A static spherically symmetric perfect fluid solution to model the interior of stars

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Received 1 January 2022; accepted 9 February 2022

An exact solution for modeling the interior of stars with perfect fluid is presented, the geometry of their interior is described by a static and spherically symmetric regular space-time. The hydrostatic functions are physically acceptable for the compactness rate $u = GM/c^2R \in (0, 0.3183497]$, the speed of sound is a monotonically decreasing function, positive and lower than the speed of light, which implies that the condition of causality is not violated, meanwhile the stability of the solution is guaranteed due to the adiabatic index $\gamma > 3.08387$ and it is a monotonically increasing function. The analysis of the solution is presented graphically for specific values of the compactness on the interval $u \in [0.2509338, 0.3183497]$ with the minimum value of this interval associated to the neutron star PSR J0348+0432, for observational data which generates the maximum compactness when the radius is minimal $R = 12.062$ km and the mass is maximum $M = 2.05$ $M_\odot$, generating a value of the central density $\rho_c = 7.520589 \times 10^{17}$ kg/m^3.

Keywords: General relativity; exact solutions; perfect fluid; relativistic compact stars.

DOI: https://doi.org/10.31349/RevMexFis.69.030701

1. Introduction

One of the investigation themes that maintains it's relevancy since the proposal of Einstein's theory of general relativity is the one related to exact interior solutions, due to their usefulness for modeling stellar objects. A task which is not simple, with the construction of exact solutions for chargeless perfect fluids being more complicated when comparing it with the case of charged perfect fluid or solutions with anisotropic pressures. In the last century a number of exact solutions with perfect fluid were presented, however, most of them are not physically acceptable [1–10]. Which contrasts with the case of charged perfect fluid or fluid with anisotropic pressures for which, parting from a physically acceptable solution with perfect fluid, we can obtain a wide array of physically acceptable solutions, besides the inherent importance of finding exact solutions with perfect fluid, there is an additional interest since these could be used as seeds to obtain charged or anisotropic models, one path for this is through the Minimal Geometric Deformation method [11–14]. Although its construction can also be done by other alternatives, according to the objectives of the investigation work intended to be developed [15–21]. Given the difficulty of obtaining exact solutions to Einstein's equations with a perfect fluid and with a state equation, in a static and spherically symmetric space time, it becomes necessary to use numeric methods to describe the solutions to these equations in a graphic manner [22–25]. As such, in most cases, the physically acceptable exact solutions that are constructed suppose a specific form of the metric function g_{tt} or g_{rr}, and not a state equation, and although in some cases a condition is proposed, as it occurs in the Tolman solutions [26], this is the equivalent of giving a metric function. Until a bit over five years ago, in the works related with regular solutions with perfect fluid and in which it is specified the component g_{tt} [27–29], it was supposed $g_{tt}(r) = (1 + ar^2)^n$ with n an integer or a rational number, recently new physically acceptable solutions have been presented in which $g_{tt}(r)$ no longer has this structure [30–34], but instead the coefficient of two functions, one of these being $g_{tt} = -S(5 + 4ar^2)^2/(1 + ar^2)$ [34]. Combining the previous works that reflect the possibility of having physically acceptable solutions with $g_{tt}(r) = -S(1 + ar^2)^n$ for different values of n, it's natural to investigate if there exist physically acceptable exact solutions with $g_{tt} = -S[(5 + 4ar^2)^2/(1 + ar^2)]^n$ for different values of n. For the case with $n = 1$, discussed previously [34], it was shown that the density and the pressure are monotonically decreasing functions and that the speed of sound is monotonically increasing, with maximum compactness rate $u = GM/c^2R = 0.2660858316$ and it was applied for the star PSR J0348+0432. In this work we show that for $n = 3$ we also have a physically acceptable exact solution with a compactness value greater than the case $n = 1$, as such the new solution would be useful for representing a broader spectrum of stars.

