Effect of fractional analysis on magnetic curves

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In this present paper, the effect of fractional analysis on magnetic curves is researched. A magnetic field is defined by the property that its divergence is zero in three dimensional Riemannian manifold. We investigate the trajectories of the magnetic fields called as t-magnetic, n-magnetic and b-magnetic curves according to fractional derivative and integral. As it is known, there are not many studies on a geometric interpretation of fractional calculus. When examining the effect of fractional analysis on a magnetic curve, the conformable fractional derivative that best fits the algebraic structure of differential geometry derivative is used. This effect is examined with the help of examples consistent with the theory and visualized for different values of the conformable fractional derivative. The difference of this study from others is the use of conformable fractional derivatives and integrals in calculations. Fractional calculus has applications in many fields such as physics, engineering, mathematical biology, fluid mechanics, signal processing, etc. Fractional derivatives and integrals have become an extremely important and new mathematical method in solving various problems in many sciences.

Keywords: Magnetic curve; vector fields; fractional derivative; conformable fractional derivative.

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1. Introduction

Magnetic curves have many applications in physics and differential geometry and play an important role in these areas. When a charged particle enters a magnetic field, the Frenet vectors of this particle are affected by this magnetic field and with this effect a force which is called the Lorenz force occurs. Thus, the particle starts to follow a trajectory in this magnetic field thanks to Lorenz force. This trajectory is called a magnetic curve. The motion of a particle entering the magnetic field with the effect of the Lorenz force is explained as; if the tangent vector field T is parallel to the magnetic field, the Lorentz force will be zero, so the particle moves parallel to the magnetic field. If the tangent vector field T is perpendicular to the magnetic field, the Lorentz force is maximum and the particle moves in a circle in the magnetic field. If the tangent vector field T is at a constant angle with the magnetic field, the particle follows a helical trajectory under the influence of the Lorentz force, [1]. These curves have attracted the attention of many authors in different disciplines. For this reason, many studies have been carried out by considering these curves in different ways [2–9].

On the other hand, fractional analysis means derivative and integral accounts that are not integers. The phrase fractional derivative first appears in a letter sent by Leibniz to L'Hospital in 1695, [10]. In this letter, Leibniz is asked L'Hospital a question, "Can integer order derivatives be extended to fractional order derivatives?" Afterwards, this subject, which attracted the attention of many mathematicians, took part in many studies [11–16]. Today, the subject of fractional analysis become very popular and study by many researchers in different fields [17–20]. Since it is believed to be the better modelling the physical systems with fractional order derivative, they have many studies on this subject. Because, the classical derivative is beneficial to model the physical systems locally but the fractional order derivative is beneficial to model physical systems globally. Fractional analysis have many applications in many branches of science in recent years. The study of this subject by many mathematicians is led to the emergence of many different definitions of fractional derivatives and integrals. Riemann-Liouville, Caputo, Cauchy, and conformable fractional derivatives and integrals are just a few of these definitions. Different fractional derivative and integral definitions naturally brought with them different properties. For example, the derivative of zero is not constant for many types of fractional derivatives, except for the conformable fractional derivative and Caputo fractional derivative. Moreover, except for the conformable fractional derivative, other fractional derivatives do not have features such as the derivative of the product, the derivative of the quotient, or the chain rule, as in the classical sense [21]. In addition, the conformable fractional derivative is the local fractional derivative, unlike the Riemann-Liouville and Caputo fractional derivative. Conformable fractional derivative has many critical aspects, as it is equivalent to a simple change of variables for differentiable functions [22]. However, the effect of conformable fractional derivatives and integrals on some physical phenomena is worth investigating. It will be interesting that fractional derivatives do not have a geometric interpretation as in the classical sense. However, there are many mathematicians investigating the effect of fractional calculus on differential geometry [23–25].

In this study, the effects of conformable fractional derivatives and integrals on magnetic curves are investigated. In addition, a geometric inference is tried to be obtained with the help of examples. Moreover, we are visualize their images for different fractional values using the Mathematica program. The difference of this study from others is the use of conformable fractional derivatives and integrals in calculations. Fractional derivatives and integrals are more precision than ordinary derivatives and integrals because they give more accurate results. So, fractional calculus has applications in many different fields such as physics, engineering, mathematical biology, etc. This article is a complicated study that includes differential geometry, physics and fractional analysis fields. So, we hope that it will contribute to those working in these fields.

2. Preliminaries

2.1. Basic definitions and theorems of differential geometry

In section definitions and theorems, the curves in $\mathbb{R}^3$ will be introduced in a nutshell.

Definition 1. *Let the curve $x(s)$ be given in n-dimensional Euclidean space with (I,α) coordinate neighborhood. The arc length of the curve x from a to b, is calculated as*

$$s=\int_b^a \|x'(s)\|dt, \quad s\in I,$$

which is the length between the points $x(a)$ and $x(b)$ of the curve. The parameter s is said to be arc-length.

Theorem 1. *Let $x=x(s)$ be a regular unit speed curve in the Euclidean $3-$space where s measures its arc length. Also, let $T=x'$ be its unit tangent vector, $N=T'/\|T'\|$ be its principal normal vector and $B=T\times N$ be its binormal vector. The triple $\{T,N,B\}$ be the Frenet frame of the curve x. Then the Frenet formula of the curve is given by*

$$\begin{pmatrix} T'(s)\\ N'(s)\\ B'(s)\end{pmatrix}=\begin{pmatrix} 0 & \kappa(s) & 0\\ -\kappa(s) & 0 & \tau(s)\\ 0 & -\tau(s) & 0\end{pmatrix}\begin{pmatrix} T(s)\\ N(s)\\ B(s)\end{pmatrix}, \quad (1)$$

where $\kappa(s)=\left\|d^2x/ds^2\right\|$ and $\tau(s)=\langle dN/ds,\ B\rangle$ are curvature and torsion of x respectively [26].

Definition 2. *Let $x:I\subset\mathbb{R}\to E^3$ be a unit speed curve in Euclidean 3-space E^3. If any U fixed direction with the unit tangent vector of the curve x makes a fixed angle, the curve x is called the general helix [27]. The most well-known characterization of the helix curve is $\tau/\kappa=$ constant (Lancret theorem)* [26].

Definition 3. *Let $x:I\subset\mathbb{R}\to E^3$ be a unit speed curve in Euclidean 3-space E^3. If any U fixed direction with the principal unit normal vector of the curve x makes a fixed angle, the curve x is called the slant helix. Izumiya and Takeuchi obtain a necessary and sufficient condition for a curve to be slant helix: a curve is an oblique propeller if its geodetic curvature and the principal normal satisfy the expression*

$$\frac{\kappa^2}{(\kappa^2+\tau^2)^{3/2}}\left(\frac{\tau}{\kappa}\right)' \quad (2)$$

is constant function [28].

2.2. Basic definitions and theorems of conformable fractional calculus

In this part, some basic definitions and theorems of conformable fractionally derivative and integral are given.

Definition 4. *Let us give a function $f:[0,\infty)\to R$. Then the conformable fractional derivative for f of order α is defined by*

$$D_\alpha(f)(s)=\lim_{\varepsilon\to 0}\frac{f(s+\varepsilon s^{1-\alpha})-f(s)}{\varepsilon}$$

for all $s>0,\ 0<\alpha<1$. If f is α-differentiable in some $(0,a)$, $a>0$ and $\lim_{s\to 0^+}f^{(\alpha)}(s)$ exist, then define $f^{(\alpha)}(0)=\lim_{s\to 0^+}f^{(\alpha)}(s)$, [29].

Theorem 2. *Let $f:[0,\infty)\to R$ be a function. If a function f is α-differentiable at $s_0>0,\ 0<\alpha<1$, then f is continuous at s_0*, [29].

Accordingly, it is easily visible that the conformable fractional derivative provides all the properties given in the theorem below.

Theorem 3. *Let $f,g:[0,\infty)\to R$ be α-differentiable at each $s>0$, $0<\alpha<1$. Then*

(1) $D_\alpha(af+bg)(s)=aD_\alpha(f)(s)+bD_\alpha(g)(s)$, *for all* a, $b\in R$.

