Bohr postulates derived from the toroidal electron model

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The quantization of the electron orbits in the Bohr atom is revisited. The toroidal electron model, in which electron charge is described by Schwinger electromagnetic wave orbiting the electron mass, offers a natural explanation for the orbit quantization. As a consequence, the four Bohr postulates can be directly derived from the toroidal electron structure. A physical meaning for the Rydberg constant is also proposed.

Keywords: Toroidal electron model; Schwinger electromagnetic wave; Bohr postulates.

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1. Introduction

In 1913, Niels Bohr proposed four postulates which describe the behavior of the electrons in atoms [1]. The postulates predicted the quantization of the electron orbits around the nucleus of the atoms. Due to its simplicity and excellent agreement with the radiation spectrum emitted by hydrogen atoms, including the theoretical prediction of the Rydberg constant reported two decades before [2], the Bohr model became the basis for the Quantum Mechanics [3].

One of the most important consequences was the wave-particle duality described by de Broglie, which led to the uncertainty principle and provided the probabilistic characteristics to describe the motion of the so-called quantum particles [3].

Although, the probabilistic behavior is nowadays well accepted, one aspect is still to be clarified, more specifically, the structure of the quantum particles which make them to show the wave-particle duality.

Due to this, during the last decade many authors have devoted efforts to understand the structure of the electron [4-6] and other quantum particles [7]. Some of the models have suggested that these particles show the *Zitterbewegung* behavior [6-12]. One of the most interesting is the electron structure in which electron moves in a helical way [4,12,13].

In a previous work, the helical motion of the electron has been suggested to do with the electromagnetic wave related to the Schwinger limits [10]. In fact, electron charge appears as a consequence of the electromagnetic wave orbiting the electron mass with the Compton radius ($r_C = \lambda_C/2\pi$), where λ_C is the Compton wave length of the electron. This led us to understand a free electron at rest as the electron charge moving at the speed of light orbiting the electron mass at rest, which seems to agree with the electron structure reported by Rivas [11,12].

Furthermore, when orbiting a proton forming the Bohr atom, the electron moves in a helical way in a toroidal structure (see Refs. [10,13]).

In order to verify how far the semi-classical electron toroidal model provides good agreement with accepted models, in this letter we compare it with the Bohr postulates.

2. Model

In Fig. 1a) is displayed a free electron at rest as described in the toroidal electron model [10], in which the charge is related to the Schwinger electromagnetic wave (blue circumference) and the mass is at rest (gray circle).

In such a situation, the electron cannot have its velocity increased in the plane of the Fig. 1a) since electron charge is already at the speed of light. Thus, the only way to change the electron energy is make it to move in the perpendicular direction to the Schwinger wave plane (blue circumference) [10,14]. This will imply in a helical motion, as shown in Fig. 1b). In fact, if the electron is under the influence of an electric potential V, it will move the electron mass (m_e) to a given velocity predicted by $eV = p^2/2m_e = m_e v^2/2$, where p and e is the electron linear momentum and the electron charge. On the other hand, if the electron is near to a positive charge, such as a proton creating a Bohr atom [3], it orbits the proton due to a centripetal force having a helical motion, as described in previous works (see the Figs. 2 and 4 of the Refs. [10,13], respectively). In both cases, there will be an increase in the kinetic energy of the electron mass added to a change in the electron energy related to the motion of the Schwinger wave, which must be associated to the potential energy of the electron charge, as will be shown later.

Figure 2 shows fine details on the helical motion for three different orbits in the Bohr atom.

