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Inflation in an R^2-corrected $f(R)$ gravity model

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We conducted an analysis of the inflationary scenario within the $f(R)$ gravity framework, focusing on the Gogoi-Goswami model defined by the parameters $\alpha > 0$, $\beta > 0$, and the characteristic curvature constant R_c. This model exhibits a potential in the Einstein frame characterized by $V \propto \phi^p$. The spectral index for this model is given by $n_s = 1 - (p+2)/2N$, while the tensor-to-scalar ratio is $r = 4p/N$, where N denotes the e-folding number at horizon crossing. Although this model aligns with the Planck 2018 observational data within a narrow range, specifically $1.10 \leq p \leq 1.25$ for $N = 50$, it becomes increasingly difficult to find an appropriate value for p when $N \geq 54$. To overcome this limitation, we propose incorporating an R^2 correction term from the Starobinsky model to enhance the inflationary predictions. Our analysis indicates that this correction improves the model's performance when optimal parameters are selected, specifically by setting $x_0 = R_0/R_c \ll 1$ (with R_0 representing the scalar curvature during the late-time accelerated expansion), $\alpha_{\text{max}} = \mathcal{O}(1)$, and introducing a parameter γ related to the R^2 term within the range $-0.024600 < \gamma < 0$. The parameter x_0 establishes a connection between α and β via the de Sitter solution of the model. Additionally, the parameter R_c can be estimated similarly to that in the Starobinsky model, as $R_c \simeq (1.3 \times 10^{-5}/\kappa)^2$.

Keywords: $f(R)$ gravity; inflation; Planck 2018 observation.

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1. Introduction

The theory of General Relativity (GR) marked a significant breakthrough in the 20th century, fundamentally transforming our understanding of the universe. Einstein's famous field equations revealed the intricate relationship between spacetime and matter, laying the foundation for the standard model of Big Bang cosmology, which describes a homogeneous, isotropic, and expanding universe.

However, the standard model faces several challenges, including the flatness problem, the horizon problem, and the issue of magnetic monopoles. To address these issues, Alan Guth introduced the concept of inflation in 1980 [1]. This theory proposes that the early universe experienced a brief period of rapid expansion, triggered by the decay of a false vacuum state into a true vacuum in a supercooled universe. While inflation successfully resolves the aforementioned problems, Guth acknowledged that it leads to an inhomogeneous universe.

An alternative to Guth's inflationary model is the slow-roll inflation scenario, in which the universe's expansion is driven by a scalar field that gradually rolls toward the minimum of its effective potential [2-5]. This scenario not only addresses the issues associated with the Big Bang model but also results in a homogeneous universe, as the scalar field (inflaton) rolls slowly.

While General Relativity (GR) has been highly successful in describing the universe, it faces significant challenges, particularly its lack of renormalizability and the inability to be conventionally quantized [6]. These limitations have led to efforts to modify GR by adding additional terms to the Einstein-Hilbert action, resulting in what are now widely known as $f(R)$ gravity theories. These modifications involve replacing the Ricci curvature scalar, R, with an arbitrary function of R. The corresponding field equations can be derived by varying the action with respect to the metric tensor, using approaches such as the metric formalism, Palatini formalism [7], or the metric-affine formalism [8].

Starobinsky [9] made a significant contribution to our understanding of the early universe by predicting inflation through the inclusion of higher-order curvature terms in the action of GR. This prediction remains consistent with recent observations from the Planck mission [10]. On the other hand, Hu and Sawicki [11] proposed a model that accounts for the accelerated expansion of the late universe without the need for a cosmological constant. Various $f(R)$ gravity models have since been explored to explain phenomena such as inflation [12-15], late-time acceleration [16-18], or both epochs [19,20].

In 2020, Gogoi and Goswami [21] introduced a new $f(R)$ gravity model featuring arccot and $\exp$ correction terms, which they proposed to describe gravitational wave phenomena. This model has successfully passed solar system tests [22] and meets the viability criteria for $f(R)$ gravity [23,24]. It has been shown to produce gravitational waves, and it is constrained by the GW170817 events. However, the inflationary scenario of this model has not yet been thoroughly investigated. The scalar potential associated with the model is of the form $V \propto \phi^p$, which falls outside the inflationary constraints set by the 2σ CL-Planck observational data for $p \geq 2/3$ [10]. In this study, we examine the inflationary scenario of the model by incorporating an additional correction term from Starobinsky's model, as discussed in Ref. [25].

This paper is organized as follows: In Sec. 2, we provide a brief review of $f(R)$ gravity and introduce the model we propose. In Sec. 3, we analyze the inflationary scenario of the model under the constraints provided by Planck 2018 observations [10]. Finally, in the last Sec. 4, we summarize our findings and discuss potential directions for future research.

2. $f(R)$ Gravity

2.1. Field equations

The modified Einstein-Hilbert action of $f(R)$ gravity model in the Jordan frame is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{f(R)}{2\kappa^2} + \mathcal{L}_m\right], \tag{1}$$

where $f(R)$ is an arbitrary function of the Ricci scalar R, and $\mathcal{L}_m$ is the Lagrangian of matter. Here, $\kappa^2 = 8\pi G = M\mathrm{pl}^{-2}$, with G representing the Newton gravitational constant, and Mpl the reduced Planck mass ($c = \hbar = 1$). Varying the action in Eq. (1) with respect to the metric $g_{\mu\nu}$ yields the field equation:

$$f_{,R}(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_{,R}(R) = \kappa^2 T_{\mu\nu}. \tag{2}$$

In this equation, the subscript notation $f_{,R}$ denotes the derivative of the function f with respect to the Ricci scalar R, and $\Box = \nabla_\mu\nabla^\mu$ is the d'Alembert operator. The left-hand side of the equation accounts for the curvature-related terms, while the right-hand side corresponds to the matter content, represented by the energy-momentum tensor:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}}. \tag{3}$$

For a perfect fluid, the energy-momentum tensor $T_{\mu\nu}$ simplifies to $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$, where ρ is the energy density, p is the pressure, and u_μ is the four-velocity of the fluid.

