Bright soliton of Stochastic perturbed Biswas-Milovic equation with cubic-quintic-septic law having multiplicative white noise

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For the first time, the adopted stochastic form of the perturbed Biswas-Milovic equation with cubic-quintic-septic law having spatio-temporal and chromatic dispersion in the presence of multiplicative white noise in Itô sense was presented and examined. The Biswas-Milovic equation models numerous physical phenomena occurring in optical fiber. We analyzed the optical soliton solutions of the stochastic model with the aid of a subversion of the new extended auxiliary equation method. Furthermore, we investigated the evaluation of the noise impacts and the effects of some model parameters on the dynamics of the generated soliton. Finally, graphical depictions of the derived soliton types were represented for some solution functions. The stochastic model and the derived results will contribute to the comprehension of the nonlinear dynamics of pulse propagation in optical fibers which has great importance for the advancement of optical communication engineering.

Keywords: Stochastic model; Wiener process; Noise strength; Brownian motion.

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1. Introduction

The nonlinear stochastic partial differential equations (SPDEs) consist of stochastic derivatives or noise terms. The noise terms represent the random fluctuations or uncertainties inherent in the system being modeled. SPDEs are of great importance in the modeling of events with optical fibers, especially in optical communication. The existence of stochastic terms gives rise to extra challenges and needs specialized approaches for their analysis and solution. So, interpreting physically the SPDEs, optical soliton solutions of SPDEs have lately become the focus of attention in the literature. In particular, adopted stochastic models are constantly being developed in the literature in terms of models developed in the fields of telecommunication, fibers, nano-fibers, optics, and optoelectronics due to the importance of optical solitons.

The nonlinear Schrödinger equations (NLSEs) have a principal role in engineering and various fields [1]. Therefore, various models based on the Schrödinger equation have been developed in the literature [2-9]. The stochastic NLSEs which contain the stochastic term with multiplicative noise have also a substantial role in engineering and scientific fields. So, stochastic examinations on NLSE have been especially performed utilizing analytic approaches in recent times. Some of these are as follows: Neirameh and Eslami examined stochastic chiral NLSE in Ref. [10]. Zayed *et al*., studied the stochastic Sasa-Satsuma equation involving a multiplicative noise [11]. Cakicioglu *et al*. introduced the stochastic dispersive Schrödinger-Hirota equation with parabolic law nonlinearity [12]. Zayed *et al*., discussed (2+1)-dimensional NLSE with spatio-temporal dispersion (STD) [13]. Secer gained the stochastic optical soliton of NLSE with Kerr law nonlinearity in Ref. [14]. In Ref. [15], Mohamed *et al.*, generated new optical soliton solutions of the perturbed stochastic NLSE having generalized anti-cubic nonlinearity. Stochastic solitons to Biswas-Arshed equation with multiplicative white noise were earned in Refs. [16, 17], which includes the Wick-type stochastic NLSE with the conformable derivative. [18] presents the stochastic dark solitons to higher-order NLSE. Additionally, optical soliton solutions of various forms of NLSEs have been derived taking into account different conditions in Refs. [3,19-34].

The Biswas-Milovic equation (BME) [30], a generalized version of the NLSE, is of key significance in modeling the wave propagation of soliton transmission over long distances. Moreover, various versions of the BME have been improved and examined in the literature. Some of these are (2+1)-dimensional BME having Kerr, power, and parabolic law nonlinearities [35], (2+1) and (3+1)-dimensional BME [36], BME with STD and parabolic law [37], BME in presence dual-power law nonlinearity and multiplicative white noise via Itô calculus [38], the perturbated BME with Kudryashov's law of refractive index [39], BME having Kerr law, parabolic law, power law, and dual power-law nonlinearities [40], BME in presence quadratic-cubic and parabolic nonlinearities [41], time fractional BME [42, 43], and BME with STD [44].

