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Soliton solutions to the 3-dimensional KdV and modified 3-dimensional KdV equations

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In this study, we successfully employed the Tanh-Coth method alongside the Riccati equation transformation to derive exact soliton solutions for both the three-dimensional Korteweg-de Vries (3D KdV) equation and its modified variant. This analytical approach enabled the systematic reduction of the complex nonlinear partial differential equations to solvable ordinary differential equations. By assuming a traveling wave transformation and expressing the solution in terms of hyperbolic tangent and hyperbolic cotangent functions, solutions of the Riccati equation, we obtained a variety of solitary wave profiles, including kink, anti-kink, and localized pulse solutions. Graphical representation for some of the obtained solutions is portrayed to show the nature of the kink, anti-kink and localized pulse solution in 3D, contours and 2D respectively, by choosing suitable values of parameters.

Keywords: NLPDEs; three-dimensional KdV; modified three-dimensional KdV; Tanh-Coth method with Ricatti equation

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1. Introduction

Nonlinear evolution equations (NLEEs) occur in many sciences, engineering and industrialized applications which include physics of plasma, diffusion process, chemical reactions, condensed matter, electro/magnetic, physics of solid state, neural state, optical fibers and so on [15]. The propagation of wave in fluid like dispersive fluids, are mathematically modelled with the aid of nonlinear hyperbolic PDEs, whose soliton solution can be bright or dark or whose solutions can be bright-dark soliton in nature [12]. NLEEs give a far great challenge in an effort to obtain exact solution. In recent years, the need for soliton solution in higher dimensional fluid flow media has grown exponentially. KdV-like equations are NLPDEs that possesses dispersion and nonlinearity phenomena which are crucial in NLE models. With the presence of solitons in bubbly inviscid fluid flow, was initially taken theoretically by Van-Wijngarden in the late 60's [18], which stands for the idea that there must exist a delicate symmetrical balance between dispersion and nonlinearity [14]. Solitons are classes of solutions of some NLEEs such as the class of KdV equations, sine-Gordon (SG) equation, class of nonlinear Schrodinger equations [13]. Bright and dark soliton are also called the non-topologically and the topologically soliton solution in the case or situation of nonlinearity of optics [4].

Recently, there are so many approaches which were originated for getting the exact solution of NLEEs, which include the approach of inverse scattering [7], sine/cosine approach [15], the simple equation approach [10], 1st integral approach [20], method of extended-tanh [16,17], approach of Adomian decomposition [1], method of F-expansion [8,22], (G'/G)-expansion approach [2], Darboux/Backlund transformations [19], Miura transformation approach [9], method of Jacobi elliptic function [21] and many more. Recently, a well-organized generation of weak nonlinear long wave in bubbly flow of fluid which was described by 3-dimensional KdV equation only if there is a neglect of viscosity [11]. The 3-dimensional KdV is integrable [11].

While the Tanh-Coth method has been applied to lower-dimensional or simpler nonlinear equations in the past, its application to 3D KdV and modified 3D KdV equations represents a novel approach. The Tanh-Coth method is an effective technique for finding soliton solutions of nonlinear differential equations, but its application to higher-dimensional problems, especially involving complex nonlinearities, is less common [24].

The Tanh-Coth method uses hyperbolic functions (tangent and cotangent) to transform and solve nonlinear equations. While its utility is established for 1D and 2D problems, using it for 3D KdV equations and their modified versions is a new application. The systematic approach this method provides could make it easier to obtain exact soliton solutions in 3D, which would otherwise be very challenging to find. The study might explore new ways to handle the nonlinear terms and higher-dimensional complexities that have been difficult to approach using previous methods [24].

This study is arranged in the manner as follows: in section two, we provide a brief background of the 3-dimensional KdV and the modified 3-dimensional KdV equations. In Sec. 3, we give a brief description of Tanh-Coth approach. In Sec. 4, we apply the tanh-coth approach to the 3-dimensional KdV and the modified 3-Dimensional KdV equations. In Sec. 5, we give a graphical display of some of the obtained solutions. In Sec. 6, we give a discussion for the portrayed figures. In Sec. 7, we give a concluding remark of the study.

2. The Three-Dimensional KdV and Modified Three-Dimensional KdV Equations

In recent years, the 3-dimensional KdV equation in bubbly fluid have received huge attention. It was utilized and analyzed qualitatively [5]. Its t-dependent variable coefficients was considered in which the exact soliton solution and its auto-Backlund transformation were provided [6]. The 3-dimensional KdV equation is given as [24]

$$(v_t + vv_x + v_{xxx})_x + v_{yy} + v_{zz} = 0, \tag{1}$$

where x, y, z, and t are variables which are independent. Also, x, y, z, stand for the spatial coordinate and t is time.