(2) $D_\alpha(s^p)=ps^{p-\alpha}$, *for all* $p\in R$.

(3) $D_\alpha(\lambda)=0$, *for all constant functions* $f(s)=\lambda$.

(4) $D_\alpha(fg)(s)=f(s)D_\alpha(g)(s)+g(s)D_\alpha(f)(s)$.

(5) $D_\alpha(\frac{f}{g})(s)=\frac{f(s)D_\alpha(g)(s)-g(s)D_\alpha(f)(s)}{g^2(s)}$.

(6) *If f is a differentiable function, then* $D_\alpha(f)(s)=s^{1-\alpha}\frac{df(s)}{ds}$, [29].

Theorem 4. *Let $f,g:[0,\infty)\to R$ be $\alpha-$differentiable at $s_0>0$, $0<\alpha<1$. If $(f\circ g)$ is $\alpha-$differentiable and for all s with $s\neq 0$ and $f(s)\neq 0$, the equation*

$$D_\alpha(f\circ g)(s)=f(s)^{\alpha-1}D_\alpha f(s)D_\alpha(g)(f(s)),$$

is provided, [30].

Definition 5. *Let $f:[a,\infty)\to R$ be a function. The expression*

$$I_\alpha^a f(s)=I_1^a f(s^{\alpha-1}f)=\int_a^s\frac{f(x)}{x^{1-\alpha}}dx$$

is called a conformable fractional integral, where $\alpha>0$, [30].

Theorem 5. *Let $f:[a,\infty)\to R$ be a function. Then for all $s>0$ the following equation exists,* [30]

$$D_\alpha I_\alpha^a f(s)=f(s).$$

2.3. Basic definitions and theorems of magnetic field and curves

In this subsection, some basic definitions and theorems of magnetic field and magnetic curve are introduced.

Let M be a $(n \geq 2)$-dimensional oriented Riemannian manifold. The Lorenz force of a magnetic field F on M is defined to be a skew symmetric operator ϕ given by

$$g(\phi(X), Y) = F(X, Y), \tag{3}$$

for all $X, Y \in \chi(M)$, where $\chi(M)$ is the space of vector fields. The magnetic trajectories of F are curves x on M which satisfy the Lorenz equation

$$\nabla_{x'} x' = \phi(x'). \tag{4}$$

The mixed product of the vector fields $X, Y, Z \in \chi(M)$ is defined by

$$g(X \times Y, Z) = dv_g(X, Y, Z). \tag{5}$$

Let V be a Killing vector field on M and $F_v = \imath_v dv_g$ be the corresponding Killing magnetic field, where $\imath$ is denoted the inner product. Then, the Lorentz force of the F_v is

$$\phi(X) = V \times X. \tag{6}$$

Consequently, the Lorentz force equation may be written as

$$\nabla_{x'} x' = V \times x'. \tag{7}$$

A unit speed curve x is a magnetic trajectory of a magnetic field V if and only if V can be written along x as

$$V = \omega(s)T(s) + \kappa(s)B(s), \tag{8}$$

where the function $\omega(s)$ associated with each magnetic curve will be called its quasislope measured with respect to the magnetic field V, [31].

Proposition 1. *Let $x : I \subset R \rightarrow M^3$ be a curve in a 3D oriented Riemannian Manifold (M^3, g) and V be a vector field along the curve x. One can take a variation of x in the direction of V, say, a map $\Gamma : I \times (-\varepsilon, \varepsilon) \rightarrow M^3$ which satisfies $\Gamma(s, 0) = x(s)$, $(d\Gamma/ds)(s, k) = V(s)$. In this setting, we have the following funtions:*

(1) *The speed funtion $v(s, k) = \|(d\Gamma/ds)(s, k)\|$,*

(2) *The curvature function $\kappa(s, k)$* of $x(s)$,

(3) *The torsion function $\tau(s, k)$* of $x(s)$. *The variations of those functions at $k = 0$* are

$$V(v) = \left(\frac{dv}{dk}(s, k)\right)|_{k=0} = g(\nabla_t V, T)v, \tag{9}$$

$$V(\kappa) = \left(\frac{d\kappa}{dk}(s, k)\right)|_{k=0} = g(\nabla_T^2 V, N) - 2\kappa g(\nabla_T V, T) + g(R(V, T), N), \tag{10}$$

$$V(\tau) = \left(\frac{d\tau}{dk}(s, k)\right)|_{k=0} = \left(\frac{1}{\kappa} g(\nabla_T^2 V + R(V, T)T, B)\right)_s + \kappa(\nabla_T V, B) + \tau g(\nabla_T V, T) + g(R(V, T)N, B), \tag{11}$$

where R is the curvature tensor of M^3, [31].

Proposition 2. *Let $V(s)$ be the restriction to $x(s)$ of a Killing vector field, say V of M^3, then,* [31]

$$V(v) = V(\kappa) = V(\tau) = 0.$$

Definition 6. *Let $x : I \subset R \rightarrow M^3$ be a curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M. We call the curve x is a T-magnetic curve if the tangent vector field of the curve satisfy the Lorentz force equation, that is,* [31]

$$\nabla_{x'} T = \phi(T) = V \times T.$$

Definition 7. *Let $x : I \subset R \rightarrow M^3$ be a curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M. We call the curve x is a N-magnetic curve if the normal vector field of the curve satisfy the Lorentz force equation, that is,* [32]

$$\nabla_{x'} N = \phi(N) = V \times N.$$

Definition 8. *Let $x : I \subset R \rightarrow M^3$ be a curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M. We call the curve x is a B-magnetic curve if the binormal vector field of the curve satisfy the Lorentz force equation, that is,* [32]

$$\nabla_{x'} B = \phi(B) = V \times B.$$

2.4. Basic definitions and theorems of conformable fractional curves

In this part of the preliminaries section, we present brief information about conformable curves using conformable fractional derivative.

Definition 9. *Let $x = x(s)$ be a curve. If $x : (0, \infty) \rightarrow R^3$ is $\alpha-$differentiable curve, then x is called a conformable curve in R^3*, [33].

Definition 10. *Let $x : (0, \infty) \rightarrow R^3$ be a conformable curve in R^3. Velocity vector of x is determined by*

$$\frac{D_\alpha(x)(s)}{s^{1-\alpha}}, \tag{12}$$

for all $s \in (0, \infty)$, [33].

Definition 11. *Let $x : (0, \infty) \rightarrow R^3$ be a conformable curve in R^3. Then the velocity function v of x is defined by*

$$v(s) = \frac{\|D_\alpha(x)(s)\|}{s^{1-\alpha}},$$

for all $s \in (0, \infty)$, [33].

Definition 12. *Let $x:(0,\infty)\to R^3$ be a conformable curve in R^3. The arc length function s of x is defined by*

$$s(s_0)=I_\alpha^0\left\|D_\alpha(x)(s_0)\right\|,$$

for all $s_0\in(0,\infty)$. If $v(s)=1$ for all $s_0\in(0,\infty)$, it's said that x has unit speed, [33].

Conclusion 1. *Let $x:(0,\infty)\to R^3$ be a conformable curve in R^3. The concepts velocity vector, velocity function and arc length function obtained according to conformable fractional derivative are equivalent to the standard concepts.*

Definition 13. *Let x be a conformable curve. If $D_\alpha(x)(s)\neq 0$ for all $s\in(0,\infty)$, x is called a conformable regular curve,* [33].