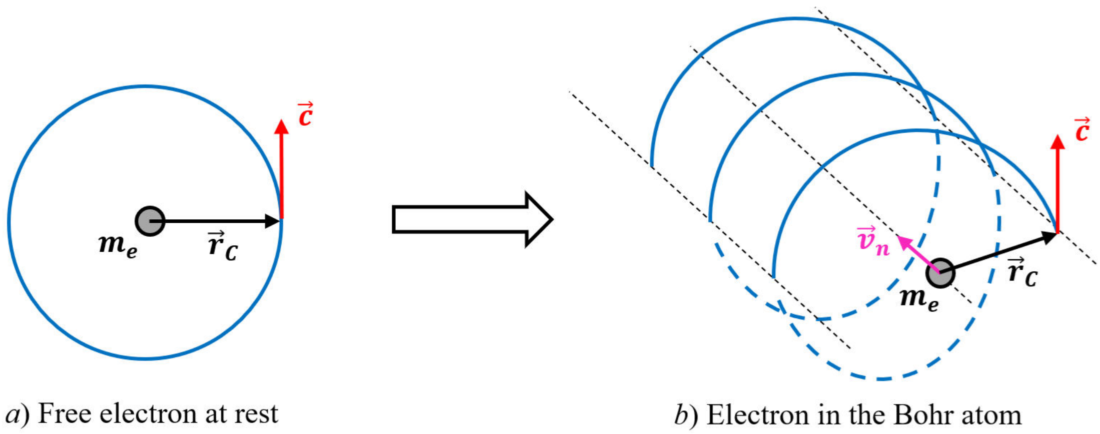

FIGURE 1. a) Free electron at rest in the toroidal electron model, in which electron charge is related to the Schwinger electromagnetic wave (blue circumference) with Compton radius ($r_C = \lambda_C/2\pi$) orbiting the electron mass at rest (gray circle). In b) is shown a fraction of the electron helical motion orbiting a proton. In such a case, the electron mass gets kinetic energy given by $K = m_e v_n^2/2$ and the Schwinger wave is drifted in its perpendicular direction. The figure is not in scale (see caption of the Fig. 2).

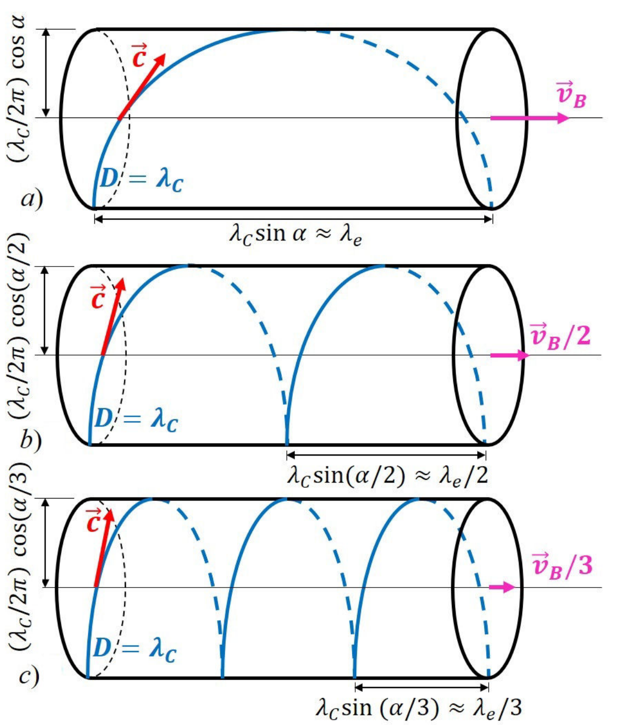

FIGURE 2. Fractions of the helical motion of the electron charge in three different orbits of the Bohr atom. (a) It is shown only one spiral ($n = 1$) within the pass given by $\lambda_e \approx \alpha\lambda_C$. In such a case, the first Bohr orbit has $v_n = v_B$ and the circumference is related to $\lambda_B = N\lambda_e$, where $N = 1/\alpha^2 \approx 18,879$ spirals [15]. In (b) and (c) are shown the number of spiral turns within the λ_e length for two different orbital velocities. The higher the number n, the lower the velocity v_n. The figure is not in scale, since the diameter in y-axis is near 44 [$= 2(\lambda_C/2\pi)/\lambda_e = 1/\alpha\pi$] times bigger than the length in x-axis (λ_e).

As described previously [13], the length (D) traveled by the Schwinger wave is constant no matter the spiral pass (P). This is given by

$$D^2 = 4\pi^2 r^2 + P^2, \tag{1}$$

where $r = (\lambda_C/2\pi)\cos(\alpha/n)$ and $P = \lambda_C \sin(\alpha/n)$, *n* is a integer number related to the electron orbit, and α is the Sommerfeld fine structure constant [15], providing $D = \lambda_C$.

Noting that $\alpha \ll 1$, then the pass for the first orbit is very close to λ_e $(= \lambda_C \sin\alpha \approx \lambda_C\alpha)$, which is related to the electron classical radius by $\lambda_e = 2\pi r_e$ which means that the first Bohr orbit $(n = 1)$ has almost 18,879 spirals $(N = 1/\alpha^2$ and $\alpha = 1/137.035999$ [13, 15]) with radius $r_C = \lambda_C/2\pi$ $(\cos\alpha \approx 1)$ and pass $P \approx \lambda_e$ [16], in agreement with the previous result [10,13].