The organization of this work is as follows: the Sec. 2 is dedicated to the presentation of the equations that describe a static sphere with perfect fluid and to the construction of the solution starting from the metric function g_{tt}. In the Sec. 3 we mention the required conditions in the center, in the interior and on the surface for a solution of Einstein's system of equations with perfect fluid to be physically acceptable, starting from the conditions, we determine the intervals of validity of the solution. In the Sec. 4 a representation and graphic analysis for some values of the compactness rate is done, showing that it is physically acceptable, due to the hydrostatic functions being monotonically decreasing, that the condition of causality is not violated and that the solution is stable. In the Sec. 5 we do a comparison between the cases $n = 1$ (that was shown previously [34]) and $n = 3$ (developed in this work), as well as a discussion of the outstanding characteristics of the new solution.

2. The system

The type of matter that we consider in the description of the star's interior is given by a perfect fluid, as such the energy-momentum tensor is described by:

$$T_{\mu\nu} = c^2\rho u_\mu u_\nu + P(u_\mu u_\nu + g_{\mu\nu}), \tag{1}$$

where ρ is the energy density, P is the pressure and u^μ the four velocity components. Meanwhile the geometry is static and spherically symmetric, as such, the line element can be represented by:

$$ds^2 = -y(r)^2dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

The relation between the hydrostatic functions and the metric components is given by Einstein's equations, $G_{\mu\nu} = kT_{\mu\nu}$, which leads us to the following system of ordinary coupled differential equations:

$$kc^2\rho = -\frac{B'}{r} + \frac{1-B}{r^2}, \tag{3}$$

$$kP = \frac{2By'}{ry} - \frac{1-B}{r^2}, \tag{4}$$

$$kP = \frac{(ry'' + y')B}{ry} + \frac{(ry' + y)B'}{2ry}, \tag{5}$$

with $k = 8\pi G/c^4$ and $'$ denotes the derivative with respect to the radial coordinate r. As well as, the equation of conservation $\nabla_\mu T^\mu{}_\nu = 0$ gives origin to the well known Tolman-Oppenheimer-Volkov (TOV) equation [35,36]:

$$P' = -\frac{\left(P + c^2\rho\right)y'}{y}, \tag{6}$$

although this last one is not an independent equation, since it can be obtained from the system of Eqs. (3) - (5). Being this the set of equations for which we will obtain the solution starting from a function $y(r)$.

2.1. The solution

Starting from a given solution $y(r)$ facilitates the integration of the system because when we compare the Eqs. (4) and (5) we obtain a linear first order non homogeneous differential equation, although this does not guarantee that it will admit an analytical solution nor that the solution to the system of equations is physically acceptable. We can have an exact solution but it can occur that when the solution is not regular, or even when the solution is regular, that the condition of causality is violated [1] or that the density and pressure are not both positive [37]. In this work, based in a metric potential $y_{\text{old}}(r)$ that was used before and allows to describe stars with a compactness rate $u = GM/c^2R \leq 0.2660858316$ [34], we suppose a new function $y(r) = y_{\text{old}}(r)^3$:

$$y(r) = S\left(\frac{5 + 4\,ar^2}{\sqrt{1 + ar^2}}\right)^3, \tag{7}$$

where S and a are constants. From the isotropy in the pressures, subtracting the Eqs. (4) and (5) and substituting the function y given by (7) it results:

$$B' - \frac{2(25 + 90ar^2 + 82a^2r^4 - 32a^4r^8)B}{(1 + ar^2)(5 + 4ar^2)(1 + 2ar^2)(5 + 8ar^2)r} + \frac{2(1 + ar^2)(5 + 4ar^2)}{(1 + 2ar^2)(5 + 8ar^2)r} = 0, \tag{8}$$

the integration of this equation leads us to:

$$B(r) = 1 - \frac{[101612 + 719063ar^2 + S_1(r)]ar^2}{(5 + 8ar^2)^3(5 + 4ar^2)^4} + \frac{64(1 + 2ar^2)(1 + ar^2)^5 S_2(r)ar^2}{3(5 + 8ar^2)^{7/2}(5 + 4ar^2)^4}, \tag{9}$$

with

$$S_1(r) = 2053262a^2r^4 + 3056936a^3r^6 + 2519264a^4r^8 + 1094720a^5r^{10} + 196608a^6r^{12},$$

