Investigating the effects of phase-space non-commutativity coordinates on the modified Deng-Fan Yukawa potential model and thermodynamic properties in 3D(NR-NCPS) and 3D(NR-QM) symmetries

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An approximate new bound state solution of the three-dimensional deformed Schrödinger equation under the deformed phase-space symmetries for the modified Deng-Fan Yukawa potential model that is obtained from the combination of the corresponding expression in three-dimensional non-relativistic quantum mechanics symmetries and some central terms $[\exp(-\alpha \mathbf{r})/\mathbf{r}(\mathbf{1}-\exp(-\alpha \mathbf{r}))$, $\exp(-\mathbf{2}\alpha \mathbf{r})/\mathbf{r}(\mathbf{1}-\exp(-\alpha \mathbf{r}))^{\mathbf{2}}$, $\exp(-\mathbf{3}\alpha \mathbf{r})/\mathbf{r}(\mathbf{1}-\exp(-\alpha \mathbf{r}))^{\mathbf{3}}$, $\exp(-\alpha \mathbf{r})/\mathbf{r}^{\mathbf{2}}$, $\exp(-\alpha \mathbf{r})/\mathbf{r}^{\mathbf{3}}$ and $\mathbf{1}/\mathbf{r}^{\mathbf{4}}]$ coupled with the infinitesimal non-commutativity vector $\mathbf{\Theta}$ and the angular momentum operator $\mathbf{L}$. With the help of the parametric generalized Bopp's shifts method, the independent time perturbation theory method, and an approximation scheme, the analytical energies of the studied were obtained for both symmetries, for different quantum numbers. The new non-relativistic energy equation under the studied potential for the homogenous diatomic molecules (HODMs) (H_2, I_2); the heterogeneous diatomic molecules (CO, HCl, LiH); the neutral transition metal hydrides (ScH, TiH, VH, CrH); the transition-metal lithide (CuLi); the transition-metal carbides (TiC, NiC); the transition metal nitrite (ScN) and the transition metal fluoride (ScF) and in the presence of deformation phase-space are dependent on the discrete atomic quantum numbers (j, l, s and m), the dissociation energy, the equilibrium bond length, and the screening parameter (r_e, D_e, and α), the deformation phase parameters (P_p^{nc} and S_p^{nc}). The new resulting energy equation is utilized to calculate spin-averaged mass spectra of the heavy mesons under the studied potential and Deng-Fan Yukawa potential model in three-dimensional non-relativistic quantum mechanics and 3it's extended symmetries. Furthermore, we have calculated the partition function, from which thermodynamic properties such as mean energy, specific heat capacity, entropy, and free energy are derived in both three-dimensional non-relativistic quantum mechanics and the deformed phase-space symmetries symmetries. Notably, the two special cases, representing the modified Yukawa potential and the modified Deng-Fan potential were treated in extended phase-space symmetry for energies and thermodynamic properties. Our current study promises to apply to different areas of physics in various domains, including atomic and molecular physics.

Keywords: Deng-Fan Yukawa potential; phase-space deformation; generalised Bopp's shift method; Schrödinger equation.

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1. Introduction

Generally, Deng-Fan potential (DFP) model introduced by Deng and Fan in 1957 [1] is used to describe electromagnetic transitions and interactions existing between homogeneous and heterogeneous diatomic molecules [2]. Oyewumi *et al.* [3] (2013) solved the Schrödinger equation (SE) with the Deng-Fan molecular potential using the Nikiforov-Uvarov (NU) method and obtained the approximate analytical bound state energy eigenvalues and the corresponding wave functions of the homogenous diatomic molecules (HODMs) (H_2, I_2); the heterogeneous diatomic molecules (HEDMs) (CO, HCl, LiH); the neutral transition metal hydrides (NMHs) (ScH, TiH, VH, CrH); the transition-metal lithide (TML) (CuLi); the transition-metal carbides (TMC) (TiC, NiC); the transition metal nitrite (TMN) (ScN) and the transition metal fluoride (TMF)(ScF). Ikot *et al.* (2021) [4] solved the Klein-Gordon equation (KGE) with the DFP using the NU-functional-analysis in higher dimensions and by employing the improved Pekeris-type approximation scheme, obtained the relativistic and nonrelativistic energy spectra of the DFP of hydrogen chloride (HCl) and lithium hydride (LiH) diatomic molecules. Njoku *et al.* (2022) [5] investigated the analytical solution of the SE with the shifted DFP within the parametric NU formalism and applied it to the ro-vibrational energies of nine diatomic molecules, H_2, CO, LiH, HCl, ScH, ScN, TiH, ScF and I_2 for both low and high-lying states for both $l = 0$ and $l \neq 0$. On the other hand, in theoretical nuclear physics, the Yukawa potential (YP) (also known by a screened Coulomb potential) that was birth in 1935 [6, 7] is recognized as a phenomeno logical central potential between two protons and neutrons [8, 9]. The YP has been applied extensively, appearing as an significant model [5-7] for defining the theoretical framework of the nuclear force medium and explaining the intuitive physical picture, as well as for deriving other forms that are similar in appearance but distinct in substance [10]. Cari *et al.* studied the interisland absorption coefficients and the changes in refractive index in spherical quantum dots using Deng-Fan Yukawa [11]. Other studies in the literature were found to be related to the Cari *et al.* investigation [12–16]. We introduce a newly suggested potential, we called it the modified Deng-Fan Yukawa potential model (MDF-YP) which created by combining the Deng-Fan Yukawa potential model with a few central terms connected with the angular momentum operator $\mathbf{L}$ and the infinitesimal non-commutativity vector $\mathbf{\Theta}$ that product resulting to the impact of phase-space deformation. Motivated by the work of [11], we suggest the NC effect on the bound state energies and thermodynamic properties of the modified Deng-Fan Yukawa potential model arising from the deformed phase-space in the context of three-dimensional non-relativistic non-commutative phase-space (3D(NR-NCPS)) symmetries. To our knowledge, no literature review has been done on this kind of investigation. This work will focus on the modified Deng-Fan Yukawa potential model in the 3D(NR-NCPS) symmetries framework. The combined potentials $V_{dy}(\mathbf{d})$ under investigation are represented as

$$V_{dy}(\mathbf{d}) = V_{dy}(r) - \frac{1}{2r}\frac{\partial V_{dy}(r)}{\partial r}\mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right). \quad (1)$$

Here the Deng-Fan Yukawa potential (DF-YP), which of the form in this work, in three-dimensional non-relativistic quantum mechanics (3D(NR-QM)) regimes, is represented as [11]:

$$V_{dy}(r) = D_e\left(1 - \frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 - V_0\frac{\exp(-\alpha r)}{r}, \quad (2)$$

where $b = \exp(\alpha r_e) - 1$, r_e is the molecular bond length, D_e is the dissociation energy. The internuclear separations in the 3D-NCPS and 3D-QM symmetries are represented by $\mathbf{d}$ and r, respectively. The scalar product of the infinitesimal non-commutativity vector Θ and the angular momentum operator $\mathbf{L}$ yields the coupling $\mathbf{L}.\mathbf{\Theta} \equiv \overrightarrow{\mathbf{L}}.\overrightarrow{\mathbf{\Theta}}$. As for the symbol $O\left(\Theta^2\right)$, it means ignoring the terms that start from Θ^2 and above. It is worth noting previous studies that are directly and indirectly related to the topic of our current research. Let us first refer to our personal research in the frameworks of relativistic and non-relativistic NC quantum mechanics symmetries [17–25]. It is well known that the 3D(NR-QM) is based on the non-commutativity of the momentums $p_\mu^{(s,h,i)}(p_\mu^s,\ p_\mu^h(t),\ p_\mu^i(t))$ and the corresponding generalized coordinates $x_\mu^{(s,h,i)}(x_\mu^s,\ x_\mu^h(t),\ x_\mu^i(t))$ only. While its extension in 3D(NR-NCPS) symmetries based on other postulates; the first new additive postulate correspond to the non-commuting of position-position (See Eq. (4)) and the non-commuting of momentum-momentum operators (See Eq. (5)) [26–31]. Formally, 3D(NR-NCPS) symmetries can be divided into three categories: the first class correspond to non-commutative space-space (NCSS), the second class correspond to non-commutative phase-phase (NCPP), while the third class corresponds non-commutative phase-space (NCPS). The work of Connes [32–34] and Seiberg-Witten [35] was an important tool in developing the new concepts of NCQM theory to find applications with a physical context, particularly in quantum field theory. It should be noted that Chaturvedi *et al.* in 1993 [36, 37] first formulated non-relativistic NCQM. Several centuries ago, the program for unifying forces began to collect all electric forces and magnetic forces within the framework of Maxwell's equations, or what is known as electromagnetism, which describes charges changing with time. This program developed in the last century, especially around 1967, to include weak interactions, and the old unification model became an expression of electroweak interactions (the Glashow, Salem and Weinberg model). Through the other success of including strong interactions, the unification model includes three fundamental interactions, especially after the Higgs was confirmed. The biggest problem is that gravitational forces are not involved in this program. Naturally, the new theory of 3D(NR-NCPS) symmetries is the strongest candidate for solving this major problem in the field of unifying all four cosmic forces. The following outlines the remainder of the paper: An overview of the 3D-SE within the Deng-Fan Yukawa potential model framework is given in Sec. 2. Section 3 investigates the three-dimensional deformed Schrödinger equation using the well-recognized generalized Bopp's shifts approach to determine the MDF-YP model's effective potential. Additionally, we determine the corrected non-relativistic energy produced by the influence of the perturbed effective potential $Z_{dy}^{pert}(r)$ of the MDF-YP model using conventional perturbation theory. Under the MDF-YP model, we get the global modified energies for non-relativistic particles, including the homogenous diatomic molecules (HODMs) (H_2, I_2); the heterogeneous diatomic molecules (HEDMs) (CO, HCl, LiH); the neutral transition metal hydrides (NMHs) (ScH, TiH, VH, CrH); the transition-metal lithide (TML) (CuLi); the transition-metal carbides (TMC) (TiC, NiC); the transition metal nitrite (TMN) (ScN) and the transition metal fluoride (TMF)(ScF). Section 4 studies MDF-YP model homogeneous and heterogeneous composite systems in 3D(NR-NCPS) symmetries. The impact of phase-space deformation on the thermal characteristics of the modified Deng-Fan Yukawa potential, including partition function, mean energy, free energy, specific heat, and entropy, is the subject of Sec. 5, the special cases related to energy in the extended phase-space framework also include the overall thermodynamic properties as a particular case in the extended phase-space through appropriate substitutions for each case. Finally, succinct closing remarks are provided in the last part.

2. A summary of SE in the 3D(NR-QM) symmetry using the Deng-Fan Yukawa potential model

In the frameworks of 3D(NR-QM) symmetry, it is helpful to remember the eigenvalues and corresponding eigenfunctions under the influence of the Deng-Fan Yukawa potential to build a physical model describing a physical system that interacted with the MDF-YP model in 3D(NR-NCPS) regimes. The radial SE for the Deng-Fan Yukawa potential model can be written as follows:

$$\left(\frac{d^2}{dr^2}+2\mu\left(E_{nl}^{dy}-D_e\left(1-\frac{b\exp\left(-\alpha r\right)}{1-\exp\left(-\alpha r\right)}\right)^2+V_0\frac{\exp\left(-\alpha r\right)}{r}-\frac{l(l+1}{2\mu r^2}\right)\right)R_{nl}\left(r\right)=0. \tag{3}$$

The HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) are composed of two particles (m_1 and m_2) that have a reduced mass μ equal to $\sum_{n=1}^{(2/\pi)} m_n/\sum_{n=1}^{2} m_n$. The value E_{nl}^{dy} are the non-relativistic eigenvalues, (n,l) are represent the principal quantum number and spin-orbit quantum number, respectively. Cari *et al.*, in Ref. [11], apply the NU method to obtain the expression of the radial part $R_{nl}\left(r\right)$ as a function of the hypergeometric polynomials (hypergeometric polynomials) as follows:

$$R_{nl}\left(r\right)=N_{nl}s^{-\sqrt{\Lambda_{nl}}}\left(1-s\right)^{\frac{1}{2}+\frac{1}{2}\sqrt{1+4\epsilon_{nl}}}\ {}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right). \tag{4}$$

Here, the variable s is equal to $\exp\left(-\alpha r\right)$ while Λ_{nl}, ϵ_{nl} and N_{nl} are given by:

$$\begin{cases} \Lambda_{nl}=\frac{2\mu}{\alpha^2}\left[D_e-E_{nl}^{dy}+\frac{l(l+1)\alpha^2}{2\mu}\right], \\ \epsilon_{nl}=l(l+1)+\frac{2\mu}{\alpha^2}D_eb^2, \\ b_{nl}=-n-2\sqrt{\frac{2\mu}{\alpha^2}\left[D_e-E_{nl}^{dy}+D_e\left(2b+\frac{V_0\alpha}{D_e}\right)+D_eb^2\right]}, \\ N_{nl}=\frac{\Gamma\left(n+2+2\sqrt{\Lambda_{nl}}\right)}{n!\Gamma\left(2+2\sqrt{\Lambda_{nl}}\right)}\left[\frac{2\alpha n!\Gamma\left(n+2+2\sqrt{\Lambda_{nl}}\right)\Gamma\left(2n+2+2\sqrt{\Lambda_{nl}}\right)}{2^{2\left(\sqrt{\Lambda_{nl}}+1+\frac{1}{2}\sqrt{1+4\epsilon_{nl}}\right)}\Gamma\left(n+1+2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+2\sqrt{\Lambda_{nl}}\right)}\right]^{1/2}. \end{cases} \tag{5}$$

Since the Deng-Fan Yukawa potential model has an isotropic property (depended only to r), it allows the known forms' complete complex non-relativistic wave function solution $\Psi\left(r,\Omega_3,t\right)$ of the known forms $(R_{nl}\left(r\right)/r)Y_m^l(\Omega_3)\exp(-iE_{nl}^{dy}t)$ with $-\left|l\right|\leq m\leq+\left|l\right|$. Hence, we can conclude the complete complex wave function $\Psi\left(r,\Omega_3,t\right)$ in usual 3D-RQM symmetries as,

$$\Psi\left(r,\Omega_3,t\right)=N_{nl}\exp\left(-iE_{nl}^{dy}t\right)\frac{s^{-\sqrt{\Lambda_{nl}}}}{r}\left(1-s\right)^{\frac{1}{2}+\frac{1}{2}\sqrt{1+4\epsilon_{nl}}}\ {}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)Y_l^m\left(\Omega_3\right). \tag{6}$$

While the corresponding values E_{nl}^{dy} of the Deng-Fan Yukawa potential model in 3D(NR-QM) regimes can be represented in a closed and compact form as

$$E_{nl}^{dy}=Q_a-Q_b\left(n+\gamma+\frac{Q_c}{n+\gamma}\right)^2=Q_a-2Q_bQ_c-\left(\frac{Q_bQ_c^2}{\rho^2}+\ Q_b\rho^2\right), \tag{7}$$

with Q_a, Q_b and Q_c are respectively:

$$\begin{cases} Q_a=D_e+\frac{\alpha^2l(l+1)}{2\mu},\ Q_b=\frac{\alpha^2}{8\mu}, \\ Q_c=\frac{\alpha^2l(l+1)}{2\mu}-D_e\left(2b+\frac{V_0\alpha}{D_e}\right)-D_eb^2, \\ \gamma=\frac{1}{2}+\frac{1}{2}\sqrt{1+4\left(l\left(l+1\right)+\frac{2\mu}{\alpha^2}D_eb^2\right)}\ \text{and}\ \ \rho=n+\gamma. \end{cases} \tag{8}$$

It is helpful in briefly studying the total energy of each of the fundamental states E_{0l}^{dy} corresponding to the quantum numbers ($n=0,l,m$), as well as the first excited state E_{1l}^{dy} corresponding to the quantum numbers ($n=1,l,m$), where Eq.(24) in these two cases becomes as follows:

$$\begin{cases} E_{0l}^{dy}=Q_a-Q_b\left(\gamma+\frac{Q_c}{\gamma}\right)^2, \\ E_{1l}^{dy}=Q_a-Q_b\left(1+\gamma+\frac{Q_c}{1+\gamma}\right)^2. \end{cases} \tag{9}$$

When an electron transitions from an excited state, described as a quantum state ($n=1,l,m$), to the fundamental state, described as a quantum state ($n=0,l,m$), it will emit or absorb electromagnetic radiation, the frequency ω_{nl}^{dy} of which is determined by the following relation:

$$\omega_{nl}^{dy}=\left|E_{1l}^{dy}-E_{0l}^{dy}\right|. \tag{10}$$

A simple calculation gives the emitted or absorbed electromagnetic radiation ω_{nc}^{dy} in the context of 3D(NR-QM) symmetry as follows:

$$\omega_{nl}^{dy}=Q_b\left[\left(1+\gamma+\frac{Q_c}{1+\gamma}\right)^2-\left(\gamma+\frac{Q_c}{\gamma}\right)^2\right]. \tag{11}$$

The following section will investigate the MDF-YP model in 3D(NR-NCPS) symmetries.

