Vanishing into the Bulk: A Higher-Dimensional explanation for the disappearance of PHL 293B-LBV

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In the current work, the possible explanations for the disappearance of PHL 293B-LBV using higher-dimensional physics, effective field theory, and interactions with dark matter or cosmic strings are studied. It has been proposed that PHL 293B-LBV's disappearance is a consequence of matter leakage into extra dimensions, which are derived through braneworld physics, effective field theory, and interactions with dark matter or cosmic strings. Under the resonance ($m_\Phi \approx m_\Psi \sim 10^{-18}$ GeV, $g_{\text{int}} \sim 0.012$), the star's mass (50 $M_\odot$) decays in ~ 1 day ($\Gamma \sim 1.16 \times 10^{-5}\,\text{s}^{-1}$). This predicts lensing shifts ($\Delta\theta \sim 8 \times 10^{-6}$ arcsec) and orbital perturbations ($\Delta v \sim 60$ cm/s), which are testable by LSST and Euclid, with KK graviton bounds ($h < 10^{-40}$) and seems consistent with non-detection. The solutions obtained in this work suggest the existence of bound states as well as possible resonances at specific energy levels. Those are determined by g_{int}, m_Φ, and m_Ψ. From an effective field theory perspective, low-energy parameters such as coupling constants and mass terms are discussed. This work suggests that the star's disappearance could be a consequence of fundamental physics rather than an astrophysical anomaly. A dispersion relation that governs matter leakage from the brane into the bulk is derived. Also, it has been shown that an imaginary component in the energy spectrum leads to an exponential decay of the star's observable mass. This phenomenon explains the absence of usual astrophysical events such as supernova explosions or black hole formation. It has been found that unique and rare conditions affected the PHL 293B, but not the nearby stars.

Keywords: Brane cosmology; PHL 293B-LBV; cosmic strings; higher dimensional solutions; Kaluza klein solutions; missing stars.

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1. Introduction

The sudden disappearance of massive stars without producing any observational end-of-life signatures, such as supernovae or black holes, questions our understanding of stellar evolution and astrophysics. Recently the observation of PHL 293B, a low-metallicity dwarf galaxy located approximately 75 million light-years away in the constellation Aquarius, exhibits this kind of mysterious disappearance [1-3]. The dwarf galaxy consists of a luminous blue variable (LBV) star, which is one of the most massive and luminous types known. Between 2011 and 2019, astronomers observed that the spectral signatures of the LBV star were missing [4-7]. In such disappearance, neither supernova explosion nor other high-energy events were detected, which are used except when such massive stars go off. The sudden disappearance of PHL 293B-LBV raises various questions about the evolution and fate of massive stars. There may be some possible explanations for this problem including extreme dust obscuration, direct collapse into a black hole, or dramatic intrinsic variability, which remain insufficient to explain this sudden disappearance. For the same problem, earlier we had attempted a possible solution in terms of cosmic strings and dark matter aspect [8].

Some observational challenges exist in this phenomenon. The PHL 293B-LBV's disappearance without typical end-of-life signatures challenges the usual models of stellar evolution. Also, there are no gravitational effects or no changes in the nearby regions were observed. In this study, we discuss numerous theoretical solutions which will be able to explain the disappearance event. Solutions including higher-dimensional physics, interactions with dark matter or dark energy fields, and the influence of topological defects such as cosmic strings are discussed.

Higher-dimensional frameworks, like Braneworld cosmology, provide possible solutions not only for this case but also for broader puzzles, such as the nature of dark energy and earlier stages of the universe. A Brane is a 4-dimensional (3 spatial + 1 temporal dimensions) hypersurface where particles and fields are confined and the bulk is a higher-dimensional spacetime [9-11]. Matter fields are in general confined to the brane due to localization (*e.g.*, the potential wells in the extra dimensions). However, under certain conditions, matter can propagate into the bulk [12]. By combining braneworld scenarios and higher-dimensional spacetime theories, we study whether the star's matter could have transitioned into extra dimensions, without producing any observational consequences.

The disappearance of PHL 293B-LBV joins a list of astrophysical anomalies, such as Betelgeuse's unexpected dimming [13-15] that queries the present explanation. This study introduces a distinct approach to propose a mechanism where stellar matter transits into extra dimensions, and that can be testable via subtle gravitational signatures. The present work derives equations for the brane field Φ and bulk field Ψ, by including the interaction term. We have derived the self-energy correction to the brane field due to its coupling with bulk modes. The decay rate Γ for the transition of brane matter into bulk mode is evaluated. Finally, we have mapped parameters to map the theoretical decay timescale with astronomical observations. This work aims to narrow down the gap between theoretical physics and observational astrophysics by presenting novel solutions for the disappearance of PHL 293B-LBV.

2. Matter leakage in braneworld models

In braneworld models, our observable universe is a 3-dimensional brane embedded in a higher-dimensional bulk [16-18]. Matter is usually confined to the brane, but under certain conditions, it might propagate into the bulk. We initially model the mass leakage by the assumption of a simple 5D spacetime, where fields Φ and Ψ interact at the brane. To understand this scenario more clearly we can have an assumption as the 5-dimensional spacetime is flat.

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \tag{1}$$

Another assumption can be considered as both Φ and Ψ have quadratic potentials.

$$V(\Phi) = \frac{1}{2}m_\Phi^2\Phi^2, \quad V_\Psi(\Psi) = \frac{1}{2}m_\Psi^2\Psi^2. \tag{2}$$

Here Φ is a field that suggests matter confined to the brane (which is associated with the star's matter) and Ψ represents the bulk field that interacts with Φ, corresponding to energy or mass transfer from the brane into the extra-dimensional bulk. Further, it can be assumed that solutions can be separated into brane and bulk components. Fourier transform for the brane coordinates x^μ is performed as,

$$\Phi(x^\mu, y) = \int \frac{d^4k}{(2\pi)^4}e^{ik_\mu x^\mu}\tilde{\Phi}(k_\mu, y), \tag{3}$$

$$\Psi(x^\mu, y) = \int \frac{d^4k}{(2\pi)^4}e^{ik_\mu x^\mu}\tilde{\Psi}(k_\mu, y), \tag{4}$$

where g_{int} is the coupling constant that represents the strength of the interaction between the bulk field Φ and the brane field Ψ. In momentum space, the equations for $\tilde{\Phi}(k_\mu, y)$ is written as,

$$\left(-k^2 + \partial_y^2 - m_\Phi^2\right)\tilde{\Phi}(k, y) = -g_{\text{int}}\delta(y)\tilde{\Psi}(k, 0). \tag{5}$$

