OPERADOR DE COMPTON.

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RESUMEN.

Se calcula un nuevo operador de cuarto orden de la Electrodiná mica Cuántica, que es el único que no ha sido estudiado de los que aparecen al calcular las correcciones radiativas al efecto Compton.

I. Introducción.

Al calcular las correcciones radiativas de cuarto orden al efecto Compton, se encuentra que una de las gráficas de Feynman correspondientes a este efecto es irreducible a los operadores que han sido calculados. Es necesario entonces calcular un nuevo operador, que se ha llamado "operador de divértice", correspondiente a la gráfica de Feynman dada en la fig. 1.

Se ha llamado operador de divértice porque la auto-energía del electrón comprende dos vértices en la gráfica de Feynman correspondiente.

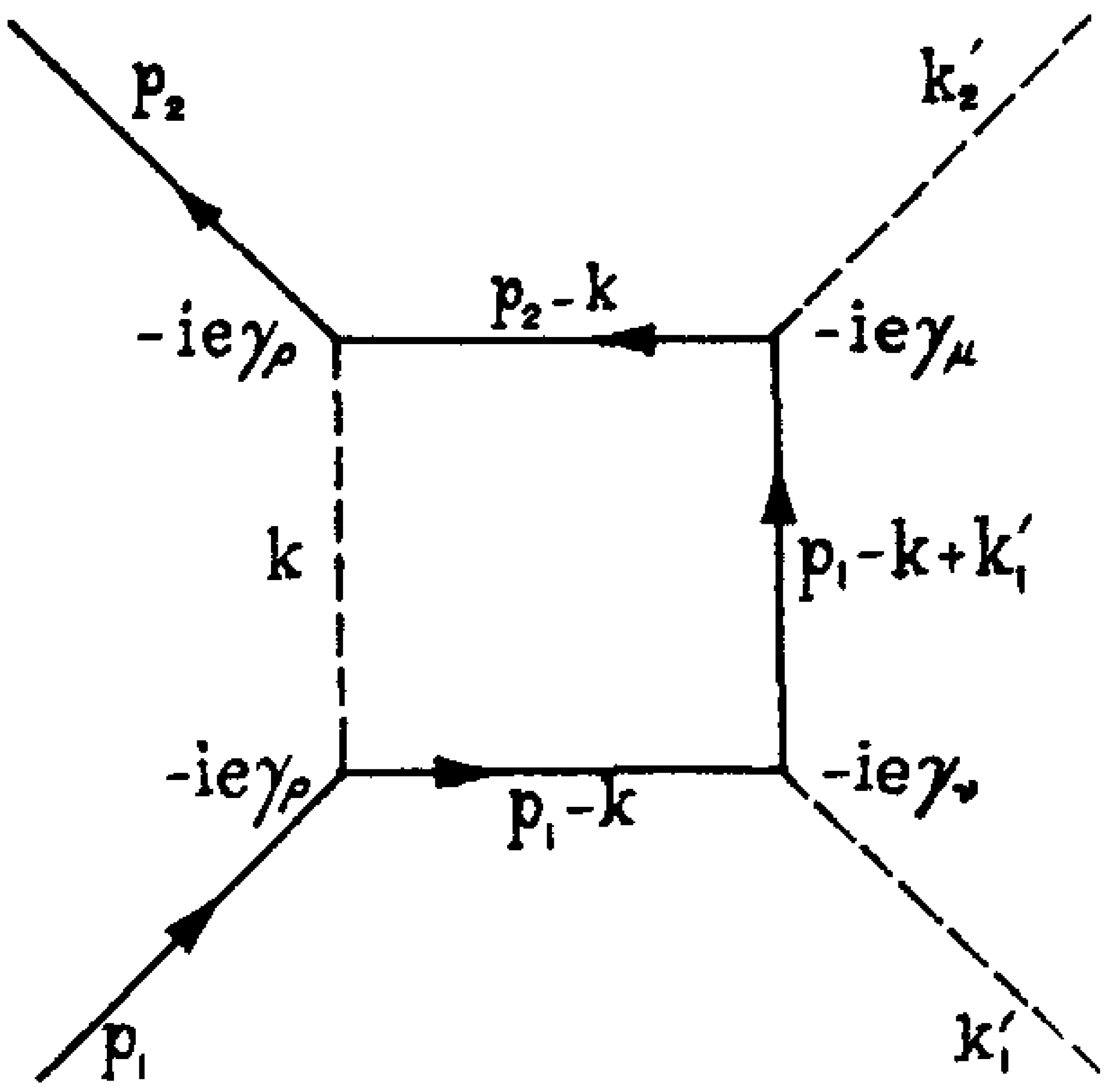

Fig. I.

II. Cálculo del operador.

Aplicando las reglas de Dyson, este operador está dado por:

$$C_{\mu\nu}(D,p_1,p_2,k_1,k_2) = -\frac{2\alpha}{(2\pi)^7}\int (dk)\ \gamma_\rho \frac{\not{p}_2-\not{k}-\varkappa}{(p_2-k)^2+\varkappa^2}\ \gamma_\mu \frac{\not{p}_1+\not{k}_1-\not{k}-\varkappa}{(p_1+k_1-k)^2+\varkappa^2}\gamma_\nu \times \frac{\not{p}_1-\not{k}-\varkappa}{(p_1-k)^2+\varkappa^2}\ \gamma_\rho\ \frac{1}{k^2+\lambda^2} \qquad (1)$$

Introduciendo las variables auxiliares t,u,v, se tiene que:

$$\frac{1}{abcd} = \int_0^1 3u^2du \int_0^1 2u\ dv \int_0^1 dt\ [\{[at+b(1-t)]\ v+c(1-v)\}\ u+d(1-u)]^4 \qquad (2)$$

Poniendo

$$a = (p_2-k)^2+\varkappa^2$$

$$b = (p_1+k_1-k)^2+\varkappa^2$$

$$c = (p_1-k)^2+\varkappa^2 \qquad (3)$$

$$d = k^2+\lambda^2$$

$$N_{\mu\nu} = \gamma_\rho\ (\not{p}_2-\not{k}-\varkappa)\ \gamma_\mu\ (\not{p}_1+\not{k}_1-\not{k}-\varkappa)\ \gamma_\nu\ (\not{p}_1-\not{k}-\varkappa)\ \gamma_\rho$$

se tiene que la ecuación (1) queda:

$$C_{\mu\nu} = -\frac{12\alpha}{(2\pi)^7}\int_0^1\int_0^1\int_0^1\int \frac{u^2 v\, du\, dv\, dt\, (dk)\, N_{\mu\nu}}{[\{k-[(k_1-k_2t)\, v + p_1]\, u\}^2 + C^2]^4} \qquad (4)$$

donde

$$C^2 = [(p_2^2-p_1^2)\, tv + (p_1+k_2u)^2 tv + (p_1+k_1)^2 (1-t)\, v - (k_1-k_2t)^2\, uv^2$$

$$-(p_1^2+k_2^2)v + (u+k_1^2 v)uv + (p_1^2+\varkappa^2) - (p_1+k_1v)^2 u]\; u + \lambda^2 (1-u) \qquad (4a)$$

Haciendo el cambio de variable:

$$k \longrightarrow k + [(k_1-t\, k_2)\, v + p_1]\, u \qquad (5)$$

se tiene que:

$$\not p_2-\not k \longrightarrow i\gamma\,(P_1-k)\;, \qquad \not p_1+\not k_1-\not k \longrightarrow i\gamma\,(P_2-k)$$

$$\not p_1-\not k \longrightarrow i\gamma\,(P_3-k) \qquad (6)$$

donde

$$P_1 = -up_1 + p_2 + uv\,(tk_2-k_1)$$

$$P_2 = (1-u)\, p_1 + (1-uv)\, k_1 + t\, uv\, k_2 \qquad (6a)$$

$$P_3 = (1-u)\, p_1 + uv\,(t\, k_2 - k_1)$$

La ecuación (4) queda:

$$C_{\mu\nu} = -\frac{12\alpha}{(2\pi)^7}\int_0^1\int_0^1\int_0^1\int (dk)\,\frac{u^2 v\, du\, dv\, dt\, N_{\mu\nu}}{(k^2+C^2)^4} \qquad (7)$$

$$N_{\mu\nu} = \gamma_\rho\ [i\gamma\ (P_1-k)\ \varkappa]\ \gamma_\mu\ [i\gamma\ (P_2-k)-\varkappa]\ \gamma_\nu\ [i\gamma\ (P_3-k)-\varkappa]\ \gamma_\rho$$

$$= n^0_{\mu\nu} + n^1_{\mu\nu\sigma}k_\sigma + n^2_{\mu\nu\sigma\tau}k_\sigma k_\tau + n^3_{\mu\nu\sigma\tau\omega}k_\sigma k_\tau k_\omega \qquad (7a)$$

donde

$$n^0_{\mu\nu} = \gamma_\rho\ (i\gamma\ P_1-\varkappa)\ \gamma_\mu\ (i\gamma\ P_2-\varkappa)\ \gamma_\nu\ (i\gamma\ P_3-\varkappa)\ \gamma_\rho$$

$$n^2_{\mu\nu\sigma\tau} = \gamma_\rho\ [\gamma_\sigma\gamma_\mu\gamma_\tau\gamma_\nu\ (i\gamma\ P_3-\varkappa) + \gamma_\sigma\gamma_\mu\ (i\gamma\ P_2-\varkappa)\ \gamma_\nu\gamma_\tau \qquad (7b)$$

$$+ (i\gamma\ P_1-\varkappa)\ \gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\tau]\ \gamma_\rho$$

Los términos que contieren a $n^1_{\mu\nu\sigma}$ y $n^3_{\mu\nu\sigma\tau\omega}$ no interesan, puesto que dan contribución nula al integrar con respecto a k.

Introduciendo la convención:

$$\Gamma_{\alpha\beta\delta\dots\omega} = \gamma_\alpha\gamma_\beta\gamma_\delta\dots\gamma_\omega$$

los términos de (7a) se pueden poner en la forma siguiente:

$$n^0_{\mu\nu} = -i\ \Gamma_{\rho\sigma\mu\tau\nu\omega\rho}\ P_{1\sigma}\ P_{2\tau}\ P_{3\omega} + \varkappa\ [\Gamma_{\rho\sigma\mu\tau\nu\rho}\ P_{1\sigma}\ P_{2\tau} +$$

$$+ \Gamma_{\rho\sigma\mu\nu\tau\rho}\ P_{1\sigma}\ P_{3\tau} + \Gamma_{\rho\mu\sigma\nu\tau\rho}\ P_{2\sigma}\ P_{3\tau}]$$

$$+ i\varkappa^2\ [\Gamma_{\rho\sigma\mu\nu\rho}\ P_{1\sigma} + \Gamma_{\rho\mu\sigma\nu\rho}\ P_{2\sigma} + \Gamma_{\rho\mu\nu\sigma\rho}\ P_{3\sigma}]$$

$$- \varkappa^3\ \Gamma_{\rho\mu\nu\rho}\ .$$

$$n^{I}_{\mu\nu} = n^1_{\mu\nu\sigma}\ k_\sigma$$

$$n^{II}_{\mu\nu} = n^2_{\mu\nu\sigma\tau}\ k_\sigma k_\tau = -\tfrac{1}{4}\ k^2\ [\Gamma_{\rho\sigma\mu\sigma\nu\tau\rho}\ P_{1\tau} + \Gamma_{\rho\sigma\mu\tau\nu\sigma\rho}\ P_{2\tau} +$$