The field Eq. (2) leads to the following trace equation:

$$f_{,R}(R)R + 3\Box f_{,R}(R) - 2f(R) = \kappa^2 T, \tag{4}$$

where $\Box$ is the d'Alembert operator. When $f(R) = R - 2\Lambda$, both Eq. (2) and Eq. (4) reduce to the familiar conditions of General Relativity (GR). Notably, the term $3\Box f_{,R}(R)$ vanishes in GR. However, in $f(R)$ gravity, this term introduces an additional scalar degree of freedom known as the scalaron [9]. We define the scalaron field as $\phi = f_{,R}(R)$, allowing us to rewrite Eq. (4) as [17,21]:

$$\Box\phi = \frac{dV_{\text{eff}}}{d\phi}, \tag{5}$$

where V_{eff} is the effective potential given by

$$\frac{dV_{\text{eff}}}{d\phi} = \frac{1}{3}\left[2f(R(\phi)) - R(\phi)\phi\right] + \frac{1}{3}\kappa^2 T. \tag{6}$$

In the absence of matter ($T = 0$), V_{eff} simplifies to V. For a stationary condition, where $dV_{\text{eff}}/d\phi = 0$, the second derivative of the potential at $\phi = \phi_0 = f_{,R}(R_0)$ provides the mass of the scalaron field:

$$m_\phi^2 = \frac{1}{3}\left[\frac{f_{,R}(R_0)}{f_{,RR}(R_0)} - R_0\right]. \tag{7}$$

This equation describes the stability of the scalaron field, which is crucial for generating the late-time expansion of the universe. In the early universe, however, the scalaron may be in a quasi-stable state [26]. It is important to ensure that the mass term remains finite, which requires that $f_{,RR}(R_0) \neq 0$. Moreover, for a specific value of R_0, we can derive the de Sitter solution from Eq. (6), and the stability condition $m_\phi^2 > 0$ from Eq. (7).

2.2. Scalar-tensor equivalence

The equivalence between $f(R)$ gravity and scalar-tensor theory has been the subject of much interest (see Refs. [6,27,28] for some reviews). Metric $f(R)$ gravity can be expressed in the form of Brans-Dicke theory [29] by introducing an auxiliary scalar field χ. Hence, we can write the action in Eq. (1) as follows:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[f(\chi) + f_{,\chi}(\chi)(R - \chi)\Big] + \int d^4x\mathcal{L}_m, \tag{8}$$

where $\mathcal{L}_m$ is the matter Lagrangian. By varying this equation with respect to the scalar field χ, we obtain the following equation:

$$f_{,\chi\chi}(\chi)(R - \chi) = 0. \tag{9}$$

From Eq. (7), we have $f_{,\chi\chi}(\chi) \neq 0$, which implies $R = \chi$. Using the previous definition of the scalaron field as $\phi = f_{,\chi}(\chi)$, the action in Eq. (8) can be expressed as follows:

$$S = \frac{1}{2\kappa^2}\int d^4\sqrt{-g}[\phi R - U(\phi)] + \int d^4x\mathcal{L}_m, \tag{10}$$

where $U(\phi)$ is a scalar field potential given by:

$$U(\phi) = \chi(\phi)\phi - f(\chi(\phi)). \tag{11}$$

The action in Eq. (10) for metric $f(R)$ gravity is equivalent to Brans-Dicke theory [29] for $\omega_{BD} = 0$ and $\kappa^2 = 1$ [6,30]. Meanwhile, for $f(R)$ with Palatini formalism, the equivalence is found for $\omega_{BD} = -3/2$.

In the Einstein frame, the action (1) can be obtained using the conformal transformation of the metric:

$$\tilde{g}_{\mu\nu} = f_{,R}(R) g_{\mu\nu}. \tag{12}$$

Thus, at $\mathcal{L}_m = 0$, the action is given by [27,31]:

$$S = \int d^4x \sqrt{-\tilde{g}} \Big[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \nabla_\mu \phi_E \nabla_\nu \phi_E - V(\phi_E) \Big], \tag{13}$$

where ϕ_E is the canonical scalar field given by:

$$\phi_E = \sqrt{\frac{3}{2}} \frac{1}{\kappa} \ln(f_{,R}(R)), \tag{14}$$

and the scalar potential is given by:

$$V(\phi_E) = \frac{U(R)}{2\kappa^2 f_{,R}(R)^2} = \frac{R f_{,R}(R) - f(R)}{2\kappa^2 f_{,R}(R)^2}. \tag{15}$$

The value of $\tilde{R}$ is calculated using the transformed metric $\tilde{g}_{\mu\nu}$, and ∇_μ is the covariant derivative operator. The mass squared term of the scalaron field in this frame is given by:

$$m^2_{\phi_E} = \frac{d^2 V(\phi_E)}{d\phi_E^2} = \frac{1}{3} \left[\frac{1}{f_{,RR}(R)} + \frac{R}{f_{,R}(R)} - \frac{4 f(R)}{f_{,R}(R)^2} \right]. \tag{16}$$

Both the Jordan and Einstein frames share equivalent dynamical properties. Anisotropic singularities in both frames can be avoided if $f_{,R}(R) > 0$ and $f_{,RR}(R) > 0$ [32]. It is convenient to study the inflationary dynamics of $f(R)$ gravity in the Einstein frame.