$$S_2(r) = C - 191\sqrt{3}\arctan\left[\frac{\sqrt{3}\sqrt{5 + 8\,ar^2}}{1 + 4ar^2}\right] + 4131\,\text{arctanh}\left[\frac{\sqrt{5 + 8\,ar^2}}{3 + 4\,ar^2}\right],$$

and C is the integration constant. Once determined the function B and from the function y proposed by the substitution of these in the Eqs. (3) and (4) arriving to

$$kc^2\rho(r) = \frac{6H(r)a^2r^4 + 3\left(25 + 105\,ar^2 + 142\,a^2r^4 + 72\,a^3r^6\right)(1-B)}{(1+ar^2)(1+2\,ar^2)(5+4\,ar^2)(5+8\,ar^2)\,r^2}, \tag{10}$$

$$kP(r) = \frac{6a\left(3+4\,ar^2\right)}{(5+4\,ar^2)(1+ar^2)} - \frac{\left(1+4\,ar^2\right)\left(5+7\,ar^2\right)(1-B)}{(1+ar^2)(5+4\,ar^2)\,r^2}, \tag{11}$$

where $H(r) = 13 + 24\,ar^2 + 16\,a^2r^4$. It's important to note that from the function B given by (9) we have that the term $(1-B)/r^2$ is regular, which guarantees the regularity of the density and the pressure in the center. Also, by the rule of the chain, we obtain the speed of sound:

$$v^2 = \frac{\partial P(\rho)}{\partial \rho} = \frac{dP(r)}{dr}\bigg/\frac{d\rho(r)}{dr},$$

the remaining expressed in the form:

$$\frac{v^2(r)}{c^2} = \frac{(5+8ar^2)(3+4ar^2)\left[B\,S_3(r) - (5+4ar^2)^2(1+ar^2)^2\right]}{B\,S_4(r) - (75+124ar^2+64a^2r^4)(5+4\,ar^2)^2(1+ar^2)^2}, \tag{12}$$

where

$$S_3(r) = 25 + 45ar^2 - 140a^2r^4 - 360a^3r^6 - 224a^4r^8,$$
$$S_4(r) = 1875 + 11475ar^2 + 26880a^2r^4 + 32440\,a^3r^6 + 20800\,a^4r^8 + 5760\,a^5r^{10}.$$

The speed of sound will be of use in the analysis of the conditions required for the solution to be physically acceptable and, particularly, in relation to the non violation of the causality. For a solution to be physically acceptable it's necessary that some criteria of regularity are met for the geometry, for the hydrostatic functions as well as conditions of behaviour in the interior and on the surface, these will be numbered in the following section.

3. Criteria for physical acceptability

The conditions that allow us to determine if an exact solution to Einstein's equations is physically acceptable have been stated in different works, their essential content can be classified in conditions on the regularity of the geometry and from the sources of matter, as well as in: the behaviour of the hydrostatic variables, energy conditions, stability conditions and causality conditions [1,38]:

(a) **Regularity conditions**. The magnitude of the static Killing vector field $\xi = \partial/\partial t$ must satisfy $g(\xi,\xi) = -y(r)^2 < 0$, $\forall r \le R$, where R is the radius of the star. The geometry and physical quantities must be regular $\forall r \le R$. In particular, from the regularity of the Kretschmann scalar near the center we get that the behavior of the metric components satisfy:

$$B(r) \approx 1 + \alpha r^2 + 0(r^4), \quad B' \approx \beta r + 0(r^3),$$
$$y(r) \approx \mu + \nu r^2 + 0(r^4), \quad y' \approx \sigma r + 0(r^3),$$

where α, β, μ, ν and σ are nonzero parameters.

(b) **Behavior of hydrostatic functions**. The pressure and density must be finite and positive, with their maximum value on the center and monotonically decreasing towards the boundary of the fluid sphere, *i.e.*, $\rho' < 0$ and $P' < 0$ for $r \in (0,R)0$ and

$$P(0) > 0, \quad P'(0) = 0, \quad P''(0) < 0,$$
$$\rho(0) > 0, \quad \rho'(0) = 0, \quad \rho''(0) < 0,$$

also, the pressure must be nullified on the surface $P(R) = 0$.

