

World personal income distribution evolution measured by purchasing power parity exchange rates

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Received 9 September 2024; accepted 19 May 2025

The evolution of global income distribution from 1988 to 2018 is analyzed using purchasing power parity exchange rates and well-established statistical distributions. This research proposes the use of two separate distributions to more accurately represent the overall data, rather than relying on a single distribution. The global income distribution was fitted to log-normal and gamma functions, which are standard tools in econophysics. Despite limitations in data completeness during the early years, the available information covered the vast majority of the world's population. Probability density function (PDF) curves enabled the identification of key peaks in the distribution, while complementary cumulative distribution function (CCDF) curves highlighted general trends in inequality. Initially, the global income distribution exhibited a bimodal pattern; however, the growth of middle classes in highly populated countries such as China and India has driven the transition to a unimodal distribution in recent years. While single-function fits with gamma or log-normal distributions provided reasonable accuracy, the bimodal approach constructed as a sum of log-normal distributions yielded near-perfect fits.

Keywords: Econophysics; income distribution; gamma function; log-normal distribution.

DOI: <https://doi.org/10.31349/RevMexFis.71.061701>

1. Introduction

The increasing interconnectedness of the global economy over the past four decades has sparked intense interest in understanding how income is distributed across the world's population. This interest stems from the growing realization that globalization has often been accompanied by rising income and wealth inequalities [1,2]. Inasmuch as both income and wealth distributions, and their respective inequalities, go to the heart of any society's viewpoints on issues regarding egalitarianism and social opportunity, the relationship between globalization processes and its impact on the income inequality has attracted a lot of recent interest, becoming in fact an important research topic among economists and econophysicists [3-5]. In particular, the characterization of income distributions yield critical information for determining richness, the gap between rich and poor and societies' well being rates at any *gross domestic product* (GDP) level [6].

Research aimed at determining the overall behaviour of income distributions were initially focused on some countries and regions and within limited time period intervals. More recently such an approach, although much more detailed than earlier studies, still focuses on particular countries and regions, which renders such studies basically fragmented if one considers the worldwide income distribution [5] and,

therefore, do not present a general scenario of its global situation. This is a clearly desirable goal in order to advance our understanding of the income distribution dynamic evolution at the world scale.

The aim of establishing a global income distribution must, however, rely upon the combination of as many national household surveys as possible because there is no global household survey of individual incomes. To include all countries is not an easy task, as discussed by Milanovic [7,8], and more recently by Anand and Segal [9], because most of the first works focusing on the world income distribution are studies of international inequality in the sense that they calculated what would be inequality in the world if it were populated by representative individuals from all countries, that is, by people having the mean income of their countries.

More accurate representations of the world income distributions were constructed afterwards from assembling income distributions of countries obtained by using income surveys or tax data. As mentioned by Milanovic [8], global inequality is a relatively recent topic because in order to calculate it one needs to have data on national income distributions for most of the countries in the world, or at least for most of the populous countries. Only from the early to mid 1980s that such data became available for China, the Soviet Union and its

constituent republics, as well as large parts of Africa. Nevertheless, the problem of data homogeneity, which ensures that variables are defined the same way as much as possible, has been a difficult one in this area since its inception.

The world income, or expenditure distribution, for 1988 and 1993 was calculated in Ref. [7] for individuals based entirely on household surveys from 91 countries adjusted for differences in *purchasing power parity* (PPP) between countries covering about 84% of world population and 93% of world GDP. In similar works [10,11] income shares for a number of countries were approximated using income shares of “similar” countries. More recently, making use of household income data from more than 130 countries the evolution of the global income distribution between 2008 and 2013 after the financial crisis was analyzed [12]. For a comprehensive review of many recent aspects of the global distribution of income, including conceptual and methodological issues, inequality and global poverty, we refer to the works of Milanovic [8,13-17] and Anand and Segal [9,18-21].

This paper aims at fitting the global income distribution data over several years and studying its evolution. Here we follow the tradition of economists, physicists and mathematicians who have sought to characterize the distribution of income in countries by a mixture of known statistical distributions [5]. Our approach here is to try to characterize the changes in time of the individual income distribution in the world as a whole by means of known statistical distributions with the smallest possible number of parameters.

The ultimate aim of studies on income and wealth distribution must be to reveal the inner dynamics of both quantities by expressing them in terms of time evolving differential equations [5]. So, the ultimate aim must be to identify the mechanisms at work so that some further theoretical work clarifies and enhances our understanding of what we observe [3]. However, income distribution is a subject that was unfortunately very much neglected by mainstream academic economics for a very long time [22], and whose revival basically happened on the onset of the 21st century [5], so the present research level of this subject still very much remains in the stage of data collecting and analyzing in order to see which basic conclusions can be reached from the data in order to try to point out possible future theoretical endeavours. This is particularly true of global income distribution, which means that the present study is very much focused on this initial research stage.

The plan of the paper is as follows. Section 2 is devoted to briefly review some models of wealth and income distributions used by economists, econophysicists and other scientists that will be employed in the present approach. The exponential-like distributions such as the gamma and log-normal distributions. Since our data are at household-level (micro) data, then in Sec. 3 we briefly present the limitations of the databases we employed in terms of their sources, standardization, drawbacks and advantages, together with the convenience of PPP to compare overall consumption and income between nations. In Sec. 4 we present our results of

world income distribution between 1988 and 2018 measured by PPP in US dollars. Section 5 is devoted to our conclusions.

2. Modeling income and wealth distributions used in econophysics

It is fair to state that at present there is a consensus among most, if not all, researchers devoted to the income distribution problem that the richest stratum of a country income distribution, that is, its upper end segment, is well represented by a power law as Vilfredo Pareto argued over a century ago [23]. However, the distributive characterization of the not so rich still remains an open problem. Different authors proposed different fitting functions to characterize the income distribution of the vast majority of populations, but until the turn of the 21st century little more has been done than trying different functional fits in relatively limited number of countries or group of countries [4,5].

Among the early attempts at different functional fits one should recall the work of Robert Gibrat, who in 1931 had already indicated that the Pareto law is only valid for the high income range, whereas for the small and middle income ranges he suggested the log-normal probability density as a better descriptor. He also proposed a law of proportionate effect, which states that a small change in a quantity is independent of the quantity itself [24].

An important, and much more recent, work in this respect was the analysis of the income distribution data of the USA as studied by Silva and Yakovenko [25]. It revealed the co-existence of two social classes as far as functional fitting is concerned: the large majority of the population is characterized by a quasi-exponential distribution, and the very small upper income segment exhibits the Pareto power-law distribution with characteristic fat tails. They argued that there is a similar-to-physics energy conservation law such that in the income distribution problem translates itself as conservation of money. This means that the middle and lower income populations of the USA are described by an exponential function whose interpretation is of being a Boltzmann-Gibbs distribution which entails such a conservative money quantity.

Silva and Yakovenko also considered currency transactions as being equivalent to elastic molecular collisions in a gas particle, where in principle all the conserved energy, in this case money, would be transferred from one particle, or agent, to another in an one-to-one interaction, or transaction, without money loss [26]. The income distribution data of Mexico [27], the European Union [28], and more than 60 countries [29] also present similar two-classes structure.

Chakrabarti and collaborators [4,5,30] extended this kinetic collision model to include savings, which then better reflects real economic transactions, yielding, in the case of a constant saving fraction for all agents, a stationary distribution very similar to the gamma function.