For $\tilde{\Psi}(k_\mu, y)$, it is written as,

$$\left(-k^2 + \partial_y^2 - m_\Psi^2\right)\tilde{\Psi}(k, y) = -g_{\text{int}}\delta(y)\tilde{\Phi}(k, 0). \tag{6}$$

Away from the Brane ($y \neq 0$), the equations can be solved for $\tilde{\Phi}(k, y)$ and $\tilde{\Psi}(k, y)$, Hence in the bulk ($y \neq 0$), the delta functions vanish, and the equations simplify to:

$$\left(-k^2 + \partial_y^2 - m_\Phi^2\right)\tilde{\Phi}(k, y) = 0, \tag{7}$$

$$\left(-k^2 + \partial_y^2 - m_\Psi^2\right)\tilde{\Psi}(k, y) = 0. \tag{8}$$

The Eqs. (7) and (8) are homogeneous differential equations whose solutions are exponential functions. The general solutions are obtained as,

$$\tilde{\Phi}(k, y) = A(k)e^{\lambda_\Phi y} + B(k)e^{-\lambda_\Phi y}, \tag{9}$$

$$\tilde{\Psi}(k, y) = C(k)e^{\lambda_\Psi y} + D(k)e^{-\lambda_\Psi y}, \tag{10}$$

where

$$\lambda_\Phi = \sqrt{k^2 + m_\Phi^2}, \quad \lambda_\Psi = \sqrt{k^2 + m_\Psi^2}. \tag{11}$$

For normalization, it is required that the fields vanish at infinity.

$$\tilde{\Phi}(k, y \to \pm\infty) \to 0, \quad \tilde{\Psi}(k, y \to \pm\infty) \to 0. \tag{12}$$

This results in,

$$A(k) = 0, \quad C(k) = 0. \tag{13}$$

Hence the solutions can be simplified into,

$$\tilde{\Phi}(k, y) = B(k)e^{-\lambda_\Phi |y|}, \tag{14}$$

$$\tilde{\Psi}(k, y) = D(k)e^{-\lambda_\Psi |y|}. \tag{15}$$

The absolute value appears here due to the fact that the equations are symmetric under $y \to -y$, and the delta functions are located at $y = 0$. The equations can be integrated across an infinitesimal interval around $y = 0$ to tackle the delta functions, especially for $\tilde{\Phi}$, from $y = -\epsilon$ to $y = \epsilon$.

$$\int_{-\epsilon}^{\epsilon} dy\left(-k^2 + \partial_y^2 - m_\Phi^2\right)\tilde{\Phi}(k, y) = -g_{\text{int}}\tilde{\Psi}(k, 0). \tag{16}$$

After simplification, for both ϕ and ψ can be written as,

$$\left[\partial_y\tilde{\Phi}(k, y)\right]_{y=-\epsilon}^{y=\epsilon} = -g_{\text{int}}\tilde{\Psi}(k, 0), \tag{17}$$

$$\left[\partial_y\tilde{\Psi}(k, y)\right]_{y=-\epsilon}^{y=\epsilon} = -g_{\text{int}}\tilde{\Phi}(k, 0). \tag{18}$$

The fields are continuous across $y = 0$, but their derivatives have fluctuations due to the delta functions. The discontinuities in the derivatives can be calculated as,

$$\left[\partial_y\tilde{\Phi}(k, y)\right]_{y=0^+}^{y=0^-} = \partial_y\tilde{\Phi}(k, 0^+) - \partial_y\tilde{\Phi}(k, 0^-). \tag{19}$$

But since the solutions are symmetric, it can be written as,

$$\partial_y\tilde{\Phi}(k, 0^+) = -\lambda_\Phi B(k), \tag{20}$$

$$\partial_y\tilde{\Phi}(k, 0^-) = \lambda_\Phi B(k), \tag{21}$$

$$\partial_y\Phi(k, 0^+) - \partial_y\Phi(k, 0^-) = -2\lambda_\phi B(k). \tag{22}$$

Hence the discontinuity is calculated as,

$$\left[\partial_y \tilde{\Phi}(k,y)\right]_{y=0^+}^{y=0^-} = -\lambda_\Phi B(k) - (\lambda_\Phi B(k)) = -2\lambda_\Phi B(k), \tag{23}$$

$$\left[\partial_y \tilde{\Psi}(k,y)\right]_{y=0^+}^{y=0^-} = -2\lambda_\Psi D(k). \tag{24}$$

From the discontinuities, the matching conditions are calculated as,

$$-2\lambda_\Phi B(k) = -g_{\text{int}}\tilde{\Psi}(k,0), \tag{25}$$

$$-2\lambda_\Psi D(k) = -g_{\text{int}}\tilde{\Phi}(k,0). \tag{26}$$

But at $y=0$ the boundary conditions result to,

$$\tilde{\Phi}(k,0) = B(k), \tag{27}$$

$$\tilde{\Psi}(k,0) = D(k). \tag{28}$$

The substitution of Eqs. (27) and (28) into Eqs. (25) and (26) leads to the matching conditions as

$$-2\lambda_\Phi B(k) = -g_{\text{int}}D(k), \tag{29}$$

$$-2\lambda_\Psi D(k) = -g_{\text{int}}B(k). \tag{30}$$

Substitution of$D(k)$ from Eq. (29) into Eq. (30) gives,

$$\frac{4\lambda_\Phi\lambda_\Psi B(k)}{g_{\text{int}}} = g_{\text{int}}B(k). \tag{31}$$

Hence, the dispersion relation is obtained as,

$$4\lambda_\Phi\lambda_\Psi = g_{\text{int}}^2. \tag{32}$$

The substitution of Eq. (11) into the dispersion relation in Eq. (32) gives,

$$\sqrt{(k^2+m_\Phi^2)(k^2+m_\Psi^2)} = \frac{g_{\text{int}}^2}{4}. \tag{33}$$

After squaring and expanding both sides, it becomes

$$k^4 + (m_\Phi^2+m_\Psi^2)k^2 + m_\Phi^2 m_\Psi^2 = \left(\frac{g_{\text{int}}^2}{4}\right)^2. \tag{34}$$