Definition 14. *Let $x=x(s)$ be a regular unit speed conformable curve in $3D$ Riemannian manifold where s measures its arc length. Also, let $t=D_\alpha(x)(s)s^{\alpha-1}$ be its unit tangent vector, $n=D_\alpha(t)(s)/\|D_\alpha(t)(s)\|$ be its principal normal vector and $b=t\times n$ be its binormal vector. The triple $\{t,n,b\}$ be the conformable Frenet frame of the curve $x(s)$. Then the conformable Frenet formula of the curve is given by*

$$\begin{pmatrix} D_\alpha(t)(s)\\ D_\alpha(n)(s)\\ D_\alpha(b)(s)\end{pmatrix} = \begin{pmatrix} 0 & \kappa_\alpha(s) & 0\\ -\kappa_\alpha(s) & 0 & \tau_\alpha(s)\\ 0 & -\tau_\alpha(s) & 0\end{pmatrix}\begin{pmatrix} t(s)\\ n(s)\\ b(s)\end{pmatrix}, \tag{13}$$

where $\kappa_\alpha(s)=\|D_\alpha(t)(s)\|$ and $\tau_\alpha(s)=\langle D_\alpha(n)(s),b(s)\rangle$ are curvature and torsion of x, respectively.

Conclusion 2. *Let $x=x(s)$ be a regular unit speed conformable curve where s measures its arc length. The following relation exists between the curvature and torsion of x according to Frenet frame and the conformable curvature and torsion of x according to conformable Frenet frame as*

$$\kappa_\alpha=s^{1-\alpha}\kappa, \tag{14}$$

$$\tau_\alpha=s^{1-\alpha}\tau. \tag{15}$$

Conclusion 3. *Let $x=x(s)$ be a regular unit speed conformable curve where s measures its arc length. As can be seen from Eq. (13), when x is a unit speed curve, the conformable derivative has no effect on the Frenet frame, so the Frenet elements do not undergo any change. However, considering Eqs. (14) and (15), the curvature and torsion of the x curve has changed under the conformable fractional derivative.*

3. Main results

3.1. Fractional t-magnetic curves

In this subsection, we define the fractional t-magnetic curve with a conformable fractional derivative focus. We are also obtained some characterizations of this curve.

Definition 15. *Let $x:I\subset R\to M^3$ be a conformable curve in 3D oriented Riemannian space (M^3,g) and F be a magnetic field on M. If the vector area of the tangent curve of x with respect to the conformable frame satisfies the Lorenz force equation, the curve x is called fractional t-magnetic curve, that is*

$$\frac{D_\alpha t(s)}{s^{1-\alpha}}=\phi(t)=V\times t.$$

Proposition 3. *Let $x:I\subset R\to M^3$ be a unit speed fractional t-magnetic curve in 3D oriented Riemannian space (M^3,g) and F be a magnetic field on M with the conformable frame elements $\{t,n,b,\kappa_\alpha,\tau_\alpha\}$. Then, we have the Lorenz force according to conformable frame as*

$$\begin{pmatrix}\phi(t)\\ \phi(n)\\ \phi(b)\end{pmatrix}=\begin{pmatrix} 0 & \kappa_\alpha(s) & 0\\ -\kappa_\alpha(s) & 0 & \Omega_1(s)\\ 0 & -\Omega_1(s) & 0\end{pmatrix}\begin{pmatrix} t(s)\\ n(s)\\ b(s)\end{pmatrix}, \tag{16}$$

where Ω_1 is a certain function.

Proof. Let $x:I\subset R\to M^3$ be a unit speed fractional t-magnetic curve in 3D oriented Riemannian space (M^3,g) and F be a magnetic field on M with the conformable frame elements $\{t,n,b,\kappa_\alpha,\tau_\alpha\}$. Since $\phi(t)\in Sp\{t,n,b\}$, we get

$$\phi(t)=\lambda_1 t+\mu_1 n+\sigma_1 b$$

and thus

$$\begin{aligned}\lambda_1&=g(\phi(t),t)=0,\\ \mu_1&=g(\phi(t),n)=g(\kappa_\alpha n,n)=\kappa_\alpha,\\ \sigma_1&=g(\phi(t),b)=0.\end{aligned}$$

From the above equations, we can write

$$\phi(t)=\kappa_\alpha n.$$

Similarly, we can easily calculate that

$$\begin{aligned}\phi(n)&=-\kappa_\alpha t+\Omega_1 b,\\ \phi(b)&=-\Omega_1 n.\end{aligned}$$

This completes the proof. ■

Proposition 4. *Let x be a unit speed fractional t-magnetic trajectory of a magnetic field V if and only if V can be written along the curve x as*

$$V=\Omega_1 t+\kappa_\alpha b. \tag{17}$$

Proof. Let x be a unit speed fractional t-magnetic trajectory of a magnetic field V. Using Proposition 3 and equation (6), we can easily see that

$$V=\Omega_1 t+\kappa_\alpha b.$$

This completes the proof. ■

Theorem 6. *Let x be a unit speed fractional t-magnetic trajectory and V be a Killing vector field on a simply connected space form $\left(M^3, g\right)$. Then the following equations exist*

$$\Omega_1 = c, \qquad c \in R, \qquad \left(\kappa_\alpha^2 \left[\frac{1}{2}\Omega_1 - \tau_\alpha\right]\right)' = 0,$$

and

$$\left[\frac{1}{\kappa_\alpha}\left(\Omega_1 \kappa_\alpha \tau_\alpha - \kappa_\alpha \tau_\alpha^2 + (1-\alpha)\, s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha'' + C\kappa_\alpha\right)\right]' + \kappa_\alpha \kappa_\alpha' = 0$$

where C is the curvature of the Riemanian space M^3.

Proof. Let V be a magnetic field in a Riemanian 3D manifold. If the $\alpha-$th conformable fractional derivative of Eq. (17) is taken with respect to s and conformable frame formulas are applied, we have

$$\begin{aligned} D_\alpha V &= D_\alpha(\Omega_1 t) + D_\alpha(\kappa_\alpha b), \\ D_\alpha V &= s^{1-\alpha}\Omega_1' t + (\Omega_1 \kappa_\alpha - \kappa_\alpha \tau_\alpha)\, n + s^{1-\alpha}\kappa_\alpha' b. \end{aligned} \tag{18}$$

It can be easily seen that if $V(v) = 0$ of Proposition 1, the case is $g(D_\alpha V, t) = 0$. So, if this equation is used in the above equation,

$$D_\alpha V = (\Omega_1 \kappa_\alpha - \kappa_\alpha \tau_\alpha)\, n + s^{1-\alpha}\kappa_\alpha' b,$$

is obtained. If the conformable derivative of the above equation with respect to s is taken once again from the $\alpha-$th order and conformable frame formulas are applied, we have

$$\begin{aligned} D_\alpha^2 V = &\left(s^{1-\alpha}\Omega_1' \kappa_\alpha + s^{1-\alpha}\Omega_1 \kappa_\alpha' - s^{1-\alpha}\kappa_\alpha' \tau_\alpha - s^{1-\alpha}\kappa_\alpha \tau_\alpha'\right) n \\ &+ (\Omega_1 \kappa_\alpha - \kappa_\alpha \tau_\alpha)\,(-\kappa_\alpha t + \tau_\alpha b) + (1-\alpha)\, s^{1-2\alpha}\kappa_\alpha' b + s^{2-2\alpha}\kappa_\alpha'' b - s^{1-\alpha}\kappa_\alpha' \tau_\alpha n. \end{aligned} \tag{19}$$

If the above equation is adjusted, we get

$$\begin{aligned} D_\alpha^2 V = &\left(\kappa_\alpha^2 \tau_\alpha - \kappa_\alpha^2 \Omega_1\right) t + \left(s^{1-\alpha}\Omega_1' \kappa_\alpha + s^{1-\alpha}\Omega_1 \kappa_\alpha' - 2s^{1-\alpha}\kappa_\alpha' \tau_\alpha - s^{1-\alpha}\kappa_\alpha \tau_\alpha'\right) n \\ &+ \left(\Omega_1 \kappa_\alpha \tau_\alpha - \kappa_\alpha \tau_\alpha^2 + (1-\alpha)\, s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha''\right) b. \end{aligned} \tag{20}$$