In general, the bulk of the lower distribution stratum of both income and wealth can be fitted by exponential, log-

normal and gamma distributions. Nevertheless, contrary to the lower regions which remain basically unchanged for both income and wealth, apart from the different functional fits, the Pareto tail slope exhibits changes in time, a behaviour that could be possibly explained by the complex processes of creation and destruction of money through investments, credit, financial derivatives, big stock market crisis, etc, features which are much more clearly related to the Pareto tail because these processes are basically from where the rich people extract their income and wealth [5].

2.1. Distribution functions

The *cumulative distribution function* (CDF) is defined as follows,

$$F(m) = \int_0^m P(m') dm', \quad (1)$$

where P signifies the *probability distribution function* (PDF), also known as *probability density*. In the present context m represents monetary value. The complement of Eq. (1) defines the *complementary cumulative distribution function* (CCDF), which may be written as below [5],

$$\bar{F}(m) = \int_m^\infty P(m') dm' = 1 - F(m). \quad (2)$$

This is a very useful quantity to study income distribution because it provides valuable insights on the data it represents by offering better visualization of tail behaviour, which in turn highlights rare events given by extreme values in the dataset, that is, far from the mean. In addition, several CCDFs plots provide helpful comparisons on how the tails behave, allowing the assessment of the heavier or lighter ones. Discussing the income distribution by means of the CCDF provides a meaningful way to comprehend the probability of values greater than or equal to a given threshold m , facilitating a deeper understanding of the data's behaviour and tail characteristicsⁱ.

2.2. Log-normal distribution

This is basically a normal function whose independent variable x scales as $\ln x$. That is, the log-normal distribution is a normal one of the logarithm of x [5,33]. So, the probability density of the normal function scaled that way may be written as,

$$N(\ln x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{(\ln x - \mu)^2}{2\sigma^2} \right], \quad (3)$$

where the parameters μ and σ are respectively the mean value of the logarithmic variable and its variance, that is, $\mu = \langle \ln x \rangle$ and $\sigma = \langle (\ln x - \mu)^2 \rangle$. A change of variables produces,

$$N(\ln x) d(\ln x) = \left[\frac{N(\ln x)}{x} \right] dx. \quad (4)$$

For equal probabilities under the normal and log-normal densities, incremental areas should also be equal, that is, $N(\ln x) d(\ln x) = N_l(x) dx$. This means that the probability density of the log-normal distribution is given by,

$$N_l(x) = \frac{N(\ln x)}{x} = \frac{1}{x\sigma\sqrt{2\pi}} \exp \left[-\frac{(\ln x - \mu)^2}{2\sigma^2} \right]. \quad (5)$$

2.3. Gamma distribution

The income distribution of the less than rich can also be reasonably well fitted by the gamma distribution [5,34,35]. This is a three-parameter function whose probability density reads as,

$$f(x) = \left[\frac{A}{\Gamma(n)m^n} \right] x^{(n-1)} e^{-(x/m)}, \quad (6)$$

where A , n , and $1/m$ are, respectively, the normalizing, shape, and rate parameters.

2.4. Distributions constructed as sums

During the process of data fitting we found useful to summing up two log-normal or gamma distributions with different parameter values. Hence, the bi-gamma PDF is written as,

$$f(x) = A_1 x^{(n_1-1)} \left[\frac{e^{-(x/m_1)}}{\Gamma(n_1)m_1^{(n_1-1)}} \right] + A_2 x^{(n_2-1)} \left[\frac{e^{-(x/m_2)}}{\Gamma(n_2)m_2^{(n_2-1)}} \right], \quad (7)$$

whereas the bi-log-normal PDF yields,

$$f(x) = \frac{A_1}{x\sigma_1} \exp \left[-\frac{(\ln x - \mu_1)^2}{2(\sigma_1)^2} \right] + \frac{A_2}{x\sigma_2} \exp \left[-\frac{(\ln x - \mu_2)^2}{2(\sigma_2)^2} \right]. \quad (8)$$

Statistical distributions such as the log-normal and gamma distributions are well suited for modelling income and wealth because of their ability to capture the natural variability of economic systems. The log-normal distribution effectively represents middle- and low-income segments, where incomes grow multiplicatively through investment or other economic processes. In contrast, the gamma distribution is particularly well suited for modelling lower-income populations because of its exponential decay but it is vanishing at the origin. The introduction of a bimodal fit, where two distributions are combined, allows for a more accurate representation of the data by taking into account the coexistence of different income groups within global populations.

3. Database

The GDP is a widely used monetary measure of the market value of all final goods and services produced in a period of time, often annually. There are two ways to measure GDP: nominally or via PPP. The first way, nominal or market value GDP, or GDP at exchange rate, occurs when the GDP of countries in their corresponding currencies are converted into a single currency, like, for example, into the *United States dollar* (USD). The second measure is GDP at PPP (GDP-PPP), when a “basket of goods” comprising a wide range of goods and services is priced equally in different countries and territories and by taking into account exchange rate.

In what follows, for brevity reasons, when we write countries, it is understood that we refer to both countries and territories. The so-called “international dollar” would buy in a given country a comparable amount of goods and services a USD would buy in the US according to PPP data. Although estimating the PPP across countries is not an easy task, it is accepted that PPP measures are generally regarded as better and more stable way than market values to compare overall consumption and income among nations. The GDP-PPP of developing countries is in general, higher than their nominal GDP, so the per capita income gap between rich and poor countries is reduced under PPP values.

The empirical data used here were obtained from two sets of data: Lakner and Milanovic [36] and Roser [37]. As it will be shown below, our main fitting results were derived from the former’s database, whereas the latter’s one was employed to subtract the income distributions of China and India in order to show how important these countries populations are to fill the “global middle-class” valley as time passes.

All aforementioned data do not include the entire world population because it has a 60k USDs upper cutoff limit that considered the inflation adjusted year 2011. In addition, the income values were measured in each country according to PPP in USD. Milanovic’s data [36] were measured in 2011 PPP USD and Roser’s data [37] in 2005 PPP USD. Even so, despite this limitation the available data included the vast majority of the world’s population. The results of our analysis will demonstrate how simple statistical models can effectively capture the dynamic evolution of income distributions

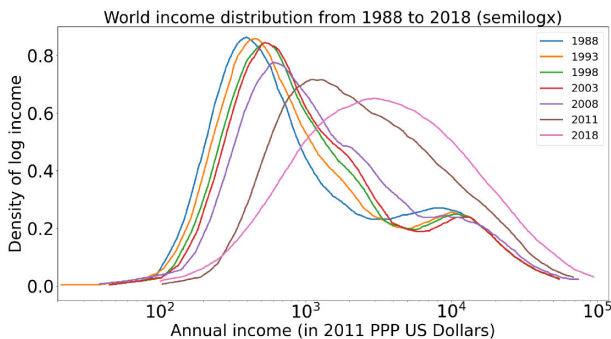


FIGURE 1. Milanovic income distribution from 1988 to 2018.