Hence the quadratic equation in k^2 is written as,

$$k^4 + ak^2 + b = 0, \tag{35}$$

where,

$$a = m_\Phi^2 + m_\Psi^2, \tag{36}$$

$$b = m_\Phi^2 m_\Psi^2 - \left(\frac{g_{\text{int}}^2}{4}\right)^2. \tag{37}$$

With quadratic relation

$$k^2 = \frac{-a \pm \sqrt{a^2-4b}}{2}$$

the discriminant are computed as,

$$\Delta = a^2 - 4b = (m_\Phi^2+m_\Psi^2)^2 - 4\left[m_\Phi^2 m_\Psi^2 - \left(\frac{g_{\text{int}}^2}{4}\right)^2\right]. \tag{38}$$

Further simplification leads to,

$$\Delta = (m_\Phi^2 - m_\Psi^2)^2 + g_{\text{int}}^2. \tag{39}$$

Since $g_{\text{int}}^2 \geq 0$, the discriminant Δ is always remain positive.

Also k^2 is computed as,

$$k^2 = \frac{-a \pm \sqrt{\Delta}}{2} = \frac{-(m_\Phi^2+m_\Psi^2) \pm \sqrt{(m_\Phi^2-m_\Psi^2)^2 + g_{\text{int}}^2}}{2}. \tag{40}$$

Since k^2 must be real and non-negative, we need to consider the physical root. There are two possible cases for this solution set.

Case 1: If g_{int}^2 is large enough, the expression inside the square root dominates, and k^2 can be negative, which will lead to imaginary k.

Case 2: If $m_\Phi = m_\Psi$ and g_{int}^2 is small, $\Delta = g_{\text{int}}^2$, and we will obtain

$$k^2 = \frac{-2m_\Phi^2 \pm g_{\text{int}}}{2}. \tag{41}$$

Which may yield negative k^2. To understand the disappearance of the star, we need to consider the time dependence of Φ on the brane ($y=0$). The time-dependent Φ field on the brane is written as,

$$\Phi(x^\mu, y=0) = \int \frac{d^4k}{(2\pi)^4} e^{ik_\mu x^\mu}\tilde{\Phi}(k, y=0), \tag{42}$$

$\tilde{\Phi}(k,0) = B(k)$, and $B(k)$ have to be determined by the matching conditions and dispersion relation. Let us assume that the dispersion relation yields a pole at $k^0 = \omega(k)$, then:

$$\Phi(t,\vec{x},y=0) \propto e^{-i\omega(k)t + i\vec{k}\cdot\vec{x}}. \tag{43}$$

If $\omega(k)$ has an imaginary part, then the field will decay exponentially with time. The imaginary part of $\omega(k)$ corresponds to the decay rate Γ is written as,

$$\omega(k) = \omega_{\text{real}}(k) - i\Gamma. \tag{44}$$

Hence field will have the kinetics as,

$$\Phi(t,\vec{x},y=0) \propto e^{-\Gamma t} e^{-i\omega_{\text{real}}(k)t + i\vec{k}\cdot\vec{x}}. \tag{45}$$

The Eq. (45) indicates that the amplitude of Φ on the brane decreases exponentially over time, which will lead to the disappearance of the star's matter from our observable universe. The coupling between Φ and Ψ is related to the energy that is about to transfer from the brane into the bulk and the decay rate Γ indicates its flow. Even though the energy density on the brane decreases, the total energy (brane + bulk) is conserved due to the exchange mediated by g_{int}. For the decay rate Γ to be prominent that will lead to observable disappearance, the coupling g_{int} should be very large. The action S for the gravitational field and matter is written as,

$$S = \int d^4x\, dy \sqrt{-g^{(5)}} \times \left[\frac{1}{2\kappa_5^2} R^{(5)} + \mathcal{L}_{\text{bulk}} + \delta(y)\left(-\sigma + \mathcal{L}_{\text{brane}}\right)\right]. \tag{46}$$

Here κ_5^2 is the 5-dimensional gravitational constant, $R^{(5)}$ is the 5-dimensional Ricci scalar, $\mathcal{L}_{\text{bulk}}$ is the Lagrangian for bulk fields, σ is the brane tension and $\mathcal{L}_{\text{brane}}$ is the Lagrangian for brane-confined matter fields. The star's matter can be represented as a scalar field such as $\Phi(x^\mu, y)$. Then the action for Φ is written as,

$$S_\Phi = \int d^4x\, dy \sqrt{-g^{(5)}} \times \left[-\frac{1}{2} g^{AB} \partial_A \Phi \partial_B \Phi - V(\Phi) - \delta(y) g_{\text{int}} \Phi \Psi\right]. \tag{47}$$

Here $A, B = 0, 1, 2, 3, 5$ (with 5 representing the y coordinate), $V(\Phi)$ is the potential for Φ, Ψ is a bulk field that interacts with Φ and g_{int} is the coupling constant for the interaction. Then the equation of motion for Φ is written as,

$$g^{AB} \nabla_A \nabla_B \Phi - \frac{\partial V}{\partial \Phi} = -g_{\text{int}} \delta(y) \Psi. \tag{48}$$

Initially, it is assumed that Φ is confined to the brane at $y = 0$, and solutions can be discussed where Φ propagates into the bulk ($y \neq 0$). The coupling term $-g_{\text{int}}\delta(y)\Phi\Psi$ indicates the interaction between brane-confined matter and bulk fields. With possible astrophysical conditions such as high energy densities and specific field configurations, this interaction will become more important. As a result, this may lead to, energy transfer in which the energy from Φ is transferred to Ψ. This will allow Φ to acquire momentum in the y-direction. Also, this leads to propagation into bulk, in which Φ begins to propagate into the extra dimension. As the result, the star will disappear from our 4D brane.