2.3. The $f(R)$ model

We consider the $f(R)$ gravity model proposed by Ref. [21], given by the following equation:

$$f_0(R) = R - \frac{\alpha}{\pi} R_c \,\mathrm{arccot}\left(\frac{R_c^2}{R^2} \right) - \beta R_c \left[1 - \exp\left(-\frac{R}{R_c} \right) \right], \tag{17}$$

where α and β are dimensionless positive constants, and R_c is a curvature characteristic constant with the same dimensions as R. This model was proposed to study the characteristics of gravitational waves (GWs) through the scalar polarization mode arising from extra degrees of freedom in $f(R)$ gravity. The model provides a good explanation of the solar system test and late-time acceleration. However, the viability of the model under other cosmological constraints, such as early-time expansion in the inflationary scenario, still needs to be elaborated. This article aims to explain the inflationary phase of the model.

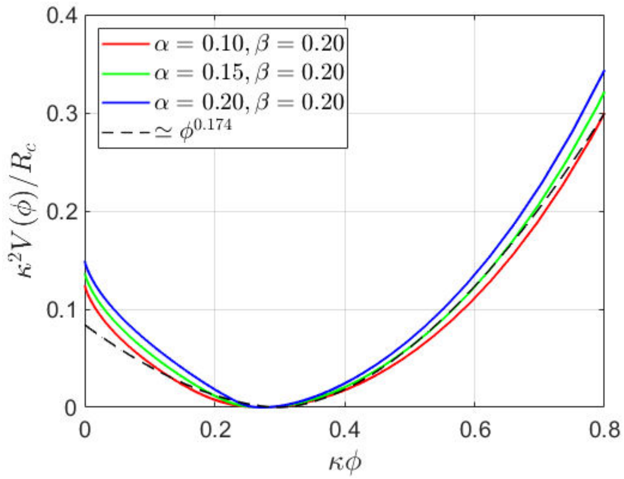

FIGURE 1. The scalar potential in the Einstein frame of the original model can be approximated as $V(\phi) \propto (\kappa\phi)^p$. To achieve sufficient inflation, the parameter p should lie within the range $1.10 \leq p \leq 1.25$ for $N = 50$.

To fully grasp the discussion on inflation in the Gogoi-Goswami model, it is crucial to have a solid understanding of slow-roll inflation, which is covered in Sec. 3. Readers are encouraged to review that section to effectively follow the forthcoming discussion.

The slow-roll inflation scenario in this model can be analyzed using the scalaron potential in the Einstein frame, as defined in Eq. (15). Figure 1 illustrates the scalar potential for various parameter values. With the parameters specified by the model, the scalar potential in the Einstein frame takes the form $V(\phi) \propto (\kappa\phi)^p$. For this type of potential, the spectral index is given by $n_s = 1 - (p+2)/(2N)$, and the tensor-to-scalar ratio is $r = 4p/N$, where N represents the number of e-foldings at horizon crossing, typically in the range of $N \approx 50 - 60$ [33]. To align with the Planck 2018 observational data, which determined the spectral index of scalar perturbations to be $n_s = 0.9649 \pm 0.0042$ with an upper limit on the tensor-to-scalar ratio of $r < 0.1$, the parameter p must be set within the range $1.10 \leq p \leq 1.25$ for $N = 50$. However, it becomes challenging to find a suitable value of p for $N \geq 54$ that satisfies the Planck constraints. Consequently, the Planck 2018 observational data [10] disfavors this type of potential.

Even though the model described by Eq. (17) provides a good explanation for late-time acceleration, we propose adding an extra R^2 correction term to account for the inflationary phase. The R^2 term becomes relevant in regions of high curvature, where the model's behavior aligns with that proposed by Starobinsky [9], which successfully explains the inflationary phase. This approach is also discussed in Ref. [25]. The modified model is given by:

$$f(R) = R - \frac{\alpha}{\pi} R_c \operatorname{arccot}\left(\frac{R_c^2}{R^2}\right) - \beta R_c \left[1 - e^{\left(-\frac{R}{R_c}\right)}\right] - \gamma \frac{R^2}{R_c}, \tag{18}$$

where γ is a constant.

Before exploring the inflationary scenario of this model, it is essential to test it against several criteria to ensure its viability as an $f(R)$ gravity model. This step is crucial because the addition of the γ term to the original model could impact its viability. We consider the following conditions [34]:

$$\lim_{R\to 0} f(R) = 0, \qquad \lim_{R\to\infty} f(R) \propto R^2. \tag{19}$$

The first condition corresponds to a flat Minkowski solution, while the second condition generates inflationary expansion. Our model (18) satisfies these conditions for any values of α, β, and $\gamma < 0$. It can be inferred that the α and β parameters dominate in the small curvature region, while the γ parameter dominates in the large curvature region.

To avoid an antigravity regime, the $f(R)$ gravity model must satisfy the condition $f_{,R}(R) > 0$. For our model, this condition takes the form:

$$1 - \beta e^{-x} - 2\gamma x - \frac{2\alpha x}{\pi\left(x^4+1\right)} > 0, \tag{20}$$

where $x = R/R_c$. Meanwhile, to ensure stable cosmological perturbation, the condition $f_{,RR}(R) > 0$ must hold, which is given by:

$$\beta\, e^{-x} - 2\gamma + \frac{2\alpha\left(3x^4-1\right)}{\pi\left(x^4+1\right)^2} > 0. \tag{21}$$

Both conditions are easily satisfied when $\gamma < 0$ and x is large, as the γ term becomes dominant. However, as $x \to 0$, both conditions require further investigation, as the contributions from α and β can no longer be neglected, potentially leading to a violation of these conditions.