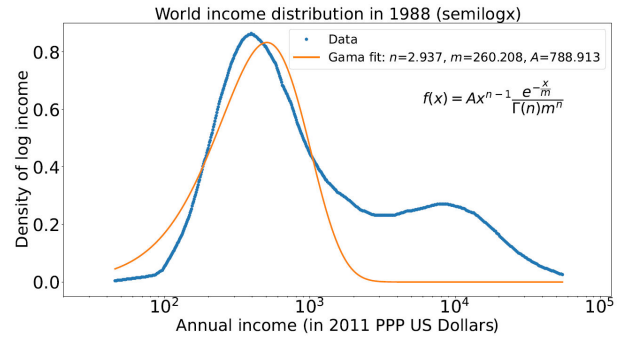


FIGURE 2. Gamma fit for Milanovic income distribution for year 1988, $R^2 = 0.69068$.

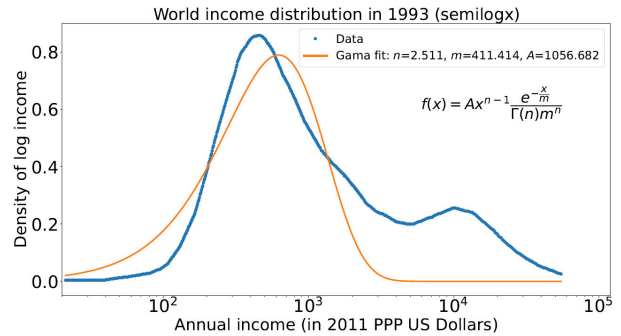


FIGURE 3. Gamma fit for year 1993, $R^2 = 0.73948$.

over time. The following section delves into these results, highlighting key patterns and through our fitting methods.

4. Results

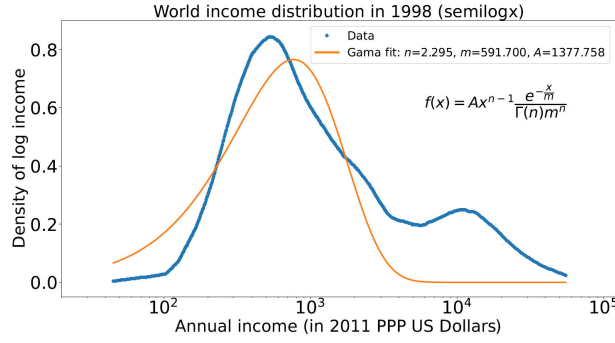
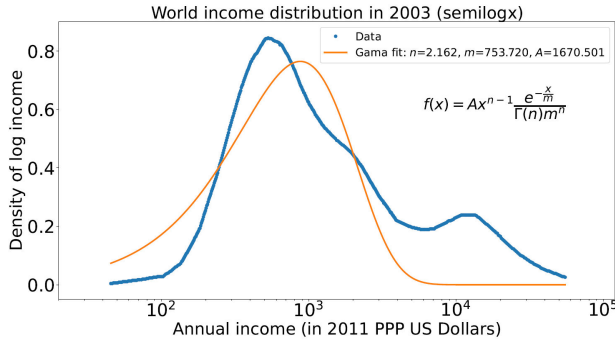
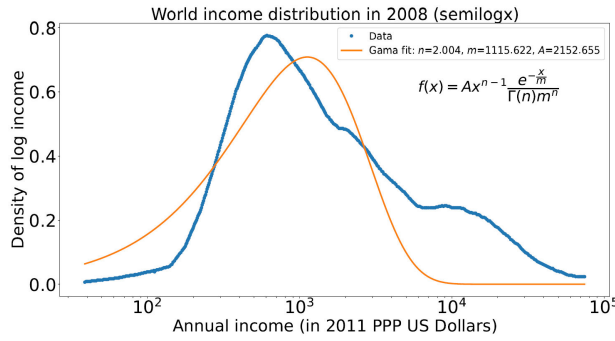
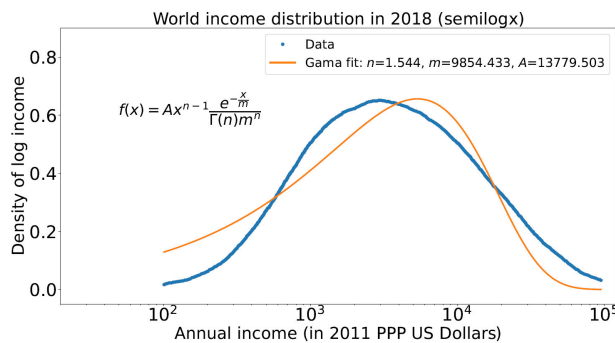
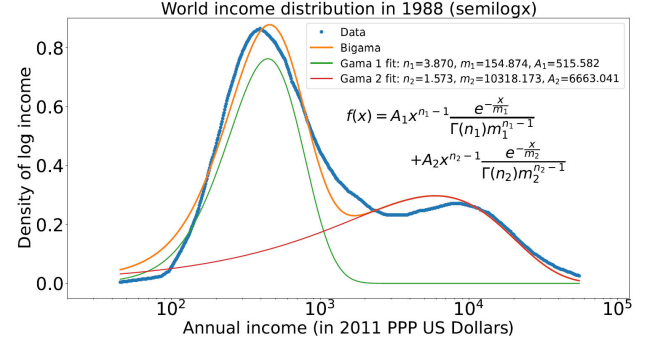
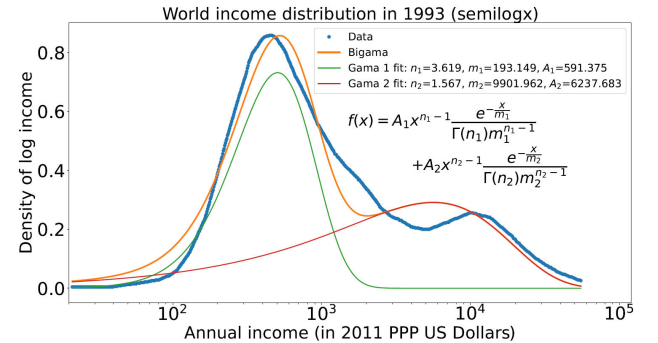
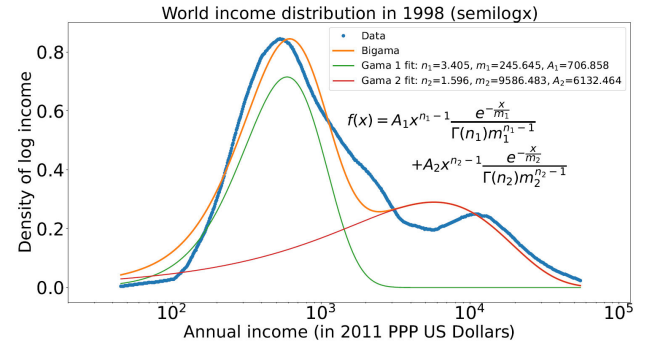
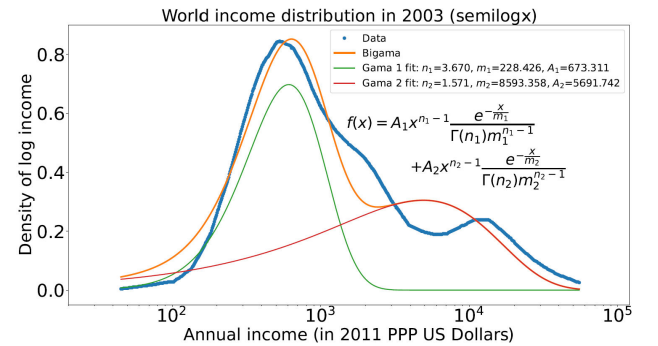
Data fitting using the distribution functions discussed above were carried out with all available data. The results are presented below grouped by the respective function used to fit the data. In all figures the term “semilogx” at the top indicates that there is a logarithmic scale at the x -axis.

