We study the possible existence of bound states of three \( \Omega_{bbb} \) baryons. We consider only \( S \) wave interactions and we start from recent lattice QCD results which give a strongly attractive potential between two \( \Omega_{bbb} \) baryons in the \( ^1S_0 \) channel. We analyze different scenarios. At baryonic level, the \( \Omega_{bbb} \Omega_{bbb} \) interaction could be understood to be basically spin-independent, so that the two contributing channels, \( ^1S_0 \) and \( ^5S_2 \), would have a very similar interaction. This baryonic analysis leads to the existence of bound states in the three-body system. At the quark level, repulsive effects would appear in the \( ^5S_2 \) channel, making it more repulsive than the \( ^1S_0 \) channel. We study the effect of such repulsion.

**Keywords:** Tribaryons; quark model; baryon-baryon interaction; Pauli effects; faddeev equations.

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1. Introduction

Recently, there have been several interesting developments on the possible existence of bound states of two and three \( \Omega \) baryons. For example, Ref. [1], using a one-boson-exchange (OBE) model found bound states of the systems \( \Omega_{ccc} \Omega_{ccc} \) and \( \Omega_{bbb} \Omega_{bbb} \). Reference [2] derived a \( \Omega \Omega \) interaction based on lattice QCD. Similarly, Ref. [3], a lattice QCD calculation with nearly physical light-quark masses, derived the \( \Omega_{ccc} \Omega_{ccc} \) interaction in the \( ^5S_2 \) channel. They obtain a bound state with a binding energy of 5.68 MeV. More recently, Ref. [4] performed a lattice QCD calculation of the \( \Omega_{bbb} \Omega_{bbb} \) system finding a very deep bound state in the \( ^1S_0 \) channel, with a binding energy of 81 MeV. The energy of the bound states of the two-body systems would be the threshold of any possible three-body bound state. Finally, Ref. [5], using the existing lattice QCD interactions for the different \( \Omega \Omega \) systems, investigated the three-body systems \( \Omega \Omega \Omega \), \( \Omega_{ccc} \Omega_{ccc} \), and \( \Omega_{bbb} \Omega_{bbb} \Omega_{bbb} \). They found that none of the three-body systems binds. However, making use of the OBE interactions of Ref. [1] the \( \Omega \Omega \Omega \) system develops a bound state.

In this work, we investigate whether the \( \Omega_{bbb} \Omega_{bbb} \Omega_{bbb} \) system is bound. The \( \Omega_{bbb} \) baryon has spin \( 3/2 \) and no isospin, so that the two-body system can have total spin \( S_i = 0, 1, 2, \) and \( 3 \). However, the states \( S_i = 1 \) and \( S_i = 3 \) are not allowed in \( S \)-wave by the Pauli principle, so that one is left with only the states \( S_i = 0 \) and \( S_i = 2 \). In Ref. [4], they obtained the \( \Omega_{bbb} \Omega_{bbb} \) interaction only for the channel \( S_i = 0 \), so that we will have to discuss the situation of the channel \( S_i = 2 \). We will use some hypotheses deduced either at the baryon level or at the quark level about the \( \Omega_{bbb} \Omega_{bbb} \Omega_{bbb} \) interaction.

We carry out our study within the formalism of the non-relativistic Faddeev equations for three identical particles considering only \( S \) waves. We start our discussion of the three-body system considering only the \( S_i = 0 \) two-body channel and afterwards we analyze the effect of the \( S_i = 2 \) two-body channel for the three-body bound state.

2. Single channel Faddeev problem

The Faddeev equations for three identical particles are

\[
T = 2t_i G_0 T ,
\]

where \( t_i \) is the \( t \)-matrix of the two-body system,

\[
t_i = V + V G_0 t_i ,
\]

where \( V \) is the two-body interaction in the \( ^1S_0 \) channel and \( G_0 \) is the propagator for three free particles.