To fully grasp the behavior of the model as $x \to 0$, it is essential to consider the late-time de Sitter solution. This solution corresponds to the condition $dV/d\phi = 0$, which can be expressed as [35,36]:

$$2f(R_0) - R_0 f_{,R}(R_0) = 0, \tag{22}$$

where R_0 represents the de Sitter curvature. The stability of this de Sitter solution is characterized by [37]:

$$\frac{f_{,R}(R_0)}{f_{,RR}(R_0)} > R_0. \tag{23}$$

Assuming $R_0 = R_c$, Eq. (22) yields a relation for β:

$$\beta = \frac{e((\pi-2)\alpha - 2\pi)}{2\pi(3-2e)} \cong -0.202697\alpha + 1.115621. \tag{24}$$

The stability condition from Eq. (23) results in:

$$\frac{2}{R_c\pi^2}\left(\alpha + \beta e^{-1}\pi - \frac{\pi}{2}\right)\left(\alpha + \beta e^{-1}\pi - 2\pi\gamma\right) < 0. \tag{25}$$

By substituting the expression for β from Eq. (24), we can determine the range of α:

$$-1.6838 + 8.2054\gamma < \alpha < 0.3675. \tag{26}$$

This range is similar to that of the original model, but includes an additional term dependent on γ. Equation (26) implies that $\gamma < 0.25$ for consistency. However, the original model's late-time stable de Sitter solution requires $\gamma < 0$. For instance, setting $\gamma = -0.0122$ slightly lowers the lower bound of α to 0.1 and increases the upper bound of β to 0.02.

When $R_0 \neq R_c$, the de Sitter solution is obtained if

$$\frac{2\alpha x_0^2}{x_0^4+1} + \pi\left[\beta e^{-x_0}(x_0+2) + x_0 - 2\beta\right] = 2\alpha \operatorname{arccot}(x_0^{-2}), \tag{27}$$

where $x_0 \equiv R_0/R_c$. We find that the γ term does not appear in the de Sitter solution, so the parameter bounds remain consistent with those of the original Gogoi-Goswami model [21]. The solution for β is given by

$$\beta\left((-2 + (x_0+2)e^{-x_0})\pi(x_0^4+1)\right) = (-2\alpha x_0^4 - 2\alpha)\arctan(x_0^{-2}) + \alpha(\pi x_0^4 - 2x_0^2 + \pi) - \pi(x_0^5 + x). \tag{28}$$

The stability condition for the de Sitter solution, assuming $R_c > 0$, is expressed as

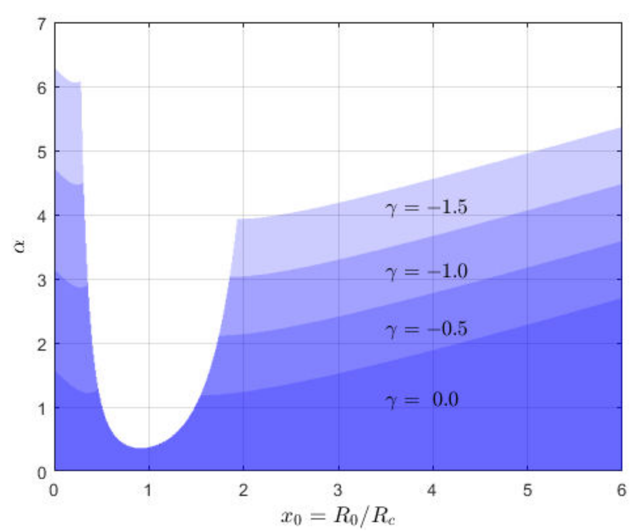

FIGURE 2. Relationship between the parameter α and x_0 for various values of γ. For example, when $\gamma = -0.5$, the corresponding line indicates the upper bound of α as x_0 changes.