(c) **Energy conditions**. In addition to the intuitive physical requirements mentioned above, the interior solution should satisfy either:

- The Strong Energy Condition: $c^2\rho + 3P \ge 0$, $c^2\rho + P \ge 0$ or
- The Dominant Energy Condition: $\rho \ge 0$ and $c^2\rho \ge |P|$

(e) **Causality condition**. The speed of sound must not exceed the speed of light, which implies

$$v(r)^2 = \frac{dP(\rho)}{d\rho} \in [0, c^2].$$

(f) **Stability condition**. In order to have an equilibrium configuration the matter must be stable and, as a required condition, the relativistic adiabatic index

$$\Gamma = \frac{P + c^2\rho}{P}\frac{dP}{d\rho} > \frac{4}{3}, \qquad \forall\, r \in [0,R].$$

(g) **Matching condition**. On the surface of the star $r = R$, the interior solution should match continuously with the exterior region described by the Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \quad r \geq R,$$

where M represents the total mass inside the fluid sphere. Which implies the continuity of $y^2(r)$ and $B(r)$ across the boundary $r = R$.

These conditions allow us to fixate and determine the values and intervals of the constants present in the solution as well as determining the validity of our solution as a stellar model.

It is of interest to mention that according to the Petrov classification [39–41], the interior solutions for static and spherically symmetrical spacetimes are Petrov type D or Petrov type O, since Ψ_2 is the only Weyl scalar different than zero. For the case in which we have a perfect fluid, taking into account the Eqs. (4) y (5), we arrive to:

$$\Psi_2 = -\frac{B'}{6r} - \frac{1}{3r^2}(1 - B). \tag{13}$$

If $\Psi_2 \neq 0$ the solution is type D and if $\Psi_2 = 0$ the solution is type O. Imposing $\Psi_2 = 0$ we obtain $B(r) = 1 - ar^2$ and replacing in the difference of the Eqs. (4) and (5) it results:

$$y(r) = C + D\sqrt{1 - ar^2}, \tag{14}$$

which correspond to the interior Schwarzschild solution, this being the only interior solution, in a static and spherically symmetrical spacetime with perfect fluid, which is Petrov type 0 and as such it is the only one that is conformally flat [42, 43]. The rest of the interior solutions with perfect fluid are Petrov type D just like the exterior Schwarzschild solution.

3.1. Condition on the model

The calculation of the Kretschmann scalar will allow us to affirm that the geometry is regular, however, given the extension of it we only show the form of the behaviour for the metric functions in the vicinity of the center:

$$y(r) = 25S[5 + \frac{9}{2}ar^2 + O\left(r^4\right)],$$

$$B(r) = 1 - \frac{4\left[76209\sqrt{5} - 16S_2(0)\right]}{234375\sqrt{5}} ar^2 + O\left(r^4\right).$$

And the non-existence of the event horizon will be shown graphically in the following section. The conditions for the hydrostatic functions will generate for us a series of inequalities for the intervals of validity, of the constants a and C. The evaluation of the density, pressure and speed of sound implies

$$kc^2\rho(0) = \frac{304836}{78125}a - \frac{64\sqrt{5}S_2(0)}{390625}a > 0, \tag{15}$$

$$kP(0) = \frac{179638}{78125}a + \frac{64\sqrt{5}S_2(0)}{1171875}a > 0, \tag{16}$$

$$0 \leq \frac{v(0)^2}{c^2} = \frac{64S_2(0) - 726711\sqrt{5}}{5\left[64S_2(0) - 101711\sqrt{5}\right]} \leq 1. \tag{17}$$