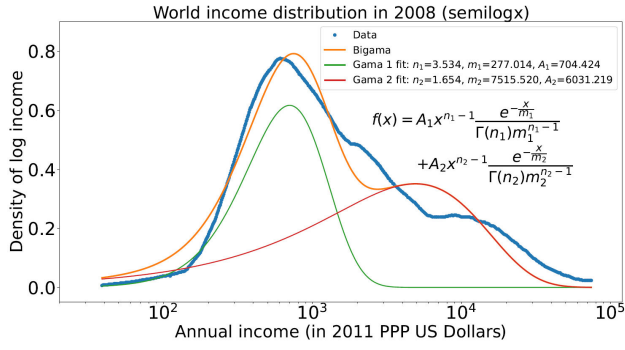
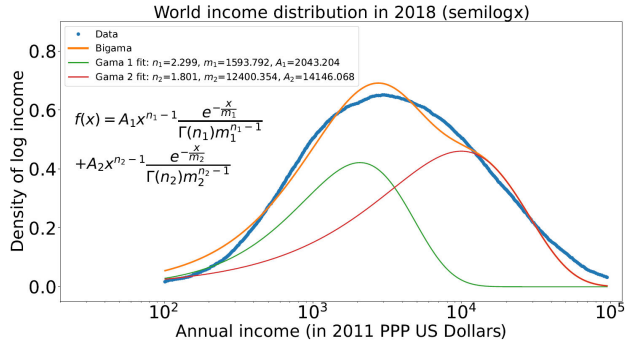
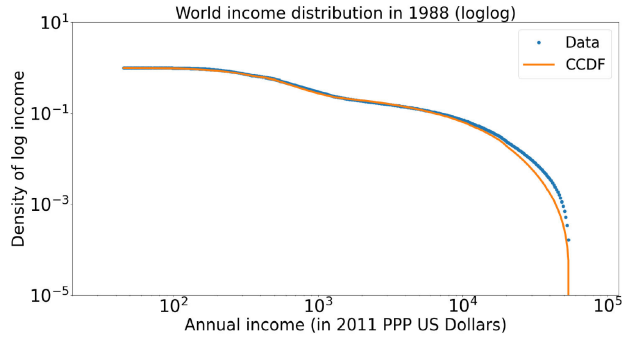
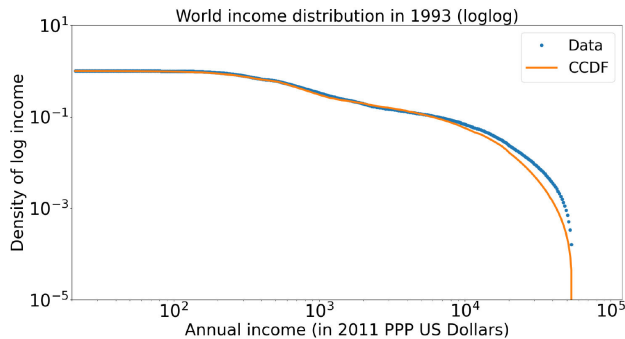
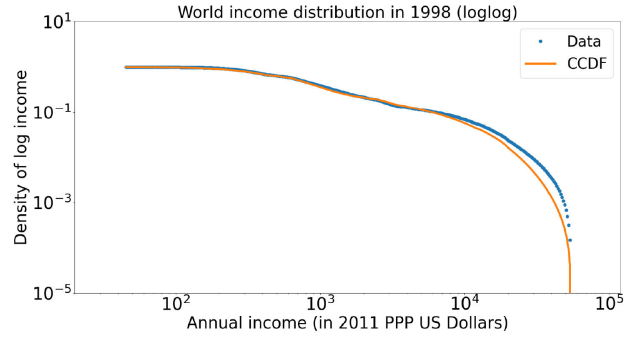
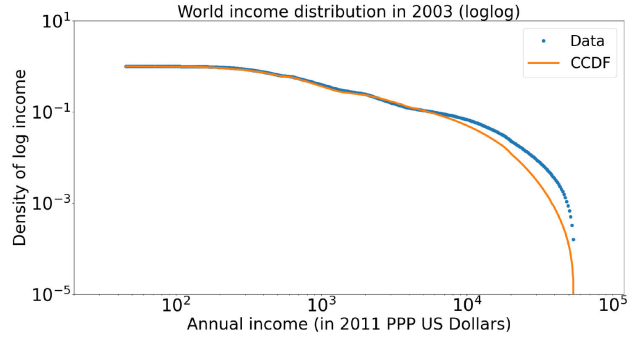
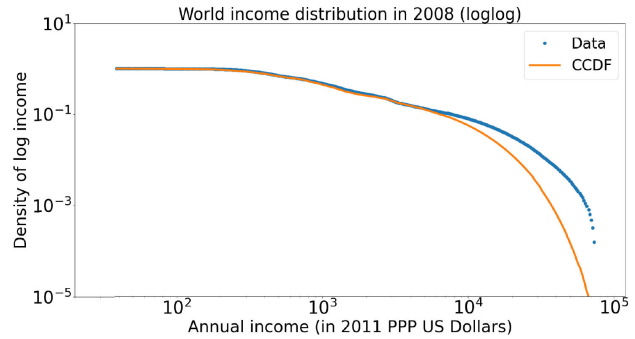
4.1. Gamma and bi-gamma fits

Figure 1 shows the world income PDF in 2011 PPP USD from 1988 to 2018 obtained by Milanovic. Two results can be clearly noticed from the plots as time passes: the distribution shifts to the right and the valley tends to disappear.

Figures 2-7 show that a single gamma distribution only matches the first peak. The adjustment parameter R^2 is around 0.72 except for year 2018 when it is 0.85. These plots also show that a single gamma distribution only matches the first peak. The adjustment parameter R^2 is around 0.72 except for year 2018 when it is equal to 0.85. Table I shows the values of R^2 of all the fittings we present here.

Figures 8-13 show basically the same data as in previous figures now fitted with bi-gamma functions, either separately or together. The plots show that each one fits well some portion of the data and that taking them together provides a better

FIGURE 4. Gamma fit for year 1998, $R^2 = 0.72081$.FIGURE 5. Gamma fit for year 2003, $R^2 = 0.73558$.FIGURE 6. Gamma fit for year 2008, $R^2 = 0.72134$.FIGURE 7. Gamma fit for year 2018, $R^2 = 0.85627$.FIGURE 8. Bi-Gamma fit for Milanovic income distribution in 1988, $R^2 = 0.96115$.FIGURE 9. Same as Fig. 8 but for 1993, $R^2 = 0.9499$.FIGURE 10. Same as Fig. 8 but for 1998, $R^2 = 0.94064$.FIGURE 11. Same as Fig. 8 but for 2003, $R^2 = 0.93372$.