In this paper, we aim to explore the optical soliton solutions of the stochastic perturbed BME with cubic-quintic-septic law in the presence of STD and chromatic dispersion having multiplicative white noise in Itô sense via subversion of the new extended auxiliary equation method (SAEM246) [45].

Governing model

The adopted stochastic form of perturbed Biswas-Milovic equation with cubic-quintic-septic law having chromatic dispersion and spatio-temporal in the presence of multiplicative white noise in Itô sense is put forth, for the first time, as:

$$\begin{aligned} i\left(\vartheta^n\right)_t+\alpha\left(\vartheta^n\right)_{xt}+\beta\left(\vartheta^n\right)_{xx}+(c_1|\vartheta|^2+c_2|\vartheta|^4 \\ +c_3|\vartheta|^6)\vartheta^n=i(\lambda\left(|\vartheta|^2\vartheta^n\right)_x+\tau\left(|\vartheta|^2\right)_x\vartheta^n \\ +\mu|\vartheta|^2\left(\vartheta^n\right)_x)-\sigma(\vartheta^n-i\alpha(\vartheta^n)_x)\,W_t(t), \end{aligned} \tag{1}$$

in which n is BME to NLSE generalization parameter, α and β symbolize the STD and group velocity dispersion coefficients, c_1, c_2, c_3 express cubic, quintic, and septic law of nonlinearities of self-phase modulation, respectively. Furthermore, τ is defined as a self-frequency shift coefficient whereas λ, and μ are the coefficients of self-steepening and nonlinear terms, respectively. Herein, σ stands for the noise strength, and $W(t)$ is the standard Wiener process which is expressed as [46]:

$$W(t)=\int_0^t\Phi(\eta)dW(\eta),\ \ \eta<t, \tag{2}$$

where η is defined as the stochastic variable. In particular, stochastic studies on equations modeling optical soliton behavior come into prominence and become widespread in recent years. However, stochastic studies on the BME are scarcely any. Regarding the perturbed BME, there has been no research reflected in the literature. In this aspect, as far as we know, Eq. (1) has not been introduced before in the literature.

The paper is arranged into five sections. Section 2 expresses the mathematical analysis of Eq. (1). We describe SAEM246 and its utilization to gain the soliton solutions of the stochastic model in Sec. 3. Section 4 summarizes the main findings and their significance. In Sec. 5, conclusions are given.

2. Mathematical analysis of Eq. (1)

The following transformation is taken into account for the model in Eq. (1):

$$\begin{aligned} \vartheta(x,t)&=\mathcal{U}(\zeta)\,e^{i\phi(x,t)}, \\ \phi(x,t)&=-kx+\omega t+\sigma(W(t)-\sigma t)+\varphi_0, \\ \zeta&=x-\nu t, \end{aligned} \tag{3}$$

in which $\vartheta(x,t)$ is the soliton pulse profile, $\mathcal{U}(\zeta)$ states real function which represents amplitude, φ_0 is phase constant, and k,ω,ν are frequency, wave number, and speed, which are non-zero real values. Inserting Eq. (3) into Eq. (1), the imaginary and real parts of the resultant equation are presented in the following structure, respectively:

$$n\left(\sigma\alpha(n-1)\frac{dW(t)}{dt}+\left(\left(k\nu-\sigma^2+\omega\right)\alpha-2\beta k\right)n-\nu\right)\mathcal{U}'\mathcal{U}^{n+1}-((\lambda+\mu)n+2\lambda+2\tau)\mathcal{U}'\mathcal{U}^{n+3}=0, \tag{4}$$