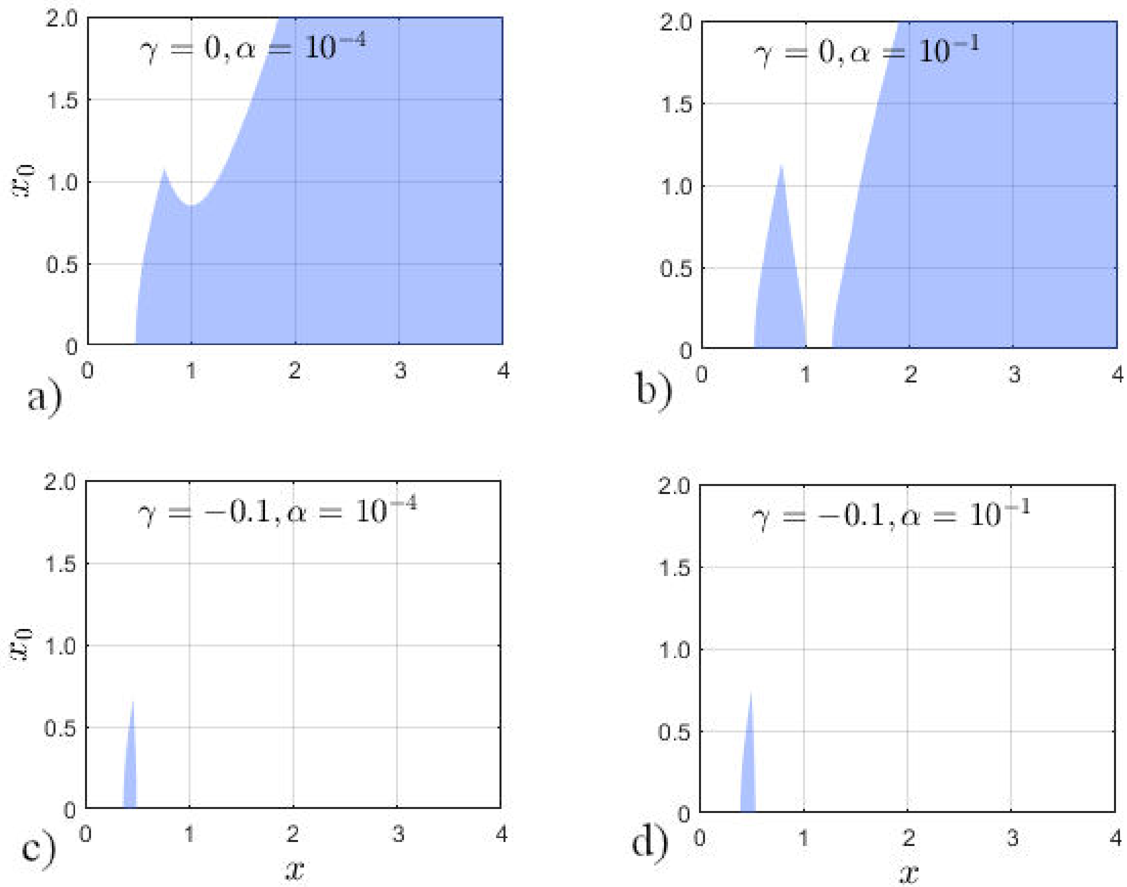

FIGURE 3. The relationship between x_0 and x for specific values of α and γ is illustrated in the plot. This shows that to maintain a small x_0, both the parameter γ should be negative and α should be sufficiently small.

$$\begin{aligned}&\left(\pi\beta(x_0+1)(x_0^4+1)^2e^{-x_0}-\pi(x_0^4+1)^2+8\alpha x_0^5\right)\\&\times\left(\pi\beta(x_0^4+1)^2e^{-x_0}-2\pi\gamma(x_0^4+1)^2+2\alpha(3x_0^4-1)\right)\\&\quad<0.\end{aligned} \tag{29}$$

As shown in Eq. (25), the term with γ introduces a deviation to the solution at the boundaries of α and β, with a factor of $(x_0^4+1)^2$. It can be inferred that a small negative value of γ is required to maintain consistency with the Gogoi-Goswami model.

Figure 2 illustrates the relationships between the parameters α, x_0, and γ derived from the de Sitter solution and its stability. In this context, β is expressed in terms of α and x_0 using Eq. (28). For each γ, the permissible values of $\alpha \geq 0$ are constrained by the line corresponding to that γ value.

The relationship between x_0 and x for specific values of γ and α can be determined by examining the de Sitter solution, its stability, the antigravity condition $f_{,R} > 0$, and the stability of cosmological perturbations $f_{,RR} > 0$. This relationship is depicted in Fig. 3. The Ricci scalar in de Sitter space, R_0, is proportional to the cosmological constant Λ (with $\Lambda \approx 1.1056 \times 10^{-52}\text{m}^{-2}$), which governs the late-time expansion of the universe. Consequently, if R_c represents the energy scale during inflation, $x_0 = R_0/R_c \to 0$. To achieve this, α and γ must be selected to drive $x_0 \to 0$. Figure 3 shows that this condition is met when $\gamma < 0$ and α is sufficiently small.

3. Inflation of the model

The dynamics of slow-roll inflation are governed by a scalar field ϕ and an effective potential $V(\phi)$ in the Einstein frame. The Friedmann equations [5] are commonly used to study these dynamics, given by:

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi} + V(\phi)\right], \tag{30}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}(\phi). \tag{31}$$

Here, $H \equiv \dot{a}/a$ is the Hubble parameter, a is the expansion scale factor, and $\dot{\phi}$ indicates the derivative with respect to time t. Slow-roll inflation occurs when the acceleration term in Eq. (31) is small ($\ddot{\phi} \ll 3H\dot{\phi}$), and can be neglected, leading to the condition:

$$\dot{\phi} \simeq -\frac{V_{,\phi}(\phi)}{3H}. \tag{32}$$

Other slow-roll parameters, ϵ_V and η_V, can also be defined as:

$$\epsilon_V = \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}(\phi)}{V(\phi)}\right)^2, \tag{33}$$

$$\eta_V = \frac{1}{\kappa^2}\frac{V_{,\phi\phi}(\phi)}{V(\phi)}, \tag{34}$$

which must satisfy $\epsilon_V, \eta_V \ll 1$ for inflation to occur [4,10].

The e-folding number N is another important quantity that determines the slow-roll inflation scenario, representing the number of expansions that occur. It can be expressed as [5]:

$$N \equiv \ln(\frac{a_f}{a_i}) \simeq -\int_{\phi_i}^{\phi_f}\frac{V(\phi)}{V_{,\phi}(\phi)}d\phi, \tag{35}$$

where a_i and a_f are the initial and final values of the scale factor. Inflation ends when ϵ_V and η_V approach 1, and the e-folding number is typically between $N = 50$ and 60.

