$\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$ tribaryons

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We study the possible existence of bound states of three Ω_{bbb} baryons. We consider only S wave interactions and we start from recent lattice QCD results which give a strongly attractive potential between two Ω_{bbb} baryons in the $^1S_0$ channel. We analyze different scenarios. At baryonic level, the $\Omega_{bbb}\Omega_{bbb}$ interaction could be understood to be basically spin-independent, so that the two contributing channels, $^1S_0$ and $^5S_2$, would have a very similar interaction. This baryonic analysis leads to the existence of bound states in the three-body system. At the quark level, repulsive effects would appear in the $^5S_2$ channel, making it more repulsive than the $^1S_0$ channel. We study the effect of such repulsion.

Keywords: Tribaryons; quark model; baryon-baryon interaction; Pauli effects; faddeev equations.

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1. Introduction

Recently, there have been several interesting developements on the possible existence of bound states of two and three Ω baryons. For example, Ref. [1], using a one-boson-exchange (OBE) model found bound states of the systems $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$. Reference [2] derived a $\Omega\Omega$ interaction based on lattice QCD. Similarly, Ref. [3], a lattice QCD calculation with nearly physical light-quark masses, derived the $\Omega_{ccc}\Omega_{ccc}$ interaction in the $^1S_0$ channel. They obtain a bound state with a binding energy of 5.68 MeV. More recently, Ref. [4] performed a lattice QCD calculation of the $\Omega_{bbb}\Omega_{bbb}$ system finding a very deep bound state in the $^1S_0$ channel, with a binding energy of 81 MeV. The energy of the bound states of the two-body systems would be the threshold of any possible three-body bound state. Finally, Ref. [5], using the existing lattice QCD interactions for the different $\Omega\Omega$ systems, investigated the three-body systems $\Omega\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$. They found that none of the three-body systems binds. However, making use of the OBE interactions of Ref. [1] the $\Omega\Omega\Omega$ system develops a bound state.

In this work, we investigate whether the $\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$ system is bound. The Ω_{bbb} baryon has spin $3/2$ and no isospin, so that the two-body system can have total spin $S_i = 0, 1, 2$, and 3. However, the states $S_i = 1$ and $S_i = 3$ are not allowed in S-wave by the Pauli principle, so that one is left with only the states $S_i = 0$ and $S_i = 2$. In Ref. [4], they obtained the $\Omega_{bbb}\Omega_{bbb}$ interaction only for the channel $S_i = 0$, so that we will have to discuss the situation of the channel $S_i = 2$. We will use some hypotheses deduced either at the baryon level or at the quark level about the $\Omega_{bbb}\Omega_{bbb}$ $^5S_2$ interaction.

We carry out our study within the formalism of the non-relativistic Faddeev equations for three identical particles considering only S waves. We start our discussion of the three-body system considering only the $S_i = 0$ two-body channel and afterwards we analyze the effect of the $S_i = 2$ two-body channel for the three-body bound state.

2. Single channel Faddeev problem

The Faddeev equations for three identical particles are

$$T = 2t_i G_0 T \ , \tag{1}$$

where t_i is the t-matrix of the two-body system,

$$t_i = V + V G_0 t_i \ , \tag{2}$$

where V is the two-body interaction in the $^1S_0$ channel and G_0 is the propagator for three free particles.

We use the complete set of basis states $|i\rangle$,

$$|i\rangle = |p_i q_i((s_j, s_k)S_i, s_i)J\rangle \ , \tag{3}$$

with p_i and q_i the standard Jacobi momenta, s_i, s_j, and s_k the spins of the three particles, S_i the total spin of the pair jk, and J the total spin of the three-body system. In this basis the Faddeev equation (1) becomes,

$$\langle i| T |\phi_0\rangle = 2 \langle i| t_i |i'\rangle \langle i'| j\rangle G_0 \langle j| T |\phi_0\rangle \ , \tag{4}$$

where the explicit form of the integral equation in momentum space is given in the Appendix.