$$\begin{aligned} &\left(\sigma\left(n-1\right)\left(\alpha kn-1\right)\frac{d}{dt}W(t)-n\left(\left(\left(\sigma^2-\omega\right)\alpha+k\beta\right)kn-\sigma^2+\omega\right)\right)\mathcal{U}^{n+2}+c_2\mathcal{U}^{n+6} \\ &+\left(c_1-k\left(\lambda+\mu\right)n\right)\mathcal{U}^{n+4}+c_3\mathcal{U}^{n+8}+n\mathcal{U}''\mathcal{U}^{n+1}-n\left(n-1\right)\left(\alpha\nu-\beta\right)\mathcal{U}^n\left(\mathcal{U}'\right)^2=0, \end{aligned} \tag{5}$$

where $U=\mathcal{U}(\zeta)$ and Eq. (4) presents the following constraints:

$$\nu=\frac{\left(\alpha\sigma^2-\alpha\omega+2\beta k\right)n}{\alpha kn-1},\ \ \tau=-\frac{\left(\lambda+\mu\right)n+2\lambda}{2}. \tag{6}$$

Herein, we need to add the following comment regarding Eq. (4). Considering the first term of Eq. (4), we derive the following expression as an algebraic condition:

$$\left(\sigma\alpha(n-1)\frac{dW(t)}{dt}\right)\mathcal{U}'\mathcal{U}^{n+1}=0. \tag{7}$$

In Eq. (7), $\mathcal{U}'$ and $\mathcal{U}^{n+1}$ cannot be zero because $\mathcal{U}$ is nonzero and has the second-order derivative as a solution function. Since $W(t)$ has a first-order derivative as the Wienerr process, it is not possible for $dW(t)/dt$ to be zero. Because σ is noise strength and α is the model parameter, they are not zero. Therefore, n must be 1. So, it can be simply interpreted that the cubic-quintic-septic form of Eq. (1) is considered as the NLSE form for BME. Moreover, this evaluation is presented for the first time in this study. So, by considering the $n=1$, Eq. (5) turns into the following form:

$$\begin{aligned} &-\left(\left(\left(\sigma^2-\omega\right)\alpha+k\beta\right)k-\sigma^2+\omega\right)\mathcal{U}^3+c_2\mathcal{U}^7 \\ &+\left(c_1-k\left(\lambda+\mu\right)\right)\mathcal{U}^5+c_3\mathcal{U}^9+\mathcal{U}''\mathcal{U}^2=0. \end{aligned} \tag{8}$$

Balancing the terms $\mathcal{U}''\mathcal{U}^2$, $\mathcal{U}^9$ in (8) by considering Eqs. (13), (14), the balance constant is determined as $J{=}1/3$.

So, we obtain the requirement to perform the following transformation:

$$\mathcal{U}(\zeta) = \mathcal{V}^{1/3}(\zeta). \tag{9}$$

Rearrangement of Eq. (8) and using Eq. (9) yields the following nonlinear ordinary differential equation:

$$\begin{aligned} &9\left(c_1 - (\lambda+\mu)\,k\right)\mathcal{V}(\eta)^{\frac{8}{3}} + 9c_2\mathcal{V}^{\frac{10}{3}} + 9(c_3\mathcal{V}^2 - \beta\,k^2 \\ &\quad + \alpha\left(\omega - \sigma^2\right)k + \sigma^2 - \omega)\mathcal{V}^2 \\ &\quad + 2\left(\alpha\nu - \beta\right)(\mathcal{V}')^2 - 3\left(\alpha\nu - \beta\right)\mathcal{V}\mathcal{V}'' = 0. \end{aligned} \tag{10}$$

For integrability of Eq. (10), we must set,

$$c_1 = k\left(\lambda + \mu\right),\; c_2 = 0. \tag{11}$$

As a result, the NLODE in Eq. (10) is formed as follows:

$$\begin{aligned} &9\left(c_3\mathcal{V}^2 + \alpha(\omega-\sigma^2)k - \beta k^2 + \sigma^2 - \omega\right)\mathcal{V}^2 \\ &\quad + 2\left(\alpha\nu - \beta\right)(\mathcal{V}')^2 - 3\left(\alpha\nu - \beta\right)\mathcal{V}\mathcal{V}'' = 0. \end{aligned} \tag{12}$$

Balancing the terms $\mathcal{V}''\mathcal{V}$, $\mathcal{V}^4$ in Eq. (12), the balance constant is determined as positive integer.