The modified 3-dimensional KdV is the cubic nonlinear type of the 3-dimensional KdV equation. The modified three-dimensional KdV equation is given as [24]

$$\left(v_t + v^2 v_x + v_{xxx}\right)_x + v_{yy} + v_{zz} = 0, \tag{2}$$

where the direction of propagation of the wave is taken to be in the x, y and z directions and for the measurement of the time of the propagation of the wave, we use t.

3. Description of the Approach

Here, let's take a 3-dimensional NLPDE of the form

$$v_t = \Xi\left(v, v^{\cdot}{}_{x}, v^{\cdot}{}_{y}, v^{\cdot}{}_{z}, v^{\cdot}{}_{xx}, v^{\cdot}{}_{yy}, v^{\cdot}{}_{zz}, ...\right), \tag{3}$$

now, upon employing the transformations

$$v(x,y,z,t) = v(\xi), \quad \xi = ax + by + cz - rt, \tag{4}$$

where a, b, c, r are constants.

Equation (3) can be transformed or reduced into an ODE

$$\begin{aligned} -rv' = \Xi(&v\left(\xi\right), av'\left(\xi\right), bv'\left(\xi\right), cv'\left(\xi\right), \\ &a^2 v''\left(\xi\right), b^2 v''\left(\xi\right), c^2 v''\left(\xi\right), ...) \end{aligned} \tag{5}$$

where the independent variable consists of new variables $\xi = ax + by + cz - rt$, where a, b, c are constants and r is the speed of the wave, x, y, z and t are respectively the spatial coordinates and time. The ODE (5) is then integrated to reduce the derivatives involved and where integration have been done, constants are always considered to be zero. The ODE is therefore simplified and solved by the tanh-coth approach [23], which admits the usage of the following finite expansions

$$v(\xi) = a_0 + \sum_{i=1}^{n} a_i Y^i(\xi) + \sum_{i=1}^{n} b_i Y^{-i}(\xi), \tag{6}$$

together with the Riccati equation

$$Y'(\xi) = RY^2(\xi) + QY(\xi) + P, \tag{7}$$

By the use of variable change

$$\begin{aligned} Y'(\xi) &= RY^2(\xi) + QY(\xi) + P, \\ \frac{dv}{d\xi} &= \left(P + QY + RY^2\right)\frac{dv}{dY}, \\ \frac{d^2v}{d\xi^2} &= (Q + 2RY)\left(P + QY + RY^2\right)\frac{dv}{dY} \\ &\quad + \left(P + QY + RY^2\right)^2 \frac{d^2v}{dY^2}, \end{aligned} \tag{8}$$

where P, Q, and R are to be provided, and a_i, b_i ($i = 0, 1, \ldots, n$) are constants that are to be obtained. The positive integer n can be obtained by considering the homogeneous balancing between the derivatives of higher order and the highly nonlinear term that appears in Eq. (5). Plugging (6) into (5) and making use of (7) and (8) gives an equation in terms of $Y^i(\xi)$. Equating the coefficient of the same powers of $Y^i(\xi)$ to zero, we obtain a system of algebraic equations in P, Q, R, a_i, b_i, a, b, c, and r.

For this study, we shall consider using the following solution of Riccati equation: If $P = Q = 1$ and $R = 0$, then $Y(\xi) = e^{\xi} - 1$, and if $Q = 0$, $R = -1/2$, and $P = 1/2$, then $Y(\xi) = \coth(\xi) \pm \operatorname{csch}(\xi)$. The reason being, using Riccati conditions, the solution is better constrained, reducing the degrees of freedom in finding the right form of the solution. This leads to faster and more stable convergence, particularly in cases where the nonlinear dynamics of the system align well with the Riccati framework [24].

4. Application of the approach

Here, we shall apply the approach to the three-dimensional KdV and the modified three-dimensional KdV equations.

