# Errata to 'Local available quantum correlations for Bell diagonal states and Markovian decoherence'

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In this brief erratum, we complete the analysis presented previously in [RMF 64 (2018) 662-670] regarding the quantifiers of the classical correlations and the so-called local available quantum correlations for Bell diagonal states. A correction is introduced in their previous expressions once two cases within the optimizations are included.

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In Ref. [1], the analytical results of the correlation quantifiers related to the so-called local available quantum correlations (LAQC) [2] for the family of Bell diagonal states [3] were presented. These states are written in the Bloch representation as

$$\rho^{BD} = \frac{1}{4}\left(\mathbb{1}_4 + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i\right), \qquad (1)$$

where the coefficients $c_i \in [-1,1]$ are such that $\rho^{BD}$ is a well-behaved density matrix (*i. e.* has non-negative eigenvalues) and $\sigma_i$ are the well-known Pauli matrices.

The classical correlations quantifier defined in Ref. [2] can be written in terms of the $R_{ij}(\theta_A, \phi_A, \theta_B, \phi_B)$ coefficients that define the optimal computational basis as

$$\mathcal{C}(\rho_{AB}) = \min_{\substack{\theta_A,\phi_A \\ \theta_B,\phi_B}} \left\{ \sum_{i,j} R_{ij}(\theta_A, \phi_A, \theta_B, \phi_B) \times \log_2\left[\frac{R_{ij}(\theta_A, \phi_A, \theta_B, \phi_B)}{R_i(\theta_A, \phi_A) R_j(\theta_B, \phi_B)}\right]\right\}. \qquad (2)$$

Since Bell diagonal (BD) states have null local Bloch vector, it is straightforward that they are invariant under subsystem exchange **A** $\leftrightarrow$ **B**. Therefore, only two angles, $\theta$ and $\phi$, are necessary, and the coefficients $R_{ij}(\theta, \phi)$ are given by

$$\begin{aligned} R_{ij}(\theta, \phi) = &\frac{1}{4}[1 + (-1)^{i+j} c_3] \\ &+ (-1)^{i+j}\frac{1}{2}\cos^2\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right) \\ &\times\left[(c_1 + c_2) + \cos(2\phi)(c_1 - c_2) - 2c_3\right], \end{aligned} \qquad (3)$$

with $R_{00}(\theta, \phi) = R_{11}(\theta, \phi)$, $R_{01}(\theta, \phi) = R_{10}(\theta, \phi)$, and $R_i = 1/2$.

The minimization in (2) leads to three different cases:

I For $\theta = 0$ and $\phi = 0$:

$$R_{00}(0, 0) = \frac{1}{4}(1 + c_3) \quad R_{01}(0, 0) = \frac{1}{4}(1 - c_3). \qquad (4)$$

II For $\theta = \pi/2$ and $\phi = 0$:

$$\begin{aligned} R_{00}\left(\frac{\pi}{2}, 0\right) &= \frac{1}{4}(1 + c_1) \\ R_{01}\left(\frac{\pi}{2}, 0\right) &= \frac{1}{4}(1 - c_1). \end{aligned} \qquad (5)$$

III For $\theta = \pi/2$ and $\phi = \pi/2$:

$$\begin{aligned} R_{00}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) &= \frac{1}{4}(1 + c_2), \\ R_{01}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) &= \frac{1}{4}(1 - c_2). \end{aligned} \qquad (6)$$

Therefore, by defining

$$c_m \equiv \min\left\{|c_1|, |c_2|, |c_3|\right\}, \qquad (7)$$

we can write the classical correlations quantifier (2) as

$$\mathcal{C}\left(\rho^{BD}\right) = \frac{1 + c_m}{2}\log_2(1 + c_m) + \frac{1 - c_m}{2}\log_2(1 - c_m). \qquad (8)$$

The above expression is the same as Eq. (33) in [1] but now the minimization achieved for $\theta = \pi/2$ and $\phi = 0$ when $c_m = |c_1|$ has been included.