To constrain our model, we used the latest Planck observation data [10]. This required the use of the important parameters, the tensor-to-scalar ratio (r) and the scalar spectral index (n_s), which can be inferred from Eqs. (33) - (34) as follows:

$$r = 16\epsilon_V, \tag{36}$$

$$n_s = 1 - 6\epsilon_V + 2\eta_V. \tag{37}$$

The values of these parameters given by Ref. [10] are $r < 0.11$ and $n_s = 0.9649 \pm 0.0042$. The Planck 2018 constraint on r is equivalent to an upper bound on the energy scale of inflation when the pivot scale exits the Hubble radius, given by

$$V_* = \frac{3\pi^2 A_s}{2} r M_{\rm pl}^4 < (1.6\times 10^{16}\text{GeV})^4 \quad (95\% \ \text{CL}), \tag{38}$$

where A_s is the scalar power spectrum amplitude. This relation implies an upper bound on the Hubble parameter during inflation of

$$H_* < 2.5\times 10^{-5} M_{\rm pl}. \tag{39}$$

We examined the slow-roll inflation scenario of the model described by Eq. (18) in the Einstein frame. In this context, the inflaton is represented by the canonical scalar field given by:

$$\phi(x) = \sqrt{\frac{3}{2}}\frac{1}{\kappa}\ln\left(1 - \frac{2\alpha x}{\pi\,(x^4+1)} - \beta\,\mathrm{e}^{-x} - 2\gamma x\right), \tag{40}$$

where $x = R/R_c$. The corresponding scalar potential is

$$V(x) = \frac{R_c}{\kappa^2}\frac{A(x)}{B(x)}, \tag{41}$$

with

$$A(x) = \alpha\,\mathrm{arccot}\left(x^{-2}\right) - \beta\pi\,(x+1)\,\mathrm{e}^{-x} + \pi\left(\beta - \gamma\,x^2\right) - \frac{2\alpha\,x^2}{x^4+1}, \tag{42}$$

and

$$B(x) = \frac{8}{\pi}\left(\frac{\beta\pi\,\mathrm{e}^{-x}}{2} + \left[\gamma x - \frac{1}{2}\right]\pi + \frac{\alpha x}{x^4+1}\right)^2. \tag{43}$$

Figure 4 illustrates the relationship between the potential V and the inflaton ϕ. For any given values of α, x_0, and γ, the effective potential exhibits a flat region, which is essential for achieving slow-roll inflation. The parameter γ significantly influences the potential, as it sets the energy scale during inflation and thus affects the inflaton's behavior. In

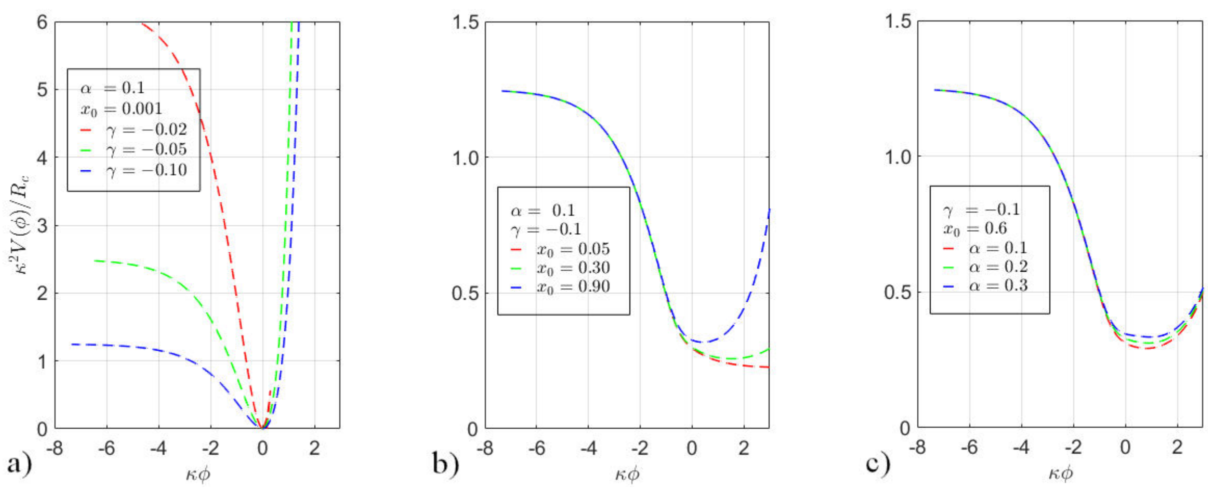

FIGURE 4. The figure displays a parametric plot of $\phi(R)$ and $V(\phi(R))$ for our model (18). The plots show that the parameters α and x_0 are associated with the minima, whereas γ determines the overall height of the potential.

contrast, the parameters α and x_0 determine the potential's behavior beyond the flat region, impacting the end of inflation and the minima of the effective potential where reheating occurs.

For a qualitative analysis, we focus on the flat region of the scalar potential $V(\phi)$. In this regime, the γ term dominates compared to the α and β terms, particularly when $R \gg R_c$. Under these conditions, we approximate $f(R) \approx R - \gamma(R^2/R_c)$, leading to the scalaron field $\kappa\phi \approx \sqrt{(3/2)}\ln\left(1 - 2\gamma[R/R_c^2]\right)$. In this regime, $R \approx [R_c/2\gamma]\left(1 - e^{-\sqrt{2/3}\kappa\phi}\right)$. Thus, the inflaton potential can be approximated as:

$$V \approx \frac{-R_c}{8\gamma\kappa^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\phi}\right)^2. \qquad (44)$$

This potential resembles the Starobinsky type, characterized by $f(R) \approx R - \gamma(R^2/R_c)$. For effective inflation, this potential necessitates $-(\gamma/R_c) \approx (1/6M^2)$, with $M \sim 1.3 \times 10^{-5}/\kappa \sim 3 \times 10^{13}$ GeV [25,38]. Therefore, a reasonable estimate for γ in this model is obtained by setting $R_c \approx M^2$, which yields $-\gamma \approx 1/6 \approx 0.167$. This estimate will be compared with our numerical results.

We conducted numerical simulations to explore the inflationary dynamics of the model given by Eq. (18). Our findings indicate that the inflationary properties of the model are significantly influenced by the parameters α, x_0, and γ. The parameter x_0 is derived by substituting β using the de Sitter solution relation. Since x_0 is associated with the Ricci scalar R_0 of the current spacetime, which is proportional to the cosmological constant Λ and responsible for late-time acceleration, it is crucial to choose a sufficiently small value for x_0. In our simulations, we set $x_0 \leq 0.1$. To maintain a small x_0, the model's viability requires $\gamma < 0$ and α to be sufficiently small. In our calculations, γ was varied with $\gamma \leq -10^{-4}$, where this upper limit is used to represent the value $\gamma \to 0$.