We use the complete set of basis states \( |i\rangle \),

\[
|i\rangle = |p_i q_i ((s_j, s_k) S_i, s_i) J\rangle ,
\]

with \( p_i \) and \( q_i \) the standard Jacobi momenta, \( s_i \), \( s_j \), and \( s_k \) the spins of the three particles, \( S_i \) the total spin of the pair \( jk \), and \( J \) the total spin of the three-body system. In this basis the Faddeev equation (1) becomes,

\[
\langle i | T | \phi_0 \rangle = 2 \langle i | t_i | i' \rangle \langle i' | j \rangle G_0 \langle j | T | \phi_0 \rangle ,
\]

where the explicit form of the integral equation in momentum space is given in the Appendix.
The recoupling coefficient,
\[ \langle i' | j \rangle = \langle p_i^j | p_j q_j \rangle \times \langle (s_j, s_k) S_i, s_i | (s_k, s_i) S_j, s_j \rangle, \]  
(5)
is of great interest. The space part \( \langle p_i^j | p_j q_j \rangle \) is positive definite [6]; however, the spin part is
\[ \langle (s_j, s_k) S_i, s_i | (s_k, s_i) S_j, s_j \rangle = \]  
\[ (-1)^{s_i + s_j - j} \sqrt{(2S_i + 1)(2S_j + 1)} \times W(s_j, s_k, J, s_i, S_i, S_j). \]  
(6)
Since \( S_i = S_j = 0 \) and \( s_i = s_j = s_k = J = 3/2 \) one gets,
\[ \langle (s_j, s_k) S_i, s_i | (s_k, s_i) S_j, s_j \rangle = \]  
\[ (-1)^{2s_i} \frac{1}{2s_j + 1} = -\frac{1}{4}. \]  
(7)
which is a negative number, so that it effectivly changes the nature of the two-body interaction from attractive to repulsive such that no bound state can be obtained in a one-channel calculation.

The result of Eq. (7) is a direct consequence of the Pauli principle and it applies for all systems with three identical fermions, i.e., particles with spin half-integer, like the case of three neutrons where,
\[ \langle (s_j, s_k) S_i, s_i | (s_k, s_i) S_j, s_j \rangle = \]  
\[ (-1)^{2s_i} \frac{1}{2s_j + 1} = -\frac{1}{2}. \]  
(8)
It is worth noting that in the three-neutron case there is only a two-body channel, \( S_i = 0 \), if one includes only \( S \) waves, so that there is no possibility for a three-neutron bound state with any interaction. However, in the case of the three omegas besides the \( S_i = 0 \) channel one also has the \( S_i = 2 \) two-body channel.

3. Baryonic level \( ^5S_2 \Omega \Omega \Omega \) interaction
A two-body interaction acting in \( S \)-waves contains only central and spin-spin terms, since terms like spin-orbit, tensor, etc., act only for nonzero orbital angular momentum.

The phenomenological description of the spectra of mesons and baryons in a nonrelativistic approach is based in a two-body potential between quarks [8, 9]. In Ref. [8] such potential was taken to be,
\[ V(r) = -\frac{\kappa}{r} + \lambda r - \Lambda + \frac{\kappa}{m_i m_j} \frac{e^{r/p}}{r} + \frac{1}{r_0} \delta_i \cdot \delta_j, \]  
(9)
and similarly in Ref. [9]. The four terms in the r.h.s. of Eq. (9) are, respectively, the Coulomb term, the linear confinement term, the constant term, and the spin-spin term. This interaction is able to reproduce reasonably well the masses and other properties of all the existing mesons and baryons [9].

The Yukawa function in the spin-spin term is an extended delta function which becomes a delta function if \( r_0 \to 0 \). This form of the spin-spin interaction is suggested by the non-relativistic reduction of the one-gluon-exchange diagram [10],
\[ H = \frac{2\alpha_s}{3m_i m_j} \frac{8\pi}{3} \delta^3(\vec{r}) \vec{S}_i \cdot \vec{S}_j, \]  
(10)
where \( m_i \) and \( \vec{S}_i \) are the mass and spin operator of particle \( i \). As one can see in Eqs. (9) and (10), the spin-spin term is inversely proportional to \( m_i m_j \) so that in the case of baryons with identical quarks it goes as \( 1/m_i^2 \). Thus, using the masses for the light quarks \( n \approx 0.3 \text{ GeV} \) and for the heavy quark \( b \approx 5 \text{ GeV} \) from Refs. [8, 9], one gets that the spin-spin term in the case of the \( b \bar{b} \) interaction is about 250 times smaller than that of the \( n \bar{n} \) interaction and therefore it is negligible. This means that the interaction between two \( b \) quarks is basically independent of the spin.