Also, $P'(0) = 0$, $\rho'(0) = 0 \quad (v(0)^2)' = 0$. So, if the inequalities are satisfied (15)- (17) and also that the second derivatives in the origin of the density and the pressure are negative, *i.e.*,

$$\rho''(0) = \frac{6\left[64\sqrt{5}S_2(0) - 508555\right]}{390625kc^2}a^2 < 0, \tag{18}$$

$$P''(0) = \frac{6\left[64S_2(0) - 726711\sqrt{5}\right]}{390625\sqrt{5}k}a^2 < 0. \tag{19}$$

the requirement that these functions have a maximum value in the center would be met. From the condition that the pressure must be zero on the surface of the star $P(R) = 0$, Eq. (4) valued in $r = R$, we express C in terms of $w = aR^2$:

$$C = \frac{3\sqrt{5+8w}\left[H_2 + 64(150236w + 36864w^2)w^5\right]}{64\,(5+7w)\,(1+4w)\,(1+2w)\,(1+w)^4} + 191\sqrt{3}\arctan\left[\frac{\sqrt{3(5+8w)}}{1+4w}\right] - \operatorname{arctanh}\left[\frac{\sqrt{5+8w}}{3+4w}\right], \tag{20}$$

where $H_2 = -898190 + 5887971w - 14385882w^2 - 14277600w^3 + 438528w^4 + 12575808w^5$. From the expression (20) for C and the set of inequalities (15)-(17) we obtain the interval of validity for the parameter $w = aR^2 \in (0, 0.655607717]$. From this set of inequalities, the one that restricted the maximum possible value of w was obtained from imposing that the speed of sound is lower than the speed of light in the center of the star. The behaviour of the solution in the interior is shown in a graphic manner in the following section.

4. Graphic representation of the solution

The type of stellar object that the model represents is determined by the compactness value $u = GM/c^2R$ and in our case it is obtained from imposing the geometry's continuity condition on the surface, specifically of the component g_{rr}, resulting in:

$$u(w) = \frac{GM}{c^2R} = \frac{1}{2}[1 - B(R)] = \frac{3w\,(3+4w)}{(5+7w)(1+4w)}, \tag{21}$$

this is a monotonically increasing function and it's maximum value in the interval of validity of the solution is

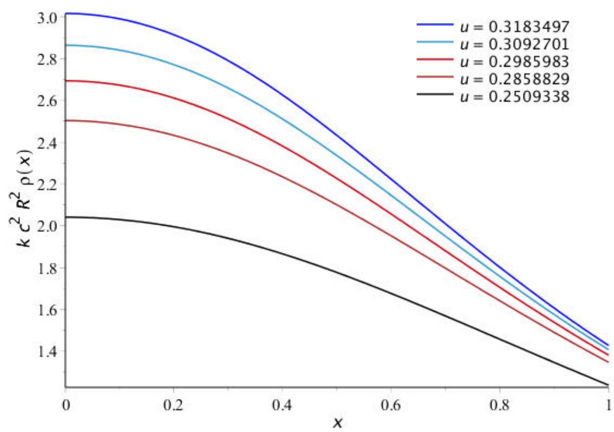

FIGURE 1. Behaviour of the density for different compactness values.

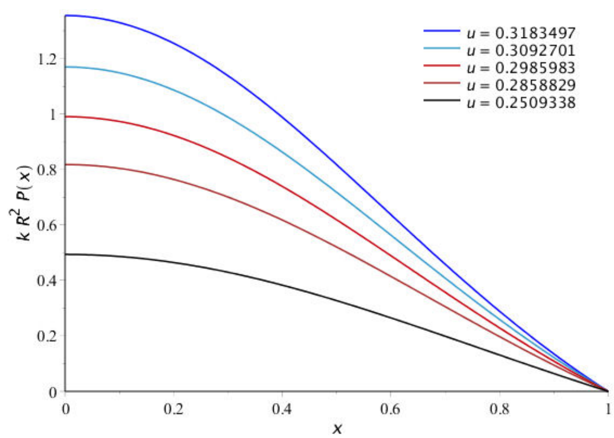

FIGURE 2. Pressure for different values of the compactness u.

