

Simulation of the 2E Technique on neutron multiplicity measurement as a function of fragment mass in spontaneous fission of ^{252}Cf

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A Monte Carlo simulation algorithm to investigate the measurement of the average prompt neutron multiplicity as a function of pre-neutron mass, $\bar{\nu}(A)$, for fragments from the spontaneous fission of ^{252}Cf is presented. The input data consist of experimental measurements of the kinetic energy and mass distributions obtained by Göök *et al.*, and the values of $\bar{\nu}(A)$ calculated using the FIFRELIN model by Piau *et al.*, $\bar{\nu}_{\text{th}}(A)$. We analyze the output curve, $\bar{\nu}_{\text{sim}}(A)$, obtained by simulation of the 2E technique, which should ideally match $\bar{\nu}_{\text{th}}(A)$. However, we find that $\bar{\nu}_{\text{sim}}(A)$ exhibits a maximum value at $A \approx 122$, close to mass symmetry, while $\bar{\nu}_{\text{th}}(A)$ has a maximum at $A \approx 118$. Additionally, we observe that $\bar{\nu}_{\text{sim}}(A) > \bar{\nu}_{\text{th}}(A)$ for $A < 90$ and $A > 169$, respectively. We attribute this discrepancy to inaccuracies in the relationship between provisional mass and the pre-neutron mass used in the 2E technique for data processing in each fission event.

Keywords: Spontaneous fission; californium 252; neutron emission; fragment distribution; neutron multiplicity.

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1. Introduction

The fission process begins with the fissile nucleus, culminating at the scission point, where the nuclear interaction fades, and two complementary fragments with proton and nucleon numbers (Z, Z') and (A, A') whose excitation energies are XE and XE' , respectively, are formed [1–3]. The study of fission dynamics requires knowledge of the distribution of these variables [4], which is not directly accessible due to the particle emission from fragments before they reach the detectors [5].

An intermediate step would involve measuring the distribution of kinetic energy values acquired by the fragments due to the Coulomb repulsion they experience after the scission point (KE, KE') . Additionally, it is essential to measure the excitation energy of the fragments $(XE, \text{ and } XE')$, which is expressed in the decay through particle emission. With these values, the available energy of the reaction can be calculated [6]:

$$\begin{aligned} Q &= TKE + TXE \\ &= (KE + KE') + (XE + XE'), \end{aligned} \quad (1)$$

where $TKE = KE + KE'$ is the total kinetic energy and $TXE = XE + XE'$ is the total excitation energy.

However, for each fission event, the fragments arrive at the detectors after having emitted (n, n') prompt neutrons [7]. Thus, the post-neutron values of the kinetic energy of the fragments (e, e') , respectively, result from mass loss and recoil effects due to neutron emission. The final mass and kinetic energy of one fragment may be measured using a mass separator as the LOHENGRIN at the ILL Institute in Grenoble, France [8], but the emitted neutrons are not detected.

The so-called double energy (2E) technique consists in measuring (e, e') to calculate the pre-neutron mass. However

it would be necessary to detect all neutrons emitted by the fragment, which represents an unresolved issue in terms of efficiency [9]. The recoil effect from the emission of those neutrons is not fully accessible.

Göök *et al.* [10] and Al-Adili *et al.* [11] have employed the double kinetic energy (2E) technique to calculate the average prompt neutron multiplicity as a function of fragment mass, denoted as $\bar{\nu}_{\text{ex}}(A)$ for the spontaneous fission of ^{252}Cf . On the other hand, there are results from calculations based on theoretical models that compute the values of $\bar{\nu}(A)$, in this work referred to as $\bar{\nu}_{\text{th}}(A)$. Among these is the FIFRELIN model used by Piau *et al.* [12].

In this study, a simulation algorithm has been employed to investigate the disparity between the results obtained through the 2E measurement technique and the values of the average prompt neutron multiplicity as a function of pre-neutron mass, $\bar{\nu}_{\text{th}}(A)$, assumed as real, for fragments from the spontaneous fission of ^{252}Cf .