Let us now consider the spin-spin interaction in the case of two \( \Omega_{b\bar{b}} \) baryons. In a quark model description, the baryon-baryon interaction can be deduced from the quark-quark interaction following a well-known procedure [11, 12]. In the case where the interactions are restricted to \( S \) waves, if the quark-quark interaction is almost independent of the spin, the baryon-baryon interaction will also be almost independent of the spin [12]. This would imply a purely central \( \Omega_{b\bar{b}} \Omega_{b\bar{b}} \) interaction, so that \( V_{\Omega_{b\bar{b}}\Omega_{b\bar{b}}}(1S_0) \sim V_{\Omega_{b\bar{b}}\Omega_{b\bar{b}}}(5S_2) \) and, thus, one can use the \( S_i = 0 \) interaction of Ref. [4] also for \( S_i = 2 \).

4. Two-channel Faddeev problem
The explicit form of the Faddeev equations for the two-channel problem in momentum space is also given in the Appendix. If one includes the two channels \( S_i = 0 \) and \( S_i = 2 \), the spin recoupling coefficients of Eq. (6) are,
\[ \langle 0 | 0 \rangle = -\frac{1}{4}, \quad \langle 0 | 2 \rangle = -\frac{\sqrt{5}}{4}, \]  
\[ \langle 2 | 0 \rangle = -\frac{\sqrt{5}}{4}, \quad \langle 2 | 2 \rangle = 3 \frac{1}{4}. \]  
(11)
The recoupling coefficient \( \langle 2 | 2 \rangle \) is positive and much larger than the coefficient \( \langle 0 | 0 \rangle \), so that if the interaction in the channel \( S_i = 2 \) is equal to that of the channel \( S_i = 0 \), one would expect to get a bound state.

Following the conclusion of the previous section we start by taking \( V_{\Omega_{b\bar{b}}\Omega_{b\bar{b}}}(5S_2) = V_{\Omega_{b\bar{b}}\Omega_{b\bar{b}}}(1S_0) \). We find in this case that the ground state and one excited state of the three-body \( \Omega_{b\bar{b}}\Omega_{b\bar{b}}\Omega_{b\bar{b}} \) system, that lie at \(-393.8 \text{ MeV} \) and \(-94.9 \text{ MeV} \), respectively \((-363.9 \text{ MeV} \) and \(-81.0 \text{ MeV} \), respectively, if one includes the Coulomb interaction).
5. Quark level $^5S_2 \Omega \Omega$ interaction

As it has been discussed in the literature [11, 14–16] there appears quark Pauli blocking for particular channels of some two-baryon systems. This has been discussed in detail, for example, for the $\Sigma N$ system [16] or for the $\Delta N$ and $\Delta \Delta$ systems [11, 14, 15]. Pauli blocking translates into repulsive cores. See, for example, Fig. 3 of Ref. [15]. The $\Omega_{bbb}\Omega_{bbb}$ two-baryon belongs to the same flavor multiplet as the $\Delta\Delta$ system and thus it also shows Pauli blocking for the $^5S_2$ channel, which means a repulsive core. In order to simulate these effects we will take the $S_i = 2$ interaction as,

$$V_2(r) = \begin{cases} V_0(r) & r > r_0, \\ V_0(r) - A[r_0/r_0] - A[r_0/r_0]^2 & r \leq r_0, \end{cases}$$

where $A$ is a constant and $r_0 = 0.104$ fm is the radius where $V_0$ is minimum (see Fig. 1). The form (12) guarantees that $V_2(r)$ and $dV_2(r)/dr$ are continuous at $r = r_0$. This transformation has the effect of pushing upwards the short-range part of the interaction, thereby reducing the attraction. We show this behavior in Fig. 1 for the special cases $A = 1$ and $A = 2$ as compared with the $A = 0$ case, which corresponds to the model of Ref. [4]. Thus, our final model of the $\Omega_{bbb}\Omega_{bbb}$ interaction contained in Eq. (12) and Fig. 1, has the $^5S_2$ interaction less attractive than the $^1S_0$ interaction which is similar to what is obtained in the OBE model of Ref. [1].