3. A subversion of the new extended auxiliary equation method (SAEM246)

The SAEM246 [45] is derived from the subversion of well known auxiliary equation [47-49]. The method offers that Eq. (12) has a solution as the following truncated series form:

$$\mathcal{V}(\zeta) = \sum_{j=0}^{J}\Lambda_j\mathcal{N}^j(\zeta),\; \Lambda_J \neq 0, \tag{13}$$

in which Λ_j are real constants, J is the positive integer balance constant to be computed considering the Eqs. (12), (13), and (14) which is obtained as 2. The function $\mathcal{N}(\zeta)$ admits:

$$\left(\frac{d\mathcal{N}}{d\zeta}\right)^2 = \rho^2\mathcal{N}^2(\zeta)\left[1 - \chi_1\mathcal{N}^2(\zeta) - \chi_2\mathcal{N}^4(\zeta)\right], \tag{14}$$

in which ρ, χ_1, χ_2 are nonzero real values. One of the solutions for Eq. (14) is given with the following structure [45]:

$$\mathcal{N}\left(\zeta\right) = \sqrt{\frac{16e^{2\rho\zeta}}{(e^{2\rho\zeta} + 4\chi_1)^2 + 64\chi_2}}. \tag{15}$$

Considering that J is 2, Eq. (13) is rewritten in the following form:

$$\mathcal{V}\left(\zeta\right) = \Lambda_0 + \Lambda_1\mathcal{N}\left(\zeta\right) + \Lambda_2\mathcal{N}^2\left(\zeta\right),\;\; \Lambda_2 \neq 0. \tag{16}$$

Inserting Eq. (16) and its corresponding derivatives in Eq. (12) and taking the identical various powers of $\mathcal{N}\left(\zeta\right)$ as zero, the following system is earned:

$$\begin{aligned} &\mathcal{N}^0(\zeta): \Lambda_0^2\Omega_1 = 0, \\ &\mathcal{N}^1(\zeta): \left(\rho^2\Omega_2 - 6c_3\Lambda_0^2 + 6\Omega_1\right)\Lambda_0\Lambda_1 = 0, \\ &\mathcal{N}^2(\zeta): \left(\rho^2\Lambda_1^2 + 12\rho^2\Lambda_0\Lambda_2\right)\Omega_2 - 9c_3\Lambda_0^2(\Omega_3 + 4\Lambda_1^2) - 9\Omega_1\Omega_3 = 0, \\ &\mathcal{N}^3(\zeta): \left(15\rho^2\Omega_2 - 18c_3\Lambda_0^2 - 18\Omega_1 - 8\rho^2\right)\Lambda_1\Lambda_2 - 6\left(\rho^2\chi_1\Omega_2 + 6c_3\Omega_3\right)\Lambda_0\Lambda_1 = 0, \\ &\mathcal{N}^4(\zeta): 2\rho^2\Omega_2\left(2\Lambda_2^2 - 4\chi_1\Lambda_1^2 - 9\chi_1\Lambda_0\Lambda_2\right) - 9c_3\left(\Lambda_0^2\Lambda_2^2 + 8\Lambda_0\Lambda_1^2\Lambda_2 + \Omega_3^2\right) - 9\Lambda_2^2\Omega_1 = 0, \\ &\mathcal{N}^5(\zeta): 16\rho^2\chi_1\Omega_2\Lambda_1\Lambda_2 + 9\rho^2\chi_2\Omega_2\Lambda_0\Lambda_1 + 36c_3\Lambda_0\Lambda_1\Lambda_2^2 + 36c_3\Omega_3\Lambda_1\Lambda_2 = 0, \\ &\mathcal{N}^6(\zeta): 10\rho^2\chi_1\Omega_2\Lambda_2^2 + 7\rho^2\chi_2\Omega_2\Lambda_1^2 + 24\rho^2\chi_2\Omega_2\Lambda_0\Lambda_2 + 36c_3\Lambda_0\Lambda_2^3 + 54c_3\Lambda_1^2\Lambda_2^2 = 0, \\ &\mathcal{N}^7(\zeta): \left(25\rho^2\chi_2\Omega_2 + 36c_3\Lambda_2^2\right)\Lambda_1\Lambda_2 = 0, \\ &\mathcal{N}^8(\zeta): \left(16\rho^2\chi_2\Omega_2 + 9c_3\Lambda_2^2\right)\Lambda_2^2 = 0, \end{aligned} \tag{17}$$