$$+ \Gamma_{\rho\tau\mu\sigma\nu\sigma\rho}\ P_{3\tau}) - \varkappa\ (\Gamma_{\rho\sigma\mu\sigma\nu\rho} + \Gamma_{\rho\sigma\mu\nu\sigma\rho} + \Gamma_{\rho\mu\sigma\nu\sigma\rho})]$$

$$n^{III}_{\mu\nu} = n^{3}_{\mu\nu\sigma\tau\omega}\; k_\sigma k_\tau k_\omega$$

Aplicando las reglas dadas en el apéndice para reducir $\Gamma_{\alpha\beta\delta\ldots\omega}$ cuando hay índice repetido, se tiene:

$$n^{o}_{\mu\nu} = -2i\;(\Gamma_{\mu\tau\nu\omega\sigma} - \Gamma_{\sigma\mu\omega\nu\tau} - \Gamma_{\sigma\tau\nu\omega\mu})\; P_{1\sigma}\, P_{2\tau}\, P_{3\omega}$$

$$+ 2\varkappa\; [(\Gamma_{\sigma\nu\tau\mu} + \Gamma_{\mu\tau\nu\sigma})\; P_{1\sigma}\, P_{2\tau} + (\Gamma_{\sigma\tau\nu\mu} + \Gamma_{\mu\nu\tau\sigma})\; P_{1\sigma}\, P_{3\tau}$$

$$+ (\Gamma_{\mu\tau\nu\sigma} + \Gamma_{\sigma\nu\tau\mu})\; P_{2\sigma}\, P_{3\tau}] - 2i\varkappa^2 [\Gamma_{\nu\mu\sigma}\, P_{1\sigma} + \Gamma_{\nu\sigma\mu}\, P_{2\sigma} + \Gamma_{\sigma\nu\mu}\, P_{3\sigma})$$

$$-4\; \delta_{\mu\nu}\; \varkappa^3$$

$$n^{II}_{\mu\nu} = -ik^2 (\Gamma_{\nu\mu\tau}\; P_{1\tau} + \Gamma_{\mu\tau\nu}\; P_{2\tau} + \Gamma_{\tau\nu\mu}\; P_{3\tau}) \qquad (7c)$$

Definiendo

$$Q = Q_\sigma = -u\; p_{1\sigma} + p_{2\sigma} + uv\; (t\; k_{2\sigma} - k_{1\sigma}) \qquad (8)$$

las ecuaciones (6a) quedan:

$$P_1 = Q\;, \quad P_2 = Q + k_2\;, \quad P_3 = Q + p_1 - p_2 \qquad (8a)$$

Utilizando (8a) se pueden hacer en las ecuaciones (7c) las siguientes substiciones·

$$P_{1\sigma}\; P_{2\tau} \longrightarrow \tfrac{1}{4}\; \delta_{\sigma\tau}\; Q^2 + Q_\sigma\; k_{2\tau}$$

$$P_{1\sigma}\; P_{3\tau} \longrightarrow \tfrac{1}{4}\; \delta_{\sigma\tau}\; Q^2 + Q_\sigma\; (p_{1\tau} - p_{2\tau})$$

$$P_{2\sigma}\; P_{3\tau} \longrightarrow \tfrac{1}{4}\; \delta_{\sigma\tau}\; Q^2 + Q_\sigma\; (p_{1\tau} - p_{2\tau}) + k_{2\sigma}\; (p_{1\tau} - p_{2\tau}) + k_{2\sigma}\; Q_\tau$$

$$P_{1\sigma}\; P_{2\tau}\; P_{3\omega} \longrightarrow \tfrac{1}{4}\; \delta_{\tau\omega}\; Q_\sigma Q^2 + \tfrac{1}{4}\; \delta_{\sigma\tau} Q^2 (p_{1\omega} - p_{2\omega}) + \tfrac{1}{4}\; \delta_{\omega\sigma} Q^2 k_{2\tau} + Q_\sigma k_{2\tau} (p_{1\omega} - p_{2\omega})$$