4.1. The 3-Dimensional KdV Equation

The three-dimensional KdV is given as in Eq. (1) above. By applying the transformation (4), (1) is transformed into an ODE, as seen below

$$a\left(-r\nu' + a\nu\nu' + a^2\nu'''\right)' + b^2\nu'' + c^2\nu'' = 0. \tag{9}$$

Integrating (9) two times and also by setting the constant of integration equal to zero, we obtain

$$(b^2 + c^2 - ar)\nu + \frac{a^2}{2}\nu^2 + a^4\nu'' = 0. \tag{10}$$

Balancing between ν'' and ν^2 in Eq. (10), yields $n = 2$, which enables us to get from (6), the equation

$$\nu(\xi) = a_0 + a_1 Y + a_2 Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}. \tag{11}$$

Plugging (11) into (10) and then making use of Eqs. (7) and (8), we obtain the following system of equations

$$\begin{gathered}
6a^4P^2b_2 + \frac{1}{2}a^2b_2^2 = 0,\\
2a^4P^2b_1 + 10a^4PQb_2 + a^2b_1b_2 = 0,\\
3a^4PQb_1 + \frac{1}{2}a^2b_1^2 + b^2b_2 + c^2b_2 + 4a^4Q^2b_2\\
-arb_2 + 8a^4PRb_2 + a^2a_0b_2 = 0\\
b^2b_1 + c^2b_1 + a^4Q^2b_1 - arb_1 + 2a^4PRb_1\\
+a^2a_0b_1 + 6a^4QRb_2 + a^2a_1b_2 = 0\\
b^2a_0 + c^2a_0 - ara_0 + \frac{1}{2}a^2a_0^2 + a^4PQa_1\\
+2a^4P^2a_2 + a^4QRb_1 + a^2a_1b_1\\
+2a^4R^2b_2 + a^2a_2b_2 = 0\\
b^2a_1 + c^2a_1 + a^4Q^2a_1 - ara_1 + 2a^4PRa_1\\
+a^2a_0a_1 + 6a^4PQa_2 + a^2a_2b_1 = 0\\
3a^4QRa_1 + \frac{1}{2}a^2a_1^2 + b^2a_2 + c^2a_2 + 4a^4Q^2a_2\\
-ara_2 + 8a^4PRa_2 + a^2a_0a_2 = 0\\
2a^4R^2a_1 + 10a^4QRa_2 + a^2a_1a_2 = 0,\\
6a^4R^2a_2 + \frac{1}{2}a^2a_2^2 = 0
\end{gathered}$$

Solving the above system, we obtain the following cases:

Case one

$$\begin{gathered}
a_0 = -12a^2PR, \quad a_1 = 0, \quad a_2 = 0,\\
b_1 = -12a^2PQ, \quad b_2 = -12a^2P^2,\\
r = \frac{b^2 + c^2 + a^4Q^2 - 4a^4PR}{a}.
\end{gathered} \tag{12}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_1(x,y,z,t) = -\frac{12a^2}{(-1+e^{\xi})^2} - \frac{12a^2}{-1+e^{\xi}}, \tag{13}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_2(x,y,z,t) = 3a^2 - \frac{3a^2}{(\coth[\xi] + \operatorname{csch}[\xi])^2}. \tag{14}$$

Case two

$$\begin{gathered}
a_0 = -2(a^2Q^2 + 2a^2PR), \quad a_1 = 0, \quad a_2 = 0,\\
b_1 = -12a^2PQ, \quad b_2 = -12a^2P^2,\\
r = \frac{b^2 + c^2 - a^4Q^2 + 4a^4PR}{a}.
\end{gathered} \tag{15}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_3(x,y,z,t) = -2a^2 - \frac{12a^2}{(-1+e^{\xi})^2} - \frac{12a^2}{-1+e^{\xi}}, \tag{16}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_4(x,y,z,t) = a^2 - \frac{3a^2}{(\coth[\xi] + \operatorname{csch}[\xi])^2}. \tag{17}$$

Case three

$$\begin{gathered}
a_0 = -12a^2PR, \qquad a_1 = -12a^2QR,\\
a_2 = -12a^2R^2, \qquad b_1 = 0, b_2 = 0,\\
r = \frac{b^2 + c^2 + a^4Q^2 - 4a^4PR}{a}.
\end{gathered} \tag{18}$$

Now, for $P = Q = 1$ and $R = 0$, the solution is zero. But when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_5(x,y,z,t) = 3a^2 - 3a^2(\coth[\xi] + \operatorname{csch}[\xi])^2. \tag{19}$$