in which

$$\Omega_1 = \Lambda_0^2c_3 + \alpha\left(\omega - \sigma^2\right)k - \beta\,k^2 + \sigma^2 - \omega, \qquad \Omega_2 = \left(-\frac{\alpha\left(\left(-\sigma^2 + \omega\right)\alpha - 2k\beta\right)}{k\alpha - 1} - \beta\right),$$

and

$$\Omega_3 = \left(2\Lambda_0\Lambda_2 + \Lambda_1^2\right).$$

Solving the system in Eq. (17), we get:

$$\left\{\begin{array}{c}\omega = \dfrac{\sigma^2(\Upsilon + 9\alpha^2 k^2) + 4\beta\,\rho^2(\alpha k + 1) + 9\,k^2\beta(\alpha k - 1)}{\Upsilon} \\ \chi_1 = 0, \Lambda_0 = 0, \Lambda_1 = 0, \Lambda_2 = -\dfrac{4\sqrt{c_3\Upsilon\beta\chi_2}\,\rho}{c_3\Upsilon}\end{array}\right\}. \tag{18}$$

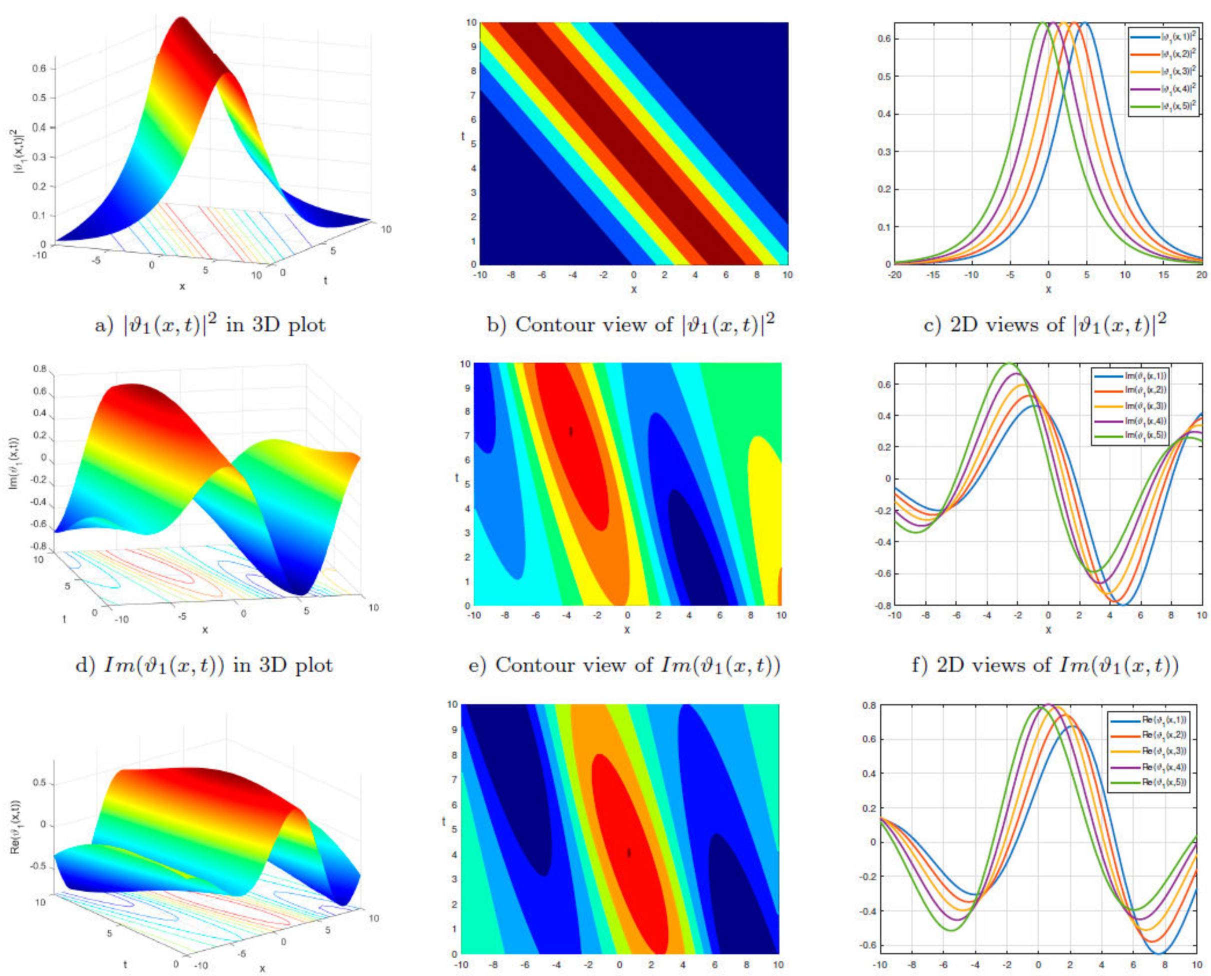
where $\Upsilon = 4\alpha^2\rho^2 + 9\alpha^2 k^2 - 18\alpha k + 9$ and $c_3\Upsilon\beta\chi_2 > 0$. Unification of Eqs. (3), (9) (15),(16) allows to reach the solution of Eq. (1):

$$\vartheta_1(x,t) = 16^{\frac{1}{3}}\left(\frac{\Lambda_2 \mathrm{e}^{2\rho\left(x + \frac{\left((\omega-\sigma^2)\alpha - 2k\beta\right)t}{\alpha k - 1}\right)}}{\left(\mathrm{e}^{2\rho\left(x+\frac{((\omega-\sigma^2)\alpha-2k\beta)t}{\alpha k-1}\right)} + 4\chi_1\right)^2 + 64\chi_2}\right)^{\frac{1}{3}} \mathrm{e}^{\mathrm{i}(-kx+\omega t+\psi_0+\sigma(W(t)-\sigma t))}, \tag{19}$$

where $\omega, \chi_1, \Lambda_2$ are given in Eq. (18).

4. Results and discussion

This section consists of various graphics indicating the effectiveness of SAEM246 in gaining soliton solutions. Figure 1 consists of nine subfigures for the parameters $\alpha = 1, \beta = c_3 = k = 0.5, \rho = 0.2, \chi_2 = 2, \psi_0 = \sigma = 0$: a) expresses the 3D square of the modulus, b) depicts the contour view, c) illustrates the 2D square of the modulus, d) indicates the 3D imaginary component,