La ecuación (7a) queda:

$$N_{\mu\nu} = -ik^2 \; [(\Gamma_{\nu\mu\sigma}+\Gamma_{\mu\sigma\nu}+\Gamma_{\sigma\nu\mu})Q_\sigma+\Gamma_{\sigma\nu\mu}(p_{1\sigma}-p_{2\sigma})+\Gamma_{\mu\tau\nu}k_{2\tau}]$$

$$+2i(\Gamma_{\sigma\tau\nu\omega\mu}+\Gamma_{\sigma\mu\omega\nu\tau}-\Gamma_{\mu\tau\nu\omega\sigma})Q_\sigma(p_{1\omega}-p_{2\omega})k_{2\tau}$$

$$+[2\varkappa(\Gamma_{\mu\omega\nu\tau}+\Gamma_{\tau\nu\omega\mu})k_{2\tau}-i\Gamma_{\omega\nu\mu}(Q^2+2\varkappa^2)](p_{1\omega}-p_{2\omega})$$

$$+8\varkappa[Q_\nu k_{2\mu}+Q_\mu k_{2\nu}+Q_\mu(p_{1\nu}-p_{2\nu})-Q_\tau k_{2\tau}\delta_{\mu\nu}]-i\Gamma_{\nu\mu\sigma}(Q^2+\varkappa^2)Q_\sigma$$

$$-i(\Gamma_{\mu\tau\nu}Q^2+2\Gamma_{\nu\tau\mu}\varkappa^2)k_{2\tau}-4i\varkappa^2\gamma_\mu Q_\nu-4\delta_{\mu\nu}\varkappa^3+n^{I}_{\mu\nu}+n^{III}_{\mu\nu}$$

Substituyendo en la ecuación anterior el valor de Q dado por (8) y agrupando convenientemente, se tiene:

$$N_{\mu\nu} = \Lambda^0_{\mu\nu}\, k^2 + u\,\Lambda^1_{\mu\nu}\,(p_1^2+\varkappa^2) + \Lambda^1_{\mu\nu}\,(p_2^2+\varkappa^2) +$$

$$+(\not{p}_2+\varkappa)\;\Lambda^2_{\mu\nu}\;(\not{p}_1+\varkappa)+(\not{p}_2+\varkappa)\Lambda^3_{\mu\nu}+\Lambda^4_{\mu\nu}(p_1+\varkappa)+\Lambda^5_{\mu\nu}+n^{I}_{\mu\nu}+n^{III}_{\mu\nu} \qquad (9)$$

donde:

$$\Lambda^0_{\mu\nu} = -2i(\not{p}_2+\varkappa)\sigma_{\mu\nu}+[2iu\;\sigma_{\nu\mu}-(1-u)\;\Gamma_{\nu\mu}]\;(\not{p}_1+\varkappa)$$

$$+2i\;\gamma_\mu[(2u-1)p_{1\nu}+p_{2\nu}]\;-2i\;\gamma_\nu(up_{1\mu}+p_{2\mu})+(1-u)\varkappa(2i\;\sigma_{\nu\mu}+\Gamma_{\nu\mu})$$

$$-i\Gamma_{\mu\tau\nu}k_{2\tau}-i(\Gamma_{\mu\nu\sigma}+\Gamma_{\mu\sigma\nu}+\Gamma_{\sigma\nu\mu})\;uv\;(t\;k_{2\sigma}-k_{1\sigma})$$

$$\Lambda^1_{\mu\nu} = ik_{2\tau}\Gamma_{\mu\tau\nu} \qquad\qquad \Lambda^2_{\mu\nu} = -2ik_{2\tau}\;\Sigma_{\mu\nu\tau}(1-u)$$

$$\Lambda^3_{\mu\nu} = 2uvk_{2\tau}(tk_{2\sigma}-k_{1\sigma})\Sigma_{\sigma\tau\nu\mu}+2i\varkappa k_{2\tau}[(u-1)(2i\;\gamma_\mu\sigma_{\tau\nu}\;+$$