FIGURE 12. Same as Fig. 8 but for 2008, $R^2 = 0.94681$.FIGURE 13. Same as Fig. 8 but for 2018, $R^2 = 0.85627$.FIGURE 14. CCDF bi-gamma for 1988, $R^2 = 0.99782$.FIGURE 15. CCDF bi-gamma for 1993, $R^2 = 0.99712$.FIGURE 16. CCDF bi-gamma for 1998, $R^2 = 0.99721$.FIGURE 17. CCDF bi-gamma for 2003, $R^2 = 0.99711$.FIGURE 18. CCDF bi-gamma for 2008, $R^2 = 0.99685$.

fit to the whole distribution. In general the fitting is better in the low “poor” region than in the “rich” one.

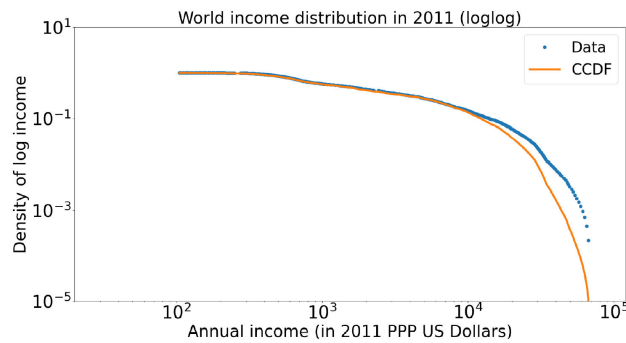
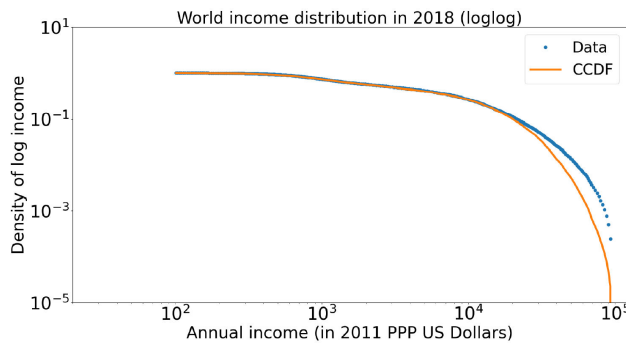
Figures 14-20 present the same data as in previous figures, but fitted to the CCDF bi-gamma to obtain better values for R^2 than the PDF fittings. It is clear how in the bi-gamma CCDF curves deviate slightly below the ones of the empirical values at the tail of the distribution, that is, in the rich region.

4.2. Log-normal and bi-log-normal fits

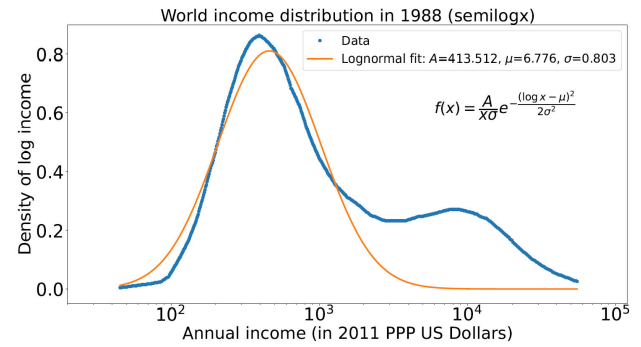
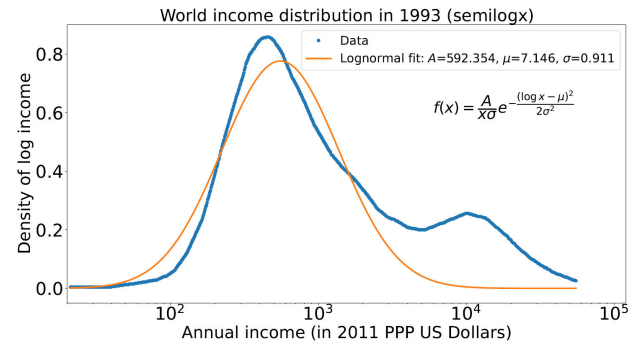
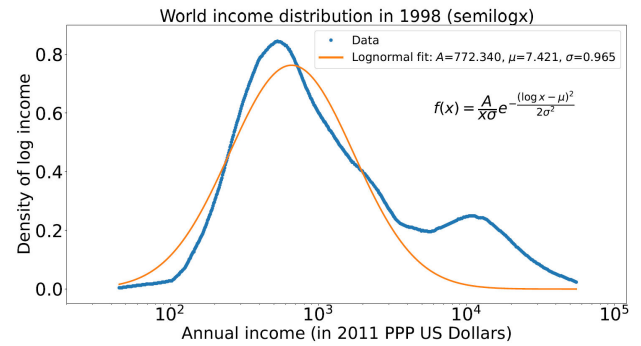
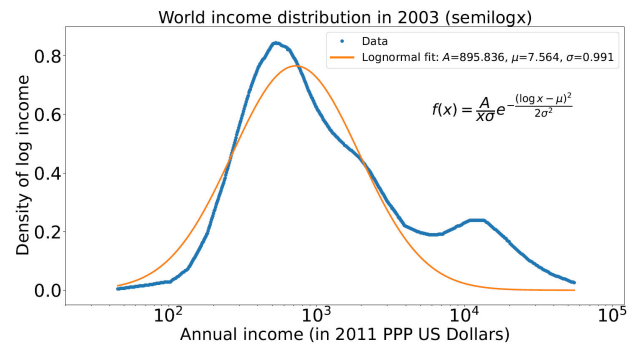
As in the case of the gamma function, the log-normal fits shown in Figs. 21-26 coincide well with all data for the first peak. Figs. 27-33 show that bi-log-normal fits are better than bi-gamma fits as shown by the R^2 values shown in Table I. Figures 34-40 present the respective bi-log-normal CCDF

