TWO MODELS FOR THE NUCLEAR PHOTOEFFECT

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RESUMEN

The Migdal-Jensen collective model and the Wilkinson shell model both give reasonable agreement with photonuclear experiments concerning the integrated cross section, the resonance energy, and the width of the resonance. We discuss possible reasons for the similarity between the predictions of the two different models. Experiments on emission of fast nucleons show that the excited state reached by electric dipole photon absorption contains a large amplitude of shell model wave functions representing single particle excitation.

We shall discuss briefly the collective two-fluid model[1] and the shell model[2] applied to calculations of moderate energy electric dipole transitions in the nuclear photoeffect. We shall be particularly interested in finding out which properties of the nuclear photoeffect are independent of, or insensitive to, the model employed. We shall also discuss tentative relations between the two models. It is clear that each model represents an extreme approximation, and that the next major step in photonuclear calculations must involve combining features of these models, and perhaps also including effects of two-body dynamical correlations, which are of importance in the high energy nuclear photoeffect[3].

Photonuclear experiments show that photon absorption occurs principally in a peak of width Γ about 5 Mev. the peak energy E_m being about 15 Mev. While earlier work[4] gave E_m proportional to $A^{-0.2}$, recent measurement of photon elastic scattering[5] give E_m proportional to $A^{-1/3}$. The width is particularly small at magic numbers of neutrons or protons. The integrated cross section $\sigma_{int} = \int \sigma \, dW$ is about 0.02 A MeV-barns, in agreement with the electric dipole sum-rule with an increase due to neutron-proton exchange forces[6]. The main features of the products of photonuclear disintegration are in agreement with calculations based on disintegration of a compound nucleus; but a few percent of the nucleons are emitted with a higher energy and a different angular distribution than predicted by the compound nucleus model. At higher energies, of order 100Mev, the coincidences and angular correlations of high energy neutrons and protons are in reasonable agreement with the predictions of the quasi-deuteron- model[3].

The collective model assumes that the electric field produces an oscillation of the proton fluid against the neutron fluid, the restoring force being provided by the symmetry energy density. Jensen[1] calculated the resonance energy E_m by calculating from the symmetry energy the phase velocity for waves of relative motion of the two fluids, and by calculating from the boundary conditions the wave-length at resonance for this two-fluid vibration. He finds $E_m \sim A^{-1/3}$, with a coefficient somewhat less than the experimental value. Jensen's model gives a summed oscillator strength $\mu_0 = \Sigma_n f_n = NZ/A$, or an integrated cross section of about 0.015 A Mev-barns, since this result is the same for all classical models[6].

Jensen's result for E_m had, we have recently learned, been derived much

earlier by Migdal, from a sum-rule calculation. The electrical susceptibility ϵ for a constant electric field[8] is proportional to $\mu_{-2} = \Sigma_n f_{on} / (E_n - E_o)^2$, where f_{on} is the oscillator strength for an electric dipole transition to a state of energy $E_n - E_o$ above the ground state. A mean energy (W_{-2}) for proton absorption is given by $W_{-2} = (\mu_o / \mu_{-2})^{1/2}$. Migdal calculated the electrical susceptibility from a simple classical treatment of the symmetry energy density giving $\epsilon = R^2 Ae^2/40K$ for a nucleus of radius R, where $K(N-Z)^2$ is the Weiszäcker symmetry energy term. However in calculating the mean energy W_{-2} he made a mistake in taking the summed oscillator strength μ_o as Z, instead of the correct classical NZ/A for internal excitation. (The remaining oscillator strength of Z^2/A appears in the process of nuclear Thomson scattering). If we put $\mu_o = NZ/A$ in Migdal's formula we get essentially Jensen's result for the resonance energy $E_m = (40\, NZ \hbar^2 K / A^2 M R^2)^{1/2}$ (Jensen gives $(34.6 NZ \hbar^2 K/A^2 MR^2)^{1/2}$.)
Migdal's work is of great significance for three reasons:

1) It was the earliest calculation of E_m, and the only calculation that was a prediction, since the experimental values of E_m first appeared in 1948.