$$+\Gamma_{\tau\nu\mu})+\Gamma_{\tau\nu\mu}-\Gamma_{\mu\nu\tau}]+4k_{2\tau}[(\Gamma_{\tau\nu}+2iu\;\sigma_{\nu\tau})\;p_{1\mu}\;-$$

$$-(\Gamma_{\mu\nu}+2iu\;\sigma_{\nu\mu})\;p_{1\tau}\;+\;(\Gamma_{\mu\tau}+2iu\;\sigma_{\tau\mu})\;p_{1\nu}]$$

$$\Lambda^4_{\mu\nu} = -2uvk_{2\tau}(tk_{2\sigma}-k_{1\sigma})\ \Sigma_{\sigma\tau\nu\mu}+2i\varkappa k_{2\tau}\ [(u-1)(2i\gamma_\mu\ \sigma_{\tau\nu} + \Gamma_{\tau\nu\mu})$$

$$+\Gamma_{\tau\nu\mu}-\Gamma_{\mu\nu\tau}] + 4\ [(\Gamma_{\tau\nu} + 2i\ \sigma_{\nu\tau})\ p_{2\mu} -$$

$$-(\Gamma_{\mu\nu}+2i\sigma_{\nu\mu})p_{2\tau}+(\Gamma_{\mu\tau}+2i\sigma_{\tau\mu})p_{2\nu}] + (u-1)\ \Gamma_{\nu\mu}\ (Q^2+2\varkappa^2)$$

$$\Lambda^5_{\mu\nu} = -2\ \varkappa^3\ \lambda^0_{\mu\nu} + \varkappa^2\ \lambda^1 + \varkappa\ \lambda^2 + \lambda^3$$

$$\lambda^0_{\mu\nu} = (1-u)\ \Gamma_{\nu\mu} + 2\ \delta_{\mu\nu}$$

$$\lambda^1_{\mu\nu} = -i(u+1)k_{2\tau}\Gamma_{\mu\tau\nu}+2i(u-1)k_{2\tau}\ \Sigma_{\mu\nu\tau} + 4i\ \gamma_\mu(p_{2\nu}-p_{1\nu})$$

$$+2ik_{2\tau}(2\ \Sigma_{\mu\nu\tau}+\Gamma_{\nu\tau\mu})+2iuv\ \Gamma_{\nu\mu\sigma}(tk_{2\sigma}-k_{1\sigma}) - 4i\ \gamma_\mu\ Q_\nu$$

$$\lambda^2_{\mu\nu} = -8k_{2\tau}[(iu\ \sigma_{\tau\mu}+\delta_{\tau\mu})p_{2\nu}+(iu\ \sigma_{\nu\tau}+\delta_{\nu\tau})\ p_{2\mu}$$

$$-(iu\ \sigma_{\nu\mu}+\delta_{\nu\mu})p_{2\tau}] - 8iuk_{2\tau}\{\sigma_{\tau\mu}p_{1\nu}+\sigma_{\nu\tau}p_{1\mu}-\sigma_{\nu\mu}p_{1\tau}\}]$$

$$-Q^2(1-u)\Gamma_{\nu\mu}-8(Q_\mu k_{1\nu}+Q_\nu k_{2\mu}) - 8Q_\tau k_{2\tau}\ \delta_{\mu\nu}$$

$$\lambda^3_{\mu\nu} = -4k_{2\tau}iuv(tk_{2\sigma}-k_{1\sigma})[(\Delta_{\sigma\tau\mu}p_{2\nu}-\Gamma_{\sigma\mu\tau}p_{1\nu})$$

$$-\Delta_{\sigma\nu\mu}p_{2\tau}-\Gamma_{\sigma\mu\nu}p_{1\tau}) + (\Delta_{\sigma\nu\tau}p_{2\mu}-\Gamma_{\sigma\tau\nu}p_{1\mu})-(\Delta_{\mu\nu\tau}p_{2\sigma}-\Gamma_{\mu\tau\nu}p_{1\sigma})]$$