Case four

$$a_0 = -2(a^2Q^2 + 2a^2PR), \quad a_1 = -12a^2QR, \tag{20}$$

$$a_2 = -12a^2R^2, \quad b_1 = 0, \quad b_2 = 0, \tag{21}$$

$$r = \frac{b^2 + c^2 - a^4Q^2 + 4a^4PR}{a}. \tag{22}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_6(x,y,z,t) = -2a^2, \tag{23}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_7(x,y,z,t) = a^2 - 3a^2(\coth[\xi] + \operatorname{csch}[\xi])^2. \tag{24}$$

4.2. The 3-Dimensional modified KdV equation

The three-dimensional modified KdV equation is given as in (2) above. By use of (4), (2) is transformed into an ODE, as seen below

$$a\left(-r\nu' + a\nu^2\nu' + a^2\nu'''\right)' + b^2\nu'' + c^2\nu'' = 0. \tag{25}$$

Integrating (23) two times and also by setting the integration constant zero, we obtain

$$(b^2 + c^2 - ar)\nu + \frac{a^2}{3}\nu^3 + a^4\nu'' = 0. \tag{26}$$

Balancing between ν'' and ν^3 in Eq. (24), yields $n = 1$, which enables us to get from (6), the equation

$$\nu(\xi) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{27}$$

Plugging (25) into (24) and then making the use of Eqs. (7) and (8), the following system of equations is realized

$$\begin{aligned}
2a^4P^2b_1 + \frac{1}{3}a^2b_1^3 &= 0\\
3a^4PQb_1 + a^2a_0b_1^2 &= 0\\
b^2b_1 + c^2b_1 + a^4Q^2b_1 - arb_1&\\
+2a^4PRb_1 + a^2a_0^2b_1 + a^2a_1b_1^2 &= 0\\
b^2a_0 + c^2a_0 - ara_0 + \frac{1}{3}a^2a_0^3&\\
+a^4PQa_1 + a^4QRb_1 + 2a^2a_0a_1b_1 &= 0\\
b^2a_1 + c^2a_1 + a^4Q^2a_1 - ara_1&\\
+2a^4PRa_1 + a^2a_0^2a_1 + a^2a_1^2b_1 &= 0\\
3a^4QRa_1 + a^2a_0a_1^2 &= 0\\
2a^4R^2a_1 + \frac{1}{3}a^2a_1^3 &= 0
\end{aligned}$$

Solving the above system, we obtain the following cases.

Case one

$$a_0 = -i\sqrt{\frac{3}{2}}aQ, \quad a_1 = -i\sqrt{6}aR, \quad b_1 = 0, \tag{28}$$

$$r = \frac{2b^2 + 2c^2 - a^4Q^2 + 4a^4PR}{2a}. \tag{29}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_8(x,y,z,t) = -i\sqrt{\frac{3}{2}}a, \tag{30}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_9(x,y,z,t) = i\sqrt{\frac{3}{2}}a(\coth[\xi] + \operatorname{csch}[\xi]). \tag{31}$$

Case two

$$a_0 = -i\sqrt{\frac{3}{2}}aQ, \quad a_1 = 0, \quad b_1 = -i\sqrt{6}aP, \tag{32}$$

$$r = \frac{2b^2 + 2c^2 - a^4Q^2 + 4a^4PR}{2a}. \tag{33}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_{10}(x,y,z,t) = \pm i\sqrt{\frac{3}{2}}a \pm \frac{i\sqrt{6}a}{-1 + e^{\xi}}, \tag{34}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_{11}(x,y,z,t) = \pm\frac{i\sqrt{\frac{3}{2}}a}{\coth[\xi] + \operatorname{csch}[\xi]}. \tag{35}$$

Case three

$$a_0 = i\sqrt{\frac{3}{2}}aQ, \quad a_1 = i\sqrt{6}aR, \quad b_1 = 0, \tag{36}$$

$$r = \frac{2b^2 + 2c^2 - a^4Q^2 + 4a^4PR}{2a}. \tag{37}$$

Now, for $P = Q = 1$ and $R = 0$, we have

$$v_{12}(x,y,z,t) = i\sqrt{\frac{3}{2}}a, \tag{38}$$

and when $P = 1/2$, $Q = 0$, $R = -1/2$, we have

$$v_{13}(x,y,z,t) = -i\sqrt{\frac{3}{2}}a(\coth[\xi] + \operatorname{csch}[\xi]). \tag{39}$$

5. Graphical representation for some of the obtained results

In this section, we represent the 3D contour and 2D graphical representation for some of the obtained results with physical description by choosing suitable values for the parameters involved.