The recoupling coefficient,

$$\langle i'|\, j\rangle = \langle p_i' q_i' |\, p_j q_j\rangle \times \langle ((s_j, s_k)S_i, s_i)J|\, ((s_k, s_i)S_j, s_j)J\rangle \ , \quad (5)$$

is of great interest. The space part $\langle p_i' q_i' |\, p_j q_j\rangle$ is positive definite [6]; however, the spin part is

$$\langle ((s_j, s_k)S_i, s_i)J|\, ((s_k, s_i)S_j, s_j)J\rangle = (-)^{S_j+s_j-J}\sqrt{(2S_i+1)(2S_j+1)} \times W(s_j s_k J s_i; S_i S_j) \ . \quad (6)$$

Since $S_i = S_j = 0$ and $s_i = s_j = s_k = J = 3/2$ one gets,

$$\langle ((s_j, s_k)S_i, s_i)J|\, ((s_k, s_i)S_j, s_j)J\rangle = (-)^{2s_j}\frac{1}{2s_j+1} = -\frac{1}{4} \ , \quad (7)$$

which is a negative number, so that it effectively changes the nature of the two-body interaction from attractive to repulsive such that no bound state can be obtained in a one-channel calculation[i].

The result of Eq. (7) is a direct consequence of the Pauli principle and it applies for all systems with three identical fermions, *i.e.*, particles with spin half-integer, like the case of three neutrons where,

$$\langle ((s_j, s_k)S_i, s_i)J|\, ((s_k, s_i)S_j, s_j)J\rangle = (-)^{2s_j}\frac{1}{2s_j+1} = -\frac{1}{2} \ . \quad (8)$$

It is worth noting that in the three-neutron case there is only a two-body channel, $S_i = 0$, if one includes only S waves, so that there is no possibility for a three-neutron bound state with any interaction. However, in the case of the three omegas besides the $S_i = 0$ channel one also has the $S_i = 2$ two-body channel.

3. Baryonic level $^5S_2$ $\Omega\Omega$ interaction

A two-body interaction acting in S-waves contains only central and spin-spin terms, since terms like spin-orbit, tensor, etc., act only for nonzero orbital angular momentum.

The phenomenological description of the spectra of mesons and baryons in a nonrelativistic approach is based in a two-body potential between quarks [8,9]. In Ref. [8] such potential was taken to be,

$$V(r) = -\frac{\kappa}{r} + \lambda r - \Lambda + \frac{\kappa}{m_i m_j}\frac{exp^{-r/r_0}}{r r_0}\vec{\sigma}_i\cdot\vec{\sigma}_j \ , \quad (9)$$

and similarly in Ref. [9]. The four terms in the r.h.s. of Eq. (9) are, respectively, the Coulomb term, the linear confinement term, the constant term, and the spin-spin term. This interaction is able to reproduce reasonably well the masses and other properties of all the existing mesons and baryons [9].

The Yukawa function in the spin-spin term is an extended delta function which becomes a delta function if $r_0 \to 0$. This form of the spin-spin interaction is suggested by the non-relativistic reduction of the one-gluon-exchange diagram [10],

$$H = \frac{2\alpha_s}{3m_i m_j}\frac{8\pi}{3}\delta^3(\vec{r})\vec{S}_i\cdot\vec{S}_j, \quad (10)$$

where m_i and $\vec{S}_i$ are the mass and spin operator of particle i. As one can see in Eqs. (9) and (10), the spin-spin term is inversely proportional to $m_i m_j$ so that in the case of baryons with identical quarks it goes as $1/m_i^2$. Thus, using the masses for the light quarks n (≈ 0.3 GeV) and for the heavy quark b (≈ 5 GeV) from Refs. [8,9], one gets that the spin-spin term in the case of the *bb* interaction is about 250 times smaller that that of the nn interaction and therefore it is negligible. This means that the interaction between two b quarks is basically independent of the spin.

Let us now consider the spin-spin interaction in the case of two Ω_{bbb} baryons. In a quark model description, the baryon-baryon interaction can be deduced from the quark-quark interaction following a well-known procedure [11,12]. In the case where the interactions are restricted to S waves, if the quark-quark interaction is almost independent of the spin, the baryon-baryon interaction will also be almost independent of the spin [12]. This would imply a purely central $\Omega_{bbb}\Omega_{bbb}$ interaction, so that $V_{\Omega_{bbb}\Omega_{bbb}}(^1S_0) \sim V_{\Omega_{bbb}\Omega_{bbb}}(^5S_2)$ and, thus, one can use the $S_i = 0$ interaction of Ref. [4] also for $S_i = 2$.