The numerical results for the inflationary parameters, specifically the tensor-to-scalar ratio r and the spectral index n_s, are presented in Fig. 5. Initially, we determine the values of x_0 and α before varying $\gamma \leq 10^{-4}$. To identify permissible values for γ, we use the criterion $\epsilon_V(\phi_f) \simeq 1$ to check whether inflation can terminate. The initial value of the scalaron field at the onset of inflation (ϕ_i) is determined by applying the e-folding number criterion, with N ranging from 50 to 60. For each e-folding number N, ϕ_i is calculated, and the tensor-to-scalar ratio $r(\phi_i)$ and the spectral index $n_s(\phi_i)$ are then computed and plotted in Fig. 5.

Figure 5 shows the numerical results for r and n_s plotted against the Planck 2018 inflationary constraints to assess the alignment of the chosen parameter values with these con-

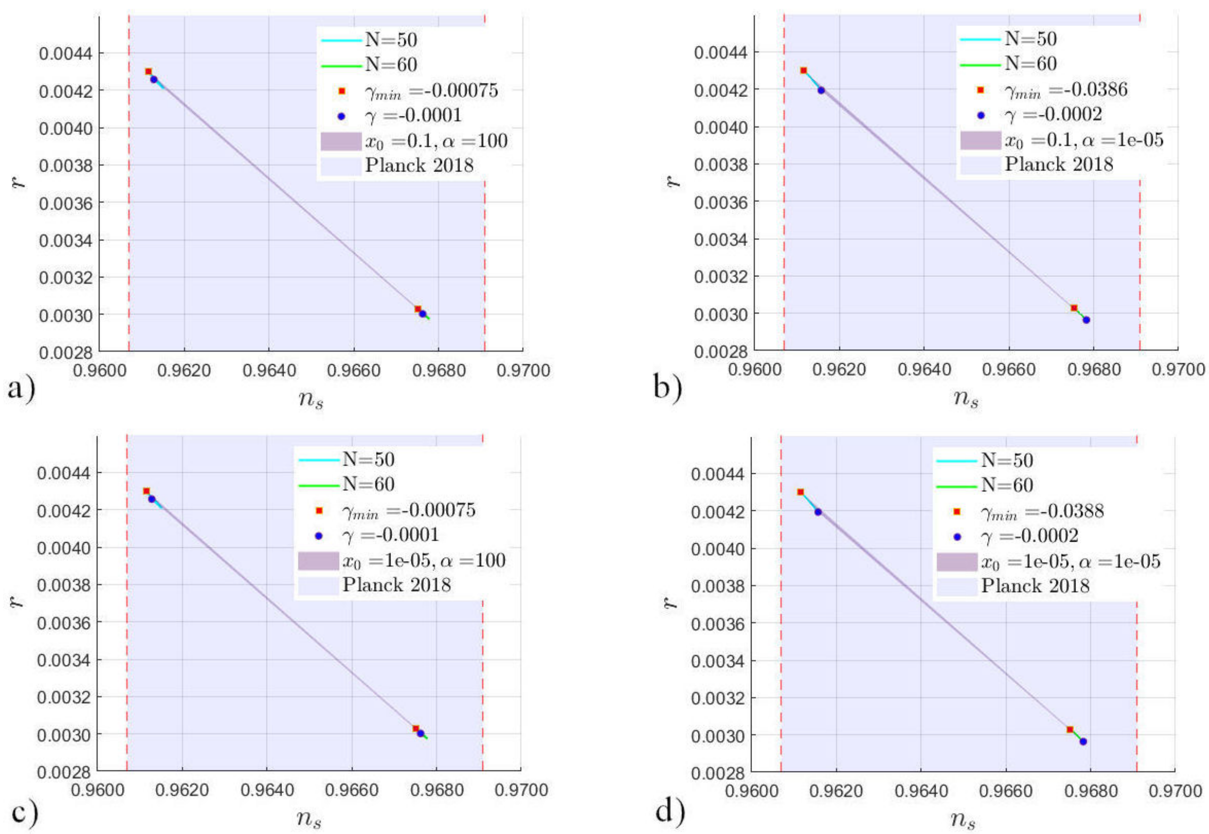

FIGURE 5. The plots display the tensor-to-scalar ratio r and the spectral index n_s for various values of the parameters x_0 and α. The parameter γ is varied, generating lines for r and n_s corresponding to e-folding numbers $N = 50$ and $N = 60$, measured from the end of inflation. The range of γ is depicted from a value represented by a blue bullet ($\simeq -10^{-4}$) to the minimum value compatible with inflation ($\gamma_{\min}$), indicated by a red square.

TABLE I. Numerical results for inflation parameters for various values of α and $x_0 \ll 1$.