Then, if $V(v) = 0$ in Proposition 1 and Eqs. (9), (10) and (11) are considered in Eq. (18), following equation is obtained

$$s^{1-\alpha}\Omega_1' = 0, \tag{21}$$

where it is clear that $s^{1-\alpha} \neq 0$. So, as can be clearly seen

$$\Omega_1 = c, c \in R.$$

Thus, the first part of the theorem is proved. Then (18) and (20) are considered with $V(\kappa) = 0$ in Proposition 1, we obtain

$$s^{1-\alpha}\Omega_1' \kappa_\alpha + s^{1-\alpha}\Omega_1 \kappa_\alpha' - 2s^{1-\alpha}\kappa_\alpha' \tau_\alpha - s^{1-\alpha}\kappa_\alpha \tau_\alpha' + g(R(V,t)t, n) = 0.$$

In particular, if M^3 has constant curvature C, then

$$g(R(V,t)t, n) = Cg(v, n) = 0,$$

and so,

$$s^{1-\alpha}\Omega_1' \kappa_\alpha + s^{1-\alpha}\Omega_1 \kappa_\alpha' - 2s^{1-\alpha}\kappa_\alpha' \tau_\alpha - s^{1-\alpha}\kappa_\alpha \tau_\alpha' = 0,$$

and

$$(\Omega_1 \kappa_\alpha)' - 2\kappa_\alpha' \tau_\alpha - \kappa_\alpha \tau_\alpha' = 0. \tag{22}$$

If the above equation is arranged, we have

$$\left(\kappa_\alpha^2 \left[\frac{1}{2}\Omega_1 - \tau_\alpha\right]\right)' = 0. \tag{23}$$

Thus, the second part of the theorem is proved. Similarly (18) and (20) are considered with $V(\tau)=0$ in Proposition 1, we obtain

$$s^{1-\alpha}\left(\frac{1}{\kappa_\alpha}\left[\Omega_1\kappa_\alpha\tau_\alpha-\kappa_\alpha\tau_\alpha^2+\{1-\alpha\}\,s^{1-2\alpha}\kappa_\alpha'+s^{2-2\alpha}\kappa_\alpha''+g\{R(V,t)t,b\}\right]\right)'+s^{1-\alpha}\kappa_\alpha\kappa_\alpha'+g(R[V,t]n,b)=0. \quad (24)$$

Hence, if M^3 has constant curvature C, then $g(R[V,t]t,b)=Cg(V,b)=C\kappa_\alpha$ and $g(R[V,t]n,b)=0$. So, we have the following equations

$$s^{1-\alpha}\left(\frac{1}{\kappa_\alpha}\left[\Omega_1\kappa_\alpha\tau_\alpha-\kappa_\alpha\tau_\alpha^2+\{1-\alpha\}\,s^{1-2\alpha}\kappa_\alpha'+s^{2-2\alpha}\kappa_\alpha''+C\kappa_\alpha\right]\right)'+s^{1-\alpha}\kappa_\alpha\kappa_\alpha'=0$$

and

$$\left(\frac{1}{\kappa_\alpha}\left[\Omega_1\kappa_\alpha\tau_\alpha-\kappa_\alpha\tau_\alpha^2+\{1-\alpha\}\,s^{1-2\alpha}\kappa_\alpha'+s^{2-2\alpha}\kappa_\alpha''+C\kappa_\alpha\right]\right)'+\kappa_\alpha\kappa_\alpha'=0.$$

So, the last part of the theorem is proved and the proof is completed. ■

Corollary 1. *Let x be a unit speed fractional t-magnetic curve in $3D$ oriended Riemanian manifold (M^3,g). If the function Ω_1 is a zero and κ_α is non-zero constant function, then the curve x is a helix or circle. Moreover, the axis of the helix is the vector field V.*

Proof. We assume that x be a fractional t-magnetic curve in $3D$ Riemann space with Ω_1 is a zero and κ_α is non-zero constant function, then from Eq. (23), we get

$$\left(\kappa_\alpha^2\left[\frac{1}{2}\Omega_1-\tau_\alpha\right]\right)'=0$$

and

$$\left(-\kappa_\alpha^2\tau_\alpha\right)'=0.$$

If necessary algebric operations are done, we obtain

$$2\kappa_\alpha'\tau_\alpha+\kappa_\alpha\tau_\alpha'=0,$$

and

$$\frac{\tau_\alpha'\kappa_\alpha-\tau_\alpha\kappa_\alpha'}{\kappa_\alpha^2}=-\frac{3\kappa_\alpha'\tau_\alpha}{\kappa_\alpha^2}.$$

Finally, if the above equation is arranged, we get

$$\left(\frac{\tau_\alpha}{\kappa_\alpha}\right)'=-\frac{3\kappa_\alpha'\tau_\alpha}{\kappa_\alpha^2}.$$

Since κ_α is non-zero constant function, we get

$$\left(\frac{\tau_\alpha}{\kappa_\alpha}\right)=\text{constant}.$$

■

Remark 1. *The conformable derivative for differentiable functions is equivalent to a simple change of variable. Precisely, $u=x^\alpha/\alpha$. It should be noted that a criticism of the conformable derivative is that, although conformable at the limit $\alpha\to 1$, it is not conformable $\alpha\to 0$. From the point of view of the assertion about the equality of the conformable derivative to a change of variables, one can say that the conformable derivative is not conformable as at the other limit $\alpha\to 0$ because t^α/α is undefined at $\alpha=0$,* [22].

3.2. Fractional n-magnetic curves

In this section, we redefine the n-magnetic curve with a conformable fractional derivative focus. We are also obtained some characterizations of this curve.

Definition 16. *Let $x:I\subset R\to M^3$ be a conformable curve in 3D oriented Riemannian space (M^3,g) and F be a magnetic field on M. If the vector area of the tangent curve with respect to the conformable frame satisfies the Lorenz force equation, the x curve is called fractional n-magnetic curve, that is*

$$\frac{D_\alpha n}{s^{1-\alpha}}=\phi(n)=V\times n.$$

Proposition 5. *Let $x : I \subset R \rightarrow M^3$ be a unit speed fractional n-magnetic curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M with the conformable frame elements $\{t, n, b, \kappa_\alpha, \tau_\alpha\}$. Lorenz force eqations in the conformable frame are written as*

$$\begin{pmatrix} \phi(t) \\ \phi(n) \\ \phi(b) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_\alpha(s) & 0 \\ -\kappa_\alpha(s) & 0 & \tau_\alpha(s) \\ -\Omega_2(s) & -\tau_\alpha(s) & 0 \end{pmatrix} \begin{pmatrix} t(s) \\ n(s) \\ b(s) \end{pmatrix}, \tag{25}$$

where Ω_2 is a certain function.

Proof. Let $x : I \subset R \rightarrow M^3$ be a unit speed fractional n-magnetic curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M with the conformable frame elements $\{t, n, b, \kappa_\alpha, \tau_\alpha\}$. Since $\phi(t) \in Sp\{t, n, b\}$, we get

$$\phi(t) = \lambda_2 t + \mu_2 n + \sigma_2 b$$

and thus

$$\begin{aligned} \lambda_2 &= g(\phi(t), t) = 0, \\ \mu_2 &= g(\phi(t), n) = -g(\phi(n), t) = \kappa_\alpha, \\ \sigma_2 &= g(\phi(t), b) = \Omega_2. \end{aligned}$$

From the above equations, we can write

$$\phi(t) = \kappa_\alpha n + \Omega_2 b.$$

Similarly, we can easily calculate that

$$\begin{aligned} \phi(n) &= -\kappa_\alpha t + \tau_\alpha b, \\ \phi(b) &= -\Omega_2 t + \tau_\alpha b. \end{aligned}$$

This completes the proof. ■

Proposition 6. *Let x be a unit speed fractional n-magnetic trajectory of a magnetic field V if and only if V can be written along the curve x as*

$$V = \tau_\alpha t - \Omega_2 n + \kappa_\alpha b. \tag{26}$$

Proof. Let x be a unit speed fractional n-magnetic trajectory of a magnetic field V. Using Proposition 3 and Eq. (6), we can easily see that

$$V = \tau_\alpha t - \Omega_2 n + \kappa_\alpha b.$$

This completes the proof. ■

Theorem 7. *Let x be a unit speed fractional n-magnetic trajectory and V be a Killing vector field on a simply connected space form $\left(M^3, g\right)$. Then the following equations exist*

$$\begin{aligned} s^{1-\alpha}\tau'_\alpha + \Omega_2\kappa_\alpha &= 0, \\ (\alpha-1)\, s^{1-2\alpha}\Omega'_2 - s^{2-2\alpha}\Omega''_2 - s^{1-\alpha}\kappa'_\alpha\tau_\alpha + \Omega_2\tau_\alpha^2 &= C\Omega_2, \\ \left(\frac{1}{\kappa_\alpha}\left[-s^{1-\alpha}\Omega'_2\tau_\alpha + s^{2-2\alpha}\kappa''_\alpha - s^{1-\alpha}\left\{\tau_\alpha\Omega_2\right\}' + \left\{1-\alpha\right\} s^{1-2\alpha}\kappa'_\alpha\right]\right)' + \kappa_\alpha\left(\kappa'_\alpha - s^{\alpha-1}\tau_\alpha\Omega_2\right) &= 0 \end{aligned}$$

where C is the curvature of the Riemanian space M^3.