The interaction alters the standard conservation law as,

$$\nabla^\mu T_{\mu\nu} = -g_{\text{int}} \Psi \partial_\nu \Phi. \tag{49}$$

This Eq. (49) results in the non-zero divergence that indicates the flow of energy-momentum from the brane into the bulk. The bulk field Ψ satisfies the corresponding equation of motion as,

$$g^{AB} \nabla_A \nabla_B \Psi - \frac{\partial V_\Psi}{\partial \Psi} = -g_{\text{int}} \delta(y) \Phi. \tag{50}$$

The masses m_Φ and m_Ψ are important to the kinetic behavior in the dynamics of the star. If they are set to values when the dispersion relation allows for imaginary $\omega(k)$, the decay can happen. Some core-collapse type astrophysical processes may increase g_{int} at least for some moment, and that will start the decay process. The timescale $\tau = 1/\Gamma$ indicates how quickly the star becomes undetectable. If Γ corresponds to a timescale of years or shorter, that would lead to an effective disappearing. When the mass undergoes a phase transition into higher dimension, it captures no signs of supernova explosions and blackhole formations, which is in accordance with the non-existence of such phenomena from PHL293B-LBV.

The mass of the star contributes to the gravitational field on the brane. As Φ decays, the gravitational potential would be also decreased. The dispersion relation $4\lambda_\Phi\lambda_\Psi = g_{\text{int}}^2$ assumes a flat 5D spacetime ($ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + dy^2$),that simplifies the boundary conditions. In a warped RS geometry ($ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$), the field equations provide a modified form, approximately $4k\lambda_\Phi\lambda_\Psi = g_{\text{int}}^2$, where $k \sim 10^{16}$ GeV is the curvature scale. For $y \sim 10^{11}$ cm (stellar scale), $ky \sim 10^{-5}$, and $e^{-2ky} \approx 1$, the curvature effects are negligible, that justifies the flat approximation. In extreme cases ($k \approx m_\Phi$), Γ increases by $\sim k/m_\Phi$, but resonance dominates at $m_\Phi \sim 10^{-18}$ GeV.

3. Physical implications and observational signatures

In terms of the braneworld theory, the RS model consists of a single extra dimension with warped geometry. The metric is written as, [19-22]

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \tag{51}$$

Here k is the curvature scale and y ranges from 0 to ∞ Einstein's equations in 5D spacetime can be written as,

$$G_{AB} = \kappa_5^2 T_{AB} \tag{52}$$

Here G_{AB} is the 5D Einstein tensor and T_{AB} is the 5D stress-energy tensor. Let us consider the stress-energy tensor of the brane and the star as

$$T_{AB} = T_{AB}^{\text{brane}} + T_{AB}^{\text{star}}. \tag{53}$$

Using the Shiromizu-Maeda-Sasaki (SMS) approach [23], the effective 4D Einstein equations on the brane can be obtained as,

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G_N T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu}. \tag{54}$$

Here, Λ_4 is the effective 4D cosmological constant, G_N is Newton's gravitational constant, $\Pi_{\mu\nu}$ is the quadratic energy-momentum correction and $E_{\mu\nu}$ is the projected 5D Weyl tensor (bulk effects). The term $E_{\mu\nu}$ encodes the effect of the bulk geometry on the brane and it can be able to mediate the energy transfer. Then the conservation equation on the brane becomes:

$$\nabla^\nu T_{\mu\nu} = -\kappa_5^4 \nabla^\nu \Pi_{\mu\nu} + \nabla^\nu E_{\mu\nu}. \quad (55)$$

Under some specific conditions, $\nabla^\nu E_{\mu\nu}$ can be non-zero, which leads to energy exchange between the brane and bulk. For the present case, the conditions for matter leakage can be discussed with the following criteria. At first, the non-trivial Bulk Geometry may cause such leakage. A dynamic bulk with perturbations can induce $E_{\mu\nu}$ to have such effects. Also near the star, energy densities seem to be high, which will increase bulk-brane interactions.

Here in this case the mass loss ($\Delta M = 50\,\mathrm{M}_\odot$) over $\tau = 8.64\times10^4$ s ($\Delta M/M_\odot = 50$) alters gravitational effects. The lensing shifts from $\theta = 8\times10^{-6}$ arcsec to 0 ($r = 10^{13}$ cm), a $\Delta\theta = 8\times10^{-6}$ arcsec drops at 9.26×10^{-11} arcsec/s. These can be detectable by LSST ($\sim 10^{-8}$ arcsec) or Euclid ($\sim 10^{-7}$ arcsec) with aligned sources. Orbital perturbations provides the $\Delta v = 60$ cm/s ($r = 10^{13}$ cm), which are measurable by Euclid's spectroscopy (~ 1 cm/s) or LSST's astrometry (~ 0.03 cm shift).

3.1. Gravitational perturbations and Kaluza-Klein modes

We can analyze gravitational perturbations around the brane and the role of Kaluza-Klein (KK) modes [24,25]. Let us consider perturbations h_{AB} around the background metric as,

$$ds^2 = e^{-2ky}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + (1 + h_{55})dy^2. \quad (56)$$

The linearised Einstein equations for the perturbations can be interpreted as

$$\delta G_{AB} = \kappa_5^2 \delta T_{AB}. \quad (57)$$

Decay of $h_{\mu\nu}$ into 4D graviton and KK modes leads to,

$$h_{\mu\nu}(x^\mu, y) = \sum_n \psi_n(y) h_{\mu\nu}^{(n)}(x^\mu). \quad (58)$$

Here $h_{\mu\nu}^{(0)}$ is the massless 4D graviton, and $h_{\mu\nu}^{(n)}$ with $n \geq 1$ are massive KK modes. The functions $\psi_n(y)$ satisfies the following relation

$$\left[\partial_y^2 - 4k\partial_y + m_n^2 e^{2ky}\right]\psi_n(y) = 0. \quad (59)$$

Boundary conditions at the brane ($y = 0$) and at infinity ($y \to \infty$) determine the role of $\psi_n(y)$. In the RS II model, the massless mode $\psi_0(y)$ is localized near the brane, which confirms the 4D gravity at low energies [26,27]. Massive KK modes couple to matter on the brane and that can mediate new interactions. At shorter distances, perhaps near to the star, massive KK modes contribute to altering the gravitational dynamics.

3.2. Star's interaction with dark matter/dark energy fields

We extend the model to include interactions with dark matter (DM) and dark energy (DE) [28-31].