The LAQC quantifier is given by

$$\mathcal{L}(\rho_{AB}) \equiv \max_{\{\Phi_1, \Phi_2\}} I(\Phi_1, \Phi_2), \qquad (9)$$

where

$$I(\Phi_1,\Phi_2) = \sum_{i,j} P(i_A, j_B, \Phi_1, \Phi_2) \times \log_2\left(\frac{P(i_A, j_B, \Phi_1, \Phi_2)}{P(i_A, \Phi_1)P(j_B, \Phi_2)}\right), \tag{10}$$

with $P(i_A, j_B, \Phi_1, \Phi_2)$ the probability distributions associated with the complementary basis [4] of $\rho_{AB}$ written in the optimal computational basis, and $P(i_A, \Phi_1)$ an $P(j_B, \Phi_2)$ are the corresponding marginal probabilities. Contrary to what is stated in [1], the density matrix of BD states does not remain invariant when written in the optimal computational basis. That is only true for Werner [5] and Werner-like states [6, 7].

The density matrix $\tilde{\rho}^{BD}$ and their corresponding $P(i, j, \Phi)$ for each $\theta$ and $\phi$, with $P(0, 0, \Phi) = P(1, 1, \Phi)$, $P(0, 1, \Phi) = P(1, 0, \Phi)$, and $P(i, \Phi) = 1/2$, are the following:

I) For $\theta = 0$ and $\phi = 0$:

$$\tilde{\rho}^{BD}=\frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1+c_3 \end{pmatrix} \tag{11}$$

and

$$P(0,0,\Phi) = \frac{1}{4}\left(1 + \frac{c_1+c_2}{2} + \frac{c_1-c_2}{2}\cos[2\Phi]\right),$$
$$P(1,0,\Phi) = \frac{1}{4}\left(1 - \frac{c_1+c_2}{2} - \frac{c_1-c_2}{2}\cos[2\Phi]\right). \tag{12}$$

II) For $\theta = \frac{\pi}{2}$ and $\phi = 0$:

$$\tilde{\rho}^{BD}=\frac{1}{4}\begin{pmatrix} 1+c_1 & 0 & 0 & c_3-c_2 \\ 0 & 1-c_1 & c_3+c_2 & 0 \\ 0 & c_3+c_2 & 1-c_1 & 0 \\ c_3-c_2 & 0 & 0 & 1+c_1 \end{pmatrix} \tag{13}$$

and

$$P(0,0,\Phi) = \frac{1}{4}\left(1 + \frac{c_3+c_2}{2} + \frac{c_3-c_2}{2}\cos[2\Phi]\right),$$
$$P(1,0,\Phi) = \frac{1}{4}\left(1 - \frac{c_3+c_2}{2} - \frac{c_3-c_2}{2}\cos[2\Phi]\right). \tag{14}$$

III) For $\theta = \pi/2$ and $\phi = \pi/2$:

$$\tilde{\rho}^{BD}=\frac{1}{4}\begin{pmatrix} 1+c_2 & 0 & 0 & c_3-c_1 \\ 0 & 1-c_2 & c_3+c_1 & 0 \\ 0 & c_3+c_1 & 1-c_2 & 0 \\ c_3-c_1 & 0 & 0 & 1+c_2 \end{pmatrix} \tag{15}$$

and

$$P(0,0,\Phi) = \frac{1}{4}\left(1 + \frac{c_3+c_1}{2} + \frac{c_3-c_1}{2}\cos[2\Phi]\right),$$
$$P(1,0,\Phi) = \frac{1}{4}\left(1 - \frac{c_3+c_1}{2} - \frac{c_3-c_1}{2}\cos[2\Phi]\right). \tag{16}$$

For each $\theta$ and $\phi$, $\Phi$ depends on $|c_1| > |c_2|$, $|c_2| > |c_3|$, or $|c_1| > |c_3|$, respectively. Therefore, as was done with the classical correlations quantifier (8), defining

$$c_M \equiv \max |c_1|, |c_2|, |c_3| \tag{17}$$

allows us to write a general expression for the LAQC quantifier that encompasses all these possibilities:

$$\mathcal{L}(\rho^{BD}) = \frac{1+c_M}{2}\log_2(1+c_M) + \frac{1-c_M}{2}\log_2(1-c_M). \tag{18}$$

As with the classical correlations quantifiers, the above expression is equivalent to the one presented in Eq. (36) of [1]. Nevertheless, this newly defined $c_M$ also includes $|c_3|$. The case of $c_M = |c_3|$ arises when the density matrix $\rho^{BD}$ is written in the optimal computational basis with $\theta = \pi/2$.

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