6. Discussion and physical description graphs

In Fig. 1, we have the surface of bell-shaped soliton in 3D, contour and 2D absolute plot of exact solution $v_1(x,y,z,t)$ for the 3-dimensional KdV equation which is given by Eq. (13). The values of the parameters involved are taken as follows: $a = 1$, $b = 1$, $c = 1$, $r = 0.5$, $y = 1$, $z = 1$. The range of values of x-axis in all the plots is taken as $[-10, 10]$, while the range of time is taken as $[0, 2]$. The 2D plot is drawn at $t = 0, 2, 4$ as shown in the figure.

In Fig. 2, we have the surface of bell-shaped soliton in 3D, contour and 2D absolute plot of exact solution $v_2(x,y,z,t)$ for the 3-dimensional KdV equation which is

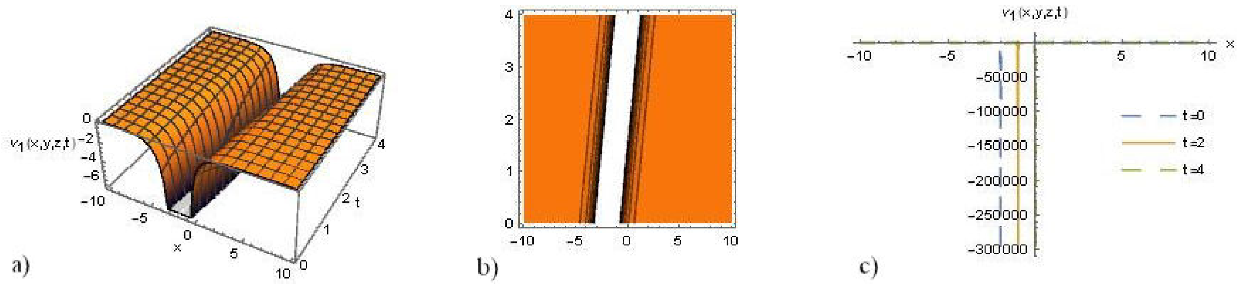

FIGURE 1. The 3D, contour and 2D absolute plots of $v_1(x, y, z, t)$.

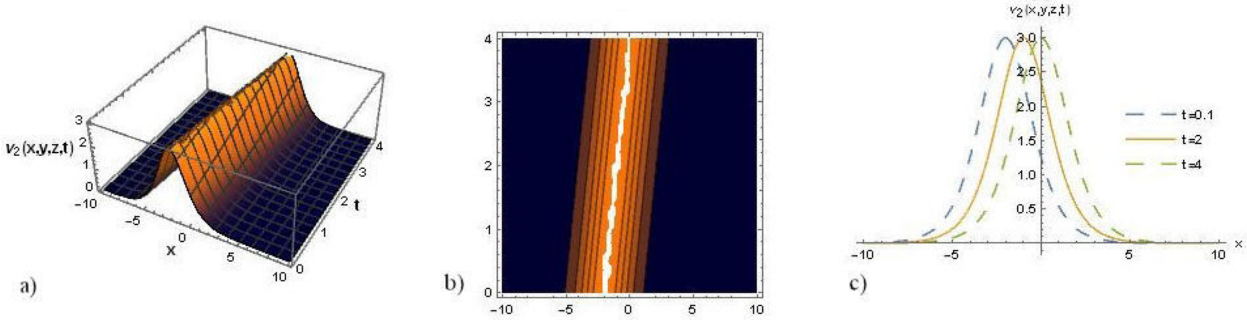

FIGURE 2. The 3D, contour and 2D absolute plots of $v_2(x, y, z, t)$.

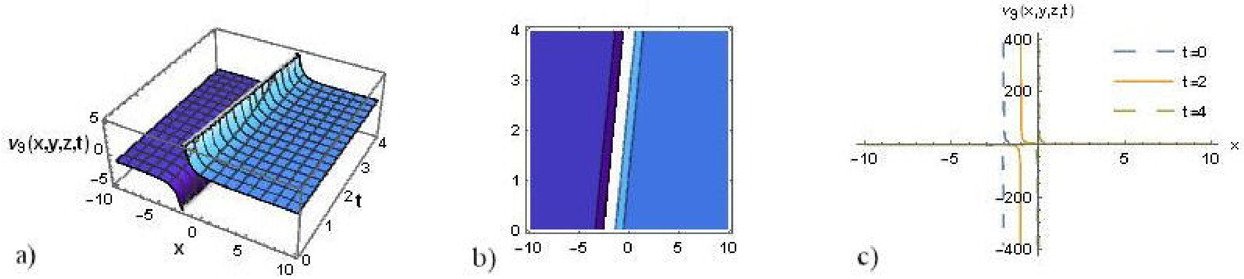

FIGURE 3. The 3D, contour and 2D imaginary plots of $v_9(x, y, z, t)$.