2. Monte Carlo simulation of $\bar{\nu}(A)$ measurement using the 2E technique

Before neutron emission, the distribution of fragment mass is characterized by the pre-neutron fragment mass yield $(Y(A))$. The complementary fragments with mass (A, A') have a total kinetic energy distribution characterized by its average $(\overline{TKE}(A))$ and its standard deviation $(\sigma_{TKE}(A))$.

For each pair of fragments with (A, A') and a total kinetic energy TKE , there is a distribution of prompt neutron number whose average is represented by $\bar{\nu}(A, TKE)$, which is approximately a linear function of TKE , having $(\partial\bar{\nu}/\partial TKE)$ as slope. The fragments with mass A emit prompt neutrons with an average kinetic energy relative to the center of mass represented by $\bar{\eta}(A)$.

The above functions have been measured by Göök *et al.* [10]. Piau *et al.* [12] have calculated the average neutron multiplicity as a function of pre-neutron mass ($\bar{\nu}_{\text{th}}(A)$).

The experimental Göök *et al.* data [10] and the function $\bar{\nu}_{\text{th}}(A)$ calculated by Piau *et al.* [12] are the input data for the simulation of the experiment measuring the average prompt neutron multiplicity, with the 2E technique.

For the simulation, we assume a Gaussian distribution of $TKE(A)$:

$$TKE = \overline{TKE} + r\sigma_{TKE}(A), \quad (2)$$

where r is a number that follows a Gaussian distribution with a mean of 0 and a standard deviation of 1.

For the neutron multiplicity, we employ the following approximate relationship:

$$\begin{aligned} \bar{\nu}_{\text{th}}(A, TKE) &= \bar{\nu}_{\text{th}}(A) \\ &\times \left(1 + \frac{\partial \bar{\nu}}{\partial TKE} (\overline{TKE} - TKE) \right). \end{aligned} \quad (3)$$

We assume that complementary fragments with masses A and A' with total kinetic energy TKE emit n and n' given by relationships:

$$n = \text{int} \left(\bar{\nu}_{\text{th}}(A) \left(1 + \frac{r}{3} \right) + 0.5 \right), \quad (4a)$$

$$n' = \text{int} \left(\bar{\nu}_{\text{th}}(A') \left(1 - \frac{r}{3} \right) + 0.5 \right), \quad (4b)$$

where r is a number that follows a Gaussian distribution with a mean of 0 and a standard deviation of 1.

Additionally, we assume that fragments with mass A associated with the total kinetic energy TKE emit isotropically n neutrons with kinetic energy $\bar{\eta}(A)$. The recoil effect due to neutron emission is then calculated to obtain the final kinetic energy values of the complementary fragments e and e' . Based on these values, the provisional mass of the two complementary fragments is calculated:

$$(\mu, \mu') = 252 \frac{(e', e)}{tke}, \quad (5)$$

where $tke = e + e'$.

3. Results

The outcome of the Monte Carlo simulation for the 2E technique measurement of $\bar{\nu}$ as a function of provisional mass, $\bar{\nu}_{\text{prov}}(\mu)$, is presented in Fig. 1.

To approximately calculate the pre-neutron values of kinetic energy, we utilize the relationship used by Göök *et al.* [10]:

$$KE \approx tke \left(\frac{\mu}{\mu - \bar{\nu}_{\text{prov}}(\mu, tke)} \right). \quad (6)$$