3. Investigate DSE solutions in the 3D(NR-NCPS) regime under the MDF-YP model

3.1. Review 3D(NR-NCPS) regime

3D(NR-NCPS) symmetries formalism, based on new algebra of self-adjoint differential operators (d_α^s, $d_\alpha^h(t)$ and $d_\alpha^i(t)$) and (π_α^s, $\pi_\alpha^h(t)$ and $\pi_\alpha^i(t)$) that come in three different kinds of Schrödinger, Heisenberg, and interaction pictures (SP, HP and IP) in three varieties. These varieties that satisfy a deformed algebra are the canonical structure variety, the Lie structure variety, and the quantum plane variety as follows (QSV, LSV and QSV, in short) [38–47]:

$$\left[d_\alpha^s,\pi_\beta^s\right]_*=\left[d_\alpha^h(t),\pi_\beta^h(t)\right]_*=\left[d_\alpha^i(t),\pi_\beta^i(t)\right]_*=i\hbar_{eff}\delta_{\alpha\beta}, \tag{12}$$

$$\left[d_\alpha^s,d_\beta^s\right]_*=\left[d_\alpha^h(t),d_\beta^h(t)\right]_*=\left[d_\alpha^i(t),d_\beta^i(t)\right]_*=i\Omega_{\alpha\beta}, \tag{13}$$

$$\left[\pi_\alpha^s,\pi_\beta^s\right]_*=\left[\pi_\alpha^h(t),\pi_\beta^h(t)\right]_*=\left[\pi_\alpha^i(t),\pi_\beta^i(t)\right]_*=i\overline{\Omega}_{\alpha\beta}, \tag{14}$$

with

$$\left(\Omega_{\alpha\beta},\overline{\Omega}_{\alpha\beta}\right)=\left(\epsilon_{\alpha\beta}\theta\equiv\theta_{\alpha\beta},\epsilon_{\alpha\beta}\overline{\theta}\equiv\overline{\theta}_{\mu\nu}\right):\epsilon_{\alpha\beta}\in\text{CSV}\ , \tag{15}$$

$$\left(\Omega_{\alpha\beta},\overline{\Omega}_{\alpha\beta}\right)=\sum_{\delta=1}h_{\mu\nu}^{\delta}\left(d_\delta^{(s,h,i)},\pi_\delta^{(s,h,i)}\right):h_{\mu\nu}^{\delta}\in\text{LSV}\ , \tag{16}$$

and

$$\left(\Omega_{\alpha\beta},\overline{\Omega}_{\alpha\beta}\right)=\sum_{\delta,\gamma=1}^{3}G_{\alpha\beta}^{\delta\gamma}\left(d_\gamma^{(s,h,i)}d_\delta^{(s,h,i)},\pi_\gamma^{(s,h,i)}\pi_\delta^{(s,h,i)}\right):G_{\alpha\beta}^{\alpha\beta}\in\text{QPV}\ . \tag{17}$$

The symbol $[Q,S]_*$ is a new commutator which means (Q_*S-S_*Q), $\theta_{\mu\nu}$ and $\overline{\theta}_{\mu\nu}$ are antisymmetric real constant (3×3) matrices, which satisfied the physical conditions $[\theta_{\mu\nu}]=\epsilon_{\mu\nu}[\theta]$ and $[\overline{\theta}_{\mu\nu}]=\epsilon_{\mu\nu}[\overline{\theta}]$ are equal to (length)^2 and (momentums)^2 ,here $(\overline{\theta},\theta)$ are the real non-commutative phase-space parameters and $\alpha_{\mu\nu}$ is the Kronecker symbol. The indices (α,β) equal to the values $(1,2,3)$, and $\epsilon_{\mu\nu}$ is just an antisymmetric tensor operator that is satisfied $\epsilon_{\mu\nu}=-\epsilon_{\nu\mu}=1$ for $\mu\neq\nu$ and $\epsilon_{\epsilon\epsilon}=0$. The effective Planck constant $\hbar_{eff}$ equal to the corresponding values $\hbar$ plus $(\theta\eta/4)\hbar$ that is present the impact of phase-space deformation on commutator $[x_\alpha^s,p_\beta^s]$ 3D(NR-QM) symmetries. For the purpose of simplifying the writing of mathematical equations, we have adopted the most commonly used natural units, corresponding to adopting both the reduced Planck constant $\hbar$ and the speed of light c in a vacuum as equal to one. In 3D(NR-NCPS) and 3D(NR-QM) symmetries, the deformed generalized coordinates $d_\alpha^{(s,h,i)}$(d_α^s, $d_\alpha^h(t)$ and $d_\alpha^i(t)$) and deformed generalizing momentums $\pi_\alpha^{(s,h,i)}$(π_α^s, $\pi_\alpha^h(t)$ and $\pi_\alpha^i(t)$) and the corresponding operators in 3D(NR-QM) symmetries ($x_\mu^{(s,h,i)}$(x_μ^s, $x_\mu^h(t)$ and $x_\mu^i(t)$) and $p_\mu^{(s,h,i)}$(p_μ^s, $p_\mu^h(t)$ and $p_\mu^i(t)$)) satisfied the uncertainty relation that corresponds to the Eq. (12) becomes:

$$\begin{cases}\left|\Delta x_\alpha^s\Delta p_\beta^s\right|=\left|\Delta x_\alpha^h\Delta p_\beta^h\right|=\left|\Delta x_\alpha^i\Delta p_\beta^i\right|\geqslant\hbar\delta_{\alpha\beta}/2\Rightarrow\\ \left|\Delta d_\alpha^s\Delta\pi_\beta^s\right|=\left|\Delta d_\alpha^h\Delta\pi_\beta^h\right|=\left|\Delta d_\alpha^i\Delta q_\beta^i\right|\geqslant\hbar_{eff}\delta_{\alpha\beta}/2\end{cases}. \tag{18}$$

Nonetheless, a new uncertainty relation is shown through Eqs. (1.4) and (1.5):

$$\begin{cases}\left|\Delta d_\alpha^s\Delta d_\beta^s\right|=\left|\Delta d_\alpha^h\Delta d_\beta^h\right|=\left|\Delta d_\alpha^i\Delta d_\beta^i\right|\geqslant\Xi_{\alpha\beta}^{(1)}\\ \left|\Delta\pi_\alpha^s\Delta\pi_\nu^s\right|=\left|\Delta\pi_\alpha^h\Delta\pi_\nu^h\right|=\left|\Delta\pi_\alpha^i\Delta\pi_\nu^i\right|\geqslant\Xi_{\alpha\beta}^{(2)}\end{cases}. \tag{19}$$

For QSV, LSV and QSV $\left(\Xi_{\mu\nu}^{(1)},\Xi_{\mu\nu}^{(2)}\right)$ are equal to, respectively:

$$\begin{cases} 2\left(\Xi_{\mu\nu}^{(1)},\Xi_{\mu\nu}^{(2)}\right)=\left(\theta,\overline{\theta}\right)\left|\epsilon_{\mu\nu}\right|, \\ 2\left(\Xi_{\mu\nu}^{(1)},\Xi_{\mu\nu}^{(2)}\right)=\left(\beta_{\mu\nu},\gamma_{\mu\nu}\right), \\ \left(\Xi_{\mu\nu}^{(1)},\Xi_{\mu\nu}^{(2)}\right)=\left(L_{\mu\nu},\alpha_{\mu\nu}\right). \end{cases} \tag{20}$$

with $\beta_{\mu\nu}/\gamma_{\mu\nu}$ and $L_{\mu\nu}/\alpha_{\mu\nu}$ are equal to the average values:

$$\begin{cases} \beta_{\mu\nu}=\left|\left\langle\sum_{\alpha}^{3}\left(f_{\mu\nu}^{\alpha}d_{\alpha}^{(s,h,i)}\right)\right\rangle\right|, \\ \gamma_{\mu\nu}=\left|\left\langle\sum_{\alpha}^{3}\left(f_{\mu\nu}^{\alpha}\pi_{\alpha}^{(s,h,i)}\right)\right\rangle\right|, \end{cases} \tag{21}$$

and

$$\begin{cases} L_{\mu\nu}=\left\langle\sum_{\alpha,\beta}^{3}\left(G_{\mu\nu}^{\alpha\beta}d_{\alpha}^{(s,h,i)}d_{\beta}^{(s,h,i)}\right)\right\rangle, \\ \alpha_{\mu\nu}=\left\langle\sum_{\alpha,\beta}^{3}\left(G_{\mu\nu}^{\alpha\beta}\pi_{\alpha}^{(s,h,i)}\pi_{\beta}^{(s,h,i)}\right)\right\rangle. \end{cases} \tag{22}$$

There is no equivalent in the current literature (3D(NR-QM) symmetries) for the novel subdivided three-uncertainty relations in Eq. (19). We have extended the modified equal-time non-commutative canonical commutation relations in 3D-NCPS symmetries to include the standard Schrödinger, Heisenberg and interaction pictures. The new deformed scalar product $(f*h)\,(x,p)$ is defined by the Weyl-Moyal $*$-product for a canonical structure variety expressed as [48–55]:

$$(f*h)\,(x,p)=(fh)\,(x,p)-\frac{i}{2}\left(\theta^{\alpha\beta}\partial_{x^{\alpha}}f\partial_{x^{\beta}}h+\overline{\theta}^{\alpha\beta}\partial_{p^{\alpha}}f\partial_{p^{\beta}}h\right)(x,p)+O\left(\overline{\theta}^{2},\theta^{2}\right), \tag{23}$$

here $\partial_{x^{\alpha}}$ and $\partial_{p^{\alpha}}$ are equal to $\partial/\partial x^{\alpha}$ and $\partial/\partial p^{\alpha}$, respectively. We must preserve new expectation relation in the 3D(NR-NCPS) regimes, respectively:

$$\begin{aligned} {}_{S}\langle\Psi\left|A\right|\Psi\rangle_{S}={}_{H}\,\langle\Psi\left|A_{qm}^{h}\left(t\right)\right|\Psi\rangle_{H}={}_{i}\,\langle\Psi\left|A_{qm}^{i}\left(t\right)\right|\Psi\rangle_{i}\Longrightarrow \\ {}_{S}^{nc}\langle\Psi\left|A_{nc}^{s}\right|\Psi\rangle_{S}={}_{H}^{nc}\,\langle\Psi\left|A_{nc}^{h}\left(t\right)\right|\Psi\rangle_{H}^{nc}={}_{i}^{nc}\,\langle\Psi\left|A_{nc}^{i}\left(t\right)\right|\Psi\rangle_{i}^{nc}. \end{aligned} \tag{24}$$

This enables the creation of two scales of space and phase cells with volumes $(\theta^{3/2},\overline{\theta}^{3/2})$. The second component $(-(i/2)$ $(\theta^{\alpha\beta}\partial_{x^{\alpha}}f\partial_{x^{\beta}}h)(x,p))$ of Eq. (23) represents the physical consequences of phase-space non-commutativity, while the third component $(-(i/2)(\overline{\theta}^{\alpha\beta}\partial_{p^{\alpha}}f\partial_{p^{\beta}}h)\,(x,p))$ represents the physical consequences of phase-phase non-commutativity.

3.2. Investigating GBSM

The principal approaches to resolving the impact of the (NR-NC) phase-space on the SE utilizing the MDF-YP model will be discussed in this subsection. The novel notions mentioned in the introduction have been identified explicitly in Eqs. (12), (13), (14), and (23) are considered in new relationships described by new non-commutative canonical commutation relations (NNCCCRs) and the concept of the Weyl-Moyal star product. We may rewrite the usual radial SE in Eq. (3) in 3D(NR-NCPS) using these data as follows:

$$\left(\frac{d^{2}}{dr^{2}}+2\mu(E_{nl}^{dy}-D_{e}\left(1-\frac{b\exp\left(-\alpha r\right)}{1-\exp\left(-\alpha r\right)}\right)^{2}+V_{0}\frac{\exp\left(-\alpha r\right)}{r}-\frac{l(l+1)}{2\mu r^{2}})\right)*R_{nl}\left(r\right)=0. \tag{25}$$

Researchers in solving the four fundamental (SE, KGE, DE and the Duffin-Kemmer-Petiau equation), including non-commutative quantum principles, rely on two equivalent methods. The first method is represented by reformulating the different new physical fields in the NC-quantum group, such as $\mathbf{\Psi}_{nl}$ (Dirac spinor), $\mathbf{\Phi}_{nl}$ (Klein-Gordon field operator), $\mathbf{e}_{\mu}^{a}$ (virbien in quantum gravity), $\mathbf{F}_{\alpha\beta}$ (electromagnetic antisymmetric tensor in $U(1)$ symmetry) and others in terms of their corresponding physical fields in the usual quantum group (Ψ_{nl}, Φ_{nl}, e_{μ}^{a}, $F_{\alpha\beta}$ and other among), in proportion to the NC parameters $\Theta(\theta_{12},\theta_{23},\theta_{13})\,/2$, which is similar to the Taylor development [55–60], whereas the second method is represented by reformulating the non-commutative operator ($\mathbf{q}$ and π) with its view of the quantum operators ($\mathbf{x}$ and $\mathbf{p}$) known When employing either of them, the physical outcomes are expected to be identical. F. Bopp introduced a new quantization rule (x and p) →($\mathbf{q} = x - (i/2)\partial_p$ and $\pi = p + (i/2)\partial_x$) instead of the standard correspondence (x and p)→($\mathbf{q} = x$ and $\mathbf{q} = p + (i/2)\partial_x$), which is known as the generalized Bopp's shift method (GBSM) [61–65]. This quantization method is known to researchers as Bopp quantization [64]. The Weyl-Moyal star product $f(x,p) * g(x,p)$ promotes GBSM by being replaced by $f(\mathbf{q} = x - (i/2)\partial_p$, $\pi = p + (i/2)\partial_x) * g(x,p)$ [65]. As a result, we may obtain transformations from the Weyl-Moyal star product ($*$) to the typical product using the MDF-YP model, as shown below.

$$\begin{cases} D_e\left(1-\frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 * R_{nl}(r) = D_e\left(1-\frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 R_{nl}(r)\,, \\ -V_0\frac{\exp(-\alpha r)}{r} * R_{nl}(r) = -V_0\frac{\exp(-\alpha d)}{d} R_{nl}(r)\,, \\ \frac{l(l+1}{2\mu r^2} * R_{nl}(r) = \frac{l(l+1}{2\mu \mathbf{d}^2} R_{nl}(r)\,, \end{cases} \tag{26}$$

and

$$\frac{p^2}{2\mu} * R_{nl}(r) = \frac{\pi^2}{2\mu} R_{nl}(r)\,. \tag{27}$$

We should inform the reader that the generalized Bopp's shift method succeeded by applying it to the four fundamental equations classified according to spin (integer, half-integer, or zero) and energy value (low or high). For SE [66–73] and the other three relativistic equations represented by the KGE [74–80], the DE [81–86], and the Duffin-Kemmer-Petiau equation [87–90], GBSM has achieved great success. It is worth noticing that GBSM allows us to reduce Eq. (25) to its new simple form:

$$\left(\frac{d^2}{dr^2} + 2\mu\left(E_{nl}^{dy} - D_e\left(1-\frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 + V_0\frac{\exp(-dr)}{d} - \frac{l(l+1)}{2\mu\mathbf{d}^2}\right)\right) R_{nl}(r) = 0. \tag{28}$$

The deformed algebraic structure (new non-commutative algebra) of covariant canonical non-commutation relations with the notion of the Weyl-Moyal star product presented in Eqs. (3), (4) and (5) reduce to simple new NNCCCRs as follows:

First, the reduced new NNCCCRs in the Schrödinger picture:

$$\begin{cases} \left[d_\alpha^s, \pi_\beta^s\right]_* = i\hbar_{eff}\alpha_{\alpha\beta} \Rightarrow \left[d_\alpha^s, \pi_\beta^s\right] = i\delta_{\alpha\beta}, \\ \left[d_\alpha^s, d_\beta^s\right]_* = i\theta_{\alpha\beta} \Rightarrow \left[d_\alpha^s, d_\beta^s\right] = i\theta_{\alpha\beta}, \\ \left[\pi_\alpha^s, \pi_\beta^s\right]_* = i\overline{\theta}_{\alpha\beta} \Rightarrow \left[\pi_\alpha^s, \pi_\beta^s\right] = i\overline{\theta}_{\alpha\beta}. \end{cases} \tag{29}$$

Second, the reduced new NNCCCRs in the Heisenberg picture:

$$\begin{cases} \left[d_\alpha^h(t), \pi_\beta^h(t)\right]_* = i\delta_{\alpha\beta} \Rightarrow \left[q_\alpha^h(t), \pi_\beta^h(t)\right] = i\delta_{\alpha\beta}, \\ \left[d_\alpha^h(t), d_\beta^h(t)\right]_* = i\theta_{\alpha\beta} \Rightarrow \left[d_\alpha^h(t), q_\beta^h(t)\right] = i\theta_{\alpha\beta}, \\ \left[\pi_\alpha^h(t), \pi_\beta^h(t)\right]_* = i\overline{\theta}_{\alpha\beta} \Rightarrow \left[\pi_\alpha^h(t), \pi_\beta^h(t)\right] = i\overline{\theta}_{\alpha\beta}. \end{cases} \tag{30}$$

Third, the reduced new NNCCCRs in the interaction picture:

$$\begin{cases} \left[d_\alpha^i(t), \pi_\beta^i(t)\right]_* = i\hbar_{eff}\delta_{\alpha\beta} \Rightarrow \left[q_\alpha^i(t), \pi_\beta^i(t)\right] = i\delta_{\alpha\beta}, \\ \left[d_\alpha^i(t), d_\beta^i(t)\right]_* = i\theta_{\alpha\beta} \Rightarrow \left[d_\alpha^i(t), q_\beta^i(t)\right] = i\theta_{\alpha\beta}, \\ \left[\pi_\alpha^i(t), \pi_\beta^i(t)\right]_* = i\overline{\theta}_{\alpha\beta} \Rightarrow \left[\pi_\alpha^i(t), \pi_\beta^i(t)\right] = i\overline{\theta}_{\alpha\beta}. \end{cases} \tag{31}$$