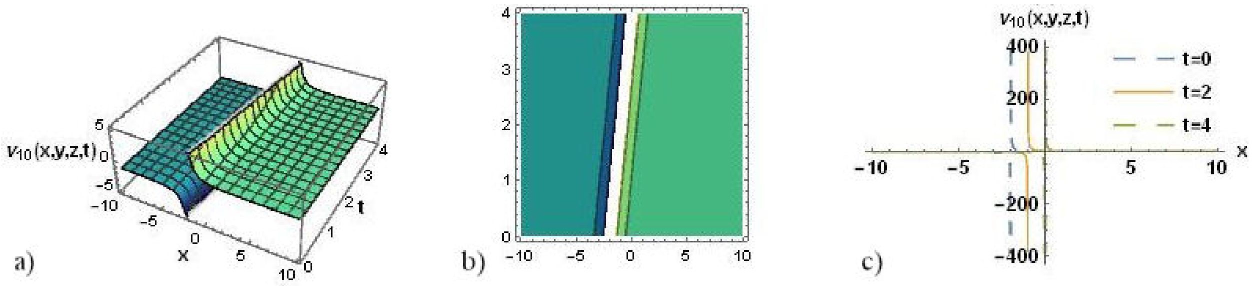

FIGURE 4. The 3D, contour and 2D imaginary plots of $v_{10-}(x, y, z, t)$.

given by Eq. (14). The values of the parameters involved are taken as follows: $a = 1$, $b = 1$, $c = 1$, $r = 0.5$, $y = 1$, $z = 1$. The range of values of x-axis in all the plots is taken as $[-10, 10]$, while the range of time is taken as $[0, 2]$. The 2D plot is drawn at $t = 0, 2, 4$ as shown in the figure.

In Fig. 3, we have the surface of kink-shaped soliton in 3D, contour and 2D imaginary plot of exact solution $v_9(x, y, z, t)$ for the modified 3-dimensional KdV equation which is given by Eq. (28). The values of the parameters involved are taken as follows: $a = 1$, $b = 1$, $c = 1$, $r = 0.5$, $y = 1$, $z = 1$. The range of values of x-axis in all the plots is taken as $[-10, 10]$, while the range of time is taken as $[0, 2]$. The 2D plot is drawn at $t = 0, 2, 4$ as shown in the figure.

In Fig. 4, we have the surface of kink-shaped soliton in 3D, contour and 2D imaginary plot of exact solution $v_{10-}(x, y, z, t)$ for the modified 3-dimensional KdV equation which is given by Eq. (30). The values of the parameters involved are taken as follows: $a = 1$, $b = 1$, $c = 1$, $r = 0.5$,

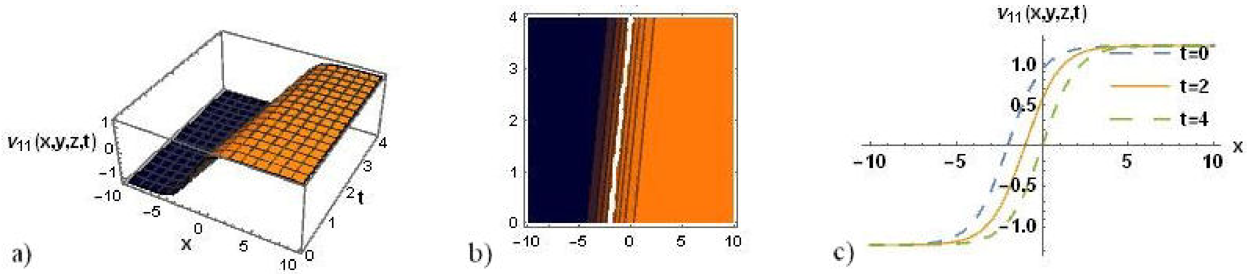

FIGURE 5. The 3D, contour and 2D imaginary plots of $v_{11+}(x, y, z, t)$.