Proof. Let V be a magnetic field in a Riemanian 3D manifold. If the $\alpha-$th conformable fractional derivative of Eq. (26) is taken with respect to s and conformable frame formulas are applied, we have

$$\begin{aligned} D_\alpha V &= D_\alpha(\tau_\alpha t) - D_\alpha(\Omega_2 n) + D_\alpha(\kappa_\alpha b), \\ D_\alpha V &= \left(s^{1-\alpha}\tau'_\alpha + \kappa_\alpha\Omega_2\right) t - s^{1-\alpha}\Omega'_2 n + \left(s^{1-\alpha}\kappa'_\alpha - \Omega_2\tau_\alpha\right) b. \end{aligned} \tag{27}$$

It can be easily seen that if $V(v) = 0$ of Proposition 1, the case is $g(D_\alpha V, t) = 0$. So, if this equation is used in the above equation, we get

$$D_\alpha V = -s^{1-\alpha}\Omega'_2 n + \left(s^{1-\alpha}\kappa'_\alpha - \Omega_2\tau_\alpha\right) b,$$

is obtained. If the conformable derivative of the above equation with respect to s is taken once again from the $\alpha-$th order and conformable frame formulas are applied, we have

$$\begin{aligned}D_\alpha^2 V &= \left(-\left(1-\alpha\right)s^{1-2\alpha}\Omega_2' n - s^{2-2\alpha}\Omega_2''\right) n - s^{1-\alpha}\Omega_2'\left(-\kappa_\alpha t + \tau_\alpha b\right)\\ &+ \left(\left(1-\alpha\right)s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha'' - s^{1-\alpha}\Omega_2'\tau_\alpha - s^{1-\alpha}\Omega_2\tau_\alpha'\right) b - \left(s^{1-\alpha}\kappa_\alpha'\tau_\alpha - \Omega_2\tau_\alpha^2\right) n.\end{aligned}$$

If the equation is arranged, we obtain

$$\begin{aligned}D_\alpha^2 V &= s^{1-\alpha}\Omega_2'\kappa_\alpha t + \left(\left(\alpha-1\right)s^{1-2\alpha}\Omega_2' - s^{2-2\alpha}\Omega_2'' - s^{1-\alpha}\kappa_\alpha'\tau_\alpha + \Omega_2\tau_\alpha^2\right) n\\ &+ \left(-s^{1-\alpha}\Omega_2'\tau_\alpha + \left(1-\alpha\right)s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha'' - s^{1-\alpha}\Omega_2'\tau_\alpha - s^{1-\alpha}\Omega_2\tau_\alpha'\right) b.\end{aligned} \tag{28}$$

Then, if $V(v) = 0$ in Proposition 1 and Eqs. (9), (10) and (11) are considered in Eq. (27), we have

$$s^{1-\alpha}\tau_\alpha' + \kappa_\alpha\Omega_2 = 0. \tag{29}$$

Thus, the first part of the theorem is proved. Then Eqs. (27) and (28) are considered with $V(\kappa) = 0$ in Proposition 1, we obtain

$$\left(\alpha-1\right)s^{1-2\alpha}\Omega_2' - s^{2-2\alpha}\Omega_2'' - s^{1-\alpha}\kappa_\alpha'\tau_\alpha + \Omega_2\tau_\alpha^2 + g(R(V,t)t, n) = 0.$$

In particular, if M^3 has constant curvature C, then

$$g(R(V,t)t, n) = Cg(V, n) = -C\Omega_2$$

and so,

$$\left(\alpha-1\right)s^{1-2\alpha}\Omega_2' - s^{2-2\alpha}\Omega_2'' - s^{1-\alpha}\kappa_\alpha'\tau_\alpha + \Omega_2\tau_\alpha^2 = C\Omega_2. \tag{30}$$

Thus, the second part of the theorem is proved. Similarly Eqs. (27) and (28) are considered with $V(\tau) = 0$ in Proposition 1, we obtain

$$\begin{aligned}&s^{1-\alpha}\left(\frac{1}{\kappa_\alpha}\left[-s^{1-\alpha}\Omega_2'\tau_\alpha + \left\{1-\alpha\right\}s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha'' - s^{1-\alpha}\left\{\Omega_2\tau_\alpha\right\}' + g(R(V,t)t, b)\right]\right)'\\ &+ s^{1-\alpha}\kappa_\alpha\kappa_\alpha' + g(R(V,t)n, b) = 0.\end{aligned} \tag{31}$$

Hence, if M^3 has constant curvature C, then $g(R(V,t)t, b) = Cg(V, b) = C\kappa_\alpha$ and $g(R(V,t)n, b) = 0$. So we obtain following

$$\left(\frac{1}{\kappa_\alpha}\left[\Omega_2\kappa_\alpha\tau_\alpha - \kappa_\alpha\tau_\alpha^2 + \left\{1-\alpha\right\}s^{1-2\alpha}\kappa_\alpha' + s^{2-2\alpha}\kappa_\alpha'' + C\kappa_\alpha\right]\right)' + \kappa_\alpha\left(\kappa_\alpha' - s^{\alpha-1}\Omega_2\tau_\alpha\right) = 0.$$

Thus, the last part of the theorem is proved and the proof is completed.■

Corollary 2. *Considering Ω_2 is a non-zero constant function, we easily see that the fractional n-magnetic curve is a curve in the Euclidean 3-space.*

Corollary 3. *Let x be a unit speed fractional n-magnetic curve in 3D oriented Riemann manifold $\left(M^3, g\right)$. If the function Ω_2 is non-zero constant, then the curve x is a slant helix. Moreover, the axis of the slant helix is the vector field V.*

Proof. We assume that x is a fractional n-magnetic curve in Euclidean 3-space with non-zero constant function Ω_2, then from (29), (30) and (31), we have

$$\Omega_2 = -s^{1-\alpha}\frac{\tau_\alpha'}{\kappa_\alpha} = s^{1-\alpha}\frac{\kappa_\alpha'}{\tau_\alpha}.$$

If the above equation is arranged, we get

$$\kappa_\alpha^2 + \tau_\alpha^2 = \text{constant}.$$

If necessary arregements are made, we obtain

$$\tau_\alpha'\kappa_\alpha - \kappa_\alpha'\tau_\alpha = -\Omega_2\left(\kappa_\alpha^2 + \tau_\alpha^2\right),$$

and

$$\Omega_2 = \frac{\kappa_\alpha^2}{\kappa_\alpha^2 + \tau_\alpha^2}\left(\frac{\tau_\alpha}{\kappa_\alpha}\right)'.$$

These complete the proof. ■

3.3. Fractional b-Magnetic Curves

In this section, we define the b-magnetic curve with a conformable fractional derivative focus. We are also obtained some characterizations of this curve.