This relation is an approximation of the formula:

$$KE = TKE \left(\frac{A}{A - n} \right), \quad (7)$$

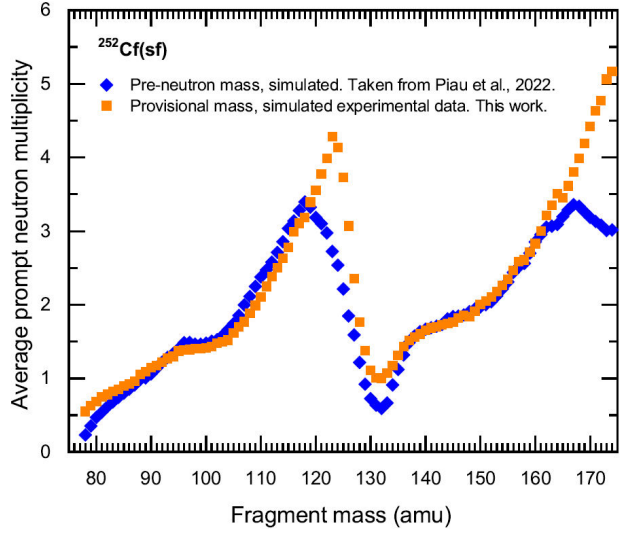


FIGURE 1. Average prompt neutron multiplicity as a function of pre-neutron mass, calculated by Piau *et al.* ($\bar{\nu}_{\text{th}}$: diamonds) [12], and as a function of provisional mass ($\bar{\nu}_{\text{prov}}$: squares) for fragments from the spontaneous fission of ^{252}Cf , obtained as the output of the Monte Carlo simulation.

which relates the pre-neutron kinetic energy to the final energy of fragments emitting n neutrons with zero kinetic energy in the center of mass frame.

From the approximate values of kinetic energy obtained using Eq. (6), we calculate the approximate values of the primary fragment masses:

$$(A, A') \approx 252 \left(\frac{KE', KE}{TKE} \right). \quad (8)$$

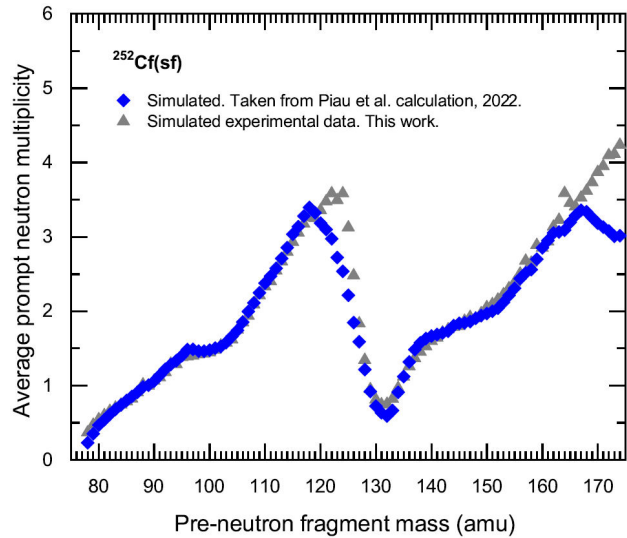


FIGURE 2. Average prompt neutron multiplicity as a function of pre-neutron mass, calculated by Piau *et al.*, ($\bar{\nu}_{\text{th}}$: diamonds) [10] for fragments from spontaneous fission of ^{252}Cf , and the results of the simulated experiment with the 2E technique ($\bar{\nu}_{\text{sim}}(A)$: triangles).