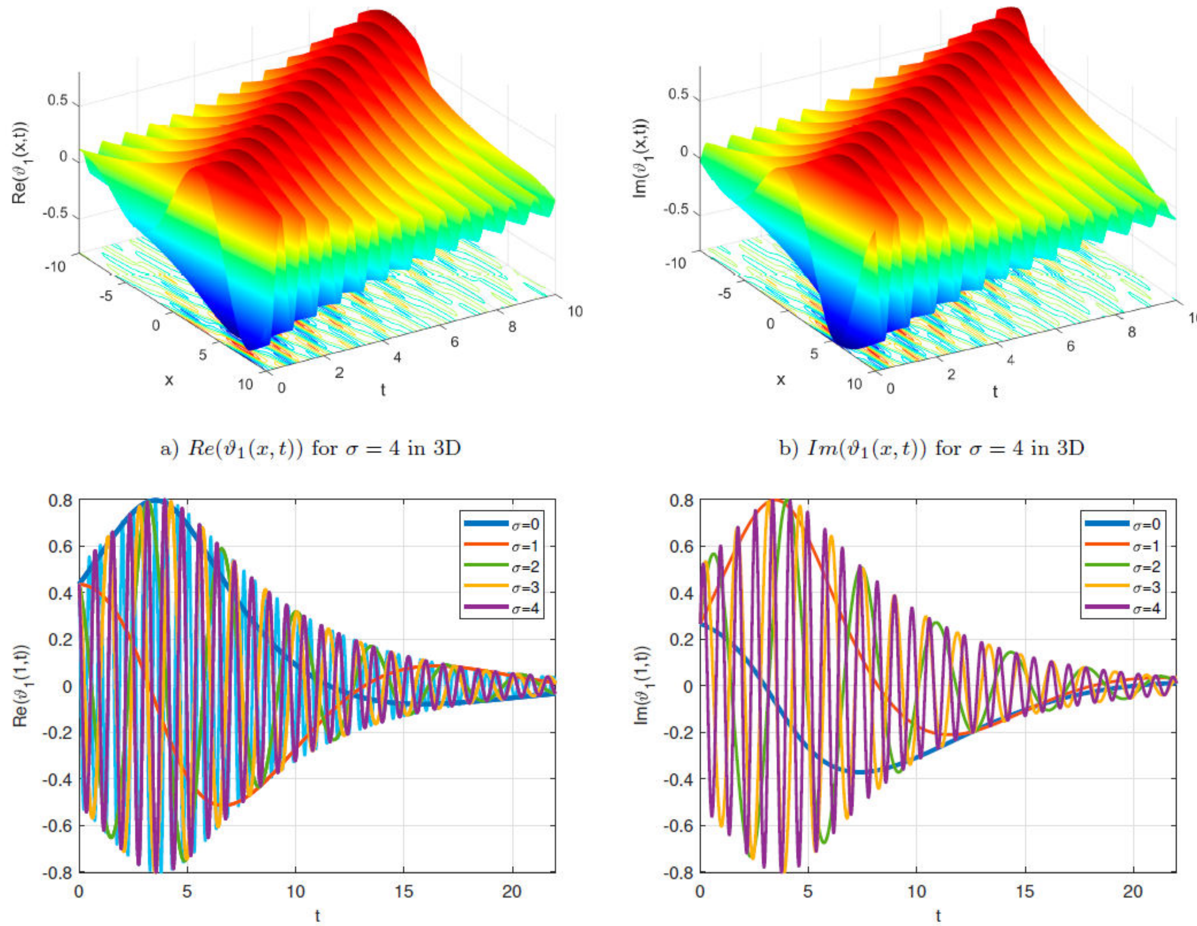


a) $|\vartheta_1(x,t)|^2$ in 3D plot b) Contour view of $|\vartheta_1(x,t)|^2$ c) 2D views of $|\vartheta_1(x,t)|^2$

d) $Im(\vartheta_1(x,t))$ in 3D plot e) Contour view of $Im(\vartheta_1(x,t))$ f) 2D views of $Im(\vartheta_1(x,t))$

g) $Re(\vartheta_1(x,t))$ in 3D plot h) Contour view of $Re(\vartheta_1(x,t))$ i) 2D views of $Re(\vartheta_1(x,t))$

