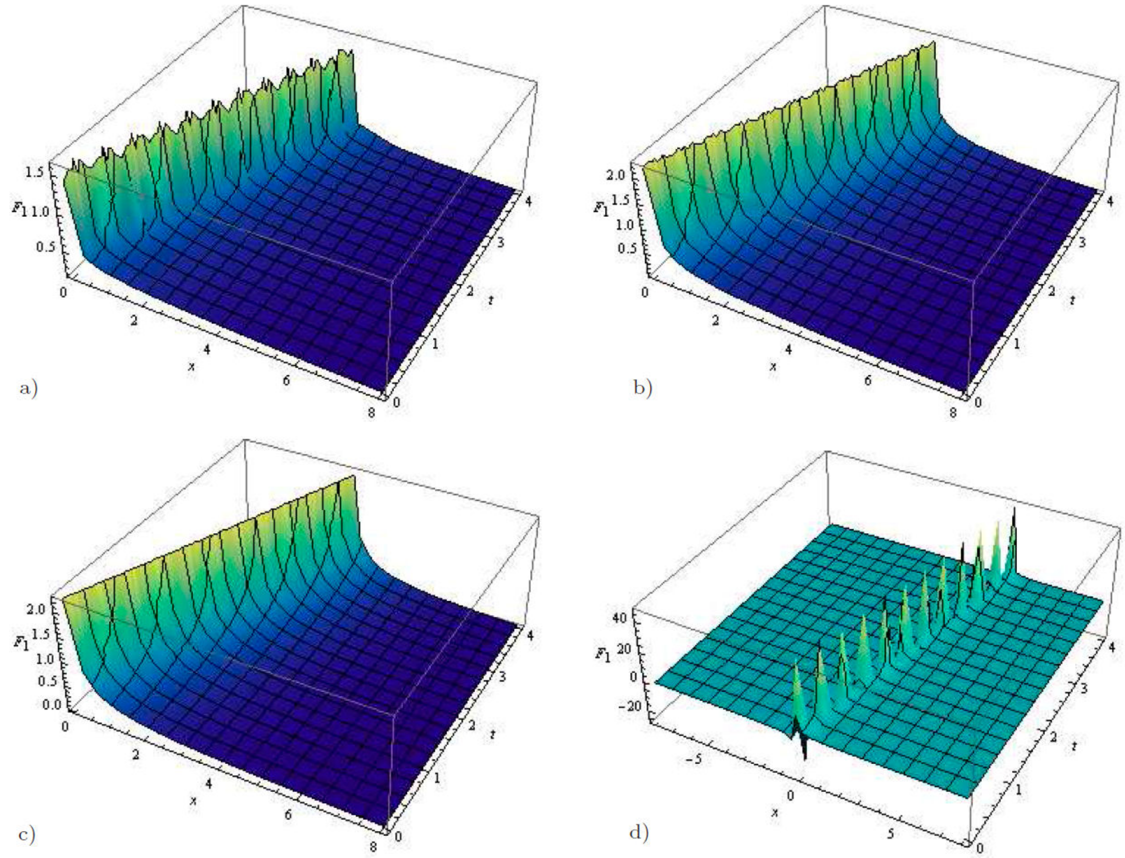

FIGURE 3. Plots of Eq. (51) for the parameters: $\delta_1 = -0.9$, $\delta_3 = -2.1$, $a = 0.8$, $e_1 = 1$, $U_0 = 0.5$ and $\alpha = 0.2, 0.5, 0.7, 1$.

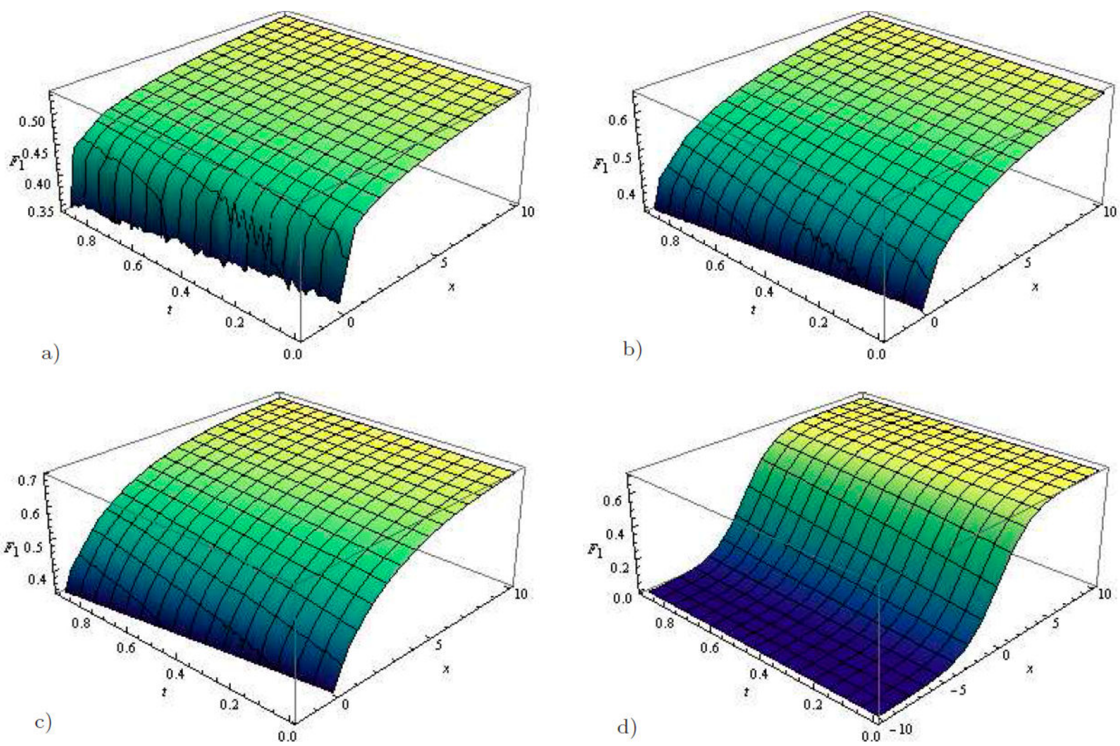

FIGURE 4. Plots of Eq. (52) for the parameters: $\delta_1 = -0.55$, $\delta_3 = -2.2$, $a = -2$, $e_1 = 1$, $U_0 = 0.5$ and $\alpha = 0.2, 0.5, 0.7, 1$.

5. Conclusion

In this article, we have obtained the optical solitons for fractal coupled NLSE in magneto-optic waveguides that have many applications to the propagation of data in optical fibers. Bright and kink solitons are retrieved by the implementation of He's semi-inverse and Painlevé methods. The semi-inverse approach is a fascinating integration scheme to deduce variational principles for various differential models. On the other hand, the Painlevé technique is compelling to find exact solutions of non-integrable nonlinear differential equations by averting their meromorphic solutions. The suitable choice of parameters enables us to discuss the fractal behavior of the system. The outcomes could be helpful in the telecommunication industry to increase transmission system output capability. The impact of fractal dimension value on solutions of the coupled system has been shown graphically, facilitating the understanding of understand the dynamics of the model. The applied methodologies may be conducive to solve a variety of problems arising in engineering and applied physics.

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