2) It obtains Jensen's result, but with fewer assumptions as to the detailed model. If we had a shell model calculation of the symmetry energy density we could obtain Jensen's result for E_m with no assumption of collective motion at this excitation energy.

3) It allows us to increase E_m by increasing μ_o for the effect of neutron-proton exchange forces[8], thus obtaining better agreement with the experimental values.

The Jensen collective model has been extended recently by Wildermuth[9], and by Okamoto[10] and Danos[11] to give more than the results for μ_o and E_m which were derived by Migdal's sum-rules using only properties of the nuclear ground state. Wildermuth calculated the width as about 6 Mev by calculation of the mean time for one of the protons in the proton fluid to be scattered by one of the neutrons in the neutron fluid. (His model is similar to that used by Lane and Wandel[12] in calculation of the opacity of the cloudy crystal ball potential.) Okamoto and Danos have independently explained the larger values of the width far from magic numbers as due to the distortion of the nucleus from a sphere to an ellipsoid. The single resonance at energy proportional to $1/R$ for a sphere of radius R is, for an unpolarized system of ellipsoids with radius R_1 and R_2, split into two resonances at energies proportional to $1/R_1$ and $1/R_2$. If they take the width for a spherical nucleus as 2 or 3 Mev, or about half that calculated by Wildermuth, they find that deformations in agreement with the Bohr-Mottelson model give reasonable agreement with experimental results for the increased width for photon absorption by non-spherical nuclei.

As first shown by Burkhardt[13], the shell model gives a sharply peaked cross section for the photoeffect, but the resonance energy is low: 9 Mev for Cu, instead of the experimental value of 18 Mev. Wilkinson and others have discussed ways of increasing the calculated value of E_m. The most likely proposal is that of assuming a velocity-dependence of the shell model potential. Sum-rule calculations[14] show that $\mu_{-1} = \Sigma_n f_{on}/(E_n - E_o)$ calculated using shell model wave functions, for a nuclear radius of $1.2\ A^{1/3}$ fermis, is in reasonable agreement with the experimental data for the Bremsstrahlung-weighted cross section $\sigma_b = \int(\sigma/W)dW$. But while shell model wave functions give a reasonable value for σ_b, they give too low a value for μ_o: thus a velocity-independent shell model potential always gives the classical value $\mu_o = NZ/A$. A fair approximation migth be the use of Van Vleck's velocity-dependent shell model potential[15], which would increase the calculated values for both μ_o and E_m. The Van Vleck value for μ_o is in fair, but not exact agreement[16], with that calculated[8] using the neutron-proton exchange force.

The width for the giant dipole resonance has been calculated by Wilkinson[2]. For a closed shell nucleus (doubly-magic) Γ is determined by the lifetime for the excited state, which consists of a single nucleon with a positive energy of some 5 Mev. Wilkinson finds this lifetime from the analysis of nucleon scattering by the cloudy crystal ball model; as an alternative one could use the Lane-Wandel calculation[12] of the cloudiness. The value of two to three Mev, is in reasonable agreement with experiment. Wilkinson explains the increased Γ far from closed shells as due to the large splitting of levels for different arrangements of many nucleons in the same configuration.

Wilkinson has also made detailed calculations as to the numbers of fast nucleons emitted, and as to their energy distribution and angular distribution. In his model, a single nucleon absorbs the full photon energy; but this nucleon is in most cases in a state of high orbital angular momentum so it would take it a long time to escape from the nucleus, and during this it is likely to make a collision with another nucleon. The absorption by the cloudiness of the potential thus usually leads to a compound nucleus which then decays in the usual manner. Wilkinson's detailed calculations are in surprisingly good agreement with experiment.