In 3D(NR-NCPS) symmetries, one possible way of implementing the algebra defined by Eqs. (29), (30) and (31) are to construct the non-commutative set of variables (d_α^s, $q_\alpha^h(t)$ and $q_\alpha^i(t)$) and (π_α^s, $\pi_\alpha^h(t)$ and $\pi_\alpha^i(t)$) from the corresponding commutative variables (x_μ^s, $x_\mu^h(t)$ and $x_\mu^i(t)$) and (p_μ^s, $p_\mu^h(t)$ and $p_\mu^i(t)$) by employing linear transformations. This can be generally done by using the Seiberg-Witten map, given by:

$$\begin{cases} d_\mu^s = x_\mu^s - \sum\limits_{\nu=1}^{3}\frac{\theta_{\mu\nu}}{2}p_\nu^s + O\left(\Theta^2\right), \\ \pi_\mu^s = p_\mu^s + \sum\limits_{\nu=1}^{3}\frac{\overline{\theta}_{\mu\nu}}{2}x_\nu^s + O\left(\overline{\theta}^2\right) \end{cases} \tag{32}$$

and

$$\begin{cases} d_{\mu}^{h}(t) = x_{\mu}^{h}(t) - \sum\limits_{\nu=1}^{3} \frac{\theta_{\mu\nu}}{2} p_{\nu}^{h}(t) + O\left(\Theta^{2}\right), \\ \pi_{\mu}^{h}(t) = p_{\mu}^{h}(t) + \sum\limits_{\nu=1}^{3} \frac{\overline{\theta}_{\mu\nu}}{2} x_{\nu}^{h}(t) + O\left(\overline{\theta}^{2}\right), \end{cases} \tag{33}$$

and

$$\begin{cases} d_{\mu}^{i}(t) = x_{\mu}^{i}(t) - \sum\limits_{\nu=1}^{3} \frac{\theta_{\mu\nu}}{2} p_{\nu}^{i}(t) + O\left(\Theta^{2}\right), \\ \pi_{\mu}^{i}(t) = p_{\mu}^{i}(t) + \sum\limits_{\nu=1}^{3} \frac{\overline{\theta}_{\mu\nu}}{2} x_{\nu}^{i}(t) + O\left(\overline{\theta}^{2}\right). \end{cases} \tag{34}$$

We have applied Einstein's term regarding the addition process in the above-mentioned equations, where the repeated indices ν once up and once down correspond to the addition process from 1 to 3.This allows us to find the operators $(\mathbf{d}^2$, $1/\mathbf{d}^2$, π^2 and $V_{dy}(\mathbf{d}))$, in the 3D(NR-NCPS) symmetries, equal to:

$$\begin{cases} \mathbf{d}^2 = r^2 - \mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right), \\ \frac{1}{2\mu\mathbf{d}^2} = \frac{1}{2\mu r^2} + \frac{\mathbf{L}.\mathbf{\Theta}}{2\mu r^4} + O\left(\Theta^2\right), \end{cases} \tag{35}$$

and

$$\begin{cases} \pi^2 = p^2 + \mathbf{L}.\overline{\theta} + O\left(\overline{\theta}^2\right), \\ V_{dy}(\mathbf{q}) = D_e\left(1 - \frac{b}{\exp(\alpha r)-1}\right)^2 - V_0\frac{\exp(-\alpha r)}{r} - \frac{1}{2r}\frac{\partial V_{dy}}{\partial r}\mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right). \end{cases} \tag{36}$$

It is worth noting that the couplings $\mathbf{L}.\mathbf{\Theta}$ and $\mathbf{L}.\overline{\theta}$ are expressed to the angular momentum operator $\mathbf{L}$ obtained from rotation of two vectors ($\mathbf{r}$ and $\mathbf{p}$) that are equal to $\sum_{i,j,k} \epsilon_{ijk}\theta^k p^j x^i$ and $\sum_{i,j,k} \epsilon_{ijk}\overline{\theta}^k p^j x^i$, respectively. On the other hand, this double coupling, which expresses the interaction of physical properties and topological deformations, are expressed by the scalar product $(L_x\theta_{12} + L_y\theta_{23} + L_z\theta_{12})/2$ and $L_x\overline{\theta}_{12} + L_y\overline{\theta}_{23} + L_z\overline{\theta}_{13}$. When we substitute Eqs. (35) and (36) into Eq. (28), we get the following like-SE:

$$\left[\frac{d^2}{dr^2} + 2\mu\left(E_{nl}^{dy} - D_e\left(1 - \frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 + V_0\frac{\exp(-\alpha r)}{r} - \frac{l(l+1)}{2\mu r^2} + 2\mu Z_{dy}^{pert}(r,\Theta)\right)\right] R_{nl}(r) = 0, \tag{37}$$

with

$$Z_{dy}^{pert}(r,\Theta) = \left(\frac{1}{2r}\frac{\partial V_{dy}(r)}{\partial r} - \frac{l(l+1)}{2\mu r^4}\right)\mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right). \tag{38}$$

The above equation combines the physical characteristics $(1/2r)(\partial V_{dy}(r)/\partial r)$ and $\pi^2/2\mu$ of the MDF-YP model with the angular momentum operator $\mathbf{L}$, as well as the topological features generated by phase-space deformations. After performing the mathematical calculations, one obtains $(1/2r)(\partial V_{dy}(r)/\partial r)$ and $\pi^2/2\mu$.

$$\begin{cases} \frac{1}{2r}\frac{\partial V_{dy}(r)}{\partial r} = D_e\frac{\alpha b\exp(-\alpha r)}{r(1-\exp(-\alpha r))} + D_e\frac{\left(\alpha b - \alpha b^2\right)\exp(-2\alpha r)}{r(1-\exp(-\alpha r))^2} - D_e\frac{\alpha b^2\exp(-3\alpha r)}{r(1-\exp(-\alpha r))^3} + \frac{\alpha V_0}{2}\frac{\exp(-\alpha r)}{r^2} + \frac{V_0}{2}\frac{\exp(-\alpha r)}{r^3}, \\ \frac{\pi^2}{2\mu} = \frac{p^2}{2\mu} + \frac{\mathbf{L}.\overline{\theta}}{2\mu} + O\left(\overline{\theta}^2\right), \end{cases} \tag{39}$$

As a result of the topological features of the deformation phase-space, the spontaneously generated term $V_{eff-nc}^{dy}(r,\Theta)$ and the global working Hamiltonian operator $H_{nc}^{dy}\left(p,r,\Theta,\overline{\theta}\right)$ that equal to the modified kinetic energy $\pi\left(p,\overline{\theta}\right)/2\mu$ plus the effective potential $V_{eff-nc}^{dy}(r,\Theta)$ of the MDF-YP model:

$$\begin{cases} H_{nc}^{dy}\left(p,r,\Theta,\overline{\theta}\right) = H_{dy}(p,r) + H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right), \\ V_{eff-nc}^{dy}(r,\Theta) = V_{eff}^{dy}(r) + Z_{dy}^{pert}(r,\Theta). \end{cases} \tag{40}$$

with

$$\begin{cases} H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)=Z_{dy}^{pert}\left(r,\Theta\right)+\frac{\mathbf{L}.\overline{\theta}}{2\mu}+O\left(\overline{\theta}^{2},\Theta^{2}\right), \\ Z_{dy}^{pert}\left(r,\Theta\right)=\frac{\mathbf{D_{e}\alpha b\exp(-\alpha r)}}{\mathbf{r(1-\exp(-\alpha r))}}\left(\mathbf{1}+\frac{\mathbf{(1-b)\exp(-\alpha r)}}{\mathbf{1-\exp(-\alpha r)}}-\frac{\mathbf{b\exp(-2\alpha r)}}{\mathbf{(1-\exp(-\alpha r))^{2}}}\right)\mathbf{L.\Theta} \\ \qquad +\left(\frac{\alpha\mathbf{V_{0}}}{\mathbf{2}}\frac{\exp(-\alpha\mathbf{r})}{\mathbf{r^{2}}}+\frac{\mathbf{V_{0}}}{\mathbf{2}}\frac{\exp(-\alpha\mathbf{r})}{\mathbf{r^{3}}}-\frac{\mathbf{l(l+1)}}{\mathbf{2\mu r^{4}}}\right)\mathbf{L.\Theta}+O\left(\Theta^{2}\right), \\ H_{dy}\left(p,x\right)=\frac{p^{2}}{2\mu}+D_{e}\left(1-\frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^{2}-V_{0}\frac{\exp(-\alpha r)}{r}. \end{cases} \tag{41}$$

We can express the global effective potential in 3D(NR-NCPS) symmetries $V_{dy}^{nc-eff}\left(r,\Theta\right)$ as a function of corresponding follows of the effective potential $V_{dy}^{eff}\left(r\right)$ in 3D(NR-QM) symmetries as:

$$V_{dy}^{nc-eff}\left(r,\Theta\right)=V_{dy}^{eff}\left(r\right)+Z_{dy}^{pert}\left(r,\Theta\right)+O\left(\Theta^{2}\right), \tag{42}$$

with

$$V_{dy}^{eff}\left(r\right)=D_{e}\left(1-\frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^{2}-V_{0}\frac{\exp(-\alpha r)}{r}+\frac{l(l+1)}{2\mu r^{2}}\ . \tag{43}$$

Furthermore, Eq. (37) cannot be solved analytically for any state $l\neq0$ because of the centrifugal terms $(\exp\left(-\alpha\mathbf{r}\right)/\mathbf{r}(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right))$, $\exp\left(-\mathbf{2}\alpha\mathbf{r}\right)/\mathbf{r}\left(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right)\right)^{\mathbf{2}}$, $\exp\left(-\mathbf{3}\alpha\mathbf{r}\right)/\mathbf{r}\left(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right)\right)^{\mathbf{3}}$, $\exp\left(-\alpha\mathbf{r}\right)/\mathbf{r^{2}}$, $\exp\left(-\alpha\mathbf{r}\right)/\mathbf{r^{3}}$ and $\mathbf{1/r^{4}})$ and the studied potential itself. In fact, the global Hamiltonian $H_{nc}^{dy}\left(p,r,\Theta,\overline{\theta}\right)$ and effective potential $V_{dy}^{nc-eff}\left(r,\Theta\right)$ given in Eq. (45) has a strong singularity $r\rightarrow0$; we need to use the suitable improved approximation of the centrifugal term proposed by Greene and Aldrich [91] and applied by Cari *et al.* [11]. The radial part of the three-dimensional deformed Schrödinger equation with the MDF-YP model contains the previous centrifugal terms since we assume $l\neq0$. However, the MDF-YP model is a kind of potential that cannot be solved exactly when the centrifugal term is taken into account unless $l=0$ it is assumed. The conventional approximation used in this paper:

$$\frac{1}{r^{2}}\approx\frac{\alpha^{2}}{\left(1-\exp\left(-\alpha r\right)\right)^{2}}=\frac{\alpha^{2}}{\left(1-s\right)^{2}}\Leftrightarrow\frac{1}{r}\approx\frac{\alpha}{1-\exp\left(-\alpha r\right)}=\frac{\alpha}{1-s}. \tag{44}$$

Thus, performing the calculations, one gets the following results:

$$\begin{cases} D_{e}\frac{\alpha b\exp(-\alpha r)}{r(1-\exp(-\alpha r))}=D_{e}b\frac{s}{(1-s)^{2}}, \\ D_{e}\frac{\left(\alpha b-\alpha b^{2}\right)\exp(-2\alpha r)}{r(1-\exp(-\alpha r))^{2}}=D_{e}b\left(1-b\right)\alpha^{2}\frac{s^{2}}{(1-s)^{3}}, \\ -D_{e}\frac{\alpha b^{2}\exp(-3\alpha r)}{r(1-\exp(-\alpha r))^{3}}=-D_{e}\alpha^{2}b^{2}\frac{s^{3}}{(1-s)^{4}}, \\ \frac{\alpha V_{0}}{2}\frac{\exp(-\alpha r)}{r^{2}}=\frac{\alpha^{3}V_{0}}{2}\frac{s}{(1-s)^{2}}, \\ \frac{V_{0}}{2}\frac{\exp(-\alpha r)}{r^{3}}=\frac{V_{0}\alpha^{3}}{2}\frac{s}{(1-s)^{3}}, \\ -\frac{l(l+1)}{2\mu r^{4}}=-\frac{l(l+1)}{2\mu}\frac{\alpha^{4}}{(1-s)^{4}}. \end{cases} \tag{45}$$

This gives the perturbative effective Hamiltonian $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$ and effective perturbed potential $V_{dy}^{pert}\left(r,\Theta\right)$ given in Eq. (46) as follows:

$$\begin{cases} H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)=\Upsilon\left(s\right)\mathbf{L.\Theta}+\frac{\mathbf{L}.\overline{\theta}}{\mathbf{2}\mu}+O\left(\overline{\theta}^{2},\Theta^{2}\right), \\ Z_{dy}^{pert}\left(r,\Theta\right)=\Upsilon\left(s\right)\mathbf{L.\Theta}+O\left(\Theta^{2}\right). \end{cases} \tag{46}$$

with

$$\begin{cases} \Upsilon\left(s\right)=\frac{a_{1}s}{(1-s)^{2}}+\frac{a_{2}s^{2}}{(1-s)^{3}}+\frac{a_{3}s^{3}}{(1-s)^{4}}+\frac{a_{4}s}{(1-s)^{3}}+\frac{a_{5}}{(1-s)^{4}}, \\ D_{e}b+\frac{\alpha^{3}V_{0}}{2}=a_{1},\quad D_{e}b\left(1-b\right)\alpha^{2}=a_{2}-D_{e}\alpha^{2}b^{2}=a_{3},\quad \frac{V_{0}\alpha^{3}}{2}=a_{4}\quad \text{and}\quad -\frac{l(l+1)\alpha^{4}}{2\mu}=a_{5}. \end{cases} \tag{47}$$

The potential under study, to become a MDF-YP model in 3D(NR-NCPS) symmetries, the Deng-Fan Yukawa potential model is expanded by including new radial terms $s/(1-s)^2$, $s^2/(1-s)^3$, $s^3/(1-s)^4$, $s/(1-s)^3$and $1/(1-s)^4$. Furthermore, the new additive part of the Hamiltonian operator $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$ is also includes two infinitesimal couplings $\mathbf{L}.\left(\boldsymbol{\Theta},\overline{\theta}\right)$ that interacted with previous radial terms. This is logical from a physical point of view because it explains the interaction between the physical properties of the studied potential **L** and the topological properties resulting from the deformation of phase-space that is described with $\left(\boldsymbol{\Theta},\overline{\theta}\right)$. This enables us to treat the new additive part of the Hamiltonian operator $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$ as a perturbation operator in the symmetries of 3D(NR-NCPS) symmetries, compared to the main potential $H_{dy}\left(p,x\right)$ (parent Hamiltonian operator); the inequality $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)\ll H_{dy}\left(p,x\right)$ has been achieved. All of the physical arguments for using time-independent perturbation theory are met. As a result, we can provide a thorough prescription for estimating the energy level of generalized $\left(n,l,m\right)^{th}$ excited states.