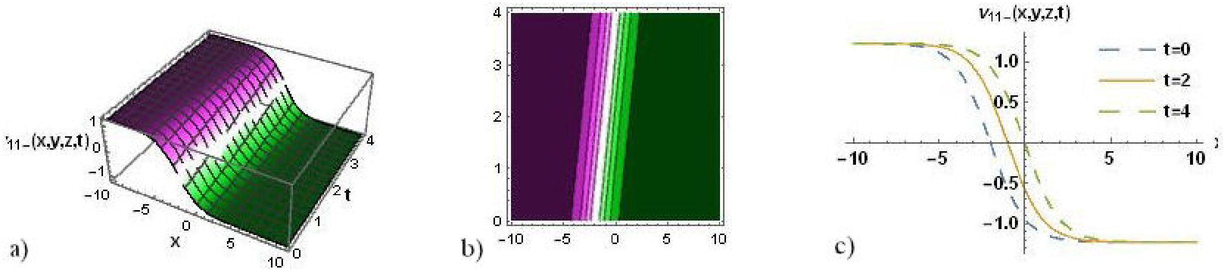

FIGURE 6. The 3D, contour and 2D imaginary plots of $v_{11-}(x, y, z, t)$.

$y = 1$, $z = 1$. The range of values of x-axis in all the plots is taken as $[-10, 10]$, while the range of time is taken as $[0, 2]$. The 2D plot is drawn at $t = 0, 2, 4$ as shown in the figure.

In Figs. 5 and 6, we have the surface of kink-shaped soliton in 3D, contour and 2D imaginary plot of exact solution $v_{11+}(x, y, z, t)$ and $v_{11-}(x, y, z, t)$ respectively, for the modified 3-dimensional KdV equation which is given by Eq. (31). The values of the parameters involved are taken as follows: $a = 1$, $b = 1$, $c = 1$, $r = 0.5$, $y = 1$, $z = 1$. The range of values of x-axis in all the plots is taken as $[-10, 10]$, while the range of time is taken as $[0, 2]$. The 2D plot is drawn at $t = 0, 2, 4$ as shown in the figure.

7. Conclusion

In this study, we successfully employed the Tanh–Coth method alongside the Riccati equation transformation to derive exact soliton solutions for both the three-dimensional Korteweg–de Vries (3D KdV) equation and its modified variant. This analytical approach enabled the systematic reduction of the complex nonlinear partial differential equations to solvable ordinary differential equations. By assuming a traveling wave transformation and expressing the solution in terms of hyperbolic tangent and hyperbolic cotangent functions, solutions of the Riccati equation, we obtained a variety of solitary wave profiles, including kink, anti-kink, and localized pulse solutions.

These explicit solutions highlight the inherent nonlinear dispersive balance present in 3D KdV-type systems, extending the traditional soliton theory to higher-dimensional settings. The method's flexibility and simplicity confirm its utility in handling complex nonlinear structures in multidimensional frameworks.

In conclusion, the Tanh–Coth method with Riccati framework proves to be a powerful and efficient analytical tool for constructing soliton solutions to multidimensional nonlinear evolution equations. The results obtained here provide deeper insight into the rich structure of nonlinear wave phenomena and offer a strong theoretical foundation for future studies in both mathematics and applied physics.

The identification and classification of bell and kink soliton solutions enrich our understanding of nonlinear systems by revealing how different solution types correspond to different physical mechanisms and boundary behaviors. The Tanh–Coth method, when applied to equations like the 3D or modified 3D KdV equations, often uncovers both bell and kink profiles depending on the nature of the transformation and parameter conditions. These structures not only offer analytical insight but also serve as building blocks in the modeling of complex physical, biological, and engineering systems.

Declarations

Ethical approval

The authors state that they did not conduct any experiments involving animals throughout the duration of this research.

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Data availability

This study did not make use of any data sets.

Conflict of interest

The authors assert that they have no conflicts of interest to disclose.

Author contributions

H.R. made noteworthy advancements in the methodology, while M.A.H. played a crucial role in enhancing, evaluating, and refining the concept. A.D. and A.T. led the investigation and managed the computational aspects adeptly. Additionally, S.S. assumed a leading position in the development of the concept and model.