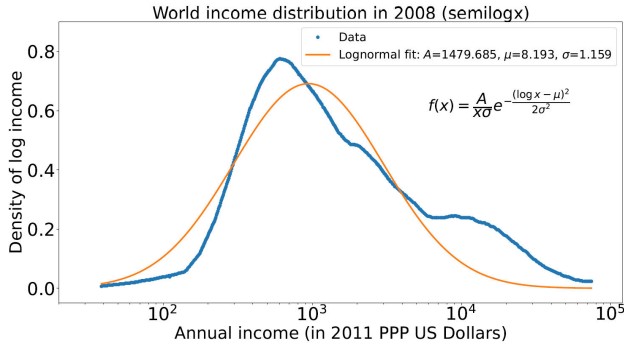
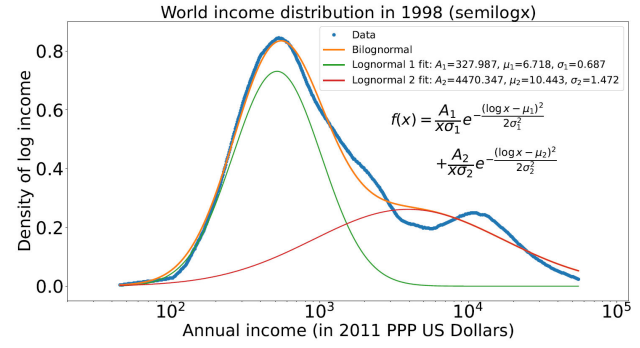
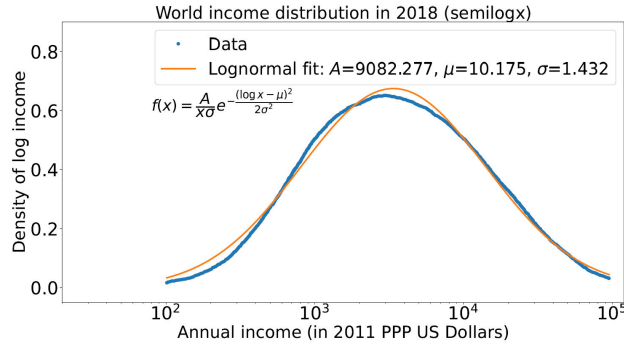
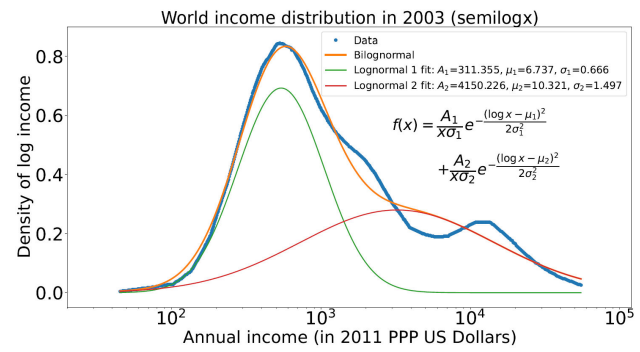
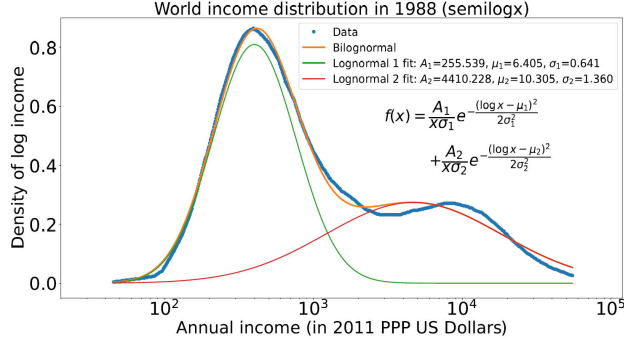
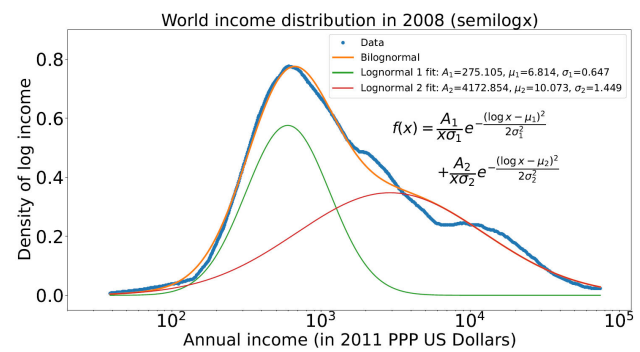
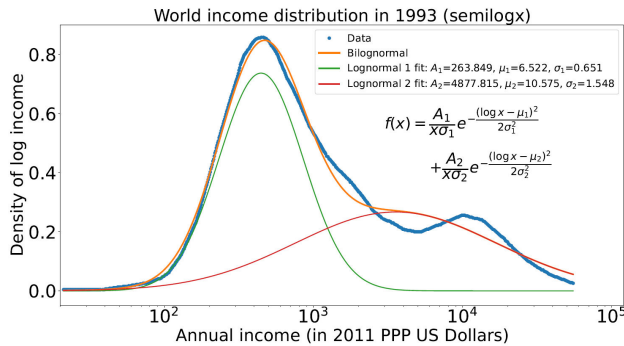
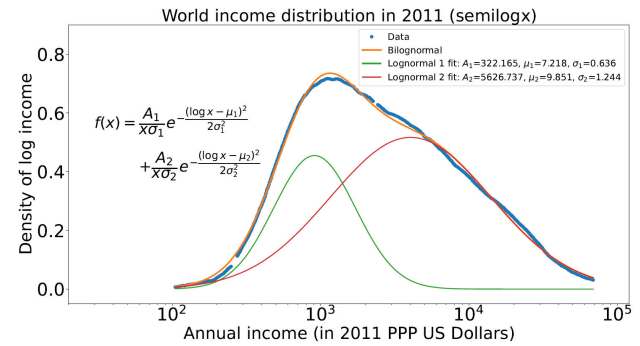
TABLE I. R^2 values of fittings.