Definition 17. *Let $x : I \subset R \to M^3$ be a conformable curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M. If the vector area of the tangent curve with respect to the conformable frame satisfies the Lorenz force equation, the x curve is called fractional b-magnetic curve, that is*

$$\frac{D_\alpha b}{s^{1-\alpha}} = \phi(b) = V \times b.$$

Proposition 7. *Let $x : I \subset R \to M^3$ be a unit speed fractional b-magnetic curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M with the conformable frame elements $\{t, n, b, \kappa_\alpha, \tau_\alpha\}$. So, Lorenz force according to the conformable frame is written as*

$$\begin{pmatrix} \phi(t) \\ \phi(n) \\ \phi(b) \end{pmatrix} = \begin{pmatrix} 0 & \Omega_3(s) & 0 \\ -\Omega_3(s) & 0 & \tau_\alpha(s) \\ 0 & -\tau_\alpha(s) & 0 \end{pmatrix} \begin{pmatrix} t(s) \\ n(s) \\ b(s) \end{pmatrix}, \tag{32}$$

where Ω_3 is a certain function.

Proof. Let $x : I \subset R \to M^3$ be a unit speed fractional b-magnetic curve in 3D oriented Riemannian space (M^3, g) and F be a magnetic field on M with the conformable frame elements $\{t, n, b, \kappa_\alpha, \tau_\alpha\}$. Since $\phi(t) \in Sp\{t, n, b\}$, we get

$$\phi(t) = \lambda_3 t + \mu_3 n + \sigma_3 b$$

and thus

$$\lambda_3 = g(\phi(t), t) = 0,$$
$$\mu_3 = g(\phi(t), n) = \Omega_3(s),$$
$$\sigma_3 = g(\phi(t), b) = 0.$$

From the above equations.,we can write

$$\phi(t) = \Omega_3 n.$$

Similarly, we can easily calculate that

$$\phi(n) = -\Omega_3 t + \tau_\alpha b,$$
$$\phi(b) = -\tau_\alpha n.$$

This completes the proof. ■

Proposition 8. *Let x be a unit speed fractional b-magnetic trajectory of a magnetic field V if and only if V can be written along the curve x as*

$$V = \tau_\alpha t + \Omega_3 b. \tag{33}$$

Proof. Let x be a unit speed fractional b-magnetic trajectory of a magnetic field V. Using Proposition 3 and Eq. (6), we can easily see that

$$V = \tau_\alpha t + \Omega_3 b.$$

This completes the proof. ■

Theorem 8. *Let x be a unit speed fractional b-magnetic trajectory and V be a Killing vector field on a simply connected space form $\left(M^3, g\right)$. Then the following equations exist*

$$s^{1-\alpha}\tau'_\alpha = 0,$$
$$\kappa'_\alpha \tau_\alpha - 2\tau_\alpha \Omega'_3 = 0,$$
$$\left(\frac{1}{\kappa_\alpha}\left[\kappa_\alpha \tau_\alpha^2 - \Omega_3 \tau_\alpha^2 + \{1-\alpha\} s^{1-2\alpha}\Omega'_3 + s^{2-2\alpha}\Omega'''_3 + C\kappa_\alpha\right]\right)' + s^{1-\alpha}\Omega'_3 \kappa_\alpha = 0.$$

Proof. Let V be a magnetic field in a Riemanian 3D manifold. If the $\alpha-$th conformable fractional derivative of Eq. (33) is taken with respect to s and conformable frame formulas are applied, we have

$$D_\alpha V = s^{1-\alpha}\tau_\alpha' t + (\kappa_\alpha\tau_\alpha - \Omega_3\tau_\alpha)\, n + s^{1-\alpha}\Omega_3' b. \tag{34}$$

It can be easily seen that if $V(v)=0$ of Proposition 1, the case is $g(D_\alpha V, t)=0$. So if this equation is used in the above on,

$$D_\alpha V = (\kappa_\alpha\tau_\alpha - \Omega_3\tau_\alpha)\, n + s^{1-\alpha}\Omega_3' b,$$

is obtained. If the conformable derivative of the above equation with respect to s is taken once again from the $\alpha-$th order and conformable frame formulas are applied, we have

$$\begin{aligned} D_\alpha^2 V &= \left(s^{1-\alpha}\kappa_\alpha'\tau_\alpha + s^{1-\alpha}\kappa_\alpha\tau_\alpha' - s^{1-\alpha}\Omega_3'\tau_\alpha - s^{1-\alpha}\Omega_3\tau_\alpha'\right) n \\ &\quad + (\kappa_\alpha\tau_\alpha - \Omega_3\tau_\alpha)(-\kappa_\alpha t + \tau_\alpha b) + (1-\alpha)\, s^{1-2\alpha}\Omega_3' b + s^{2-2\alpha}\Omega_3'' b - s^{1-\alpha}\Omega_3'\tau_\alpha n. \end{aligned}$$

If the equation is arranged, we get

$$D_\alpha^2 V = \left(\kappa_\alpha\tau_\alpha\Omega_3 - \kappa_\alpha^2\tau_\alpha\right) t + \left(s^{1-\alpha}\kappa_\alpha'\tau_\alpha - 2s^{1-\alpha}\Omega_3'\tau_\alpha\right) n + \left(\kappa_\alpha\tau_\alpha^2 - \Omega_3\tau_\alpha^2 + (1-\alpha)\, s^{1-2\alpha}\Omega_3' + s^{2-2\alpha}\Omega_3''\right) b. \tag{35}$$

Then, if $V(v)=0$ in Proposition 1 and Eqs. (9), (10) and (11) are considered in equation (34), we have

$$s^{1-\alpha}\tau_\alpha' = 0. \tag{36}$$

Thus, the first part of the theorem is proved. Then equations (34) and (35) are considered with $V(\kappa)=0$ in Proposition 1, we obtain

$$s^{1-\alpha}\kappa_\alpha'\tau_\alpha - 2s^{1-\alpha}\Omega_3'\tau_\alpha + g(R(V,t)t, n) = 0.$$

In particular, if M^3 has constant curvature C, then $g(R(V,t)t,n) = Cg(V,n) = 0$ and so following equation

$$\kappa_\alpha'\tau_\alpha - 2\Omega_3'\tau_\alpha = 0, \tag{37}$$

is obtained. Thus, the second part of the theorem is proved. Similarly if the Eqs. (34) and (35) are considered with $V(\tau)=0$ in Proposition 1, we obtain

$$\begin{aligned} s^{1-\alpha} &\left(\frac{1}{\kappa_\alpha}\left[\kappa_\alpha\tau_\alpha^2 - \Omega_3\tau_\alpha^2 + \{1-\alpha\}\, s^{1-2\alpha}\Omega_3' + s^{2-2\alpha}\Omega_3''' + g\{R(V,t)t, b\}\right]\right)' \\ &+ s^{1-\alpha}\Omega_3'\kappa_\alpha + g(R[V,t]n, b) = 0. \end{aligned}$$

Hence, if M^3 has constant curvature C, then $g(R(V,t)t,b) = Cg(V,b) = C\kappa_\alpha$ and $g(R(V,t)n,b)=0$. So, we have following

$$\left(\frac{1}{\kappa_\alpha}\left[\kappa_\alpha\tau_\alpha^2 - \Omega_3\tau_\alpha^2 + \{1-\alpha\}\, s^{1-2\alpha}\Omega_3' + s^{2-2\alpha}\Omega_3''' + C\kappa_\alpha\right]\right)' + s^{1-\alpha}\Omega_3'\kappa_\alpha = 0.$$

Thus, the last part of the theorem is proved and the proof is completed. ■

Corollary 4. *Let x be a fractional b-magnetic curve in $3D$ oriented Riemanian manifold (M^3, g). If the function Ω_3 is a constant function, then the curve x is a general helix. Moreover, the axis of the general helix is the vector field V.*

Proof. We assume that x be a unit speed fractional b-magnetic curve in Euclidean 3-space with Ω_3 is a constant function. Then from the equation (37), we get

$$\kappa_\alpha'\tau_\alpha - 2\Omega_3'\tau_\alpha = 0$$

and

$$\Omega_3' = \frac{\kappa_\alpha'\tau_\alpha}{2\tau_\alpha}.$$

Since Ω_3 is a constant function, it can be say that κ_α is a constant function. In addition, considering the equation (36), the following equation can be easily seen

$$\frac{\tau_\alpha}{\kappa_\alpha} = \text{constant}.$$

This completes the proof. ■

Example 1. Let x be a fractional t-magnetic trajectory of a magnetic field V. If the tangent vector field t is perpendicular to the magnetic field, the Lorentz force is maximum and the moves by the particle for different α values are given in Figs. 1 and 2 in the magnetic field.