$u(0.655607717) = 0.3183497451$. Meanwhile the continuity of g_{tt} allows us to determine the form of S which appears in the metric:

$$S = \frac{(1+w)^{5/2}}{(1+4\,w)\,(5+7\,w)\,(5+4\,w)^2}. \tag{22}$$

The solution is determined by the parameter $w = aR^2$ as such for our graphic description we will chose different values of it and we will define the dimensionless variable $x = r/R$, with this the center is represented by $x = 0$ and the surface by $x = 1$, meanwhile we redefine the hydrostatic functions as $kc^2R^2\rho$, kR^2P, v^2/c^2 for their dimensionless graphic representation. In the following figures we graph the functions for different compactness values, among these some stand out, as is the maximum compactness value of the model, $u = 0.3183497451$, and the value of $u = 0.2509338$ which corresponds to the neutron star PSR J0348+0432.

In the Fig. 1 we represent the density for different compactness values, from which we can observe that it is a monotonically decreasing function with its values lowering as the compactness rate decreased.

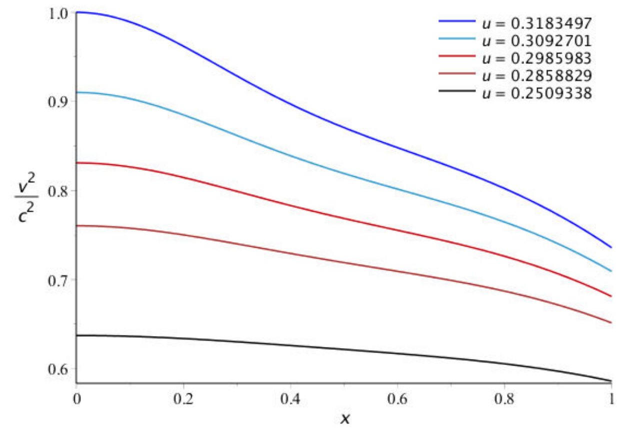

FIGURE 3. Graphic representation of the speed of sound.

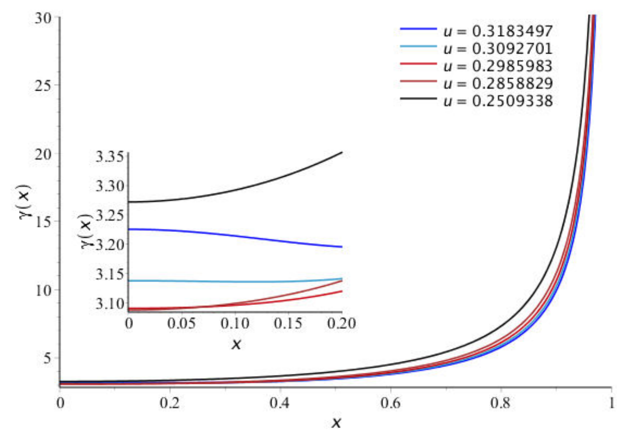

FIGURE 4. The adiabatic index for different compactness values.

The Fig. 2 shows the monotonically decreasing behaviour of the pressure as well as how it becomes zero on the surface, represented by $x = 1$.

From the Fig. 3 we observe that the speed of sound is a monotonically decreasing function, positive and lower than the speed of light. The interval of validity of the parameter w was determined by the non violation of the causality, matching the value of the speed of sound with the value of the speed of light for $u_{\max} = 0.3183497$ (or equivalently $w_{\max} = 0.655607717$), for values of $u > u_{\max}$ the condition of causality is violated. We also observe that as the compactness decreases the speed of sound is lower. The Fig. 4 shows that the adiabatic index $\gamma > 4/3$ meets the requirement for the stability of the solution, we also observe that the function γ is not monotonically decreasing (see blue colored line).

From a detailed analysis we have that the lowest value of the adiabatic index is 3.083875 and it occurs for the compactness value $u = 0.2917064$, which guarantees the stability of the solution. In relation to the energy conditions, given the positivity of the functions, we only need to verify that $c^2\rho - P \geq 0$ is satisfied. From the Figs. 1 and 2 we observe that, for a specific value of the compactness, the pressure is lower than the density, which guarantees that the requirement $c^2\rho - P \geq 0$ is met.