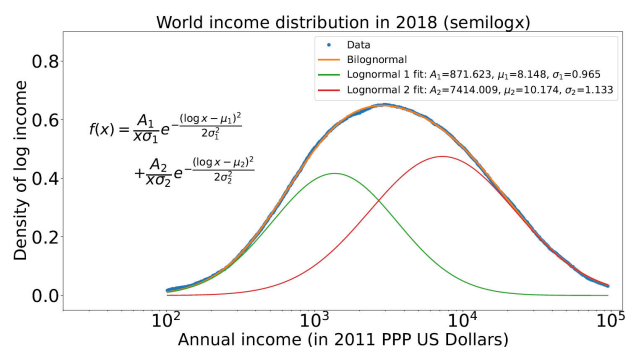
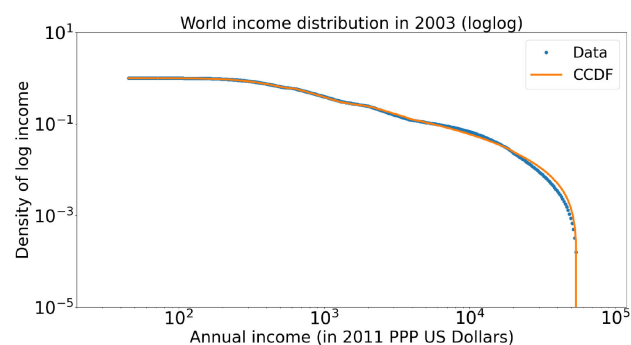
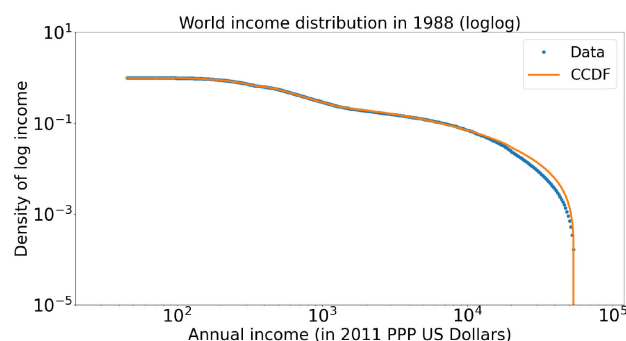
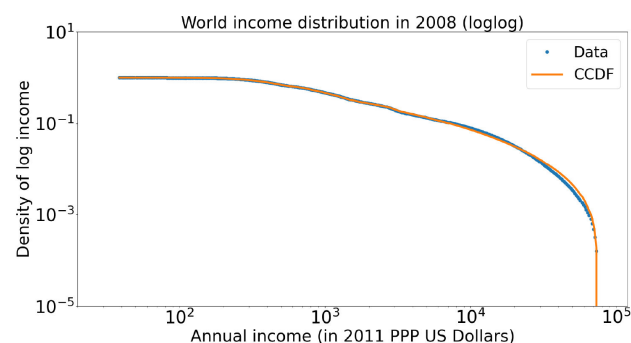
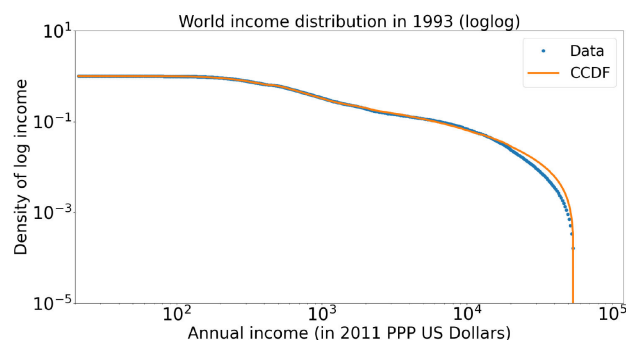
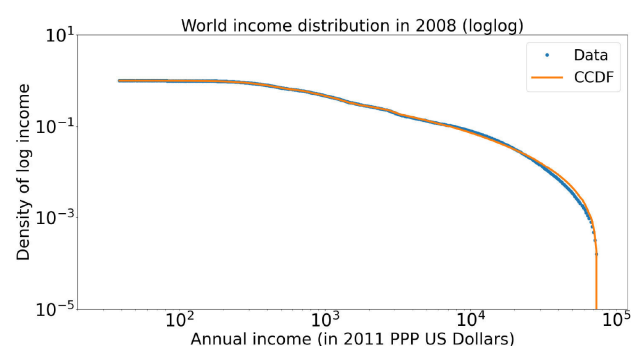
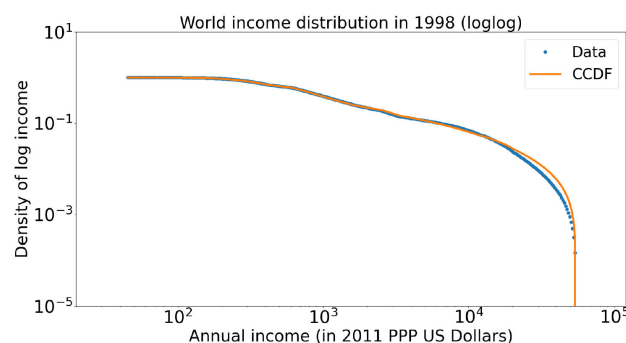
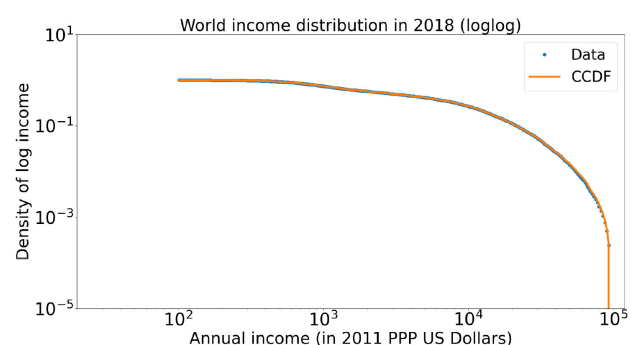
year	gamma	bi-gamma	bi-gamma CCDF
1988	0.69068	0.96115	0.99782
1993	0.73948	0.9499	0.99712
1998	0.72081	0.94064	0.99721
2003	0.73558	0.93372	0.99711
2008	0.72134	0.94681	0.99685
2011	0.69562	0.94975	0.99743
2018	0.85627	0.85627	0.99896
year	log-normal	bi-log-normal	bi-log-normal CCDF
1988	0.77358	0.99383	0.99989
1993	0.8366	0.98898	0.99987
1998	0.84092	0.98634	0.99981
2003	0.85684	0.98224	0.99976
2008	0.86455	0.99129	0.99991
2011	0.90122	0.99729	0.99995
2018	0.98991	0.99973	0.99999

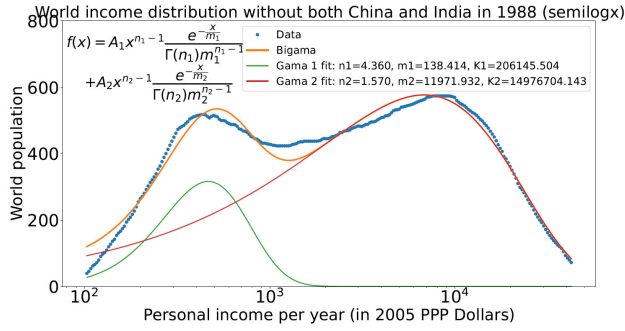
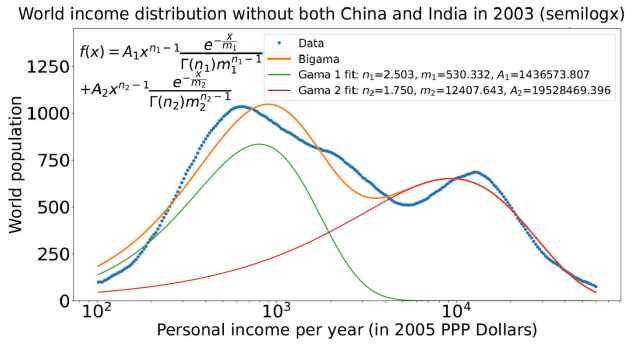
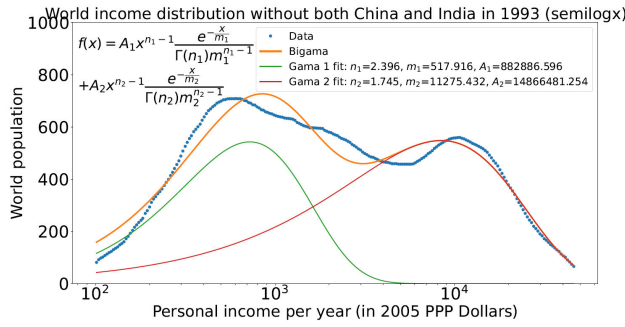
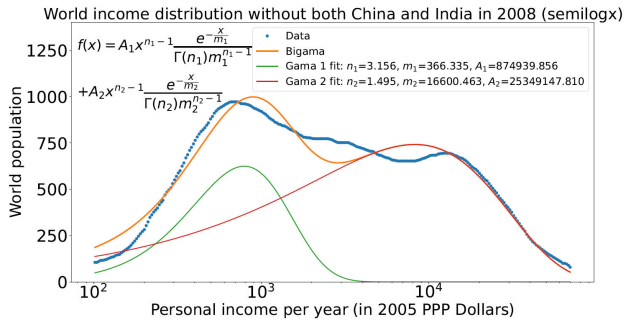
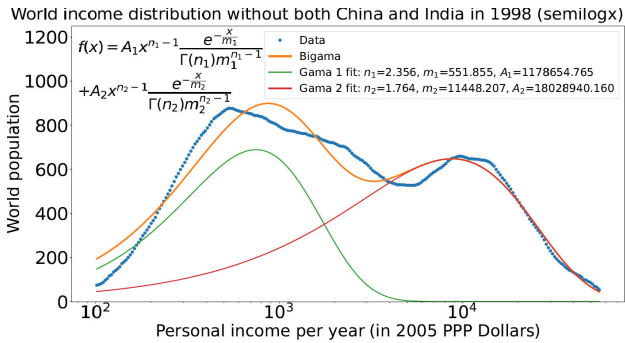
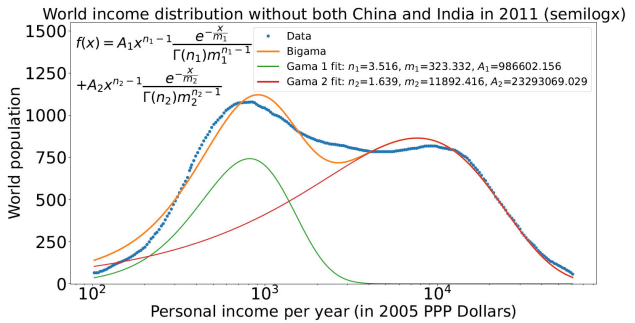
FIGURE 19. CCDF bi-gamma for 2011, $R^2 = 0.99743$.FIGURE 20. CCDF bi-gamma for 2018, $R^2 = 0.99896$.

where it is clear that this function provides a better fit at the tail of the distribution as compared to the bi-gamma CCDF ones.