4. Two-channel Faddeev problem

The explicit form of the Faddeev equations for the two-channel problem in momentum space is also given in the Appendix. If one includes the two channels $S_i = 0$ and $S_i = 2$, the spin recoupling coefficients of Eq. (6) are,

$$\langle 0|\, 0\rangle = -\frac{1}{4} \qquad \langle 0|\, 2\rangle = -\frac{\sqrt{5}}{4}$$
$$\langle 2|\, 0\rangle = -\frac{\sqrt{5}}{4} \qquad \langle 2|\, 2\rangle = +\frac{3}{4} \ . \quad (11)$$

The recoupling coefficient $\langle 2|\, 2\rangle$ is positive and much larger than the coefficient $\langle 0|\, 0\rangle$, so that if the interaction in the channel $S_i = 2$ is equal to that of the channel $S_i = 0$, one would expect to get a bound state.

Following the conclusion of the previous section we start by taking $V_{\Omega_{bbb}\Omega_{bbb}}(^5S_2) = V_{\Omega_{bbb}\Omega_{bbb}}(^1S_0)$. We find in this case that the ground state and one excited state of the three-body $\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$ system, that lie at -393.8 MeV and -94.9 MeV, respectively (-363.9 MeV and -81.0 MeV, respectively, if one includes the Coulomb interaction).

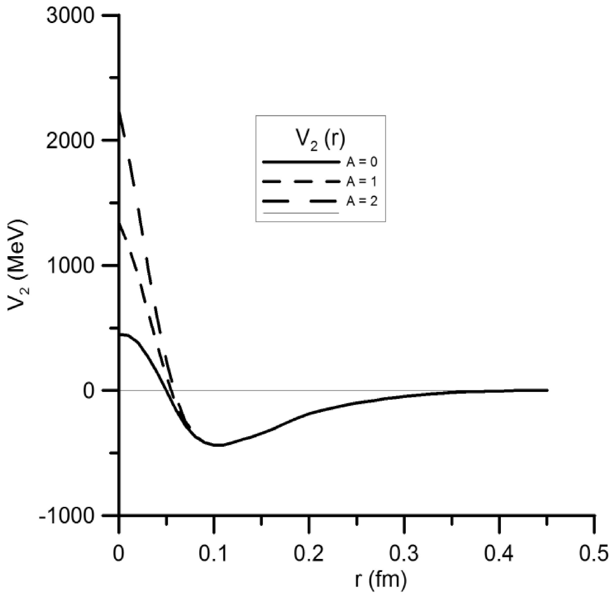

FIGURE 1. The $\Omega_{bbb}\Omega_{bbb}$ interaction V_2 for $A = 0, 1,$ and 2.

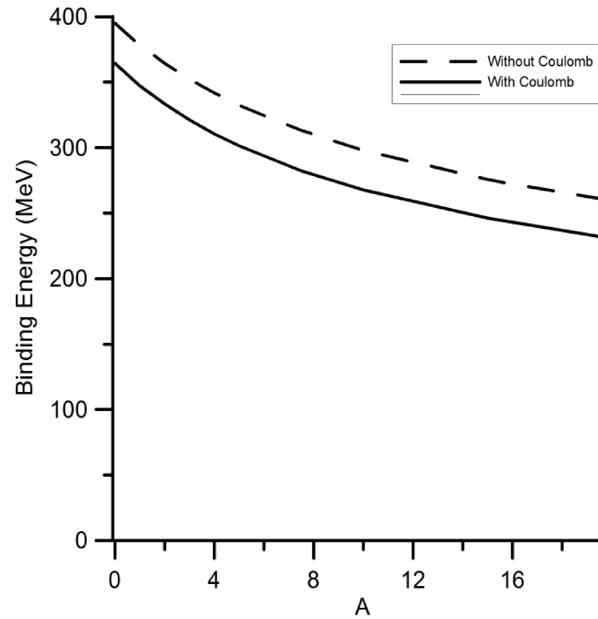

FIGURE 2. The $\Omega_{bbb}\Omega_{bbb}\Omega_{bbb}$ binding energy as a function of the parameter A.