TABLE I. Interior behavior of the physical values for the density, pressure, speed of sound and adiabatic index for the PSR J0348+0432, with $R = 12.062$ km and $M = 2.05$ M$_\odot$.

r (km)	$\rho(10^{17}$ kg/m$^3)$	$P(10^{34}$ Pa)	$v^2(c^2)$	γ
0	7.5203	1.6346	0.63733	3.2723
1.2062	7.4790	1.6100	0.63643	3.2935
2.4124	7.3505	1.5380	0.63395	3.3569
3.6186	7.1474	1.4216	0.63027	3.4781
4.8248	6.8736	1.2675	0.62618	3.6779
6.0310	6.5441	1.0833	0.62162	3.9965
7.2372	6.1740	0.87685	0.61710	4.5220
8.4434	5.7778	0.65774	0.61194	5.4424
9.6496	5.3682	0.43383	0.60551	7.3394
10.856	4.9592	0.21217	0.59718	13.135
12.062	4.5593	0	0.58613	∞

TABLE II. Comparison of the physical values between the model for $n = 1$ [34] and $n = 3$ (the model presented in this report) for the PSR J0348+0432, with $R = 12.062$ km and $M = 2.05$ M$_\odot$.

n	$\rho_c(10^{17}$ Kg/m$^3)$	$\rho_b(10^{17}$ Kg/m$^3)$	$P_c(10^{34}$ Pa)
1	12.838	3.7174	2.4209
3	7.5203	4.5593	1.6346

TABLE III. Comparison of the physical values between the model for $n = 1$ [34] and $n = 3$ (the model presented in this report) for the PSR J0348+0432, with $R = 12.062$ km and $M = 2.05$ M$_\odot$.

n	$v_c^2(c^2)$	$v_b^2(c^2)$	γ_c
1	0.49985	0.65286	1.4407
3	0.63733	0.58613	3.2723

5. Discussion and conclusions

In the previous section, by means of dimensionless functions, it has been shown that the solution has an adequate behaviour for representing the interior of the stars with compactness rate $u \leq 0.3183497$ and, in particular, we took one of the compactness values $u = 0.2509338$ associated to observational data of the star PSR J0348+0432. In the Table I, in a complementary manner, we report the physical values of the hydrostatic variables for the neutron star PSR J0348+0432.

From the Table I we can observe that the orders of magnitude from the density and pressure are of the order of magnitude characteristic for neutron stars and that the adiabatic index complies with the condition required for the stability ($\gamma > 4/3$). The choice of the neutron star PSR J0348+0432 was done with the objective of being able to realize a comparison with a model that was approached previously [34] in which the metric potential is $g_{tt} = -S[(5 + 4ar^2)^2/(1 + ar^2)]^n$ with $n = 1$, meanwhile in this report we obtained the solution for $n = 3$. From the comparative analysis between the Figs. 1-4, Tables II, III and the figures and tables from the work reported in Ref. [34], we have that both models show important differences, which are: a) the admissible compactness value is greater for $n = 3$ ($u_{\max,3} = 0.3183497 > 0.2660858316 = u_{\max,1}$), due to this in the case for $n = 3$ it would allow for the description of stellar objects with a greater compactness. b) The speed of sound is a monotonically increasing function for $n = 1$, but it is a monotonically decreasing function for $n = 3$, although both behaviours are considered physically valid, there is still the need of further discussion on this point. c) The density, both in the center and on the surface, is greater for $n = 1$ compared with the density for $n = 3$, d) The central pressure is greater for $n = 1$ than for $n = 3$.

From the previous text we have that the model presented (case $n = 3$), given it's compactness, can be applied to a greater amount of stars and according to the imposed requirement in Ref. [1] is more adequate to represent the interior of the stars, although this is still a point that has not been approached sufficiently on the literature. From the present work some questions arise that could be approached in future works, among these are determining how the form of the metric potential g_{tt} influences the behaviour of the hydrostatic variables and if there is a way to determine for which potentials the speed of sound will be a monotonically increasing or monotonically decreasing function.

Acknowledgments

We appreciate the facilities provided by the Universidad Michoacana de San Nicolás de Hidalgo and the CIC -UMSNH during the realization of this investigation as well as the CONACYT for the support given.

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