FIGURE 21. Log-normal fit for Milanovic income distribution in 1988, $R^2 = 0.77358$.FIGURE 22. Same as Fig. 21 but for 1993, $R^2 = 0.8366$.FIGURE 23. Same as Fig. 21 but for 1998, $R^2 = 0.84092$.FIGURE 24. Same as Fig. 21 but for 2003, $R^2 = 0.85684$.

FIGURE 25. Same as Fig. 21 but for 2008, $R^2 = 0.86455$.FIGURE 29. Same as Fig. 27 but for 1998 $R^2 = 0.98634$.FIGURE 26. Same as Fig. 21 but for 2018, $R^2 = 0.98991$.FIGURE 30. Same as Fig. 27 but for 2003 $R^2 = 0.98224$.FIGURE 27. Bi-log-normal fit for Milanovic income distribution in 1988, $R^2 = 0.99383$.FIGURE 31. Same as Fig. 27 but for 2008 $R^2 = 0.99129$.FIGURE 28. Same as Fig. 27 but for 1993 $R^2 = 0.98898$.FIGURE 32. Same as Fig. 27 but for 2011 $R^2 = 0.99729$.

FIGURE 33. Same as Fig. 27 but for 2018 $R^2 = 0.99973$.FIGURE 37. 2011, $R^2 = 0.99995$.FIGURE 34. 1988, $R^2 = 0.99981$.FIGURE 38. 2018, $R^2 = 0.99999$.FIGURE 35. 2003, $R^2 = 0.99976$.FIGURE 39. 1988, $R^2 = 0.96851$.FIGURE 36. 2008, $R^2 = 0.99991$.FIGURE 40. 1993, $R^2 = 0.93952$.

FIGURE 41. 1998, $R^2 = 0.93612$.FIGURE 44. 2003, $R^2 = 0.93766$.FIGURE 42. 2003, $R^2 = 0.93766$.FIGURE 45. 2008, $R^2 = 0.94921$.FIGURE 43. 2008, $R^2 = 0.94921$.FIGURE 46. 2011, $R^2 = 0.97056$.

4.3. World distribution without both China and India

Roser [37] provided the income distributions of China and India along time, and this allowed us to conveniently subtract their contribution to the global distribution after realizing these countries play a fundamental role in shaping global distribution. Figures 41-46 present these results where the Y-axis is the PDF of the fitted functions.

If we envision a scenario without the presence of these two significant demographic and economic players, a pronounced decline in the poor and middle-class values emerge, creating a noticeable “valley” in the graph. So, it seems that China and India, with their vast populations and expanding economies, act as a bridge that fills this valley, thereby generating a more uniform and comprehensive data distribution.

A comparative analysis shows that while earlier works, such as those by Milanovic, focused on single-peak representations of income distribution, they often failed to capture the complexities introduced by bimodal patterns, especially in global datasets. Similarly, many works in econophysics highlighted exponential and power-law distributions for national economies but to our knowledge, but did not address the multimodal characteristics observed in global contexts.

Therefore, our bimodal adjustments provide a new way of fitting the world income distribution along the years (qualified with R values) showing the usefulness of combining multiple distributions to model diverse economic systems effectively. Insights gained from our models can provide elegant interpretations of broader economic trends that can be improved by employing distributions with more parameters.

5. Conclusions

This paper analyzed the evolution of global income distributions over several decades using empirical datasets and fits to standard statistical functions—gamma, log-normal, and their bimodal combinations. The graphical analysis of the *probability density function* (PDF) and the *complementary cumulative density function* (CCDF) revealed clear patterns of inequality and the presence of multimodality in global income.

A key result is that single-function fits (gamma or log-normal) yield reasonably good approximations up to \$60k (2011 PPP USD), but the use of bimodal combinations significantly improves fit quality across all years studied. This suggests the global income distribution is better described as a composition of at least two subpopulations, reflecting different economic realities. The goodness-of-fit, measured by R^2 , supports this conclusion.

More recently, although there were improvements in collecting datasets for income and wealth in many countries, data for the very poor and the very rich households and individuals are still, in general, quite unreliable. Due to the above mentioned upper limit data cutoff most people from poor countries, with low per capita GDP, were included, but in places like Monaco, Qatar and others with relatively large income per capita, the datasets do not include large percentages of their respective populations. The absence of significant world high-income datasets explains why the Pareto power-law is very poorly presented, or non-existent, in various upper segments of the income distributions showed here. Therefore, world income distributions with higher upper limits should have exhibited clear power law behaviors for higher income values in addition to, possibly, second or third similar power-law behaviors in subsets of increasing incomes. For instance, the income and wealth data for billionaires as given by Forbes magazine [38] every year represents a very small subset of humans who are very important in studies of economic income and wealth inequalities related to the so-called, and much talked about, 99% vs. 1% economic disparity in the whole world.

A particularly novel finding is the structural role of China and India in shaping the global income distribution. Between

1988 and 2008, the global distribution transitioned from a bimodal to a more unimodal form, largely due to income growth in these populous nations. When these two countries are excluded, a valley between low- and middle-income peaks re-emerges, underscoring their bridging function in the global economy.

These results underscore the value of combining multiple distributions to better capture global economic heterogeneity. Moreover, they open the door to richer interpretations of macroeconomic trends and transitions in global inequality.

Importantly, such modeling approaches can be applied to study the effects of shocks or policy interventions. For example, by fitting similar distributions to post-pandemic datasets, one could quantify the impact of COVID-19 on income polarization or middle-class shrinkage in different regions—an issue already under debate in recent literature [39,40].

This work provides a simple and elegant comprehensive approach to the world income distributions fitted to commonly used functions in econophysics.

The implications of these findings pave the way for more refined studies and expanded datasets. The following section outlines potential directions to deepen our understanding of income dynamics and possible policy implications.

6. Future work

We are particularly interested in extending the present analysis employing very recent data from the LIS database [41], which contain household -and person- level data on labor income, capital income, pensions, public social benefits and private transfers, as well as taxes and contributions, demography, employment, and expenditures. So, this database will provide enough empirical results to perform many important comparative interdisciplinary analysis.

Acknowledgments

We thank R. Cayetano-Albarraquío for useful discussions. M. del C. M. acknowledges the partial financial support provided by PASPA-DGAPA-UNAM, México.

i. Equation (2) has several applications when it is integrated in time, specially in, but not limited to, medicine and engineering. In the medical literature the CCDF is known as *survival function* [31], whereas in the engineering literature it is referred as *reliability function* [32]. In these two applications the CCDF gives the probability that a patient survives, or a device remains reliable, past a certain time.

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