			$N = 50$		$N = 60$	
x_0	α	γ_{min}	n_s	r	n_s	r
1.0e-01	1.0e+02	-0.000750	0.96114476	0.00430126	0.96752005	0.00303028
1.0e-01	1.0e+01	-0.005400	0.96127743	0.00426987	0.96761423	0.00301144
1.0e-01	1.0e+00	-0.024600	0.96120678	0.00428725	0.96756413	0.00302186
1.0e-01	1.0e-01	-0.036900	0.96114711	0.00430101	0.96752106	0.00303025
1.0e-01	1.0e-03	-0.038600	0.96115527	0.00429919	0.96752763	0.00302902
1.0e-01	1.0e-05	-0.038600	0.96115651	0.00429892	0.96752849	0.00302885
1.0e-03	1.0e+02	-0.000750	0.96114478	0.00430125	0.96752004	0.00303028
1.0e-03	1.0e+01	-0.005400	0.96127769	0.00426981	0.96761442	0.00301141
1.0e-03	1.0e+00	-0.024900	0.96119895	0.00428913	0.96755751	0.00302318
1.0e-03	1.0e-01	-0.036800	0.96115935	0.00429829	0.96752944	0.00302867
1.0e-03	1.0e-03	-0.038800	0.96114238	0.00430204	0.96751831	0.00303076
1.0e-03	1.0e-05	-0.038800	0.96114598	0.00430126	0.96752083	0.00303029
1.0e-05	1.0e+02	-0.000750	0.96114478	0.00430125	0.96752004	0.00303028
1.0e-05	1.0e+01	-0.002680	0.96144495	0.00422629	0.96773567	0.00298483
1.0e-05	1.0e+00	-0.024900	0.96119895	0.00428913	0.96755751	0.00302318
1.0e-05	1.0e-01	-0.036800	0.96115936	0.00429829	0.96752944	0.00302867
1.0e-05	1.0e-03	-0.038800	0.96114238	0.00430204	0.96751831	0.00303076
1.0e-05	1.0e-05	-0.038800	0.96114598	0.00430126	0.96752083	0.00303029

straints. Due to the close proximity of the computed r and n_s values across different α and x_0 values, it is impractical to display all results in Fig. 5. Therefore, selected numerical results are summarized in Table I. From Fig. 5 and Table I, we conclude that inflation can terminate successfully ($\epsilon_V \simeq 1$) even when α is set to 10^2. The minimum value of γ (denoted γ_{min}) primarily depends on α. For instance, with $\alpha = 10^2$, $\gamma_{\text{min}} \simeq -0.000750$, while for $\alpha \ll 1$, $\gamma_{\text{min}} \simeq -0.0388$. The effect of $x_0 \ll 1$ on γ_{min} appears to be minimal.

The dependence of γ_{min} on α for $x_0 \ll 1$ can be understood by examining the properties of the scalaron potential depicted in Fig. 4. Variations in α affect the minimum height of the potential, leading to faster inflation termination for larger α due to a shallower potential minimum. To achieve sufficient inflation, a higher potential is necessary, which corresponds to a smaller $|\gamma_{\min}|$.

While selecting $\alpha = 10^2$ can yield effective inflation, the stability of the de Sitter solution must also be taken into account. Thus, a more reasonable upper limit for α when $x_0 \ll 1$ is $\alpha \in \mathcal{O}(1)$, as illustrated in Fig. 2, with $\gamma_{\text{min}} \simeq -0.024600$.

The values of the parameters x_0, α, and γ_{min} in this model do not directly influence the choice of the parameter R_c. It is crucial to note that the slow-roll parameters ϵ_V and η_V, as well as the inflation parameters r and n_s, are independent of κ and R_c. These parameters are considered solely to ensure that the inflationary potential complies with the observational constraints of Planck 2018, as specified by Eq. (38). In the Starobinsky model, $f(R) = R - \gamma(R^2/R_c)$, inflation is effectively explained by selecting $-\gamma \simeq 1/6 \simeq 0.167$ and $R_c = M^2 \simeq (1.3 \times 10^{-5}/\kappa)^2$. This model requires only minor adjustments when applied to the one under review. Inflation is well within the Planck 2018 constraints if $x_0 \ll 1$, $\alpha_{\text{max}} \in \mathcal{O}(1)$, and $\gamma = \gamma_{\text{min}} \simeq -0.024600 < 0$. The parameter choice $R_c = M^2$ remains valid for this model.

4. Conclusion

We examined the inflationary phase within the Gogoi-Goswami $f(R)$ gravity model. In this model, the potential in the Einstein frame is expressed as ϕ^p. Consequently, the spectral index is given by $n_s = 1 - (p + 2/2N)$, and the tensor-to-scalar ratio is $r = 4p/N$, where N represents the e-folding number at horizon crossing. To align with the Planck 2018 observational data, which set the spectral index of scalar perturbations at $n_s = 0.9649 \pm 0.0042$ and impose an upper limit of $r < 0.1$, we find that $1.1 \leq p \leq 1.25$ for $N = 50$ is required. However, it proves challenging to find a suitable value for p when $N \geq 54$. To address this issue, we introduced a correction term to improve the model's predictions of the inflationary phase.

Our proposed model has successfully passed viability tests. It can produce a Minkowski solution for low curvature, an inflationary phase for high curvature, and a stable de Sitter solution during late-time expansion. We have shown that the model achieves positive gravitational coupling and stable cosmological perturbations, depending on the parameters α, x_0, and γ, which ensure a stable de Sitter solution. The parameters that satisfy the viability conditions are illustrated in Figs. 2 and 3. Moreover, our model features a flat potential region influenced by the parameter $\gamma < 0$, which enables inflation. Naturally, we take $x_0 \ll 1$ so that $\alpha_{\text{max}} = \mathcal{O}(1)$ for $\gamma < 0$. Numerical analysis confirms that the model fits the Planck 2018 observational data well, with γ in the range $-0.024600 < \gamma < 0$ and $R_c \simeq (1.3 \times 10^{-5}/\kappa)^2$. Consequently, this $f(R)$ gravity model effectively explains the early universe's expansion, specifically inflation.

However, several questions remain. Since α and β (or x_0) primarily influence the potential near its minimum, their roles in the reheating phase after inflation require further investigation. Additionally, it needs to be explored whether these parameters can contribute to the reheating phase and the formation of primordial matter. These issues will be addressed in future research.

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