3.3. Non-relativistic expectation values under the MDF-YP model in 3D(NR-NCPS) regimes

Now, we want to apply the independent time standard perturbative theory and we find the non-relativistic expectation values ($\langle s/(1-s)^2\rangle_{(nlm)}^{dy}\equiv F_{dy}^1$, $\langle s^2/(1-s)^3\rangle_{(nlm)}^{dy}\equiv F_{dy}^2$, $\langle s^3/(1-s)^4\rangle_{(nlm)}^{dy}\equiv F_{dy}^3$, $\langle s/(1-s)^3\rangle_{(nlm)}^{dy}\equiv F_{dy}^4$ and $\langle 1/(1-s)^4 t\rangle_{(nlm)}^{dy}\equiv F_{dy}^5$) for the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) , taking into account the unperturbed wave functions $\Psi\left(r,\Omega_3,t\right)$ which we have seen previously in Eq. (19) in the case of 3D(NR-NCPS) symmetries. Following simple calculations, we obtain the expectation values (F_{dy}^1, F_{dy}^2, F_{dy}^3, F_{dy}^4 and F_{dy}^5) in the first order using standard perturbation theory as follows:

$$F_{dy}^1=N_{nl}^2\int_0^{+\infty}s^{-2\sqrt{\Lambda_{nl}}+1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-1}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2dr, \tag{48}$$

$$F_{dy}^2=N_{nl}^2\int_0^{+\infty}s^{-2\sqrt{\Lambda_{nl}}+2}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-2}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2dr, \tag{49}$$

$$F_{dy}^3=N_{nl}^2\int_0^{+\infty}s^{-2\sqrt{\Lambda_{nl}}+3}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-3}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2dr, \tag{50}$$

$$F_{dy}^4=N_{nl}^2\int_0^{+\infty}s^{-2\sqrt{\Lambda_{nl}}+1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-2}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2dr, \tag{51}$$

and

$$F_{dy}^5=N_{nl}^2\int_0^{+\infty}s^{-2\sqrt{\Lambda_{nl}}}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-3}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2dr. \tag{52}$$

We are introducing the change of variable $s=\exp\left(-\alpha r\right)$. This maps the region ($0\leqslant r\lessdot\infty\rightarrow 0\leqslant s\leq 1$) and allows us to obtain $dr=-dx/\alpha x$, and transform Eqs. (48), (49), (50), (51) and (52) into the following form:

$$F_{dy}^1=\frac{N_{nl}^2}{\alpha}\int_0^{+1}s^{-2\sqrt{\Lambda_{nl}}+1-1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-1}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2ds, \tag{53}$$

$$F_{dy}^2=\frac{N_{nl}^2}{\alpha}\int_0^{+1}s^{-2\sqrt{\Lambda_{nl}}+2-1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-2}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2ds, \tag{54}$$

$$F_{dy}^3=\frac{N_{nl}^2}{\alpha}\int_0^{+1}s^{-2\sqrt{\Lambda_{nl}}+3-1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-3}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2ds, \tag{55}$$

$$F_{dy}^4=\frac{N_{nl}^2}{\alpha}\int_0^{+1}s^{-2\sqrt{\Lambda_{nl}}+1-1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-2}\left[{}_2F_1\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^2ds, \tag{56}$$

and

$$F_{dy}^{5}=\frac{N_{nl}^{2}}{\alpha}\int_{0}^{+1}s^{-2\sqrt{\Lambda_{nl}}-1}\left(1-s\right)^{\sqrt{1+4\epsilon_{nl}}-3}\left[{}_{2}F_{1}\left(-n,b_{nl};1-2\Lambda_{nl};s\right)\right]^{2}ds. \tag{57}$$

The above integrals can be evaluated in two ways: either by using the formulas used by Ahmadov *et al.* [92] and Tas *et al.* [93] to obtain the general excited state directly, or by using the physical values of the principal quantum number ($n=0,1,...$) to evaluate the above integrals and then generalize the result to the general $(n,l,m)^{th}$ excited state. The first method will be less expensive and faster in signing the required solutions:

$$\int_{0}^{+1}x^{2\gamma-1}\left(1-x\right)^{\mathbf{2}(\alpha+1)}\left[{}_{2}F_{1}\left(-q,q+2\left(\alpha+\gamma+1\right);2\gamma+1;x\right)\right]^{2}dx$$
$$=\frac{q!\left(q+2\left(\alpha+1\right)/2\right)\Gamma\left(2\gamma\right)\Gamma\left(q+2\left(\alpha+1\right)\right)\Gamma\left(2\gamma+1\right)}{\left(q+2\left(\alpha+1\right)/2+2\gamma/2\right)\Gamma\left(q+2\gamma+1\right)\Gamma\left(q+2\gamma+2\left(\alpha+1\right)\right)}. \tag{58}$$

The following solutions are obtained by comparing the integrals in Eqs. (58) with Eqs. (53) - (57):

$$F_{dy}^{1}=\overline{\theta}_{nl}^{1}\frac{\Gamma\left(1-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-1\right)\Gamma\left(2-2\sqrt{\Lambda_{nl}}\right)}{\Gamma\left(n-2\sqrt{\Lambda_{nl}}+2\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\sqrt{\Lambda_{nl}}\right)}, \tag{59}$$

$$F_{dy}^{2}=\overline{\theta}_{nl}^{2}\frac{\Gamma\left(2-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\right)\Gamma\left(3-2\sqrt{\Lambda_{nl}}\right)}{\Gamma\left(n+3-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\sqrt{\Lambda_{nl}}\right)}, \tag{60}$$

$$F_{dy}^{3}=\overline{\theta}_{nl}^{3}\frac{\Gamma\left(3-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-3\right)\Gamma\left(4-2\sqrt{\Lambda_{nl}}\right)}{\Gamma\left(n+4-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\sqrt{\Lambda_{nl}}\right)}, \tag{61}$$

$$F_{dy}^{4}=\overline{\theta}_{nl}^{4}\frac{\Gamma\left(1-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\right)\Gamma\left(2-2\sqrt{\Lambda_{nl}}\right)}{\Gamma\left(n+2-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\sqrt{\Lambda_{nl}}-1\right)}, \tag{62}$$

and

$$F_{dy}^{5}=\overline{\theta}_{nl}^{5}\frac{\Gamma\left(-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+2\left(\alpha+1\right)\right)\Gamma\left(1-2\sqrt{\Lambda_{nl}}\right)}{\Gamma\left(n+1-2\sqrt{\Lambda_{nl}}\right)\Gamma\left(n+\sqrt{1+4\epsilon_{nl}}-2\sqrt{\Lambda_{nl}}+-3\right)}, \tag{63}$$

with $\overline{\theta}_{nl}^{1}$, $\overline{\theta}_{nl}^{2}$, $\overline{\theta}_{nl}^{3}$, $\overline{\theta}_{nl}^{4}$ and $\overline{\theta}_{nl}^{5}$ are equal to

$$\frac{n!\left(n+\sqrt{1/4+\epsilon_{nl}}-1/2\right)N_{nl}^{2}}{\alpha\left(n+\sqrt{1/4+\epsilon_{nl}}-\sqrt{\Lambda_{nl}}\right)},\quad \frac{n!\left(n+\sqrt{1/4+\epsilon_{nl}}-1\right)N_{nl}^{2}}{\alpha\left(n+\sqrt{1/4+\epsilon_{nl}}-\sqrt{\Lambda_{nl}}\right)},\quad \frac{n!\left(n+\sqrt{1/4+\epsilon_{nl}}-3/2\right)N_{nl}^{2}}{\alpha\left(n+\sqrt{1/4+\epsilon_{nl}}-\sqrt{\Lambda_{nl}}\right)},$$
$$\frac{n!\left(n+\sqrt{1/4+\epsilon_{nl}}-1\right)N_{nl}^{2}}{\alpha\left(n+\sqrt{1/4+\epsilon_{nl}}-1/2-\sqrt{\Lambda_{nl}}\right)}\quad \text{and}\quad \frac{n!\left(n+\sqrt{1/4+\epsilon_{nl}}-3/2\right)N_{nl}^{2}}{\alpha\left(n+\sqrt{1/4+\epsilon_{nl}}-\sqrt{\Lambda_{nl}}-3/2\right)},$$

respectively.

3.4. The MDF-YP model's effect on non-relativistic energies as a result of phase-space deformations

What stands out here is the use of our physical methods based on the principle of superposition to calculate the total values of non-relativistic energy under the MDF-YP model in 3D(NR-NCPS) symmetry. As mentioned before, the total effective potential $V_{dy}^{nc-eff}\left(r,\Theta\right)$ is the sum of three potentials $V_{nl}^{dy}\left(r,\Theta\right)$, $l\left(l+1\right)/r^{2}$and $V_{dy}^{pert}\left(r,\Theta\right)$ is responsible for the creation of total non-relativistic energies within the context of 3D(NR-NCPS) regimes. Naturally, the effective potentials $V_{nl}^{dy}\left(r\right)$ plus $l\left(l+1\right)/r^{2}$ are responsible for the non-relativistic energies E_{nl}^{dy} of SE in the Deng-Fan Yukawa potential model in 3D(NR-QM) symmetry, as shown in Eq. (23), which are dominant in the absence of phase-phase-space deformations. In 3D(NR-NCPS) symmetries, the naturally generated potentials $V_{dy}^{pert}\left(r,\Theta\right)$ due to phase-phase-space deformations will be self-sources of corrected no-relativistic energy. Given that the NC two parameters $\Theta\left(\theta_{12},\theta_{23},\theta_{13}\right)/2$ and $\overline{\theta}\left(\overline{\theta}_{12},\overline{\theta}_{23},\overline{\theta}_{13}\right)/2$ are arbitrary, we deal with them on the relevant physical need. To begin, the perturbed spin-orbit influence can be derived from the perturbed potential $V_{dy}^{pert}(r,\Theta)$ corresponding to the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF). The perturbed spin-orbit effective potentials are obtained by substituting the angular momentum L operator's coupling with the non-commutative phase-space vectors $\Theta(\theta_{12},\theta_{23},\theta_{13})/2$ and $\overline{\theta}(\overline{\theta}_{12},\overline{\theta}_{23},\overline{\theta}_{13})/2$ with the new equivalent couplings as follows:

$$\left(\mathbf{L}.\mathbf{\Theta},\mathbf{L}.\overline{\theta}\right)\rightarrow\left(\Theta,\overline{\theta}\right)\mathbf{L}.\mathbf{S}\text{ or }\left(\overrightarrow{\mathbf{L}}.\overrightarrow{\mathbf{\Theta}},\overrightarrow{\mathbf{L}}.\overrightarrow{\overline{\theta}}\right)\rightarrow\left(\Theta,\overline{\theta}\right)\overrightarrow{\mathbf{L}}.\overrightarrow{\mathbf{S}},\tag{64}$$

with $(\Theta,\bar{\theta})$ are equal to $(\sqrt{\theta_{12}^2+\theta_{23}^2+\theta_{13}^2},\ \sqrt{\bar{\theta}_{12}^2+\bar{\theta}_{23}^2+\bar{\theta}_{13}^2})$. We have oriented the spin-s of the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) to become parallels to the vector $\Theta(\theta_{12},\theta_{23},\theta_{13})/2$ and $\overline{\theta}\left(\overline{\theta}_{12},\overline{\theta}_{23},\overline{\theta}_{13}\right)/2$ which interacted with the MDF-YP model. This connection between topological and physical properties came according to our conception on the basis of the available degree of freedom. As we know, using the degree of freedom in the equations of motion is equivalent to adding the gauge fixing term to the Lagrangian. Since the degree of freedom does not apply if this term is added to the Lagrangian expression, it can be applied explicitly if we adopt the Lagrangian without this term, and this is what we did. This physical philosophy came to consider the two topological vectors ($\overrightarrow{\mathbf{\Theta}}$, $\overrightarrow{\overline{\theta}}$) as an arbitrary. The mathematical aspect of the problem is simple because when we consider two parallel vectors, we can express the proportion between them in terms of the value of the other. if we consider the vector $\overrightarrow{A}$ parallel to the vector $\overrightarrow{B}$ ($\overrightarrow{A}//\overrightarrow{B}$), thus we have $|B|\overrightarrow{A}$ equal to $|A|\overrightarrow{B}$. The expression for the perturbed Hamiltonian that we saw in Eq. (46) will then be as follows:

$$H_{so}^{dy}\left(r,\Theta,\overline{\theta}\right)=\Upsilon\left(s\right)\mathbf{L}.\mathbf{S}+\frac{\overline{\theta}}{2\mu}\mathbf{L}.\mathbf{S}.\tag{65}$$

The corresponding partially corrected energies ΔE_{dy}^{nr-so}, obtained by applying the independent time standard perturbative theory in the first order of phase-space non-commutativity parameters using unperturbed complex wave function in Eq. (19), as follows

$$\Delta E_{dy}^{nr-so}=\int\Psi^{*}\left(r,\Omega_3,t\right)H_{so}^{dy}\left(r,\Theta,\overline{\theta}\right)\Psi\left(r,\Omega_3,t\right)r^2d\Omega dr.\tag{66}$$

Here $d\Omega$ equal to $\sin(\theta)d\theta d\varphi$. Direct simplifications give

$$\Delta E_{dy}^{nr-so}=\int_0^{+\infty}R_{nl}\left(r\right)H_{so}^{dy}\left(r,\Theta,\overline{\theta}\right)R_{nl}\left(r\right)dr.\tag{67}$$

After performing the mathematical calculations, one obtains:

$$\Delta E_{dy}^{nr-so}=\left(\Theta\left\langle X\right\rangle_{(nlm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right)+\frac{\overline{\theta}}{2\mu}\right)\left\langle\mathbf{L}.\mathbf{S}\right\rangle_{(nlm)}.\tag{68}$$

Here $\left\langle X\right\rangle_{(nlm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right)$ is global expectation values that can be determined from:

$$\left\langle X\right\rangle_{(nlm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right)=\sum_{\mu=1}^{5}a_\mu F_{dy}^{\mu}\left(n,D_e,r_e,V_0,\alpha\right).\tag{69}$$

The values $F_{dy}^{\mu}\left(n,D_e,r_e,V_0,\alpha\right)$ ($\mu=\overline{1,5}$) are determine from Eqs. (59), (60), (61), (62) and (63) while the means value $\left\langle\mathbf{L}.\mathbf{S}\right\rangle_{(nlm)}$ obtained by applying the following well-known transformation:

$$\begin{pmatrix}\Theta\\ \overline{\theta}\end{pmatrix}\mathbf{L}.\mathbf{S}\rightarrow\left(\mathbf{J}^2-\mathbf{L}^2-\mathbf{S}^2\right)/2\begin{pmatrix}\Theta\\ \overline{\theta}\end{pmatrix}.\tag{70}$$

Because, in 3D(NR-NCPS) symmetry, the operators ($\widehat{\mathbf{H}}_{nc}^{dy}$, $\mathbf{J}^2$, $\mathbf{L}^2$, $\mathbf{S}^2$ and $\mathbf{J}_z$) can construct a complete set of conserved physics quantities. Thus, the eigenvalues of the operator $\left(\mathbf{J}^2-\mathbf{L}^2-\mathbf{S}^2\right)$ are equal to the values Λ for the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) are $(1/2)(j(j+1)$-$l\left(l+1\right)-s(s+1))$. The values $j\in\left[\left|l-s\right|,\left|l+s\right|\right]$ and the spin-s can be equal to $\{1/2,0,1,...\}$. Thus, a direct result, in 3D(NR-NCPS) symmetries, the partially corrected energies $\Delta E_{dy}^{nr-so}\left(n,D_e,r_e,V_0,\alpha,\Theta,\overline{\theta},j,l,s\right)\equiv$

ΔE_{dy}^{nr-so} produced by the perturbed effective Hamiltonian $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$ for the $(n,l,m)^{th}$ under MDF-YP model are determined from the following equation:

$$\Delta E_{dy}^{so}=\Lambda\left(\Theta\left\langle X\right\rangle_{(nlm)}^{dy}+\frac{\overline{\theta}}{2\mu}\right)+O\left(\Theta^{2},\overline{\theta}^{2}\right). \tag{71}$$

The influence of the magnetic perturbative potential, which causes the effect of the perturbed Hamiltonian $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$ under the MDF-YP model in the 3D(NR-NCPS) symmetries, is the second significant physical contribution. This physical action can be achieved by performing the following transitions:

$$\left(\begin{array}{c}\mathbf{L}.\mathbf{\Theta}\\ \mathbf{L}.\overline{\theta}\end{array}\right)\rightarrow\left(\begin{array}{c}\chi\\ \overline{\chi}\end{array}\right)\overrightarrow{\mathbf{L}}.\overrightarrow{\aleph}. \tag{72}$$

Here, $\aleph$ is the strength of the magnetic field caused by the influence of phase-space geometry deformation, χ and $\overline{\chi}$ are playing the role of new infinitesimal non-commutativity parameters. The physical units of $[\Theta]\equiv$ (length)^2 and $\left[\overline{\theta}\right]\equiv$ (momentum)^2 are equal to $[\chi]\,[\aleph]$ and $\left[\overline{\theta}\right]=[\overline{\chi}]\,[\aleph]$, respectively and $\overrightarrow{\aleph}=\aleph\mathbf{e}_z$. The second choice emerges from the fact that the vectors $\Theta(\theta_{12},\theta_{23},\theta_{13})/2$ and $\overline{\theta}\left(\overline{\theta}_{12},\overline{\theta}_{23},\overline{\theta}_{13}\right)/2$ are arbitrary or that the magnetic field is oriented along the (Oz) axis, which helps simplify quantitative calculations without changing the physical point of view. The expression for the perturbed Hamiltonian that we saw in Eq. (46) will then be as follows:

$$H_{mg}^{dy}\left(r,\chi,\overline{\chi}\right)=\aleph\Upsilon\left(s\right)\chi L_z+\frac{\aleph}{2\mu}\overline{\chi}L_z+O\left(\chi^{2},\overline{\chi}^{2}\right). \tag{73}$$

All of these data allow for the discovery of the new square energy shift ΔE_{dy}^{mg} (n, V_0, V_1, α, χ, $\overline{\chi}$, m) for the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) due to the perturbed Zeeman effect produced by the influence of the MDF-YP model for the $(n,l,m)^{th}$ excited state in 3D(NR-NCPS) regimes:

$$\Delta E_{dy}^{mg}=\aleph\left(\chi\left\langle X\right\rangle_{(nlm)}^{dy}+\frac{\overline{\chi}}{2\mu}\right)m+O\left(\chi^{2},\overline{\chi}^{2}\right). \tag{74}$$

After we have completed the first and second steps of self-production of energy, we will discover another very vital case under the MDF-YP model in 3D(NR-NCPS) symmetries. This new physical phenomenon is produced automatically under the influence of the perturbed Hamiltonian $H_{pert}^{dy}\left(r,\Theta,\overline{\theta}\right)$. We consider the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) undergoing rotation with angular velocity Ω. The features of this subjective phenomenon are determined by replacing the arbitrary vectors $\Theta(\theta_{12},\theta_{23},\theta_{13})/2$ and $\overline{\theta}\left(\overline{\theta}_{12},\overline{\theta}_{23},\overline{\theta}_{13}\right)/2$ with $\zeta\Omega$ and $\overline{\zeta}\Omega$. Allowing us to replace the couplings ($\mathbf{L}.\Theta$ and $\mathbf{L}.\overline{\theta}$) with ($\zeta\mathbf{L}.\Omega$ and $\overline{\zeta}\mathbf{L}.\Omega$**)**.The expression for the perturbed Hamiltonian that we saw in Eq. (46) will then be as follows:

$$H_{rot}^{dy}\left(r,\zeta,\overline{\zeta}\right)=\zeta\Upsilon\left(s\right)\mathbf{L}.\mathbf{\Omega}+\frac{\overline{\zeta}}{2\mu}\mathbf{L}.\mathbf{\Omega}+O\left(\zeta^{2},\overline{\zeta}^{2}\right). \tag{75}$$