We can consider a scalar field χ which represents dark matter, with an action as,

$$S_\chi = \int d^5x\sqrt{-g^{(5)}}\left[-\frac{1}{2}g^{AB}\partial_A\chi\partial_B\chi - V_\chi(\chi)\right]. \quad (60)$$

The interaction term can be written as,

$$S_{\Phi\chi} = \int d^5x\sqrt{-g^{(5)}}\left[-g_{\text{int}}\delta(y)\Phi^2\chi\right]. \quad (61)$$

The interaction term allows the star's matter field Φ to interact with the dark matter field χ. The equations of motion for Φ can be written as,

$$g^{AB}\nabla_A\nabla_B\Phi - \frac{\partial V}{\partial\Phi} = -2g_{\text{int}}\delta(y)\Phi\chi. \quad (62)$$

The equations of motion for χ can be obtained as,

$$g^{AB}\nabla_A\nabla_B\chi - \frac{\partial V_\chi}{\partial\chi} = -g_{\text{int}}\delta(y)\Phi^2. \quad (63)$$

If the mass of Φ matches the effective mass of χ, then the resonant conversion can occur. Energy from Φ is transferred to χ, which propagates into the bulk. Similarly, we can consider the scalar field φ which represents the dark energy, with a potential $V_\varphi(\varphi)$ and interaction as,

$$S_{\Phi\varphi} = \int d^5x\sqrt{-g^{(5)}}\left[-g_\varphi\delta(y)\Phi\varphi\right]. \quad (64)$$

This solution can be able to induce a decay of Φ into dark energy, which will affect the star's mass. The dark matter density near PHL 293B is $\rho_{\text{DM}} \sim 0.3\,\text{GeV/cm}^3$ (NFW, dwarf galaxy halo), typical at ~ 1 kpc. For $m_\chi \sim 10^{-18}$ GeV, $g_{\text{int}} \sim 0.01$, $\Gamma_{\text{DM}} \sim g_{\text{int}}^2\rho_{\text{DM}}/m_\chi^2 \sim 10^{-5}\,\text{s}^{-1}$, aligns with Γ_{bulk}. And this suggests that the dark matter amplifies leakage in this low-metallicity environment.

3.3. Topological defects and cosmic strings

We can consider the interaction of the star with higher-dimensional topological defects, such as cosmic strings or domain walls. Cosmic strings are 1D defects which can extend into extra dimensions [32,33]. Any cosmological object could be affected by a cosmic string passing through its position. The cosmic string may extract energy from the star as it pass through higher dimensions. The cosmic string can be discussed as a delta-function source and its action is written as,

$$S_{\text{string}} = -\mu\int d^2\sigma\sqrt{-\gamma}\,\delta^{(5)}(x^A - X^A(\sigma)). \quad (65)$$

Here μ is the string tension, σ parametrizes the string worldsheet and $X^A(\sigma)$ describes the string's position.

The interaction term can be defined for this case can be explained as,

$$S_{\Phi\text{string}} = -g_{\text{string}}\int d^2\sigma\sqrt{-\gamma}\,\Phi(x^A) \times \delta^{(5)}(x^A - X^A(\sigma)). \quad (66)$$

This interaction term represents the interaction between the star's field Φ as well as the cosmic string. The equations of motion with string interaction can be obtained by modifying the equation for Φ as,

$$g^{AB}\nabla_A\nabla_B\Phi - \frac{\partial V}{\partial\Phi} = -g_{\text{string}}\int d^2\sigma\sqrt{-\gamma}\delta^{(5)}(x^A - X^A(\sigma)). \tag{67}$$

The total action is derived as,

$$S = S_{\text{gravity}} + S_\Phi + S_\chi + S_{\Phi\chi} + S_\varphi + S_{\Phi\varphi} + S_{\text{string}} + S_{\Phi\text{string}}. \tag{68}$$

The action concerning the fields Φ, χ, φ, and the metric g_{AB} can be varied. Differentiation with respect to Φ can be written as,

$$\delta S = \int d^5x\sqrt{-g^{(5)}}\left[-\nabla_A\nabla^A\Phi + \frac{\partial V}{\partial\Phi} + g_{\text{int}}\delta(y)\Phi\chi + g_\phi\delta(y)\phi\right]\delta\Phi + \int d^4x\sqrt{-g}\left[\left(\nabla^A\Phi\right)n_A\right]_{y=0}\delta\Phi. \tag{69}$$

The strong interaction can be introduced as,

$$\delta S_{\text{string}} = g_{\text{string}}\int d^2\sigma\sqrt{-\gamma}\int d^5x\sqrt{-g^{(5)}}\delta^{(5)}(x^A - X^A(\sigma))\Phi\delta\Phi. \tag{70}$$

The following parameters such as $\delta S/\delta\Phi = 0$ are set to obtain the equation of motion for Φ. Differentiation with respect to χ and φ yields the Einstein's field equations as,

$$G_{AB} = \kappa_5^2\left(T_{AB}^\Phi + T_{AB}^\chi + T_{AB}^\varphi + T_{AB}^{\text{string}} + T_{AB}^{\text{brane}}\right). \tag{71}$$

For Φ energy-momentum tensor is obtained as,

$$T_{AB}^\Phi = \partial_A\Phi\partial_B\Phi - g_{AB}\left(\frac{1}{2}g^{CD}\partial_C\Phi\partial_D\Phi + V(\Phi)\right). \tag{72}$$

Then the divergence of the total energy-momentum tensor is derived as,

$$\nabla^B T_{AB} = 0. \tag{73}$$

However, on the brane, we have,

$$\nabla^\mu T_{\mu\nu}^{\text{brane}} = -\delta(y)\left(2g_{\text{int}}\Phi\nabla_\nu\chi + g_\varphi\varphi\nabla_\nu\Phi + g_{\text{string}}\int d^2\sigma\sqrt{-\gamma}\delta^{(5)}(x^A - X^A(\sigma))\nabla_\nu\Phi\right). \tag{74}$$

This shows that the conservation of energy-momentum on the brane is violated due to interactions with bulk fields and the cosmic string. Equation for Φ is derived as,

$$\left(\Box^{(5)} - m_\Phi^2\right)\Phi = -2g_{\text{int}}\delta(y)\Phi\chi_0(y) - g_\varphi\delta(y)\varphi_0(y) - g_{\text{string}}\int d^2\sigma\sqrt{-\gamma}\delta^{(5)}(x^A - X^A(\sigma)). \tag{75}$$