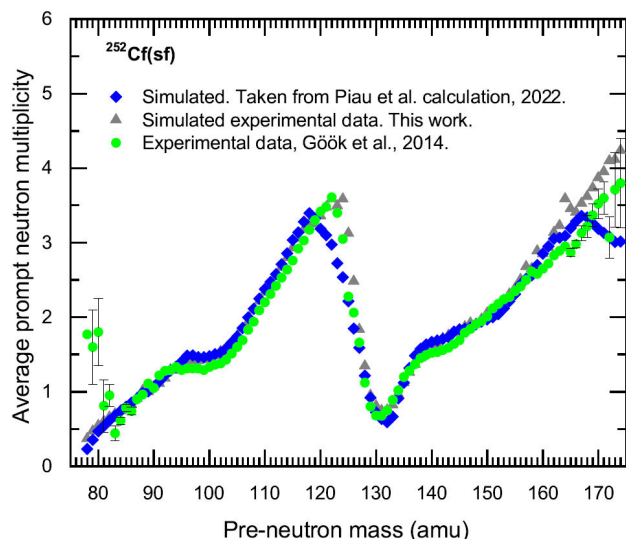


FIGURE 3. Average prompt neutron multiplicity as a function of pre-neutron mass, calculated by Piau *et al.* ($\bar{\nu}_{th}$: diamonds) [12], for fragments from spontaneous fission of ^{252}Cf ; the results of the simulated experiment with the 2E technique ($\bar{\nu}_{sim}(A)$: triangles); and the experimental curve measured by Gök *et al.* ($\bar{\nu}_{exp}(A)$: circles) [10].

With the values of n corresponding to each mass A , the $\bar{\nu}_{sim}(A)$ curve is constructed and presented in Fig. 2.

Figure 3 presents the curves $\bar{\nu}_{th}(A)$, $\bar{\nu}_{sim}(A)$, and the experimental curve $\bar{\nu}_{ex}(A)$, obtained by Gök *et al.* [10]. The curves $\bar{\nu}_{sim}(A)$ and $\bar{\nu}_{ex}(A)$ exhibit similar trends, except for the region $A < 80$, where $\bar{\nu}_{ex}(A)$ greatly surpasses $\bar{\nu}_{sim}(A)$. Both curves reach a maximum value around $A = 122$, while $\bar{\nu}_{th}(A)$ presents its maximum value at $A = 118$. Additionally, both curves have higher values than $\bar{\nu}_{th}(A)$ for $A > 169$.

4. Conclusions

Initially, we assume that the average prompt neutron multiplicity as a function of the pre-neutron mass ($\bar{\nu}_{th}(A)$) is as

calculated by Piau *et al.* [12]. Subsequently, we simulate an experiment employing the 2E technique to measure this parameter based on the provisional mass. As a result of this simulation, we obtain the curve $\bar{\nu}_{prov}(\mu)$. Figure 1 illustrates that $\bar{\nu}_{prov}(\mu)$ exhibits a peak around $\mu = 123$, whereas the theoretical primary mass-related variable, $\bar{\nu}_{th}(A)$, reaches a maximum at $A = 118$. Moreover, $\bar{\nu}_{th} < \bar{\nu}_{prov}$ for $A < 90$ and $A > 162$.

Afterward, we simulate the same experiment, this time calculating the average prompt neutron multiplicity as a function of pre-neutron mass. The result of this simulation is represented by the curve $\bar{\nu}_{sim}(A)$, which more closely approximates or aligns with the curve $\bar{\nu}_{th}(A)$ than does the curve $\bar{\nu}_{prov}(\mu)$. However, both curves are higher than the curve $\bar{\nu}_{th}(A)$ around $A = 122$ and in the mass region $A > 169$.

The discrepancy between the simulated curve $\bar{\nu}_{sim}(A)$, generated through the 2E technique simulation, and the input curve $\bar{\nu}_{th}$ is attributed to the inexactness of the relationship (6), which connects provisional mass and pre-neutron energy. A better approximation could be achieved by considering the number of neutrons n emitted by the fragment instead of $\bar{\nu}_{prov}(\mu, tke)$, representing an average value. However, even with this refinement, the relationship would still be imprecise, as it does not account for the recoil effect caused by the emission of n neutrons, which alters the kinetic energy of the emitting fragment.

If the output data from the simulation algorithm of the experiment based on the 2E technique replicate the experimental results, we can assume that the input data are compatible with the pre-neutronic characteristics of the fragments.

The simulation method enables the calculation of pre-neutron values which, when used as input in the simulation of any experiment, accurately replicate the values observed in the experimental findings. In this sense, this method can be applied in the analysis of the results of experiments based on other measurement techniques. For this purpose, the corresponding algorithm should consider the physical processes involved in these techniques.

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