In the above equation, ζ and $\overline{\zeta}$ are two real proportional constants. To make the calculations more straightforward, we choose a rotating velocity Ω parallel to the (Oz) axis Ω=$\Omega\mathbf{e}_z$. This, of course, doesn't significantly change the physical properties of the problem under study. Thus, the perturbed previously generated spin-orbit coupling operator $\mathbf{L}.\mathbf{S}$ will be transformed into a new physical form as follows:

$$H_{rot}^{dy}\left(r,\zeta,\overline{\zeta}\right)=\Omega\left(\zeta\Upsilon\left(s\right)+\frac{\overline{\zeta}}{2\mu}\right)\mathbf{L}_z+O\left(\zeta^{2},\overline{\zeta}^{2}\right). \tag{76}$$

All of this data allows for the discovery of the new corrected square energy $\Delta E_{dy}^{nr-rot}\left(n,D_e,r_e,V_0,\alpha,\zeta,\overline{\zeta},m\right)$ of the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) due to the perturbed Hamiltonian $H_{rot}^{dy}\left(r,\zeta,\overline{\zeta}\right)$, which is generated automatically by the influence of the MDF-YP model for the $(n,l,m)^{th}$ excited state in 3D(NR-NCPS) symmetries as follows:

$$\Delta E_{dy}^{rot}=\left(\zeta\left\langle V\left(r\right)\right\rangle_{(nlm)}^{dy}+\frac{\overline{\zeta}}{2\mu}\right)\Omega m+O\left(\zeta^{2},\overline{\zeta}^{2}\right). \tag{77}$$

It is essential to acknowledge that the authors of Ref. [94] investigated rotating isotropic and anisotropic harmonically confined ultra-cold Fermi gases in two and three dimensions at zero temperature; however, in this case, the rotational term was manually added to the Hamiltonian operator, while in our study, the deformation of phase-space under the MDF-YP model causes the rotation operator $H_{rot}^{dy}\left(r,\zeta,\overline{\zeta}\right)$ to appear automatically. In the symmetries of the 3D(NR-NCPS) regimes, we apply the principle of physical superposition to find the physical expression of the total non-relativistic new energies $E_{nc}^{dy}(n,D_e,r_e,V_0,\alpha,S_p^{nc},P_p^{nc},j,l,s,m) \equiv E_{nc}^{dy}$ for the HODMs (H_2, I_2); the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) under the MDF-YP model, corresponding to the generalized excited states obtained as follows

$$E_{nc}^{dy} = Q_a - Q_b\left(n+\gamma+\frac{Q_c}{n+\gamma}\right)^2 + \langle X\rangle_{(nlm)}^{dy} N_p^{sp}\left(S_p^{nc}\right) + N_p^{ph}\left(P_p^{nc}\right). \tag{78}$$

The connection between non-commutative space parameters $S_p^{nc} \equiv(\Theta,\chi,\zeta)$ non-commutative phase parameters $P_p^{nc} \equiv (\overline{\theta},\overline{\chi},\overline{\zeta})$ and the physical properties of studied system $(\Lambda,\aleph,m)$/$(\Lambda/2\mu,\aleph/2\mu,\Omega/2\mu)$ can be summarized into new representations $N_p^{sp}\left(\Theta,\chi,\zeta\right)$ and $N_p^{ph}\left(\overline{\theta},\overline{\chi},\overline{\zeta}\right)$ that given by:

$$\begin{cases} N_p^{sp}\left(\Theta,\chi,\zeta\right) = \Theta\Lambda + \left(\chi\aleph+\zeta\Omega\right)m, \\ N_p^{ph}\left(\overline{\theta},\overline{\chi},\overline{\zeta}\right) = \overline{\theta}\frac{\Lambda}{2\mu} + \left(\overline{\chi}\frac{\aleph}{2\mu}+\overline{\zeta}\frac{\Omega}{2\mu}\right)m. \end{cases} \tag{79}$$

Since we collected the partial corrective expressions ΔE_{dy}^{so}, ΔE_{dy}^{mg} and ΔE_{dy}^{rot} that we saw in Eqs. (71), (74), and (77). The first three parts (Q_a and $-Q_b\left(n+\gamma+[Q_c/n+\gamma)^2\right]$) are non-relativistic energies under the Deng-Fan Yukawa potential model obtained from equations of energy in Eq. (23) while the remaining terms in Eq. (78) represent the resulting correction produced from deformation phase-space. It is essential to point out that because we have only used corrections of the first order of infinitesimal NC-(phase-space) parameters (Θ,χ,ζ) and $\left(\overline{\theta},\overline{\chi},\overline{\zeta}\right)$, perturbation theory cannot be used to find corrections of the second order $\left(\Theta^2,\chi^2,\zeta^2\right)$ and $\left(\overline{\theta}^2,\overline{\chi}^2,\overline{\zeta}^2\right)$. It is helpful to briefly study the total energy of each of the fundamental state $E_{nc}^{dy}(n=0,D_e,r_e,V_0,\alpha,S_p^{nc},P_p^{nc},j,l,s,m) \equiv E_{nc}^{0cy}$ corresponding to the quantum numbers $(n=0,l,m)$, as well as the first excited state $E_{nc}^{dy}(n=1,D_e,r_e,V_0,\alpha,S_p^{nc},j,P_p^{nc},l,s,m) \equiv E_{nc}^{1cy}$ corresponding to the quantum numbers $(n=1,l,m)$, where Eq.(78) in these two cases becomes as follows:

$$E_{nc}^{0cy} = E_{0l}^{dy} + \langle V\rangle_{(0lm)}^{dy} N_p^{sp}\left(S_p^{nc}\right) + N_p^{ph}\left(P_p^{nc}\right), \tag{80}$$

and

$$E_{nc}^{1cy} = E_{1l}^{dy} + \langle V\rangle_{(1lm)}^{dy} N_p^{sp}\left(S_p^{nc}\right) + N_p^{ph}\left(P_p^{nc}\right). \tag{81}$$

When an electron transitions from an excited state described as a quantum state $(n=1,l,m)$ to the fundamental states, described as a quantum state $(n=0,l,m)$, it will emit or absorb electromagnetic radiation, the frequency ω_{nc}^{dy} $(n,D_e,r_e,V_0,\alpha,$ S_p^{nc}, j, l, s, $m)\equiv \omega_{nc}^{dy}$ of which is determined by the relationship:

$$\omega_{nc}^{dy} = \left|E_{nc}^{1dy} - E_{nc}^{0dy}\right|. \tag{82}$$

A simple calculation, gives the emit or absorbed electromagnetic radiation ω_{nc}^{dy}, in the context of 3D(NR-NCPS) symmetry, as follows:

$$\omega_{nc}^{dy} = \omega_{nl}^{dy} + N_p^{sp}\left(S_p^{nc}\right)\langle\Delta V\rangle_{nc}^{dy}. \tag{83}$$

Here $\omega_{nl}^{dy}(n,D_e,r_e,V_0,\alpha)$ is the emitted or absorbed electromagnetic radiation in 3D(NR-QM) symmetry (Eq. (24)),while $\langle\Delta Z\rangle_{nc}^{dy}$ is equal to:

$$\langle\Delta V\rangle_{nc}^{dy} = \langle V\rangle_{(1lm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right) - \langle V\rangle_{(0lm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right), \tag{84}$$

with

$$\begin{cases} \langle V\rangle_{(1lm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right) = \lim\limits_{n\to 1}\langle X\rangle_{(nlm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right) \\ \langle V\rangle_{(0lm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right) = \lim\limits_{n\to 0}\langle X\rangle_{(nlm)}^{dy}\left(n,D_e,r_e,V_0,\alpha\right) \end{cases}. \tag{85}$$

The term $N_p^{sp}\left(\Theta,\chi,\zeta\right)\langle\Delta V\rangle_{nc}^{dy}$ is traduce the impact of phase-space deformation on the $\omega_{nl}^{dy}(V_0,V_1,\alpha)$. This effect can vanish when NC-(phase-space) parameters $\left(S_p^{nc}\right)$ are reduced to *zero* simultaneously.

4. Study of important particular cases of MDF-YP model in 3D(NR-NCPS) symmetries

In this section, we will examine the obtained new bound state eigenvalues of the deformed Schrödinger equation with the MDF-YP model in 3D(NR-NCPS) symmetries which we have seen in Eq. (78). By suitable adjustment of the potential parameters of the Deng-Fan Yukawa potential model, we are now in the process of treating into 3D(NR-NCPS) regime:

(a) If the dissociation energy D_e reduces to zero, Eq. (1) gives the modified Yukawa potential $V_{my}(\mathbf{d})$ in 3D(NR-NCPS) symmetries as

$$V_{my}(\mathbf{d}) = -V_0 \frac{\exp(-\alpha r)}{r} - \frac{\mathbf{V_0}}{\mathbf{2}}\left(\alpha + \frac{\mathbf{1}}{\mathbf{r}}\right)\frac{\exp(-\alpha \mathbf{r})}{\mathbf{r^2}}\mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right). \tag{86}$$

From Eq. (78), we obtain the non-relativistic eigenvalues $E_{nc}^{my}(n, V_0, \alpha, S_p^{nc}, P_p^{nc}, j, l, s, m)$ for non-relativistic particles, corresponding to the generalized $(n,l,m)^{th}$ excited states in 3D-NRNCPS symmetries as:

$$\begin{aligned} &E_{nc}^{my}(n, V_0, \alpha, S_p^{nc}, P_p^{nc}, j, l, s, m) \\ &= \frac{\alpha^2 l(l+1)}{2\mu} - \frac{\alpha^2}{8\mu}\left(n + \frac{1}{2} + \frac{1}{2}\sqrt{1+4(l(l+1))} + \frac{\alpha^2 l(l+1)/2\mu}{n+\frac{1}{2}+\frac{1}{2}\sqrt{1+4(l(l+1))}}\right)^2 \\ &+ \langle V\rangle_{(nlm)}^{np}(n, V_0, \alpha) N_p^{sp} + N_p^{ph} + O\left(\left(S_p^{nc}\right)^2, \left(P_p^{nc}\right)^2\right). \end{aligned} \tag{87}$$

The first two terms are consistent with the result obtained in of Ref. [95]. The other terms are received from the impact of phase-space deformation on the potential of the modified Yukawa (See Refs. [17–24]).

The corresponding new non-relativistic expectations values $\langle V\rangle_{(nlm)}^{nc}(n, V_0, \alpha)$ of the modified negative Coulombic potential model from the following limits:

$$\langle V\rangle_{(nlm)}^{np}(n, V_0, \alpha) = \lim_{D_e\to 0}\langle X\rangle_{(nlm)}^{dy}(n, D_e, r_e, V_0, \alpha) \quad . \tag{88}$$

(b) If the potential parameter V_0 reduces to zero, Eq. (1) gives the modified Deng-Fan potential in 3D(NR-NCPS) symmetries as

$$\begin{aligned} V_{nc}(\mathbf{q}) = D_e\left(1 - \frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2 - \frac{\mathbf{D_e}\alpha\mathbf{b}\exp(-\alpha\mathbf{r})}{\mathbf{r}(\mathbf{1}-\exp(-\alpha\mathbf{r}))} \\ \times\left(\mathbf{1} + \frac{(\mathbf{1}-\mathbf{b})\exp(-\alpha\mathbf{r})}{\mathbf{1}-\exp(-\alpha\mathbf{r})} - \frac{\mathbf{b}\exp(-\mathbf{2}\alpha\mathbf{r})}{(\mathbf{1}-\exp(-\alpha\mathbf{r}))^{\mathbf{2}}}\right)\mathbf{L}.\mathbf{\Theta} + O\left(\Theta^2\right). \end{aligned} \tag{89}$$

From Eq. (78), we obtain the non-relativistic eigenvalues $E_{nc}^{df}(n, D_e, \alpha, S_p^{nc}, P_p^{nc}, j, l, s, m) \equiv E_{nc}^{df}$ for non-relativistic particles, corresponding to the generalized $(n,l,m)^{th}$ excited states in 3D-NRNCPS symmetries as:

$$\begin{aligned} E_{nc}^{df} = Q_a - Q_b\left(n + \gamma + \frac{\frac{\alpha^2 l(l+1)}{2\mu} - 2D_e b - D_e b^2}{n+\gamma}\right)^2 \langle V\rangle_{(nlm)}^{df}(n, D_e, r_e, \alpha) N_p^{sp}\left(S_p^{nc}\right) + N_p^{ph}\left(P_p^{nc}\right) \\ + O\left(\Theta^2, \overline{\theta}^2\chi^2, \overline{\chi}^2, \zeta^2, \overline{\zeta}^2\right). \end{aligned} \tag{90}$$

The first two terms are consistent with the result obtained in Eq. (89) of refs. [96,97]. The other terms are obtained from the impact of phase-space deformation on he modified Deng-Fan potential [25]. The corresponding new non-relativistic expectations values $\langle V\rangle_{(nlm)}^{df}(n, D_e, r_e, \alpha)$ of the modified positive Coulombic potential model from the following limits:

$$\langle V\rangle_{(nlm)}^{pc}(n, D_e, r_e, \alpha) = \lim_{V_0\to 0}\langle X\rangle_{(nlm)}^{dy}(n, D_e, r_e, V_0, \alpha) \quad . \tag{91}$$

5. Spin-averaged mass spectra of the heavy mesons under the MDF-YP model in 3D(NR-QM) and 3D(NR-NCPS) symmetries

In this section, we calculate the mass spectra of the heavy mesons system such as (charmonium $c\bar{c}$ and bottomonium $b\bar{b}$), that have the quark and antiquark flavor, which is the main focus of this work. In this work the Deng-Fan Yukawa potential model

model is used to investigate quark confinement, because these potentials have two distinctive features, strong interaction-asymptotic freedom and confinement. Equation (2) allow us to divide the Deng-Fan Yukawa potential model under study into two main parts $V_1^{df}(r)$ and $V_2^{yp}(r)$ that play different roles in 3D(NR-QM) symmetries. The first part is the Deng-Fan potential model:

$$V_1^{pc}(r) = D_e\left(1 - \frac{b\exp(-\alpha r)}{1-\exp(-\alpha r)}\right)^2. \tag{92}$$

Thus, this part is more singular and provides better confinement as compared to the generalized Cornell potential [98], which has a similar term (d/r^2). The second part $V_2^{yp}(r)$, similar to the Coulomb potential, has the form:

$$V_2^{yp}(r) = -V_0\frac{\exp(-\alpha r)}{r}. \tag{93}$$

This part plays the role of Coulomb force, like the Coulombic potential ($-c/r$) in the generalized Cornell potential [98].This means that the second part $V_2^{nc}(r)$ has the same behavior as the Coulombic potential. We calculate the new mass of quarkonium M_{nc}^{dy} in 3D(NR-NCPS) symmetries, by applying our following relation as in:

$$M_{nc}^{dy} = 2m_q + \begin{cases} \frac{1}{3}\left(E_{nc}^{dy-u} + E_{nc}^{dy-m} + E_{nc}^{dy-l}\right) \text{ for heavy mesons with spin-}1, \\ E_{nc}^{dy} \text{ for heavy mesons with spin-}0. \end{cases} \tag{94}$$

here m_q is the quark mass while E_{nc}^{dy-u}, E_{nc}^{dy-m}, E_{nc}^{dy-l} and E_{nc}^{dy} are the new energy eigenvalues that correspond ($j = l+1$, $s=1$), ($j=l$, $s=1$), ($j=l-1$, $s=1$) and ($j=l$, $s=0$) under the MDF-YP model in 3D(NR-NCPS) symmetries. It results from the generalization of the original relationship known in the literature [99, 100]:

$$M_{nl}^{dy} = 2m_q + E_{nl}^{dy}, \tag{95}$$

where E_{nl}^{dy} is the non-relativistic energy under the Deng-Fan Yukawa potential model which is determined by Eq. (20). We have replaced the energy eigenvalues E_{nl}^{nr} with average values $(1/3)(E_{nc}^{dy-u} + E_{nc}^{dy-m} + E_{nc}^{dy-l})$ that have spin-1 with three different values of the values j while for a spin-0, the values E_{nl}^{nr}, replaced with E_{nc}^{dy} because it represents a single value. We need to replace the factor $\Lambda(j,l,s)$ with new generalized values as follows:

$$\Lambda(j,l,s) = \begin{cases} l/2 \quad \text{For } (j=l+1, s=1), \\ -1 \quad \text{For } (j=l, s=1), \\ (-2l-2)/2 \text{ For } (j=l-1, s=1), \\ 0 \quad \text{For } (j=l, s=0). \end{cases} \tag{96}$$

Allows us to obtain (E_{nc}^{dy-u}, E_{nc}^{dy-m} and E_{nc}^{dy-l}) and E_{nc}^{dy} of the heavy mesons system such as (charmonium $c\bar{c}$ and bottomonium $b\bar{b}$) as:

1. The energy values E_{nc}^{dy-u} produced by the MDF-YP model and correspond to discrete quantum numbers $j = l+1$, $s=1$, can be expressed by the following formula:

$$E_{nc}^{dy-u} = Q_a - 2Q_bQ_c - \left(\frac{Q_bQ_c^2}{\rho^2} + Q_b\rho^2\right) + \langle X\rangle_{(nlm)}^{dy}\left[\left(\chi + \frac{\overline{\chi}}{2\mu}\right)\aleph m + \left(\zeta + \frac{\overline{\zeta}}{2\mu}\right)\Omega m + \left(\Theta + \frac{\overline{\theta}}{2\mu}\right)\frac{l}{2}\right]. \tag{97}$$

2. The energy values E_{nc}^{dy-m} produced by the MDF-YP model and correspond to discrete quantum numbers ($j=l$, $s=1$), can be expressed by the following formula:

$$E_{nc}^{dy-m} = Q_a - 2Q_bQ_c - \left(\frac{Q_bQ_c^2}{\rho^2} + Q_b\rho^2\right) + \langle X\rangle_{(nlm)}^{dy}\left[\left(\chi + \frac{\overline{\chi}}{2\mu}\right)\aleph m + \left(\zeta + \frac{\overline{\zeta}}{2\mu}\right)\Omega m - \left(\Theta + \frac{\overline{\theta}}{2\mu}\right)\right]. \tag{98}$$