We have found that the collective model and shell model give almost identical results for the integrated photonuclear cross section, similar results for the resonance energy E_m, and similar results for the width. The shell model makes detailed predictions concerning the properties of emitted fast nucleons; while the collective model implies that these fast nucleons should not be emitted. One migth wonder if the agreement between the collective and shell models concerning E_m and Γ is coincidental, or if there is a real similarity between the two contradictory models. The close agreement between calculations of σ_{int} from the two models is merely an example of the theorem that this physical quantity is insensitive to the model used; for a classical model σ_{int} is indeed independent of the model used.

To the extent that either model is regarded as being a good approximation to physical reality, the collective model and the shell model are quite different, in that they make different assumptions concerning the motion of the nucleons; a nuclear excited state in which all neutrons move in one direction and all protons move in the other is a different state from an excited state in which only one nucleon moves, the others remaining at rest. However, we can still give a partial explanation as to the similarity in the results for the resonance energy E_m in the two models. That is, we can build a partially collective model by linear combinations of many different shell model states of the same energy, or of very nearly the same energy. (For a simple harmonic oscillator shell model potential the degeneracy is exact; but the levels remain almost degenerate for a finite square well, including spin-orbit coupling[2, 13].) An appropriate linear combination of these degenerate levels will correspond to a nuclear state in which all neutrons from the top occupied shell move collectively against all protons from the top shell; i. e., about half the nucleons are moving. Thus the two very different excited nuclear states (shell model for single particle excitation and many particle collective excitation) will have the same excitation energy, E_m. (The agreement in E_m does not answer the question as to whether the state excited by absorption of a particular photon can be well represented by the linear combination of shell model levels representing a collective motion, or whether on the other hand it can be well represented by a wave function for single particle excitation). However, this collective motion does not involve the particles in inner closed shells, as is done by the Jensen collective motion. There could be coupling between the partially collective motion discussed above and the nucleons in inner closed shells, thus exciting the Jensen collective motion; but this mechanism goes beyond the simple model of a partially collective motion discussed in this paper.

We can also give a partial explanation of the approximate agreement between the shell and collective model calculation of the width Γ. In the Lane-Wandel calculation of the imaginary part of the cloudy crystal ball potential, the cloudiness varies rapidly with the difference between the nucleon energy and the nuclear Fermi energy. In the Wildermuth calculation based on the collective model, all nucleons are moving, allowing apparently many more chances for a neutron-proton collision than in the case of single nucleon excitation. However any given nucleon has a greatly reduced chance to make a collision, due to the reduction by a factor A of the nucleon excitation energy. These two differences between the mean lifetime for single nucleon excitation and that for collective motion tend to cancel, but do not cancel exactly, so that the Wilkinson (Lane-Wandel) value of 3 Mev for closed shell nuclei is appreciably different from the Wildermuth value of 6 Mev.

The increased value of Γ with increased nuclear deformation is explained simply and directly in the collective model, but only in a qualitative manner in Wilkinson's shell model. A more detailed shell model treatment, based on an elliptical potential well[17] migth make possible a detailed comparison between the two models on the question.

As stated above, the two models do have very different wave functions for the nuclear excited state reached by absorption of a particular photon. The most direct way that we know of for examining this wave function is the observation and interpretation of the emission of fast nucleons following photon absorption. Here the Wilkinson model agrees well with experimental results on the numbers of fast nucleons emitted, and on their angular distribution. We note that the emission of only a few percent of fast nucleons is consistent with complete single particle excitation, provided that we follow Wilkinson's calculation of resonance - direct emission. Emission of half as many fast nucleons as calculated by Wilkinson would indicate that about half the time a fairly pure single particle state was reached by photon absorption, and that the other half of the time a state of collective excitation was reached. (Here we imply the oversimplified model of writing the wave function of the nuclear excited state as a linear combination of Wilkinson and Jensen wave function.) This 50 - 50 mixture is probably consistent with present experiments, considering the uncertainties both in the experiments, and in the parameters in Wilkinson's calculation of the nucleon's probability of escape. But the experiments definitely show that there is an appreciable mixture of Wilkinson states of single particle excitation with the Jensen states of collective excitation.

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