$$-2iQ^2\ \gamma_\mu\ (p_{1\nu}-p_{2\nu})+4ik_{2\tau}p_1p_2[u\Delta_{\tau\nu\mu}-\Gamma_{\mu\tau\nu}]-iQ^2[uv\Gamma_{\nu\mu\tau}(tk_{2\tau}-k_{1\tau})$$

$$+\Gamma_{\mu\tau\nu}k_{2\tau}]+8iuk_{2\tau}p_{1\omega}p_{2\sigma}[\gamma_\tau(\delta_{\sigma\mu}\delta_{\omega\nu}-\delta_{\sigma\nu}\delta_{\omega\mu})$$

$$+ \gamma_\mu(\delta_{\sigma\nu}\delta_{\omega\tau}-\delta_{\sigma\tau}\delta_{\omega\nu}) + \sigma_\nu\ (\delta_{\sigma\tau}\delta_{\omega\mu}-\delta_{\omega\mu}\delta_{\omega\tau})]$$

$$\Sigma_{\mu\nu\tau} = \Gamma_{\mu\nu\tau} - \Gamma_{\mu\tau\nu} - \Gamma_{\tau\nu\mu}\ ;\quad \Sigma_{\sigma\tau\nu\mu} = \Gamma_{\sigma\tau\nu\mu}-\Gamma_{\sigma\mu\nu\tau}-\Gamma_{\mu\tau\nu\sigma}$$

$$\Delta_{\mu\nu\tau} = \Gamma_{\mu\nu\tau} - \Gamma_{\tau\nu\mu}\ .$$

Substituyendo en (7) el valor de $N_{\mu\nu}$ dado por (9) e integrando con respecto a k, se tiene:

$$C_{\mu\nu} = -\frac{i\pi\alpha}{3(2\pi)^6}\int_0^1\int_0^1\int_0^1 u^2 v\, du\, dv\, dt \times$$

$$\times \frac{1}{C^2}\{\Lambda^0_{\mu\nu} + \frac{1}{2C^2}[u\ \Lambda^1_{\mu\nu}(p_1^2+\varkappa^2) + \Lambda^1_{\mu\nu}(p_2^2+\varkappa^2) + (\not{p}_2+\varkappa)\Lambda^2_{\mu\nu}(\not{p}_1+\varkappa)$$

$$+(\not{p}_2+\varkappa)\Lambda^3_{\mu\nu} + \Lambda^4_{\mu\nu}(\not{p}_1+\varkappa) + \Lambda^5_{\mu\nu}]\}$$

donde el valor de C^2 está dado por (4a).

APENDICE.

Reducción de la matriz $\Gamma_{\alpha\beta\delta\ldots\omega}$ cuando hay índice repetido. Teniendo en cuenta que

$$\Gamma_{\mu\nu} + \Gamma_{\nu\mu} = 2\ \delta_{\mu\nu} \qquad (1A)$$

se tiene, por aplicaciones sucesivas de (1A) que:

$$\Gamma_{\tau\mu\tau} = -2\ \gamma_\mu\ ; \quad \Gamma_{\rho\mu\nu\rho} = 4\ \delta_{\mu\nu}\ ; \quad \Gamma_{\rho\mu\nu\sigma\rho} = -2\ \Gamma_{\sigma\nu\mu}$$

$$\Gamma_{\rho\mu\nu\sigma\tau\rho} = 2\ (\Gamma_{\mu\tau\sigma\nu} + \Gamma_{\nu\sigma\tau\mu})$$

$$\Gamma_{\rho\mu\nu\sigma\rho\omega\rho} = 2\ (\Gamma_{\nu\sigma\tau\omega\mu} - \Gamma_{\mu\nu\omega\rho\sigma} - \Gamma_{\mu\sigma\rho\omega\nu})$$