3. The energy values E_{nc}^{dy-l} produced by the MDF-YP model and correspond to discrete quantum numbers ($j = l-1$, $s=1$), can be expressed by the following formula:

$$E_{nc}^{dy-l} = Q_a - 2Q_bQ_c - \left(\frac{Q_bQ_c^2}{\rho^2} + Q_b\rho^2\right) + \langle X\rangle_{(nlm)}^{dy}\left[\left(\chi + \frac{\overline{\chi}}{2\mu}\right)\aleph m + \left(\zeta + \frac{\overline{\zeta}}{2\mu}\right)\Omega m - (l+1)\left(\Theta + \frac{\overline{\theta}}{2\mu}\right)\right], \tag{99}$$

while the energy values E_{nc}^{dy} produced by the MDF-YP model and correspond to discrete quantum numbers $(j=l,\, s=0)$, can be expressed as follows:

$$E_{nc}^{dy}=Q_a-2Q_bQ_c-\left(\frac{Q_bQ_c^2}{\rho^2}+\ Q_b\rho^2\right)+\langle X\rangle_{(nlm)}^{dy}\left[\left(\chi+\frac{\overline{\chi}}{2\mu}\right)\aleph m+\left(\zeta+\frac{\overline{\zeta}}{2\mu}\right)\Omega m\right]. \tag{100}$$

The new mass spectrum $M_{nc}^{dy}\left(n,D_e,r_e,V_0,\alpha,S_p^{nc},P_p^{nc}\right)$ of the heavy-light mesons systems, such as charmonium $c\bar{c}$ and bottomonium $b\bar{b}$, in 3D(NR-NCPS) symmetries under the MDF-YP model, as a function of corresponding mass spectra $M_{nl}^{dy}\left(n,D_e,r_e,V_0,\alpha\right)\equiv M_{nl}^{dy}$ in 3D(NR-QM) regime and non-commutativity (phase-space) parameters P_p^{nc} and S_p^{nc}, can be obtained by substituting Eqs. (96), (97), (98), and (99) into Eq. (93).

$$M_{nc}^{dy}=M_{nl}^{dy}+\begin{cases}\langle X\rangle_{(nlm)}^{dy}\left[\left(\chi+\frac{\overline{\chi}}{2\mu}\right)\aleph m+\left(\zeta+\frac{\overline{\zeta}}{2\mu}\right)\Omega m-\left(\Theta+\frac{\overline{\theta}}{2\mu}\right)\left(\frac{l}{2}+\frac{2}{3}\right)\right] & \text{For heavy mesons with spin-1,}\\ \langle X\rangle_{(nlm)}^{dy}\left[\left(\chi+\frac{\overline{\chi}}{2\mu}\right)\aleph m+\left(\zeta+\frac{\overline{\zeta}}{2\mu}\right)\Omega m\right] & \text{For heavy mesons with spin-0.}\end{cases} \tag{101}$$

We can express the spin-averaged mass spectra M_{nl}^{dy} of the heavy mesons system such as (charmonium $c\bar{c}$ and bottomonium $b\bar{b}$) for SE under Deng-Fan Yukawa potential model in 3D(NR-QM) symmetries by applying the law known in the literature:

$$M_{nl}^{dy}=2m_q+Q_a-2Q_bQ_c-\left(\frac{Q_bQ_c^2}{\rho^2}+\ Q_b\rho^2\right), \tag{102}$$

is extended to include δM_{nc}^{dy} in 3D(NR-NCPS) symmetries:

$$\delta M_{nc}^{dy}=\begin{cases}\langle X\rangle_{(nlm)}^{dy}\left[\left(\chi+\frac{\overline{\chi}}{2\mu}\right)\aleph m+\left(\zeta+\frac{\overline{\zeta}}{2\mu}\right)\Omega m-\left(\Theta+\frac{\overline{\theta}}{2\mu}\right)\left(\frac{l}{2}+\frac{2}{3}\right)\right] & \text{For heavy mesons with spin-1,}\\ \langle X\rangle_{(nlm)}^{dy}\left[\left(\chi+\frac{\overline{\chi}}{2\mu}\right)\aleph m+\left(\zeta+\frac{\overline{\zeta}}{2\mu}\right)\Omega m\right] & \text{For heavy mesons with spin-0.}\end{cases} \tag{103}$$

Which is sensitive to the atomic quantum numbers (j,l,s,m), potential parameters (D_e,r_e,V_0,α), and non-commutativity (phase-space) parameters P_p^{nc} and S_p^{nc} under the deformed properties of phase-space. Validity to our results examined by realization of logical physical limits:

$$\lim_{\left(S_p^{nc},P_p^{nc}\right)\to(0,0)}M_{nc}^{dy}\left(n,D_e,r_e,V_0,\alpha,S_p^{nc},P_p^{nc}\right)=M_{nl}^{dy}\left(n,D_e,r_e,V_0,\alpha\right). \tag{104}$$

6. Composite systems under MDF-YP in 3D(NR-NCPS) symmetries

In the context of deformation algebra, while studying composite systems, as molecules composed of two atoms with m_1 and m_2, it is vital to evaluate characteristics of system descriptions in 3D(NR-NCPS) symmetries under the modified Deng-Fan Yukawa potential. It was discovered that distinct deformed phase-space parameters that described the composite systems with $m_1\neq m_2$ [101–103]:

$$\left[d_\alpha^s,d_\beta^s\right]_*=\left[d_\alpha^h(t),d_\beta^h(t)\right]_*=\left[d_\alpha^i(t),d_\beta^i(t)\right]_*=i\theta_{\alpha\beta}^c, \tag{105}$$

and

$$\left[\pi_\alpha^s,\pi_\beta^s\right]_*=\left[\pi_\alpha^h(t),\pi_\beta^h(t)\right]_*=\left[\pi_\alpha^i(t),\pi_\beta^i(t)\right]_*=i\overline{\theta}_{\alpha\beta}^c. \tag{106}$$

In this case, the new NNCCCRs that we have seen in Eqs. (29), (30), and (31) will be changed to become in SP, HP, and IP as follows, respectively:

$$\begin{cases}\left[d_\alpha^s,d_\beta^s\right]_*=i\theta_{\alpha\beta}\Rightarrow\left[d_\alpha^s,d_\beta^s\right]=i\theta_{\alpha\beta}^c,\\ \left[\pi_\alpha^s,\pi_\beta^s\right]_*=i\overline{\theta}_{\alpha\beta}\Rightarrow\left[\pi_\alpha^s,\pi_\beta^s\right]=i\overline{\theta}_{\alpha\beta}^c,\end{cases} \tag{107}$$

and

$$\begin{cases}\left[d_\alpha^h(t),d_\beta^h(t)\right]_*=i\theta_{\alpha\beta}\Rightarrow\left[d_\alpha^h(t),d_\beta^h(t)\right]=i\theta_{\alpha\beta}^c,\\ \left[\pi_\alpha^h(t),\pi_\beta^h(t)\right]_*=i\overline{\theta}_{\alpha\beta}\Rightarrow\left[\pi_\alpha^h(t),\pi_\beta^h(t)\right]=i\overline{\theta}_{\alpha\beta}^c,\end{cases} \tag{108}$$

and

$$\begin{cases} \left[d_{\alpha}^{i}\left(t\right), d_{\beta}^{i}\left(t\right)\right]_{*} = i\theta_{\alpha\beta} \Rightarrow \left[d_{\alpha}^{i}\left(t\right), d_{\beta}^{i}\left(t\right)\right] = i\theta_{\alpha\beta}^{c}, \\ \left[\pi_{\alpha}^{i}\left(t\right), \pi_{\beta}^{i}\left(t\right)\right]_{*} = i\overline{\theta}_{\alpha\beta} \Rightarrow \left[\pi_{\alpha}^{i}\left(t\right), \pi_{\beta}^{i}\left(t\right)\right] = i\overline{\theta}_{\alpha\beta}^{c}. \end{cases} \tag{109}$$

In 3D(NR-NCPS) symmetries, one possible way of implementing the algebra defined by Eqs. (32), (33), and (34) are to construct the non-commutative set of variables (d_{α}^{s},$d_{\alpha}^{h}\left(t\right)$,$d_{\alpha}^{i}\left(t\right)$) and ($\pi_{\alpha}^{s}$, $\pi_{\alpha}^{h}\left(t\right)$, $\pi_{\alpha}^{i}\left(t\right)$) from the corresponding commutative variables (x_{μ}^{s}, $x_{\mu}^{h}\left(t\right)$, $x_{\mu}^{i}\left(t\right)$) and ($p_{\mu}^{s}$, $p_{\mu}^{h}\left(t\right)$, $p_{\mu}^{i}\left(t\right)$) by employing linear transformations:

$$\begin{cases} q_{\mu}^{s} = x_{\mu}^{s} - \sum\limits_{\nu=1}^{3} \frac{\theta_{\mu\nu}^{c}}{2} p_{\nu}^{s} + O\left(\Theta^{c2}\right), \\ \pi_{\mu}^{s} = p_{\mu}^{s} + \sum\limits_{\nu=1}^{3} \frac{\overline{\theta}_{\mu\nu}^{c}}{2} x_{\nu}^{s} + O\left(\overline{\theta}^{c2}\right), \end{cases} \tag{110}$$

$$\begin{cases} d_{\mu}^{h}\left(t\right) = x_{\mu}^{h}\left(t\right) - \sum\limits_{\nu=1}^{3} \frac{\theta_{\mu\nu}^{c}}{2} p_{\nu}^{h}\left(t\right) + O\left(\Theta^{c2}\right), \\ \pi_{\mu}^{h}\left(t\right) = p_{\mu}^{h}\left(t\right) + \sum\limits_{\nu=1}^{3} \frac{\overline{\theta}_{\mu\nu}^{c}}{2} x_{\nu}^{h}\left(t\right) + O\left(\overline{\theta}^{c2}\right), \end{cases} \tag{111}$$

$$\begin{cases} q_{\mu}^{i}\left(t\right) = x_{\mu}^{i}\left(t\right) - \sum\limits_{\nu=1}^{3} \frac{\theta_{\mu\nu}^{c}}{2} p_{\nu}^{i}\left(t\right) + O\left(\Theta^{c2}\right), \\ \pi_{\mu}^{i}\left(t\right) = p_{\mu}^{i}\left(t\right) + \sum\limits_{\nu=1}^{3} \frac{\overline{\theta}_{\mu\nu}^{c}}{2} x_{\nu}^{i}\left(t\right) + O\left(\overline{\theta}^{c2}\right), \end{cases} \tag{112}$$

We have applied Einstein's term regarding the addition process in the above-mentioned equations, where the repeated indices ν once up and once down correspond to the addition process from 1 to 3.This allows us to find the operators ($\mathbf{d}^{2}$, $1/\mathbf{d}^{2}$, π^{2} and $V_{dy}\left(\mathbf{d}\right)$), in the 3D(NR-NCPS) symmetries, equal to:

$$\begin{cases} \mathbf{d}^{2} = r^{2} - \mathbf{L}.\mathbf{\Theta}^{c} + O\left(\Theta^{c2}\right), \\ \frac{1}{2\mu\mathbf{d}^{2}} = \frac{1}{2\mu r^{2}} + \frac{\mathbf{L}.\mathbf{\Theta}^{c}}{2\mu r^{4}} + O\left(\Theta^{c2}\right), \\ \pi^{2} = p^{2} + \mathbf{L}.\overline{\theta}^{c} + O\left(\overline{\theta}^{c2}\right), \end{cases} \tag{113}$$

and

$$V_{dy}\left(\mathbf{d}\right) = D_{e}\left(1 - \frac{b\exp\left(-\alpha r\right)}{1-\exp\left(-\alpha r\right)}\right)^{2} - V_{0}\frac{\exp\left(-\alpha r\right)}{r} - \mathbf{D_{e}}\left(\frac{\alpha\mathbf{b}\exp\left(-\alpha\mathbf{r}\right)}{\mathbf{r}\left(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right)\right)} + \frac{\left(\alpha\mathbf{b}-\alpha\mathbf{b^{2}}\right)\exp\left(-\mathbf{2}\alpha\mathbf{r}\right)}{\mathbf{r}\left(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right)\right)^{\mathbf{2}}}\right)\mathbf{L}.\Theta^{c}$$
$$-\left(\frac{\alpha\mathbf{V_{0}}}{\mathbf{2}}\frac{\exp\left(-\alpha\mathbf{r}\right)}{\mathbf{r^{2}}} + \frac{\mathbf{V_{0}}}{\mathbf{2}}\frac{\exp\left(-\alpha\mathbf{r}\right)}{\mathbf{r^{3}}} - \mathbf{D_{e}}\frac{\alpha\mathbf{b^{2}}\exp\left(-\mathbf{3}\alpha\mathbf{r}\right)}{\mathbf{r}\left(\mathbf{1}-\exp\left(-\alpha\mathbf{r}\right)\right)^{\mathbf{3}}}\right)\mathbf{L}.\Theta^{c} + O\left(\Theta^{c2}\right). \tag{114}$$

The new couplings $\mathbf{L}.\Theta^{c}$ and $\mathbf{L}.\overline{\theta}^{c}$ are equal to $L_{x}\theta_{12}^{c} + L_{y}\theta_{23}^{c} + L_{z}\theta_{12}^{c}$ and $L_{x}\overline{\theta}_{12}^{c} + L_{y}\overline{\theta}_{23}^{c} + L_{z}\overline{\theta}_{13}^{c}$, respectively. The two non-commutativity parameters ($\theta_{\alpha\beta}^{c}, \overline{\theta}_{\alpha\beta}$) and α_{n} are equal to ($\sum_{n=1}^{2}\alpha_{n}^{2}\theta_{\alpha\beta}^{(n)}, \sum_{n=1}^{2}\alpha_{n}^{2}\overline{\theta}_{\alpha\beta}$) and $m_{n}/\sum_{n} m_{n}$, respectively, the indice ($n = 1$,2) label the particle, and ($\theta_{\alpha\beta}^{(n)}, \overline{\theta}_{\alpha\beta}$) are the parameters of non-commutativity, corresponding to the particle of mass m_{n}. As a result of the topological features of the deformation phase-space, the spontaneously new generated term $V_{eff-nc}^{dy}\left(r,\Theta^{c}\right)$ and the global working Hamiltonian operator $H_{nc}^{dy}(p,x,\Theta^{c},\overline{\theta}^{c})$ of that equal to the modified kinetic energy $\pi(p,\overline{\theta}^{c})/2\mu$ plus the effective potential $V_{eff-nc}^{dy}\left(r,\Theta^{c}\right)$ of the MDF-YP model:

$$\begin{cases} H_{nc}^{dy}\left(p,x,\Theta^{c},\overline{\theta}^{c}\right) = H_{dy}\left(p,x\right) + H_{pert}^{dy}\left(r,\Theta^{c},\overline{\theta}^{c}\right), \\ V_{eff-nc}^{dy}\left(r,\Theta^{c}\right) = V_{eff}^{dy}\left(r\right) + Z_{dy}^{pert}\left(r,\Theta_{c}\right), \end{cases} \tag{115}$$

with

$$H_{pert}^{dy}\left(r,\Theta^{c},\overline{\theta}^{c}\right) = Z_{dy}^{pert}\left(r,\Theta^{c}\right) + \frac{\mathbf{L}.\overline{\theta}^{c}}{2\mu} + O\left(\overline{\theta}^{c2},\Theta^{c2}\right), \tag{116}$$

and

$$Z_{dy}^{pert}(r,\Theta^c) = -\mathbf{D_e}\left(\frac{\alpha\mathbf{b}\exp(-\alpha\mathbf{r})}{\mathbf{r}(\mathbf{1}-\exp(-\alpha\mathbf{r}))} + \frac{(\alpha\mathbf{b}-\alpha\mathbf{b^2})\exp(-\mathbf{2}\alpha\mathbf{r})}{\mathbf{r}(\mathbf{1}-\exp(-\alpha\mathbf{r}))^{\mathbf{2}}}\right)\mathbf{L}.\Theta^c - \left(\frac{\mathbf{V_0}}{\mathbf{2}}\frac{\exp(-\alpha\mathbf{r})}{\mathbf{r^3}} - \frac{\alpha\mathbf{b^2}\mathbf{D_e}\exp(-\mathbf{3}\alpha\mathbf{r})}{\mathbf{r}(\mathbf{1}-\exp(-\alpha\mathbf{r}))^{\mathbf{3}}} - \frac{\mathbf{l}(\mathbf{l}+\mathbf{1})}{\mathbf{2}\mu\mathbf{r^4}}\right)\mathbf{L}.\Theta^c + O\left(\Theta^{c2}\right). \tag{117}$$