$$x(s) = \left(-\int s^{1-\alpha} \sin s, \int s^{1-\alpha} \cos s, 4s^{1-\alpha} \right).$$

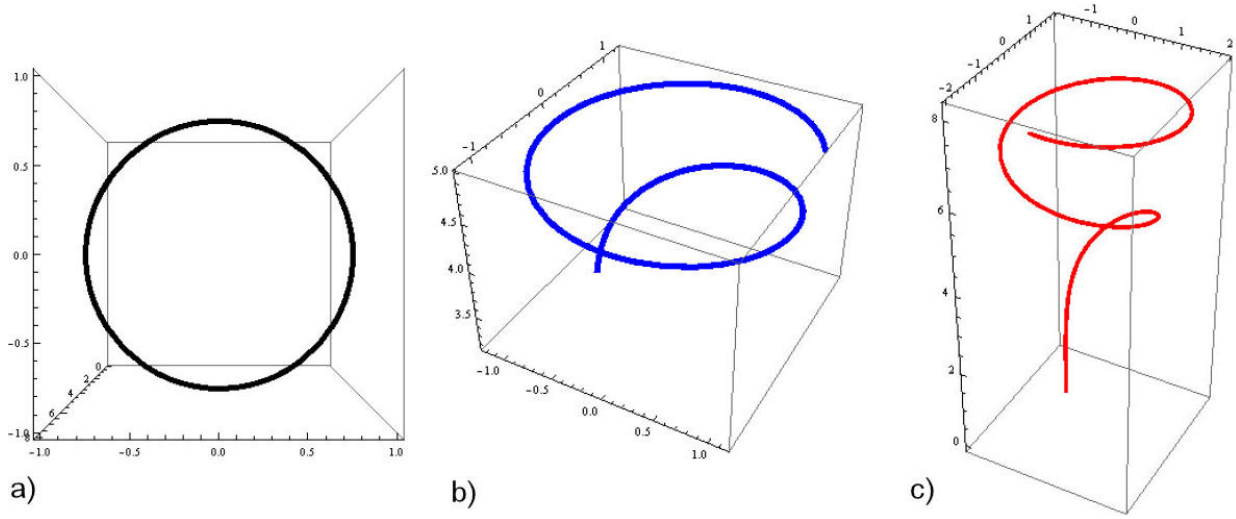

FIGURE 1. Fractional t-magnetic curve $x(s)$ for $\alpha \to 1(Black),\ \alpha = 0.9(Blue)$ and $\alpha = 0.7(Red)$, respectively.

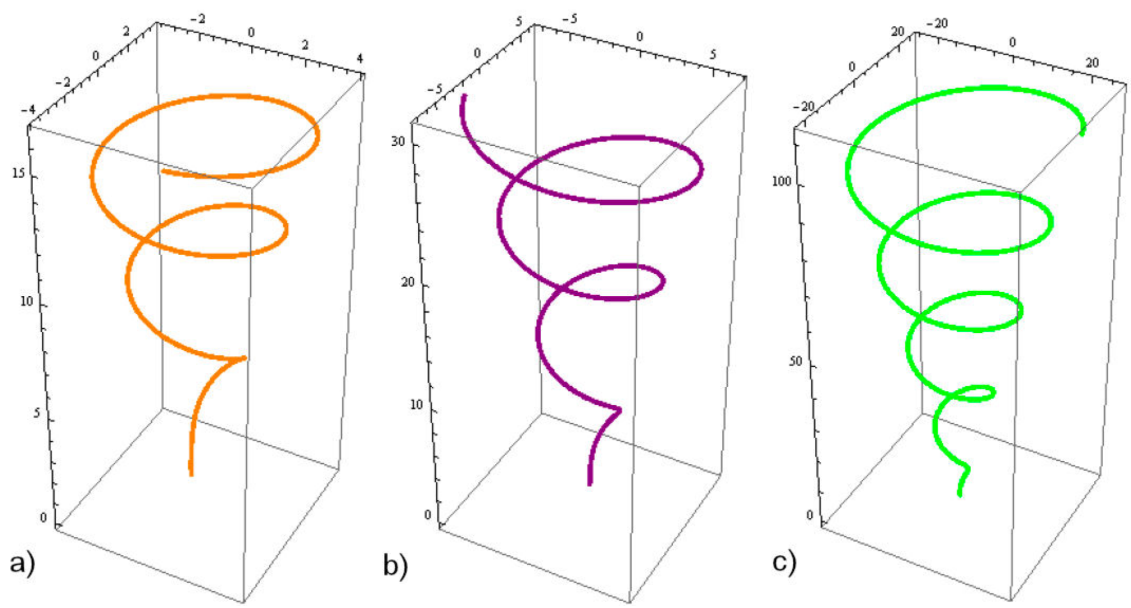

FIGURE 2. Fractional t-magnetic curve $x(s)$ for $\alpha = 0.5(Orange)$, $\alpha = 0.3(Purple)$ and $\alpha = 0.1(Green)$, respectively.

Example 2. Let x be a fractional t-magnetic trajectory of a magnetic field V. From Corollary 1, we can easily see that $\Omega_1 = 0$ and κ_α is a constant function. The figure of the t-magnetic curve for different α values are given in Figs. 3 and 4 in the magnetic field.

$$x(s) = \left(-\frac{3}{49}\int s^{1-\alpha}\sin s, \frac{4}{49}\int s^{1-\alpha}\cos s, \frac{5}{49}\int s^{1-\alpha}\right).$$

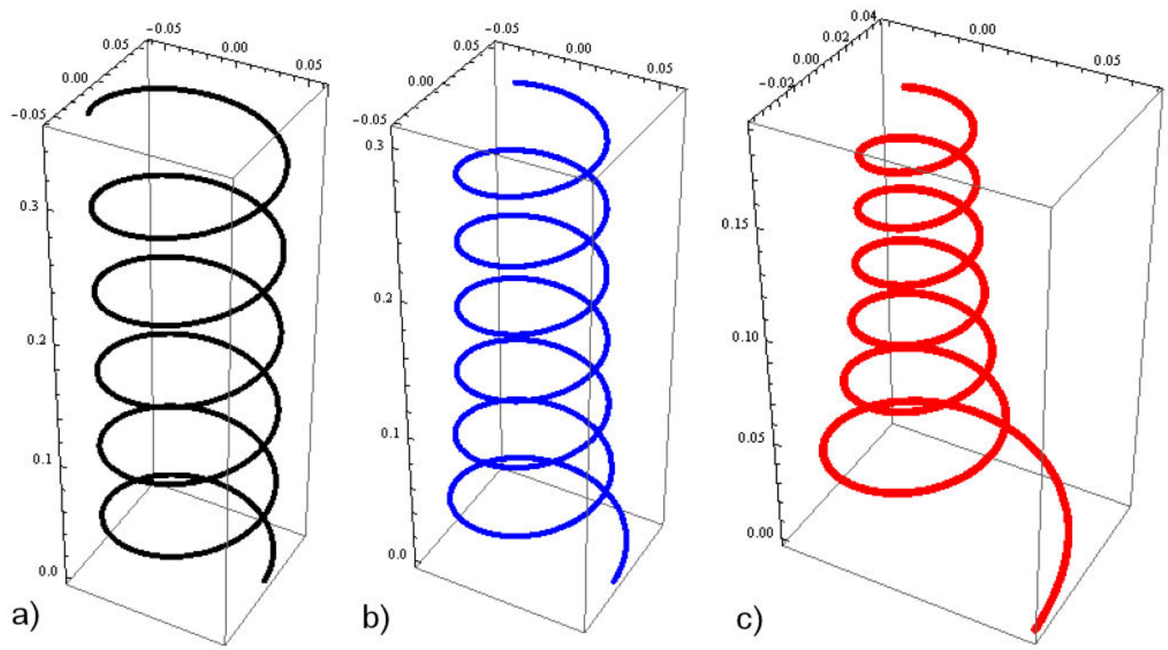

FIGURE 3. Fractional t-magnetic curve x(s) for $\alpha \to 1$(Black), $\alpha = 0.9(Blue)$ and $\alpha = 0.7(Red)$, respectively.