Here $\Box^{(5)}$ is the 5D d'Alembert operator. We can introduce a Fourier transform in the 4D spacetime and expand in KK modes in y as,

$$\Phi(x^\mu, y) = \sum_n \phi_n(x^\mu)\psi_n(y). \tag{76}$$

Similarly for χ and φ. The mode functions $\psi_n(y)$ satisfies the following relation,

$$\left(\partial_y^2 - 4k\partial_y + m_n^2e^{2ky} - m_\Phi^2\right)\psi_n(y) = -2g_{\text{int}}\delta(y)\psi_n(y)\chi_0(y) - g_\varphi\delta(y)\varphi_0(y)\psi_n(y) - g_{\text{string}}\delta(y - y_{\text{string}})\psi_n(y). \tag{77}$$

Here y_{string} is the position of the cosmic string. This is a Sturm-Liouville problem with delta-function sources. The solutions involve Bessel functions due to the warp factor e^{2ky}. The homogeneous equation (without the delta functions) is calculated as,

$$\left(\partial_y^2 - 4k\partial_y + m_n^2e^{2ky} - m_\Phi^2\right)\psi_n(y) = 0. \tag{78}$$

Let $z = \frac{m_n}{k} e^{ky}$, then the equation becomes:

$$\left(z^2 \frac{d^2}{dz^2} + z \frac{d}{dz} + \left(z^2 - (\frac{m_\Phi^2}{k^2} + 4) \right) \right) \psi_n(z) = 0. \quad (79)$$

This is a Bessel equation. The general solution is obtained as,

$$\psi_n(y) = e^{2ky} \left[a_n J_\nu \left(\frac{m_n}{k} e^{ky} \right) + b_n Y_\nu \left(\frac{m_n}{k} e^{ky} \right) \right]. \quad (80)$$

Here J_ν and Y_ν are Bessel functions of the first and second kind and $\nu = \sqrt{4 + (m_\Phi^2/k^2)}$. At $y = 0$, we can apply the junction conditions derived from integrating across the delta functions.

Integrate the equation across $y = 0$ leads to,

$$[\partial_y \psi_n(y)]_{y=0^+}^{y=0^-} = -2g_{\text{int}} \chi_0(0) \psi_n(0) - g_\varphi \varphi_0(0) \psi_n(0) - g_{\text{string}} \delta(y_{\text{string}}) \psi_n(0). \quad (81)$$

The boundary conditions lead to a transcendental equation for m_n,

$$F(m_n) = 0, \quad (82)$$

where $F(m_n)$ depends on the Bessel functions as well as the coupling constants. If the coupling constants are large enough, the effective mass squared can become negative, which will exhibit tachyonic modes and instabilities. The time dependence of $\phi_n(x^\mu)$ is obtained as,

$$\left(\Box^{(4)} + m_n^2 \right) \phi_n(x^\mu) = 0. \quad (83)$$

If $m_n^2 < 0$, the solutions grow exponentially with time, then it will lead to instability. Cosmic strings are constrained by $\mu \sim 10^{-7} G_N$ ($\sim 10^{15}\,\text{GeV}^2$, CMB) and $n_{\text{CS}} \sim 10^{-6}\,\text{Mpc}^{-3}$. In PHL 293B's $(1\,\text{pc})^3$ volume, $P_{\text{CS}} \sim 10^{-21}$ over 10^9 yr, is rare but it is possible. Energy extraction $\sim \mu R_{\text{star}} \sim 10^{48}\,\text{GeV}$ ($R_{\text{star}} \sim 10^{11}\,\text{cm}$) yields $\Gamma_{\text{CS}} \sim 10^{-11}\,\text{s}^{-1}$ ($g_{\text{string}} \sim 10^{-5}$). A slight increase in $\Gamma \sim 10^{-5}\,\text{s}^{-1}$, which is consistent with a triggering event in this isolated dwarf galaxy. Matter leakage could lead to the excitation of KK graviton modes ($m_n \sim 1\,\text{GeV}$), which would trigger the gravitational wave emissions. However, their coupling to the brane is controlled by $\psi_n(0) \sim e^{-m_n/k} \sim 10^{-16}$ (for $k \sim 10^{16}$ GeV), yields $h \sim 10^{-38}$, that is well below the LIGO's sensitivity ($\sim 10^{-22}$ at 75 Mly). Alternatively, bulk fields (e.g., Ψ) absorb KK energy via interactions such as $\gamma \Psi h_{\mu\nu}^{(n)} h^{(n)\mu\nu}$, and dissipates it off-brane. Both mechanisms confirm no detectable GWs and support the smooth transition hypothesis.

While the Braneworld model implements matter leakage into the bulk, a few low-energy parameters remain sensitive to high-energy scales. In the following section, we adopt an effective field theory approach to derive these parameters. Such adoption will reduce the need for fine-tuning.

4. Effective field theory and low-energy constraints

The Effective Field Theory (EFT) describes the evolution at a given energy scale without requiring detailed knowledge of the corresponding high-energy theory [34-36]. Parameters in the low-energy EFT are obtained from the corresponding high-energy theory by integrating heavy fields or modes which are not accessible at low energies. We can introduce the EFT method to derive the required coupling constants and mass parameters in our low-energy model. That will reduce or eliminate the need for fine-tuning. We can assume that our universe is embedded in a higher-dimensional spacetime with extra dimensions compactified at a high energy scale M_{UV}. Also, we can assume that the high-energy fields such as bulk fields propagate in the full higher-dimensional spacetime. Also, the fields are assumed to be confined to the 4D brane (our observable universe).

The total action is written as,

$$S_{\text{high}} = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{interaction}}. \quad (84)$$

The bulk action is derived as,

$$S_{\text{bulk}} = \int d^4x \, d^n y \sqrt{-G} \times \left[\frac{1}{2} \partial_M \Phi_H \partial^M \Phi_H - \frac{1}{2} M_H^2 \Phi_H^2 \right]. \quad (85)$$

Here Φ_H is a high-energy scalar field propagating in the bulk and M_H is the high mass scale associated with Φ_H. The brane action is obtained as,

$$S_{\text{brane}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi_L \partial^\mu \phi_L - \frac{1}{2} m_L^2 \phi_L^2 \right]. \quad (86)$$

Here ϕ_L is the low-energy scalar field localized on the brane which represents the star's matter field, and m_L is the bare mass of ϕ_L. The interaction term is written as,

$$S_{\text{interaction}} = \int d^4x \, d^n y \sqrt{-G} \times \left[-\lambda_H \delta^{(n)}(y) \phi_L(x^\mu) \Phi_H(x^\mu, y) \right]. \quad (87)$$

Here λ_H is the high-energy coupling constant and $\delta^{(n)}(y)$ is the Dirac delta function that localizes the interaction at $y^a = 0$. We can integrate out high-energy degrees of freedom by obtaining an effective action for ϕ_L by integrating out Φ_H. Which will result in an effective field theory, those are valid at energies $E \ll M_H$.