The main difference between previously spontaneously Hamiltonian operator and effective potential ($H_{pert}^{dy}(r,\Theta,\overline{\theta})$, $Z_{dy}^{pert}(r,\Theta)$) in Eq. (41) and the new spontaneously generated terms ($H_{pert}^{dy}(r,\Theta^c,\overline{\theta}^c)$, $Z_{dy}^{pert}(r,\Theta^c)$) in Eqs. (115) and (116) appears in the two couplings ($\mathbf{L}.\Theta$and $\mathbf{L}.\overline{\theta}$), since it is possible to move between the two binaries according to the transformation ($\mathbf{L}.\boldsymbol{\Theta}$ and $\mathbf{L}.\overline{\theta}$) $\Leftrightarrow$ ($\mathbf{L}.\boldsymbol{\Theta}^c$ and $\mathbf{L}.\overline{\theta}^c$). As for similarities, the expectations values (F_{dy}^1, F_{dy}^2, F_{dy}^3, F_{dy}^4 and F_{dy}^5) do not change, which means there is a large amount of work that does not require re-completion. Therefore, the physical relationships that express the partial corrections of energy that we saw in the previous Eqs. (71), (74), and (77) will become as follows:

$$\begin{cases} \Delta E_{dy}^{nr-so}(n,D_e,r_e,V_0,\alpha,\Theta^c,\overline{\theta}^c,l,s,m) = \Lambda\left(\Theta^c\langle X\rangle_{(nlm)}^{dy} + \frac{\overline{\theta}}{2\mu}\right) + O\left(\Theta^{c2},\overline{\theta}^{c2}\right), \\ \Delta E_{dy}^{mg}(n,D_e,r_e,V_0,\alpha,\chi^c,\overline{\chi}^c,l,s,m) = \aleph\left(\chi^c\langle X\rangle_{(nlm)}^{dy} + \frac{\overline{\chi}^c}{2\mu}\right)m + O\left(\chi^{c2},\overline{\chi}^{c2}\right), \\ \Delta E_{dy}^{rot}(n,D_e,r_e,V_0,\alpha,\zeta^c,\overline{\zeta}^c,l,s,m) = \left(\zeta^c\langle X\rangle_{(nlm)}^{dy} + \frac{\overline{\zeta}^c}{2\mu}\right)\Omega m + O\left(\zeta^{c2},\overline{\zeta}^{c2}\right). \end{cases} \tag{118}$$

In particular cases when $m_1 = m_2$ such as the homogeneous (H_2, I_2) diatomic molecules the parameters $(\theta_{\alpha\beta}^{(n)},\overline{\theta}_{\alpha\beta})$ will be identified with ordinary non-commutative parameters $(\theta_{\alpha\beta},\overline{\theta}_{\alpha\beta})$. Thus, the parameters ($\Theta$, χ, ζ) and $(\overline{\theta},\overline{\chi},\overline{\zeta})$, which are seen in Eq. (78) are changed to the new non-commutativity parameters:

$$\Lambda^{c2} = \left(\sum_{n=1}^{2}\alpha_n^2\Lambda_{12}^{(n)}\right)^2 + \left(\sum_{n=1}^{2}\alpha_n^2\Lambda_{23}^{(n)}\right)^2 + \left(\sum_{n=1}^{2}\alpha_n^2\Lambda_{13}^{(n)}\right)^2, \tag{119}$$

with Λ^{c2} can be play the roles of the square of NC-(phase-space) parameters ($\Theta^{c2}/\overline{\theta}^{c2}$, $\chi^{c2}/\overline{\chi}^{c2}$ and $\zeta^{c2}/\overline{\zeta}^{c2}$). As mentioned above, in the case of a system of two particles with the same mass $m_1 = m_2$ such as the homogeneous H_2 diatomic molecules:

$$\left(\varpi_{\mu\nu}^{(n)},\overline{\theta}_{\mu\nu}^{(n)}\right) = \left(\varpi_{\mu\nu},\overline{\upsilon}_{\mu\nu}\right). \tag{120}$$

Here $(\varpi_{\mu\nu}^{(n)},\overline{\upsilon}_{\mu\nu}^{(n)})$ can be present both $(\theta_{\mu\nu}^{(n)},\overline{\theta}_{\mu\nu}^{(n)})$, $(\chi_{\mu\nu}^{(n)},\overline{\chi}_{\mu\nu}^{(n)})$ and $(\zeta_{\mu\nu}^{(n)},\overline{\zeta}_{\mu\nu}^{(n)})$. In the end of this section, we can generalize the non-relativistic global energy E_{nc}^{dy} under the MDF-YP taking account that composite systems with different masses are described with different non-commutative parameters for the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) as:

$$E_{nc}^{dy} = Q_a + 2\Lambda_b Q_c - \left(\frac{Q_b Q_c}{\rho^2} + Q_b\rho^2\right) + \langle X\rangle_{(nlm)}^{dy}(n,D_e,r_e,V_0,\alpha)\left(N_p^{sp}\right)^c + \left(N_p^{ph}\right)^c + O\left(\left(S_p^{nc}\right)^{c2},\left(P_p^{nc}\right)^{c2}\right), \tag{121}$$

with $\left(N_p^{sp}\right)^c$ and $\left(N_p^{ph}\right)^c$ are equal to $(\Theta^c\Lambda + (\chi^c\aleph + \zeta^c\Omega)m)$ and $(\overline{\theta}^c(\Lambda/2\mu) + [\overline{\chi}^c(\aleph/2\mu) + \overline{\zeta}^c(\Omega/2\mu)]m)$, respectively.

7. Thermodynamic quantities of the MDF-YP in 3D(NR-NCPS) symmetries

The main goal of this section is to look at the thermodynamic properties (TPs) of the Deng-Fan Yukawa potential and the modified Deng-Fan Yukawa potential models in 3D(NR-QM) and 3D(NR-NCPS) symmetries. Calculating the rotational partition function $Z_{dy}^{nc}\left(n,D_e,r_e,V_0,\alpha,\beta,l,\lambda_{nc}^{dy},S_p^{nc},P_p^{nc}\right)$ is a crucial initial step in achieving this goal since it may be used to determine various thermal parameters such as specific heat capacity, internal energy, entropy, and free energy. A constant temperature T can determine the rotation-vibrational partition function by using direct summation over all potential energy levels [104–109]:

$$Z_{dy}^{nl} = \sum_{n=0}^{\lambda}\exp\left(-\beta E_{nl}^{dy}\right) \Rightarrow Z_{dy}^{nc} = \sum_{n=0}^{\lambda_{nc}^{dy}}\exp\left(-\beta E_{nc}^{dy}\right). \tag{122}$$

Here ($Z_{dy}^{nl}\left(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l\right) \equiv Z_{dy}^{nl}$, $Z_{dy}^{nc}\left(n, D_e, r_e, V_0, \alpha, \beta, l, \lambda_{nc}^{dy}, S_p^{nc}, P_p^{nc}\right) \equiv Z_{dy}^{nc}$) are the rotation-vibrational partition functions of the Deng-Fan Yukawa potential and the MDF-YP models, while and $\left(\lambda, \lambda_{nc}^{dy}\right)$ are the upper bound vibration quantum numbers (the maximum quantum numbers) in 3D(NR-QM) and 3D(NR-NCPS) symmetries, respectively, β equal to $1/K_\beta T$, with K_β is the Boltzmann constant. From the beginning of this section, we assumed that the new rotation-vibrational partition function Z_{dy}^{nc} is dependent on non-commutativity (phase-space) parameters (P_p^{nc}, S_p^{nc}), because the corresponding non-relativistic energy in these symmetries we found is related to these parameters. We obtain the parameter λ_{nc}^{dy} in 3D(NR-NCPS) as a function of corresponding values λ in 3D(NR-QM) as follows:

$$\begin{cases} \lambda = \frac{dE_{nl}^{nr}}{dn}\rfloor_{n=\lambda} = 0 \Rightarrow \lambda = -\gamma + \sqrt{|Q_c|}, \\ \frac{dE_{nc}^{dy}}{dn}\rfloor_{n=\lambda_{nc}^{dy}} = 0 \Rightarrow \lambda_{nc}^{dy} = -\gamma + \sqrt{|Q_c|} + \lambda_{per}^{dy}, \end{cases}, \tag{123}$$

with

$$\lambda_{per}^{dy} = \frac{d}{dn}\left(\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}\left(\overline{\theta}, \overline{\chi}, \overline{\zeta}\right)\right]\right)\rfloor_{n=\lambda_{per}^{dy}}. \tag{124}$$

We saw in the third paragraph that the total energy of a non-relativistic physical system E_{nc}^{dy} (Eq. (78)) in 3D(NR-NCPS) symmetries, under the influence of the MDF-YP models can be written for the case $l \neq 0$ as follows

$$E_{nc}^{dy} = E_{nl}^{dy} + \Delta E_{nl}^{dy}. \tag{125}$$

To calculate The TPs of the Deng-Fan Yukawa potential and the MDF-YP models in 3D(NR-QM) and 3D(NR-NCPS) symmetries, the rotation-vibrational energy eigenvalues E_{nl}^{dy} and the corrected energyΔE_{nl}^{dy} in 3D(NR-QM) and 3D(NR-NCPS) symmetries are expressed in a compact form as

$$\begin{cases} E_{nl}^{dy} = Q_a - 2Q_bQ_c - \left(\frac{Q_bQ_c^2}{\rho^2} + Q_b\rho^2\right), \\ \Delta E_{nl}^{dy} = \langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}. \end{cases} \tag{126}$$

In 3D(NR-NCPS) symmetries, at high temperatures in the classical limit, the modified rotation-vibrational partition function Z_{dy}^{nc} of the MDF-YP models can be represented by an integral:

$$Z_{dy}^{nl} = \int_0^{\gamma+\lambda} \exp\left(-\beta E_{nl}^{dy}\left(\rho\right)\right)d\rho \Longrightarrow Z_{dy}^{nc} = \int_0^{\gamma+\lambda_{nc}^{dy}} \exp\left(-\beta E_{nc}^{dy}\left(\rho\right)\right)d\rho. \tag{127}$$

Here ρ is equal to $(n+\gamma)$ in the classical limit. After a straightforward calculations we find the rotation-vibrational partition function Z_{dy}^{nl} of the MDF-YP models in 3D(NR-QM) symmetries as:

$$Z_{dy}^{nl} = \int_0^{\gamma+\lambda} \exp\left(-\beta Q_a + 2\beta Q_bQ_c + \beta\left(\frac{Q_bQ_c^2}{\rho^2} + Q_b\rho^2\right)\right)d\rho. \tag{128}$$

Through our observation of energy Eqs. (20) in Ref. [11] and corresponding Eq. (22) in Ref. [109] that is has the form $(K_1 - 2K_2K_3 - ([K_2K_3^2/\rho^2] + K_2\rho^2))$, it is possible to move between them from the following deplacement:

$$\begin{cases} K_1 \Longleftrightarrow Q_a, \\ K_2 \Longleftrightarrow Q_b, \\ K_3 \Longleftrightarrow Q_c. \end{cases} \tag{129}$$

This mechanism allows us to find the partition function $Z_{dy}^{nl}\left(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l\right) \equiv Z_{dy}^{nl}$ of the Deng-Fan Yukawa potential model of Eq. (20) in 3D(NR-QM) symmetries as:

$$Z_{dy}^{nl} = Z_{dy}^{nl(1)} + Z_{dy}^{nl(2)}, \tag{130}$$

with

$$\begin{cases} Z_{dy}^{nl(1)} = \frac{\exp(\beta[2Q_bQ_b - Q_a])}{4\sqrt{-Q_b\beta}}\sqrt{\pi}\exp\left(\beta Q_bQ_c\right)\text{erf}\left(\sqrt{-Q_b\beta}\lambda + \frac{Q_c\sqrt{-Q_b\beta}}{\lambda}\right), \\ Z_{dy}^{nl(2)} = \frac{\exp(\beta(2Q_bQ_b - Q_a))}{4\sqrt{-Q_b\beta}}\sqrt{\pi}\exp\left(-\beta Q_bQ_c\right)\text{erf}\left(\sqrt{-Q_b\beta}\lambda - \frac{Q_c\sqrt{-Q_b\beta}}{\lambda}\right). \end{cases} \tag{131}$$

Here erf $i\left(u\right)$ is the imaginary error function. Considering that the additive part of the energy value $\Delta E_{nl}^{dy}(n, D_e, r_e, V_0, \alpha, S_p^{nc}, P_p^{nc}) \equiv \Delta E_{nl}^{dy}$ is small compared to the main term E_{nr}^{dy}, we can make the following approximate:

$$\exp\left(-\beta E_{nc}^{dy}\right) = \exp\left(-\beta E_{nr}^{dy}\right) - \beta \Delta E_{nl}^{dy} \exp\left(-\beta E_{nr}^{dy}\right), \tag{132}$$

which gives:

$$Z_{dy}^{nc} = \int_0^{\lambda_{nc}^{dy}} \exp\left(-\beta E_{nr}^{dy}\right)\left(1 - \beta \Delta E_{nc}^{dy}\right) d\rho. \tag{133}$$

Considering the previous physical considerations, we roughly accept the terms that are proportional with infinitesimal NC-(phase-space) parameters $\left(S_p^{nc}, P_p^{nc}\right)$ in the first place only. Thus, the modified rotation-vibrational partition function Z_{dy}^{nc} of the MDF-YP model of Eq. (78) in 3D(NR-NCPS) symmetries, can be written approximatively as:

$$Z_{dy}^{nc} = Z_{dy}^{nl} - \beta\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}\right] Z_{dy}^{nr}. \tag{134}$$

Substituting Eq. (129) into Eq. (133), we have the modified rotation-vibrational partition function Z_{dy}^{nc} of the MDF-YP model as

$$Z_{dy}^{nc} = Z_{dy}^{nl(1)} + Z_{dy}^{nl(2)} - \beta\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}\right]\left(Z_{dy}^{nl(1)} + Z_{dy}^{nl(2)}\right). \tag{135}$$

Using the modified rotation-vibrational partition function Z_{dy}^{nc} in Eq. (133) of the MDF-YP model for energy equation (78), we will see the effect of the deformation in phase-space on thermodynamic values such as modified mean energy U_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), modified free energy F_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) and modified entropy S_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}). Let's start with a study of modified mean energy U_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) which is the quantity of energy required to prepare or improve the system in its internal condition. First, the effect of the deformation of phase-space on mean energy U_{dy}^{nl} (n, D_e, r_e, V_0, α, β, λ, l) for Deng-Fan Yukawa potential model is determined by applying the following formula:

$$\Delta U_{nc}^{dy} \equiv U_{nc}^{dy} - U_{dy}^{nl} = -\frac{\partial}{\partial\beta}\left[\ln Z_{dy}^{nc} - \ln Z_{dy}^{nl}\right]. \tag{136}$$

The above formula, give the effect of phase-space deformations with the MDF-YP models influence on mean energy in 3D(NR-NCPS) symmetries, as follows:

$$\Delta U_{nc}^{dy} = \frac{\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}}{1 - \beta\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}\right]}. \tag{137}$$

Thus, for the MDF-YP models, the new mean energy U_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) is equal to the corresponding values U_{dy}^{nl} (n, D_e, r_e, V_0, α, β, λ, l)$\equiv$ U_{dy}^{nl} for the Deng-Fan Yukawa potential model in 3D(NR-QM) plus the effect of the deformation of phase-space on it ΔU_{nc}^{dy} as follows:

$$U_{nc}^{dy} = U_{dy}^{nl} + \frac{\langle X\rangle_{(nlm)}^{dy}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] + N_p^{ph}}{1 - \langle X\rangle_{(nlm)}^{dy}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] - N_p^{ph}}. \tag{138}$$

A preform calculation gives the mean energy U_{dy}^{nl} for the Deng-Fan Yukawa potential model in 3D(NR-QM) symmetries as:

$$U_{dy}^{nl} = -\frac{1}{2}\frac{\Lambda_1 + \Lambda_2 + \Lambda_3}{\sqrt{\pi}\beta\lambda\left(\Upsilon^1 + \Upsilon^2\right)}, \tag{139}$$

with

$$\begin{cases} \Upsilon^1 = \exp\left(2\beta Q_b Q_c\right) \operatorname{erf}\left(\Lambda\right), \\ \Upsilon^2 = \exp\left(-2\beta Q_b Q_c\right) \operatorname{erf}\left(\Lambda\right), \\ \Lambda^- = \frac{\sqrt{-\beta Q_b}\left(\lambda^2 - Q_c\right)}{\lambda}, \end{cases} \tag{140}$$

and

$$\Lambda_1 = 8\sqrt{\pi}\beta\left(2Q_bQ_c\right)\exp\left(2\beta Q_bQ_c\right)\text{erf}\left(\Lambda^-\right) - 2\sqrt{\pi}\beta\lambda Q_a\exp\left(2\beta Q_bQ_c\right)\text{erf}\left(\sqrt{-\beta\chi_1}\left(\lambda^2+Q_c\right)\right), \tag{141}$$

$$\Lambda_2 = -2\sqrt{\pi}\beta\lambda\exp\left(-2\beta Q_bQ_c\right)\text{erf}\left(\Lambda^-\right) + 2\sqrt{-\beta Q_b}\exp\left(\beta Q_b\left(\frac{4Q_c\lambda^2+\lambda^4+Q_c^2}{\lambda^2}\right)\right)\left(\lambda^2+1\right), \tag{142}$$

$$\Lambda_3 = Q_c + 2\sqrt{-\beta Q_b}\exp\left(\beta Q_b\left(\frac{-4Q_c\lambda^2+\lambda^4+Q_c^2}{\lambda^2}\right)\right)\left(\lambda^2-Q_c\right),$$
$$-\sqrt{\pi}\lambda\exp\left(2\beta Q_bQ_c\right)\text{erf}\left(\sqrt{-\beta Q_b}\left(\lambda^2+Q_3\right)\right) - \sqrt{\pi}\lambda\exp\left(-2\beta Q_bQ_c\right)\text{erf}\left(\Lambda^-\right). \tag{143}$$

Now let's get to the effect of the deformation of phase-space on the free energy $F_{dy}^{nl}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l)$ of the MDF-YP models which obtains by applying:

$$\Delta F_{nl}^{dy} \equiv F_{nc}^{dy} - F_{dy}^{nl} = -\frac{1}{\beta}\ln Z_{dy}^{nc} - \left(-\frac{1}{\beta}\partial\ln Z_{dy}^{nr}\right). \tag{144}$$