5. Quark level $^5S_2$ $\Omega\Omega$ interaction

As it has been discussed in the literature [11, 14–16] there appears quark Pauli blocking for particular channels of some two-baryon systems. This has been discussed in detail, for example, for the ΣN system [16] or for the ΔN and $\Delta\Delta$ systems [11, 14, 15]. Pauli blocking translates into repulsive cores. See, for example, Fig. 3 of Ref. [15]. The $\Omega_{bbb}\Omega_{bbb}$ two-baryon belongs to the same flavor multiplet as the $\Delta\Delta$ system and thus it also shows Pauli blocking for the $^5S_2$ channel, which means a repulsive core. In order to simulate these effects we will take the $S_i = 2$ interaction as,

$$V_2(r) = \begin{cases} V_0(r) & r > r_0, \\ V_0(r) + A[V_0(r) & \\ \quad -V_0(r_0)]\left(1 - \frac{r}{r_0}\right)^2 & r \le r_0 \end{cases} , \quad (12)$$

where A is a constant and $r_0 = 0.104$ fm is the radius where V_0 is minimum (see Fig. 1). The form (12) guarantees that $V_2(r)$ and $dV_2(r)/dr$ are continuous at $r = r_0$. This transformation has the effect of pushing upwards the short-range part of the interaction, thereby reducing the attraction. We show this behavior in Fig. 1 for the special cases $A = 1$ and $A = 2$ as compared with the $A = 0$ case, which corresponds to the model of Ref. [4]. Thus, our final model of the $\Omega_{bbb}\Omega_{bbb}$ interaction contained in Eq. (12) and Fig. 1, has the $^5S_2$ interaction less attractive than the $^1S_0$ interaction which is similar to what is obtained in the OBE model of Ref. [1].

We show in Fig. 2 the evolution of the binding energy (with and without Coulomb) as a function of the parameter A. As one can see, the binding energy changes very slowly. We consider that values of A larger than 20 are not realistic so that we expect the binding energy to lie between 250 and 350 MeV.

Appendix A.

In order to solve the single channel three-body problem we use the method of Ref. [13], where the two-body amplitudes are expanded in terms of Legendre polynomials. Thus, the Faddeev equations for the bound state problem take the simple form,

$$T^n(q_i) = \sum_n \int_0^\infty dq_j K^{nm}(q_i, q_j; E) T^m(q_j) \ , \quad (A.1)$$

where

$$K^{nm}(q_i, q_j; E) = 2 \langle 0 | \, 0 \rangle \sum_r \tau_i^{nr}(E - q_i^2/2\nu_i) \frac{q_i^2}{2} \times \int_{-1}^{1} d\cos\theta \frac{P_r(x_i')P_m(x_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j} \ , \quad (A.2)$$

with $\langle 0 | \, 0 \rangle$ the spin recoupling coefficient (11),

$$\tau_i^{nr}(e) = \frac{2n+1}{2}\frac{2r+1}{2}\int_{-1}^{1} dx_i \times \int_{-1}^{1} dx_i' P_n(x_i) t_i(x_i, x_i'; e) P_r(x_i') \ , \quad (A.3)$$

and

$$x_i = \frac{p_i - b}{p_i + b} \ . \quad (A.4)$$

p_j and q_j are the magnitudes of the Jacobi relative momenta while η_j and ν_j are the corresponding reduced masses.

Finally, $t_i(x_i, x'_i; e)$ corresponds to the off-shell two-body t-matrix $t_i(p_i, p'_i; e)$ through the transformation (A.4), with b a scale parameter on which the solution does not depend. The off-shell two-body t-matrices are obtained by solving the Lippmann-Schwinger equation,

$$t_i(p_i, p'_i; e) = V_i(p_i, p'_i) + \int_0^\infty p''_i 2dp''_i V_i(p_i, p''_i) \times \frac{1}{e - {p''_i}^2/2\eta_j + i\epsilon} t_i(p''_i, p'_i; e) \ , \quad \text{(A.5)}$$

with $e = E - q_i^2/2\nu_i$. In order to solve the two-channel three-body problem, Eq. (A.1) becomes,

$$T_\alpha^n(q_i) = \sum_{m\beta} \int_0^\infty dq_j K_{\alpha\beta}^{nm}(q_i, q_j; E) T_\beta^m(q_j) \ , \quad \text{(A.6)}$$

with

$$K_{\alpha\beta}^{nm}(q_i, q_j; E) = 2 \langle \alpha | \ \beta \rangle \sum_r \tau_\alpha^{nr}(E - q_i^2/2\nu_i) \frac{q_i^2}{2} \times \int_{-1}^{1} d\cos\theta \frac{P_r(x'_i) P_m(x_j)}{E - p_j^2/2\eta_j - q_j^2/2\nu_j} \ . \quad \text{(A.7)}$$

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i. On the opposite side, a repulsive potential together with a negative spin recoupling coefficient does not change the nature of the interaction so as to lead to a three-body bound state, as discussed in Ref. [7] for the case of the $J = 3/2$ $\Sigma^- nn$ system.

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