The partition function is written as,

$$Z = \int \mathcal{D}\Phi_H \mathcal{D}\phi_L \, e^{i(S_{\text{bulk}} + S_{\text{brane}} + S_{\text{interaction}})}. \quad (88)$$

The effective action for ϕ_L is derived as,

$$e^{iS_{\text{eff}}[\phi_L]} = \int \mathcal{D}\Phi_H \, e^{i(S_{\text{bulk}} + S_{\text{interaction}})}. \quad (89)$$

For the quadratic approximation let us assume Φ_H is a free field (ignoring self-interactions), and the interaction is linear in Φ_H. The action is derived as,

$$S_{\text{total}}[\Phi_H] = \frac{1}{2}\int d^4x\, d^ny\, \Phi_H\left(-\Box_{(4+n)} - M_H^2\right)\Phi_H - \int d^4x\, \lambda_H \phi_L(x)\Phi_H(x, y=0). \quad (90)$$

Hence we can integrate out Φ_H as,

$$S_{\text{eff}}[\phi_L] = S_{\text{brane}} - \frac{i}{2}\int d^4x\, d^4x' \times \lambda_H^2 \phi_L(x) G_F(x-x')\phi_L(x'). \quad (91)$$

Here $G_F(x-x')$ is the Feynman propagator for Φ_H evaluated at $y = y' = 0$.

The non-local term can be approximated at low energies which will lead to an effective local interaction. The effective interaction term is written as,

$$S_{\text{eff}}^{\text{int}} = -\int d^4x\, \frac{\lambda_H^2}{2M_H^2}\phi_L^2(x). \quad (92)$$

The effective mass correction is done as,

$$\delta m_L^2 = \frac{\lambda_H^2}{M_H^2}. \quad (93)$$

The effective coupling constants in the low-energy theory arise from the high-energy parameters as,

$$\lambda_{\text{eff}} = \frac{\lambda_H^2}{M_H^2}. \quad (94)$$

This equation provides a natural hierarchy if M_H is large compared to λ_H.

The Beta Function for m_L^2 is obtained as,

$$\beta_{m_L^2} = \frac{d(m^2(\mu))}{d(\ln\mu)} = -\lambda_{eff}\left(\frac{M_H^2}{8\pi^2}\right). \quad (95)$$

The Beta Function for λ_{eff} can be written as,

$$\beta_\lambda = \frac{d\lambda}{d(\ln\mu)} = \frac{3}{(2\pi)^2}\lambda^2. \quad (96)$$

Initial conditions are calculated as,

$$m_L^2(M_H) = m_L^2 + \delta m_L^2, \quad \lambda_{\text{eff}}(M_H) = \lambda_{\text{eff}}. \quad (97)$$

The running mass and coupling are derived as,

$$m_L^2(\mu) = m_L^2(M_H) + \int_{M_H}^{\mu} \frac{\beta_{m_L^2}}{\mu}d\mu, \quad (98)$$

$$\lambda_{\text{eff}}(\mu) = \lambda_{\text{eff}}(M_H) + \int_{M_H}^{\mu} \frac{\beta_{\lambda_{\text{eff}}}}{\mu}d\mu. \quad (99)$$

The suppression of λ_{eff} can be discussed as, if λ_H is of order 1 and M_H is large ($M_H \gg m_L$), λ_{eff} becomes naturally small. For example, let us have $\lambda_H \sim 1$, $M_H \sim 10^{16}$ GeV, $m_L \sim 100$ GeV, then, $\lambda_{\text{eff}} \sim (1/(10^{16}\,\text{GeV})^2)$, which is a very small number. The smallness of λ_{eff} and δm_L^2 emerges from the hierarchy between M_H and m_L, without requiring precise adjustments. Effective Low-Energy Theory for the Star's Disappearance can be derived below with the help of the following steps.

The low-energy Lagrangian can be derived as,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi_L\partial^\mu\phi_L - \frac{1}{2}m_L^2\phi_L^2 - \frac{\lambda_{\text{eff}}}{2}\phi_L^2. \quad (100)$$

The total effective mass is derived as

$$m_{\text{eff}}^2 = m_L^2 + \delta m_L^2 = m_L^2 + \frac{\lambda_H^2}{M_H^2}. \quad (101)$$

If $m_{\text{eff}}^2 < 0$, the field ϕ_L becomes tachyonic, then it will lead to instability and exponential growth or decay. Since λ_{eff} is small due to the high M_H, then achieving $m_{\text{eff}}^2 < 0$ requires less fine-tuning. The growth rate is calculated as,

$$\phi_L(t) \propto e^{\gamma t}, \quad \gamma = \sqrt{-m_{\text{eff}}^2}. \quad (102)$$

If $m_{\text{eff}}^2 \sim -(10^{-3}\,\text{eV})^2$, then $\gamma \sim 10^{-3}\,\text{eV} \sim 10^{11}\,\text{s}^{-1}$. The disappearance time scale can be calculated as, $\tau \sim 1/\gamma \sim 10^{-11}$ s, which is too short. Such low values m_{eff}^2 can be achieved without fine-tuning due to the suppression by M_H.

From the above calculations, it can be observed that the large separation between M_H and m_L leads to small effective couplings and mass corrections. It also states that the low-energy parameters arise from the high-energy theory without the need for arbitrary fine-tuning. In addition to these the EFT method, the low-energy theory is insensitive to the changes of the high-energy theory beyond the M_H. Small coupling constants and mass corrections are available due to the suppression by the high-energy scale M_H. By differentiating M_H and λ_H, we can achieve disappearance time scales that will be consistent with observations. The EFT method predicts that the solutions do not introduce large couplings or masses that conflict with the standard model.