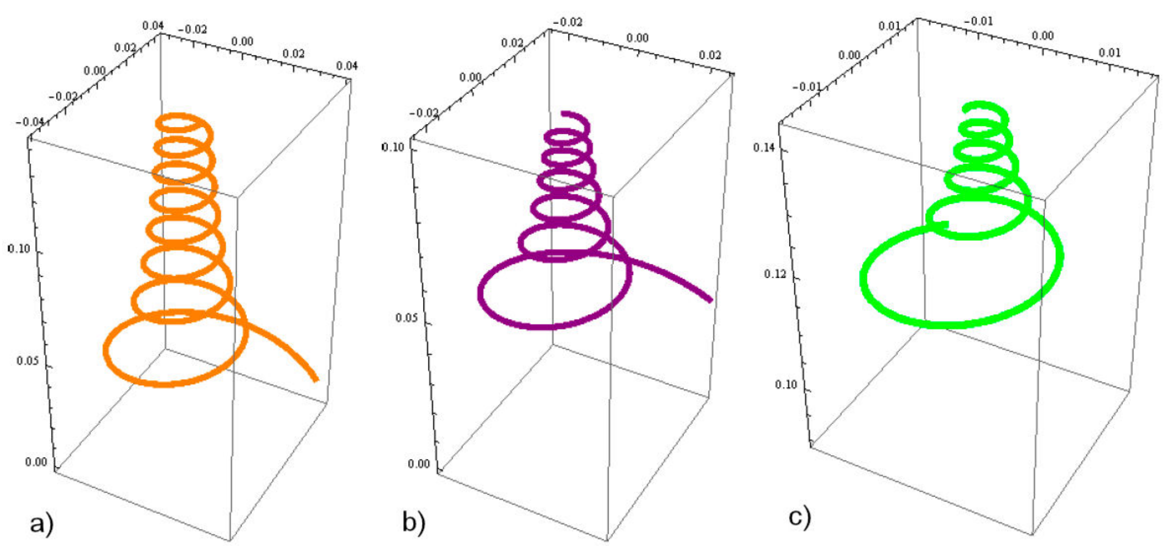

FIGURE 4. Fractional t-magnetic curve $x(s)$ for $\alpha = 0.5(Orange)$, $\alpha = 0.3(Purple)$ and $\alpha = 0.1(Green)$, respectively.

Example 3. Let x be a fractional n-magnetic trajectory of a magnetic field V. From Corollary 3, we can easily see that Ω_2 is non-zero constant. The moves by the particle for different α values are given in Figs. 5 and 6 in the magnetic field.

$$x(s) = \left(\frac{9}{16}\int s^{1-\alpha}\cos 25s + \frac{25}{16}\int s^{1-\alpha}\cos 9s,\ \frac{9}{16}\int s^{1-\alpha}\sin 25s - \frac{25}{16}\int s^{1-\alpha}\sin 9s,\ \frac{15}{8}\int s^{1-\alpha}\cos 17s\right)$$

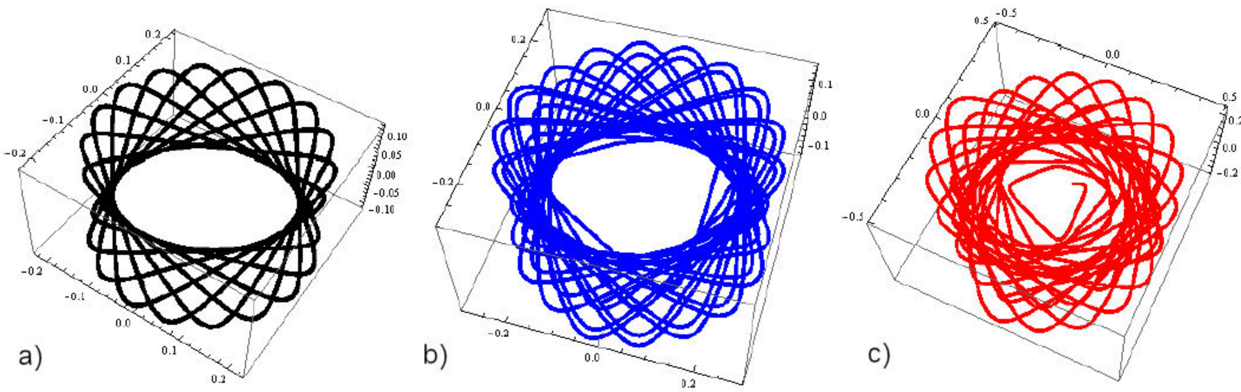

FIGURE 5. Fractional n-magnetic curve $x(s)$ for $\alpha \to 1(Black)$, $\alpha = 0.9(Blue)$ and $\alpha = 0.7(Red)$, respectively.

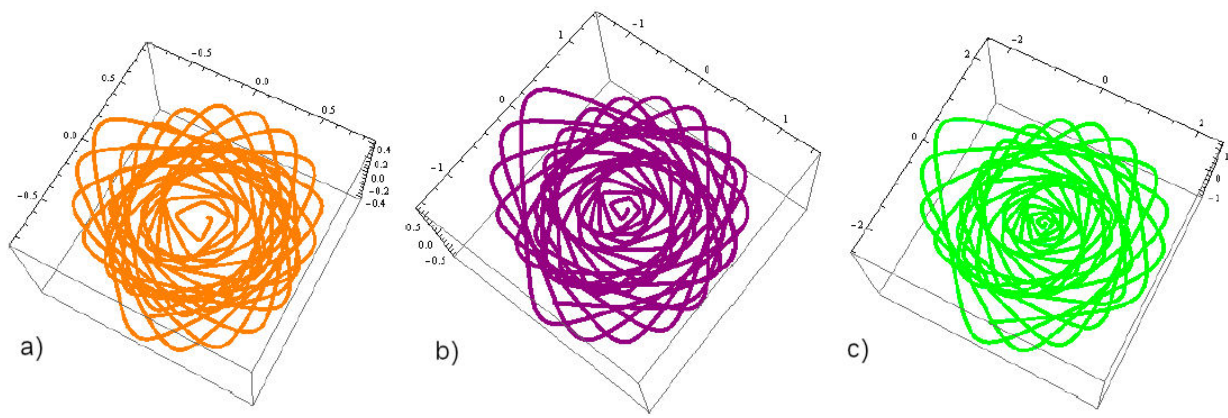

FIGURE 6. Fractional n-magnetic curve $x(s)$ for $\alpha = 0.5(Orange)$, $\alpha = 0.3(Purple)$ and $\alpha = 0.1(Green)$, respectively.

4. Conclusion

In this article, starting from the effect on the curves the effects of conformable fractional derivatives and integrals on magnetic curves are investigated. The Frenet frame has been tried to be formed with the help of conformable derivative of a unit speed conformable curve. However, as can be seen from Eq. (14), the Frenet frame of the unit speed curve is not affected by the conformable derivative, that is, the elements of the Frenet frame have not undergone any change under the conformable derivative. By U.Gözütok *et al.* are mentioned in article [33], the physical properties (velocity, speed, arc-length) of the unit speed conformable curve do not change under the conformable derivative. On the other hand, curvature and torsion concepts are one of the most important factors in determining the characterization of the curve, as those who work on the theorem of curves, which is one of the sub-branches of differential geometry, know very well. Therefore, the difference of this study from the others is that the curvature and torsion of a curve are obtained depending on the fractional derivative. As can be seen from Conclusion 2, the curvatures of the conformable curve have changed under the conformable derivative. In this study, this change in the curvature of the curve is examined and visualized with various examples to better understand the results.

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