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1. G. Adomian, “Solving frontier problems of physics: the decomposition method”, 60, Springer Science and Business Media, (2013).
2. M. A. Akbar, N. H. J. Mohd, E. M. E. Zayed, “Generalized and Improved (G'/G)-Expansion Method Combined with Jacobi Elliptic Equation”, *Commun. Theor. Phys.* **61** (2014) 669.
3. A. Biswas, “1-soliton solution of the K(m, n) equation with generalized evolution”, *Phys. Lett. A* **372** (2008) 4601.
4. G. Ebadi, A. Biswas, “The G'/G method and topological soliton solution of the K(m, n) equation”, *Commun. Nonlinear Sci. Numer. Simul.* **16** (2011) 2377-2382.
5. Y. L. Feng, W.-R. Shan, W.-R. Sun, H. Zhong, B. Tian, “Bifurcation analysis and solutions of a three-dimensional Kudryashov–Sinelshchikov equation in the bubbly liquid”, *Commun. Nonlinear Sci. Numer. Simul*. **19** (2014) 880-886.
6. X.Y. Gao, “Density-fluctuation symbolic computation on the (3+1)-dimensional variable-coefficient Kudryashov–Sinelshchikov equation for a bubbly liquid with experimental support”, *Mod. Phys. Lett. B* **30** (2016) 1650217.
7. A. Goriely, “Integrability and nonintegrability of dynamical systems”, 19, World Scientific, (2001).
8. O. Guner, A. Bekir, H. Bilgil, “A note on exp-function method combined with complex transform method applied to fractional differential equations”, *Adv. Nonlinear Anal*. **4** (2015) 201-208.
9. Z. Jun-Yi, G. Xian-Guo, “Miura transformation for the TD hierarchy”, *Chin. Phys. Lett.* **23** (2006) 1.
10. M. Kaplan, A. Bekir, A. Akbulut, E. Aksoy, “The modified simple equation method for nonlinear fractional differential equations”, *Rom. J. Phys.* **60** 1374–1383.
11. N. A. Kudryashov, D. I. Sinelshchikov, “Equation for the three-dimensional nonlinear waves in liquid with gas bubbles”, *Physica Scripta* **85** (2012) 025402.
12. Y. Sun, B. Tian, X.Y. Wu, L. Liu, Y.Q. Yuan, “Dark solitons for a variable-coefficient higher-order nonlinear Schrödinger equation in the inhomogeneous optical fiber”, *Mod. Phys. Lett. B* **31** (2017) 1750065.
13. H. Triki, A. M. Wazwaz, “Bright and dark soliton solutions for a K (m,n) equation with t-dependent coefficients”, *Phys. Lett. A* **373** (2009) 2162.
14. A. M. Wazwaz, “New solitons and kink solutions for the Gardner equation”, *Commun. Nonlinear Sci. Numer. Simul.* **12** (2007) 1395.
15. A. M. Wazwaz, “Partial differential equations and solitary waves theory”, Springer Science and Business Media, (2010).
16. A. M. Wazwaz, “The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations”, *Appl. Math. Comput.* **184** (2007) 1002.
17. L. Wazzan, “A modified tanh–coth method for solving the KdV and the KdV–Burgers’ equations”, *Commun. Nonlinear Sci. Numer. Simul.* **14** (2009) 443.
18. L. V. Wijngaarden, “On the equations of motion for mixtures of liquid and gas bubbles”, *Journal of Fluid Mechanics* **33** (1968) 465.
19. M. J. Xu, S. F. Tian, J. M. Tu, T. T. Zhang, “Bäcklund transformation, infinite conservation laws and periodic wave solutions to a generalized (2+1)-dimensional Boussinesq equation”, *Nonlinear Analysis: Real World Applications* **31** (2016) 388.
20. H. Yépez-Martínez, F. Gómez-Aguilar, I. Sosa, J. Reyes, J. Torres-Jiménez, “The Feng’s first integral method applied to the nonlinear mKdV space-time fractional partial differential equation”, *Rev. Mex. Fis.* **62** (2016) 310.
21. E. M. E. Zayed, K. A. E. Alurrfi, “A new Jacobi elliptic function expansion method for solving a nonlinear PDE describing the nonlinear low-pass electrical lines”, *Chaos, Solitons & Fractals* **78** (2015) 148.
22. S. Zhang, “Exp-function method exactly solving a KdV equation with forcing term”, *Appl. Math. Comput.* **197** (2008) 128.
23. L. Wazzan, “A modified tanh-coth method for solving the KdV and the KdV-Burgers’ equations”, *Communications in Nonlinear Science and Numerical Simulation* **14** (2009) 443.
24. Y.B. Chukkol, M. Muminov, “Bright and Dark Soliton Solutions of Three-Dimensional KdV and mKdV Equations in Inviscid Liquid with Gas Bubbles”, *Int. Jrn. Of Math. and Cpt. Sc* **15** (2020) 253.