The effect of the deformation of phase-space on the free energy $\Delta F_{nc}^{dy}\,(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc})$ of the MDF-YP models as:

$$\Delta F_{nl}^{dy} \equiv -\frac{1}{\beta}\ln\left[1-\beta\left(\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)+N_p^{ph}\right)\right]. \tag{145}$$

The new free energy (also known by the Helmholtz energy) $F_{nc}^{dy}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc})$ of the MDF-YP models in 3D(NR-NCPS) regimes is equal to the corresponding values $F_{dy}^{nl}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l)$, in 3D(NR-QM) regimes, plus the impact of phase-space deformation on it, $\Delta F_{nl}^{dy}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc})$ as follows:

$$F_{nc}^{dy} = F_{dy}^{nl} - \frac{1}{\beta}\ln\left[1-\beta\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)+N_p^{ph}\right]\right]. \tag{146}$$

On the other hand, the Helmholtz energy $F_{dy}^{nl}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l)$ in 3D(NR-QM) regimes can be derived by applying the following expression:

$$F_{dy}^{nl} = -\frac{1}{\beta}\ln\left[\frac{\exp\left(\beta\left(2Q_bQ_b-Q_a\right)\right)}{4\sqrt{-Q_b\beta}}\sqrt{\pi}\left[\Upsilon^1+\Upsilon^2\right]\right]. \tag{147}$$

The effect of phase-space deformation on the specific heat capacity $\Delta C_{nc}^{dy}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) \equiv \Delta C_{nc}^{dy}$ of the MDF-YP models is equal to the difference between their values C_{nc}^{dy} in 3D(NR-NCPS) regimes and the corresponding values $C_{dy}^{nl}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l)$, in 3D(NR-QM) regimes:

$$\Delta C_{nc}^{dy} \equiv C_{nc}^{dy} - C_{dy}^{nl} = -k\beta^2\frac{\partial\Delta U_{nc}^{dy}}{\partial\beta}. \tag{148}$$

The impact of phase-space deformation on the free energy ΔC_{nc}^{dy} of the MDF-YP models may be determined simply as follows:

$$\Delta C_{nc}^{dy} = -k\beta^2\frac{\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)+N_p^{ph}\right]^2}{\exp\left(2\beta\left[\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)+N_p^{ph}\right]\right)}. \tag{149}$$

Within the scope of our proposed approximations, this impact may be ignored since it is limited to the first order just for the values (S_p^{nc}, P_p^{nc}). In the last part, we examine how the phase-space deformation affects the entropy $S_{nc}^{dy}\,(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l)$ under the MDF-YP models. This information can be obtained by applying:

$$\Delta S_{nc}^{dy} \equiv S_{nc}^{dy} - S_{dy}^{nl} = k\beta^2\frac{\partial\Delta F_{nc}^{dy}}{\partial\beta}. \tag{150}$$

The following straightforward calculation shows how the phase-space deformation influences the entropy of the MDF-YP models:

$$\Delta S_{nc}^{dy} \equiv k\beta\frac{\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)+N_p^{ph}}{1-\beta\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda+\left(\chi\aleph+\zeta\Omega\right)m\right)-\beta N_p^{ph}}. \tag{151}$$

Therefore, in 3D(NR-NCPS) regimes, the modified entropy S_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) of the MDF-YP models is equal to the corresponding values S_{dy}^{nl} (n, D_e, r_e, V_0, α, β, λ, l) in 3D(NR-QM) plus the impact of the phase-space deformation ΔS_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) on the Deng-Fan Yukawa potential model in the following ways:

$$S_{nc}^{dy} = S_{dy}^{nl} + \frac{k\beta\left[\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right)\langle X\rangle_{(nlm)}^{dy} + N_p^{ph}\right]}{1 - \beta\langle X\rangle_{(nlm)}^{dy}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) - \beta N_p^{ph}}. \tag{152}$$

Here the entropy S_{dy}^{nl} (n, D_e, r_e, V_0, α, β, λ, l)$\equiv$ S_{dy}^{nl} of the Deng-Fan Yukawa potential in 3D(NR-QM) symmetry derived by applying the formula:

$$S_{dy}^{nl} = k\ln Z_{dy}^{nr} - k\beta\frac{\partial\ln Z_{dy}^{nr}}{\partial\beta}. \tag{153}$$

After a preform calculations we obtain the entropy S_{dy}^{nl} of the Deng-Fan Yukawa potential in 3D(NR-QM) as follows:

$$S_{dy}^{nl}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = -\frac{1}{2}\frac{\sum_{\epsilon=1}^{5}K_\epsilon}{\sqrt{\pi}\beta\lambda\left(\Upsilon^1 + \Upsilon^2\right)}, \tag{154}$$

with

$$K_1 = -2\ln\left(\exp\left(-\beta\left(-2Q_bQ_c + Q_a\right)\right)\operatorname{erf}\left(\Lambda^+\right)\right) + \exp\left(-2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^-\right), \tag{155}$$

$$K_2 = 2\beta\lambda\sqrt{\pi}\left[4\beta Q_bQ_c + Q_a\exp\left(2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^+\right)\right] + \exp\left(2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^+\right), \tag{156}$$

$$K_3 = 2\sqrt{\pi}\beta\lambda Q_a\exp\left(-2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^-\right) - 2\sqrt{-\beta Q_b}\exp\left(\beta Q_b\left(\frac{-4Q_c\lambda^2 + \lambda^4 + Q_c^2}{\lambda^2}\right)\right)\left(\lambda^2 + Q_c\right), \tag{157}$$

$$K_4 = -\sqrt{-\beta Q_b}\operatorname{erf}\left(\beta Q_b\left(\frac{-4Q_c\lambda^2 + \lambda^4 + Q_c^2}{\lambda^2}\right)\right)\left(\lambda^2 - 2Q_c\right) + 4\ln(2) - \ln\pi, \tag{158}$$

$$K_5 = \sqrt{\pi}\lambda\exp\left(2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^+\right) + \sqrt{\pi}\lambda\exp\left(-2\beta Q_bQ_c\right)\operatorname{erf}\left(\Lambda^-\right). \tag{159}$$

Here Λ^+ equal to $\sqrt{-\beta Q_b}\left(\lambda^2 + Q_C\right)/\lambda$. When the deformation of phase-space effect vanish when the simultaneous limits $\left(S_p^{nc}, P_p^{nc}\right) \to (0,0)$ is satisfied, the additive thermodynamic parts ΔZ_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), ΔU_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), ΔF_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), ΔS_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) and ΔC_{nc}^{dy} (n, D_e, r_e, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) naturally also vanish,

$$\begin{cases} \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} \Delta Z_{nc}^{dy}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = 0, \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} \Delta U_{nc}^{dy}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = 0, \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} \Delta F_{nc}^{dy}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = 0, \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} \Delta S_{nc}^{dy}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = 0, \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} \Delta C_{nc}^{dy}(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l, S_p^{nc}, P_p^{nc}) = 0. \end{cases} \tag{160}$$

Thus, all physical values in 3D(NR-NCPS) regimes, in the presence of deformation $\left(\Theta/\overline{\theta}, \chi/\overline{\chi}, \zeta/\overline{\zeta}\right) \neq (0,0,0)$, will be reverted to their initial values in 3D-(NR-QM) symmetries:

$$\begin{cases} \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} Z_{dy}^{nc}(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc}) = Z_{nl}^{dy}\left(D_e, r_e, V_0, \alpha, \beta, \lambda, l\right), \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} U_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc}) = U_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, \lambda, l), \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} F_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc}) = F_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, \lambda, l), \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} S_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc}) = S_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, \lambda, l), \\ \lim\limits_{\left(S_p^{nc}, P_p^{nc}\right)\to(0,0)} C_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, l, S_p^{nc}, P_p^{nc}) = C_{nl}^{dy}(D_e, r_e, V_0, \alpha, \beta, \lambda, l), \end{cases} \tag{161}$$

In the end, we will examine the TPs of the MDF-YP models which we have seen in Eqs. (134), (136), (144), (148) and (150) in 3D(NR-NCPS) symmetries. By suitable adjustment of the potential parameters of the Deng-Fan Yukawa potential models, we are now in the process of treating into 3D(NR-NCPS) regime:

(1) If the he dissociation energy D_e reduces to zero,Eqs. (134), (136), (144), (148) and (150) gives the impact of phase-space deformation on the induced partition function ΔZ_{nc}^{my} (n, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), the induced mean energy ΔU_{nc}^{my} (n, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), the induced free energy ΔF_{nc}^{my} (n, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), the induced entropy ΔS_{nc}^{my} (n, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}), and the induced specific heat capacity ΔC_{nc}^{my} (n, V_0, α, β, λ, l, S_p^{nc}, P_p^{nc}) for the modified Yukawa potential model as follows:

$$\begin{cases} \lim\limits_{D_e\to 0}\Delta Z_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta Z_{nl}^{my}(V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta U_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta U_{nl}^{my}(V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta F_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta F_{nl}^{my}(V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta S_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta S_{nl}^{my}(V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta C_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta C_{nl}^{my}(V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}). \end{cases} \tag{162}$$

with

$$\Delta Z_{nl}^{my} = -\beta\left[\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right)\langle X\rangle_{(nlm)}^{my} + N_p^{ph}\right]\left(\lim_{D_e\to 0} Z_{dy}^{nr}\left(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l\right)\right), \tag{163}$$

$$\Delta U_{nc}^{my} = \frac{\langle X\rangle_{(nlm)}^{my}\left[\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right] + N_p^{ph}}{1-\beta\langle X\rangle_{(nlm)}^{my}\left[\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right] - \beta N_p^{ph}}, \tag{164}$$

$$\Delta F_{nl}^{my} \equiv -\frac{1}{\beta}\ln\left[1-\beta\langle X\rangle_{(nlm)}^{my}\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right) - \beta N_p^{ph}\right], \tag{165}$$

$$\Delta C_{nl}^{my} = -k\beta^2\frac{\left[\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right)\langle X\rangle_{(nlm)}^{my} + N_p^{ph}\right]^2}{\exp\left(2\beta\left[\langle X\rangle_{(nlm)}^{my}\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right) + N_p^{ph}\right]\right)}, \tag{166}$$

and

$$\Delta S_{nc}^{my} \equiv k\beta\frac{\langle X\rangle_{(nlm)}^{my}\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right) + N_p^{ph}}{1-\beta\langle X\rangle_{(nlm)}^{my}\left(\Theta\Lambda + (\chi\aleph+\zeta\Omega)\,m\right) - \beta N_p^{ph}}. \tag{167}$$

while the non-relativistic expectations values $\langle X\rangle_{(nlm)}^{my}$ of the modified Yukawa potential model was determined in Eq. (87).

(2) If the potential parameter V_0 reduces to zero, Eqs. (134), (136), (144), (148) and (150) gives the impact of phase-space deformation on the induced partition function $\Delta Z_{nc}^{df}(n,D_e,r_e,\alpha,\beta,l,S_p^{nc},P_p^{nc})$, the induced mean energy $\Delta U_{nc}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc})$, the induced free energy $\Delta F_{nc}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc})$, the induced entropy ΔS_{nc}^{df} (n, D_e, r_e, α, β, λ, l, S_p^{nc}, P_p^{nc}), and the induced specific heat capacity ΔC_{nc}^{df} (n, D_e, r_e, α, β, λ, l, S_p^{nc}, P_p^{nc}) for the modified Deng-Fan potential model as follows:

$$\begin{cases} \lim\limits_{D_e\to 0}\Delta Z_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta Z_{nl}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta U_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta U_{nl}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta F_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta F_{nl}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta S_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta S_{nl}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}), \\ \lim\limits_{D_e\to 0}\Delta C_{nl}^{dy}(n,D_e,r_e,V_0,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}) = \Delta C_{nl}^{df}(n,D_e,r_e,\alpha,\beta,\lambda,l,S_p^{nc},P_p^{nc}). \end{cases} \tag{168}$$

with

$$\Delta Z_{nl}^{df} = -\beta\left[\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right)\langle X\rangle_{(nlm)}^{df} + N_p^{ph}\left(\overline{\theta},\overline{\chi},\overline{\zeta}\right)\right]\left(\lim_{D_e\to 0} Z_{dy}^{nr}\left(n, D_e, r_e, V_0, \alpha, \beta, \lambda, l\right)\right), \tag{169}$$

$$\Delta U_{nc}^{my} = \frac{\langle X\rangle_{(nlm)}^{my}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] + N_p^{ph}}{1 - \beta\langle X\rangle_{(nlm)}^{my}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] - \beta N_p^{ph}}, \tag{170}$$

$$\Delta F_{nl}^{my} \equiv -\frac{1}{\beta}\ln\left[1 - \beta\langle X\rangle_{(nlm)}^{df}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m + N_p^{ph}\right)\right], \tag{171}$$

$$\Delta C_{nl}^{my} = -\frac{k\beta^2\left[\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right)\langle X\rangle_{(nlm)}^{df} + N_p^{ph}\right]^2}{\exp\left(2\beta\left[\langle X\rangle_{(nlm)}^{df}\left(\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right) + N_p^{ph}\right]\right)}, \tag{172}$$

and

$$\Delta S_{nc}^{df} \equiv k\beta\frac{\langle X\rangle_{(nlm)}^{df}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] + N_p^{ph}}{1 - \beta\langle X\rangle_{(nlm)}^{df}\left[\Theta\Lambda + \left(\chi\aleph + \zeta\Omega\right)m\right] - \beta N_p^{ph}}, \tag{173}$$

while the non-relativistic expectations values $\langle X\rangle_{(nlm)}^{my}$ of the modified Yukawa potential model was determined in Eq. (90).

8. Conclusion

In this research study, we conducted an in-depth study of the 3D(NR-DSE) under the influence of the MDF-YP model within the framework of 3D(NR-NCPS) principles which we discussed in detail in the general introduction to our article. We have used the GBSM and conventional perturbation theory in 3D(NR-NCPS) symmetries. We obtained the total energy values E_{nc}^{dy} (See Eq. (78)). Where we discovered that it consists of the fundamental term E_{nl}^{dy} (Eq. (20)) resulting from a contribution of the Deng-Fan Yukawa potential model plus all corrections that produced from perturbed spin-orbital Hamiltonian, perturbed modified Zeeman Hamiltonian and perturbed rotational Hamiltonian operator ($H_{so}^{dy}\left(r,\Theta,\overline{\theta}\right)$, $H_{mg}^{dy}\left(r,\chi,\overline{\chi}\right)$ and $H_{rot}^{dy}\left(r,\chi,\overline{\chi}\right)$) (see Eqs. (58), (66), and (68)).The corrected non-relativistic energy eigenvalues seem to be influenced by the quantum numbers (n, j, l, s and m), the mixed potential depths (D_e, r_e, V_0), the screening parameter α, and the non-commutativity (phase-space) parameters ($S_{p\_}^{nc}$, P_p^{nc}). We have calculated the spin-averaged mass spectra M_{nc}^{dy} of the heavy mesons charmonium $c\bar{c}$ and bottomonium $b\bar{b}$ under the MDF-YP model in 3D(NR-QM) and 3D(NR-NCPS) symmetries (See Eq. (100)). The energy eigensolutions E_{nc}^{dy} for the HEDMs (CO, HCl, LiH); the NMHs (ScH, TiH, VH, CrH); the TML(CuLi); the TMC (TiC, NiC); the TMN (ScN) and the TMF (ScF) (See Eq. (120)). We have also calculated the thermodynamic quantities of the Deng-Fan Yukawa potential model in 3D(NR-QM) symmetries (the partition function Z_{dy}^{nl}, the mean energy U_{dy}^{nl}, the free energy F_{dy}^{nl}, and the entropy S_{dy}^{nl}, (See Eqs. (129), (138), (146) and (133), respectively). The impact of phase-space on thermodynamic quantities (the induced partition function ΔZ_{nc}^{dy}, the induced mean energy ΔU_{nc}^{dy}, the induced free energy ΔF_{nc}^{dy}, the induced specific heat capacity ΔC_{nc}^{dy} and the induced entropy ΔS_{nc}^{dy},(See Eqs. (134), (136), (144), (148), and (150), respectively) have also been examined in relation to the phase-space deformation. It has been demonstrated that (the modified partition function Z_{nc}^{dy}, the modified mean energy U_{nc}^{dy}, the modified free energy F_{nc}^{dy}, the modified entropy S_{nc}^{dy}, and the modified specific heat capacity C_{nc}^{dy}, for the MDF-YP model, are equivalent to their values in 3D(NR-QM) symmetry (the partition function Z_{dy}^{nl}, the mean energy U_{dy}^{nl}, the free energy F_{dy}^{nl}, the entropy S_{dy}^{nl}, and the specific heat capacity C_{dy}^{nl} (See Eqs. (129), (138), (146) and (153), respectively)) plus the effect of the phase-space deformation ((ΔZ_{nl}^{dy}, ΔU_{nl}^{dy}, ΔF_{nl}^{dy}, ΔS_{nl}^{dy}, and ΔC_{nl}^{dy}), (See Eqs. (134), (136), (144), (148) and (150), respectively)). We have re-treated the special cases related to energy in the extended phase-space framework to include the overall thermodynamic properties as particular cases in the extended phase-space through appropriate substitutions for each case for the modified Deng-Fan potential model and the modified Yukawa potential model. We establish the energy equations for the non-relativistic SE in 3D(NR-QM) symmetries for the simultaneous limits (S_p^{nc} and P_p^{nc}) $\to$ (0 and 0), obtained in the main Ref. [11], under a Deng-Fan Yukawa potential model.

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