Similar behavior happens for $n > 1$, in which the number of turns increases proportionally to n, making the pass to be equal to λ_e/n, since $\sin(\alpha/n) \approx (\alpha/n)$ and $\lambda_C\alpha = \lambda_e$ (see the passes at the bottom of the Fig. 2 (a) to (c)). Furthermore, such as each wave turn $(D = \lambda_C)$ spends the same time (τ) to occur no matter the pass, since the Schwinger wave travels at a speed of light $[\tau = \lambda_C/c = \lambda_e/v_B = (\lambda_e/n)/(v_B/n)]$, the higher the number of turns (n), the lower the orbital velocity $(v_n = v_B/n)$. Based upon that, it is easy to show that

$$v_n = \frac{v_B}{n} = c\frac{\alpha}{n} = \frac{e^2}{2\varepsilon_0 h}\frac{1}{n}, \tag{2}$$

where $v_B = \alpha c$ and $\alpha = e^2/2\varepsilon_0 hc$ [3,17], where ε_0 is the permittivity of the free space, h is the Planck constant, and c is the speed of light. Following this behavior, if $n \to \infty$, $v_n \to$ zero, $P =$ zero, $D = \lambda_C = 2\pi r_C$ and the free electron at rest is reached, as described in the discussion of the Fig. 1a). These results imply in the orbit quantization as a first principle, which is in complete agreement with the Bohr model [2,3]. Furthermore, this also seems to conciliate the orbit quantization with the *Zitterbewegung* physics [6-12].

Interesting is also to notice that the behavior of the Schwinger wave running in three dimensional spirals reminds, in some regards, the behavior of standing waves oscillating in a fixed length L with harmonic wavelengths (see, for instance, Ref. [18]), providing a better understanding for the orbit quantization and the wave behavior of the electron in the Bohr atom.

Furthermore, it is important to observe that the electron structure reported here avoids relativistic effects on the electron mass, since its highest velocity in the Bohr orbits is $v_B = \alpha c$ [17], whose Lorentz factor provides $m_e/m_0 = 1.000027$ ($\alpha = 1/137.035999$ [15]), where m_0 is the electron rest mass. This is an old issue, since models which consider the motion of the electron at the speed of light have never been accepted, because the electron mass cannot travel at such a speed. The toroidal electron model seems to overcome this difficulty, since electron charge can move as an electromagnetic wave at the speed of light, leaving the electron mass to move in a perpendicular direction at non-relativistic velocities (see also the final discussion in the previous paper [10]).

Once this picture is accepted, the other aspects of the orbit quantization in the Bohr atom and his postulates appear naturally, as follows.

Making the centripetal force equal to the electrical force $[F_e = m_e v^2/r = (1/4\pi\varepsilon_0)e^2/r^2]$, one can show that each quantized orbit has radius given by

$$r_n = n^2 \frac{\varepsilon_0 h^2}{\pi m_e e^2}, \tag{3}$$

which along with Eq. (1) imply in the quantization of the angular moment L and the energy U_n, such as

$$L = m_e v_n r_n = n\hbar, \tag{4}$$

and

$$U_n = U_T - U_c = K + V = -\frac{m_e e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = \frac{\alpha^2}{2\lambda_C}\frac{1}{n^2}, \tag{5}$$

respectively, where $U_T = K + V + U_C$ is the total energy, $K = m_e v_n^2/2$ is the kinetic energy related to the electron mass, $V = (1/4\pi\varepsilon_0)e^2/r_n$ is the potential energy related to the electron charge, $U_C = hc/\lambda_C$ is the Compton energy of the free electron at rest (which is also equal to $m_e c^2$), and $\lambda_C = h/m_e c$.

Furthermore, making the difference of energy between two orbits predicts the equation for the emission or absorption of photons with specific frequencies f, which are related to the radiation spectrum of the hydrogen atom as

$$\Delta U = U_n - U_m = hf = \frac{hc}{\lambda} = -\frac{hc}{\lambda_C}\frac{\alpha^2}{2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right), \tag{6}$$

providing

$$\frac{1}{\lambda} = R_y\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \tag{7}$$

where the Rydberg constant is defined as

$$R_y \equiv \frac{\alpha^2}{2\lambda_C} = \frac{1}{2N\lambda_C} = 1.097 \times 10^7 \ \text{m}^{-1}, \tag{8}$$

after using $\lambda_C = h/m_e c = 0.02426$ Å and $\alpha = 1/137.035999$. This provides a physical meaning for the Rydberg constant, in which its inverse is related to the length traveled by the $2N$ turns $(= 2/\alpha^2 \approx 37,558)$ of the Schwinger wave along the Compton circumference or it is equal to the length $2l_B$, which is related to $l_B = N\lambda_C = 4.556 \times 10^{-8}$ m, as defined in the Ref. [13].

3. Conclusion

In conclusion, the results reported in this work provide the four Bohr postulates directly from toroidal electron structure, taking the standing wave running along with the mass in the direction out-of-plane of the Schwinger wave as the basis. Furthermore, this confirms the previous result [10-13] in which the electron charge and mass are related but have different motions. As a free electron at rest, charge is due to the Schwinger electromagnetic wave traveling at the speed of light, while electron mass is at rest. The quantization of the orbits is obtained totally from first principles arguments, which is, as far as we know, the first time this is reported. As a consequence of the semi-classical model, a physical meaning is found for the Rydberg constant. The results show that the toroidal electron structure is in completely agreement with the Bohr model and opens a door to find a new view on the wave-particle duality.

Declarations

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