FIGURE 1. Various views of $\vartheta_1(x,t)$ for $\alpha = 1, \beta = c_3 = k = 0.5, \rho = 0.2, \chi_2 = 2, \psi_0 = \sigma = 0$.



a) $Re(\vartheta_1(x,t))$ for $\sigma = 4$ in 3D

b) $Im(\vartheta_1(x,t))$ for $\sigma = 4$ in 3D

c) $Re(\vartheta_1(x,t))$ in 2D

d) $Im(\vartheta_1(x,t))$ in contour plot

FIGURE 2. The various projections of $\vartheta_1(x,t)$ under the effect of noise effect.

e) demonstrates the contour view for the imaginary component, f) represents the 2D imaginary component, g) indicates the 3D real component, h) illustrates the contour view for the real component, i) depicts the 2D real component. Figure 1a) and 1c) display the bright solution of $\vartheta_1(x,t)$ in Eq. (19).

Figure 2 is devoted to the examination of the impact of σ. Figures 2a) and 2b) state the depictions of the real and imaginary parts of Eq. (19), respectively, where the effect of σ can be clearly observed. This effect is seen more clearly when the graphs drawn for $\sigma = 4$ are compared with graphs indicated in Figs. 1d)-1g) for $\sigma = 0$. The resulting effect demonstrates itself in the structure of fluctuations. However, in order to understand the impact of σ more clearly, that is, the situation occurring for increasing values of σ, 2D portraits are drawn. In Figs. 2c) and 2d), we indicate 2D projections of $Re(\vartheta_1(x,t))$ and $Im(\vartheta_1(x,t))$ for $\sigma = 0, 1, 2, 3, 4$. In Fig. 2d), firstly, the graph of Eq. (1) for $\sigma = 0$ (there is no noise effect) is depicted (blue line). This depiction basically has the same character as the wave behavior with Fig. 1i). Then, the values of $1, 2, 3$, and 4 for σ were given gradually and the $Re(\vartheta_1(x,t))$ graph was hidden each time (solid red to purple lines) in Fig. 2c). From Fig. 2c), oscillation (fluctuation) is observed with a decrease in the amplitude of the wave depending on the increasing σ values, and this fluctuation decreases as σ approaches to zero. It is possible to observe the effect for other values of σ which are not limited to the $\sigma = 4$ given in the graphic drawings. A similar examination with Fig. 2d) has been carried out for $Im(\vartheta_1(x,t))$. This study also points out a situation similar to the comments made for $Re(\vartheta_1(x,t))$. Therefore, as a general interpretation, it has been observed that for increasing values of σ, it has a fluctuating impact on the soliton in diverse values of t. Investigations on whether this effect causes a constantly increasing fluctuation depending on increasing σ values, and whether the fluctuation continues (or decreases) after a certain σ value, are some of the topics that can be researched for both the model presented in this study and other optical models.

5. Conclusion

In this research paper, the optical solitons of the stochastic perturbed Biswas-Milovic equation with cubic-quintic-septic law in the presence of STD and chromatic dispersion having by multiplicative white noise in Itô sense were scrutinized via SAEM246. We successfully reached the bright soliton.

3D, contours, and 2D depictions were additionally drawn to figure out the behavior of soliton for the generated solution. Moreover, we intended to analyze the noise influence in terms of Itô sense on soliton solution and presented the depictions of the resultants in detail. In this research paper, it has been observed that the effect of σ is indicated as a fluctuation on the soliton, this fluctuation increases depending on the increasing σ values or decreases when it approaches the value of $t \neq 0$. For both Eq. (1) and other equations, whether the σ effect will continue as a continuous fluctuation depending on the increasing σ value, whether there is a critical value where the fluctuation stops, and the analysis of the fractional order form of the Eq. (1) are open topics that can be researched in the future. Because of introducing this stochastic problem for the first time in this article, it reveals that the work will conduce to some researchers working in this field.

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