We show in Fig. 2 the evolution of the binding energy (with and without Coulomb) as a function of the parameter $A$. As one can see, the binding energy changes very slowly. We consider that values of $A$ larger than 20 are not realistic so that we expect the binding energy to lie between 250 and 350 MeV.

![Figure 1. The $\Omega_{bbb}\Omega_{bbb}$ interaction $V_2$ for $A = 0$, 1, and 2.](Image)

![Figure 2. The $\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$ binding energy as a function of the parameter $A$.](Image)

Appendix A.

In order to solve the single channel three-body problem we use the method of Ref. [13], where the two-body amplitudes are expanded in terms of Legendre polynomials. Thus, the Faddeev equations for the bound state problem take the simple form,

$$T^m(q_i) = \sum_n \int_0^\infty dq_j K^{nm}(q_i, q_j; E) T^m(q_j), \quad (A.1)$$

where

$$K^{nm}(q_i, q_j; E) = 2 \langle 0| 0 \rangle \sum_r \tau^{nr}_i (E - q_i^2/2\nu_i) \frac{q_j^2}{2}$$

$$\times \int_{-1}^1 d\cos \theta \frac{P_r(x'_i)P_m(x_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j}, \quad (A.2)$$

with $\langle 0| 0 \rangle$ the spin recoupling coefficient (11),

$$\tau^{nr}_i(e) = \frac{2\alpha + 1}{2} \frac{2r + 1}{2} \int_{-1}^1 dx_i$$

$$\times \int_{-1}^1 dx'_i P_n(x_i) t_i(x_i, x'_i; e) P_r(x'_i), \quad (A.3)$$

and

$$x_i = \frac{p_i - b}{p_i + b}. \quad (A.4)$$

$p_j$ and $q_j$ are the magnitudes of the Jacobi relative momenta while $\eta_j$ and $\nu_j$ are the corresponding reduced masses.
Finally, \( t_i(x_i, x'_i; e) \) corresponds to the off-shell two-body \( t \)-matrix \( t_i(p_i, p'_i; e) \) through the transformation (A.4), with \( b \) a scale parameter on which the solution does not depend. The off-shell two-body \( t \)-matrices are obtained by solving the Lippmann-Schwinger equation,

\[
t_i(p_i, p'_i; e) = V_i(p_i, p'_i) + \int_0^\infty p''d^3p'_i'V_i(p_i, p'_i') \times \frac{1}{e - p''^2/2\eta_j + ie),}
\]  

(A.5)

with \( e = E - q_j^2/2\nu_j \). In order to solve the two-channel three-body problem, Eq. (A.1) becomes,

\[
T^n_\alpha(q_i) = \sum_{m \beta} \int_0^\infty dq_j K_{\alpha \beta}^{nm}(q_i, q_j; E)T^m_{\beta}(q_j),
\]  

(A.6)

with

\[
K_{\alpha \beta}^{nm}(q_i, q_j; E) = 2 \langle \alpha | \beta \rangle \sum_r \sigma^{nm}_\alpha (E - q^2_i/2\nu_i)\frac{q^2_i}{2} \\
\times \int_{-1}^1 d \cos \theta P_r(x'_i)P_m(x_j).
\]  

(A.7)

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i. On the opposite side, a repulsive potential together with a negative spin recoupling coefficient does not change the nature of the interaction so as to lead to a three-body bound state, as discussed in Ref. [7] for the case of the \( J = 3/2 \) \( \Sigma^−nnn \) system.