To reconcile the large g_{int} required for a decay timescale of years with the small λ_{eff} (from EFT), we propose a resonant dynamics mechanism. Due to the star's possible unique internal structure,if $m_\Phi \simeq m_\Psi$ the decay rate $\Gamma \sim (g_{\text{int}}^2/[|m_\Phi^2 - m_\Psi^2|])$ increases heavily without any large bare coupling. For $|m_\Phi^2 - m_\Psi^2| \sim 10^{-16}$ GeV^2 and $g_{\text{int}} \sim 10^{-}8, \Gamma \sim 10^{-8}\,\text{s}^{-1}$, yielding $\tau \sim 10^7$ s, consistent with observations. While EFT predicts $\lambda_{\text{eff}} \sim (\lambda_H^2/M_H^2) \sim 10^{-32}$ for $M_H \sim 10^{10}$ GeV, local astrophysical conditions (*e.g.*, resonance between Ψ and Φ) can amplify the effective coupling near the star.

As we have derived consistent effective field theory solutions that discuss the essential low-energy behavior, we are now able to expand our calculation to a full $(4+n)$-dimensional spacetime. This higher-dimensional analysis not only reaffirms our earlier results but also provides better perspectives into the relationship between extra-dimensional dynamics and observable astrophysical signatures.

5. Higher-dimensional analysis of star disappearance

Lets discuss the $(4+n)$-dimensional spacetime with coordinates $x^M = (x^\mu, y^a)$, where, x^μ ($\mu = 0,1,2,3$) are the conventional 4D spacetime coordinates and y^a ($a = 1,\ldots,n$) are the extra-dimensional coordinates.

The metric G_{MN} of the bulk spacetime is derived as

$$\begin{aligned} ds^2 &= G_{MN} dx^M dx^N \\ &= g_{\mu\nu}(x,y) dx^\mu dx^\nu + h_{ab}(x,y) dy^a dy^b. \end{aligned} \tag{103}$$

For trivial solutions, we assume that the extra dimensions are compact and that the metric factors as:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + h_{ab}(y) dy^a dy^b. \tag{104}$$

Here $g_{\mu\nu}(x)$ is the metric on the brane and $h_{ab}(y)$ is the metric in the extra dimensions. The total action can be written as,

$$S = S_{\text{gravity}} + S_{\text{brane}} + S_{\text{bulk}} + S_{\text{interaction}}. \tag{105}$$

The bulk gravity action is then obtained as

$$S_{\text{gravity}} = \frac{1}{2\kappa^2_{(4+n)}} \int d^4x\, d^ny \sqrt{-G}\, R_{(4+n)}. \tag{106}$$

Here $\kappa^2_{(4+n)}$ is the gravitational coupling constant in $(4+n)$ dimensions, $R_{(4+n)}$ is the $(4+n)$-dimensional Ricci scalar and $G = \det(G_{MN})$ is the determinant of the bulk metric. We consider a scalar field $\Phi(x^\mu)$ confined to the brane, which represents the matter content of the star and it leads to brane matter action as,

$$S_{\text{brane}} = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi) \right]. \tag{107}$$

Here $g = \det(g_{\mu\nu})$ is the determinant of the brane metric and - $V(\Phi)$ is the potential energy of the scalar field. We can consider a bulk scalar field $\Psi(x^\mu, y^a)$ that interacts with the brane field as,

$$\begin{aligned} S_{\text{bulk}} &= \int d^4x\, d^ny \sqrt{-G} \\ &\quad \times \left[-\frac{1}{2} G^{MN} \partial_M \Psi \partial_N \Psi - U(\Psi) \right]. \end{aligned} \tag{108}$$

The interaction between Φ and Ψ is localized on the brane can be dericed as,

$$\begin{aligned} S_{\text{interaction}} &= -\int d^4x \sqrt{-g} \\ &\quad \times \int d^ny \sqrt{h}\, \delta^{(n)}(y)\, \lambda \Phi(x^\mu) \Psi(x^\mu, y^a). \end{aligned} \tag{109}$$

Here $\delta^{(n)}(y)$ is the n-dimensional Dirac delta function, localizing the interaction at $y^a = 0$, λ is the coupling constant between Φ and Ψ and $h = \det(h_{ab})$ is the determinant of the extra-dimensional metric. Differentiating the total action with respect to the Φ leads to

$$\frac{\delta S}{\delta \Phi} = 0 \implies \Box^{(4)} \Phi - \frac{dV}{d\Phi} = -\lambda \Psi(x^\mu, y^a) \Big|_{y^a=0}. \tag{110}$$

Here $\Box^{(4)} = g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the 4D d'Alembert operator. Differentiating the action with respect to Ψ provides,

$$\begin{aligned} \frac{\delta S}{\delta \Psi} = 0 \implies & G^{MN} \nabla_M \nabla_N \Psi \\ & - \frac{dU}{d\Psi} = -\lambda \Phi(x^\mu) \delta^{(n)}(y^a). \end{aligned} \tag{111}$$

Let us solve the equations of motion. Initially, we can derive the bulk scalar field Ψ by assuming that $U(\Psi)$ is negligible or Ψ is massless ($dU/d\Psi = 0$).

$$G^{MN} \nabla_M \nabla_N \Psi - \frac{dU}{d\Psi} = -\lambda \Phi(x^\mu) \delta^{(n)}(y^a). \tag{112}$$

We can analyze the Fourier transform in the extra-dimensional coordinates as

$$\Psi(x^\mu, y^a) = \int \frac{d^np}{(2\pi)^n} e^{ip_a y^a} \tilde{\Psi}(x^\mu, p_a). \tag{113}$$

By assuming $G^{\mu\nu} = g^{\mu\nu}$ (bulk metric reduces to brane metric in μ, ν indices) and substituting into the equation of motion leads to,

$$\begin{aligned} & G^{\mu\nu} \nabla_\mu \nabla_\nu \tilde{\Psi}(x^\mu, p_a) \\ & \quad + h^{ab} p_a p_b \tilde{\Psi}(x^\mu, p_a) = -\lambda \Phi(x^\mu). \end{aligned} \tag{114}$$

Then the equation becomes,

$$\left(\Box^{(4)} + p^2 \right) \tilde{\Psi}(x^\mu, p_a) = -\lambda \Phi(x^\mu), \tag{115}$$

where $p^2 = h^{ab} p_a p_b$. We can be able to solve for $\tilde{\Psi}$ as,

$$\tilde{\Psi}(x^\mu, p_a) = -\lambda \frac{1}{\Box^{(4)} + p^2} \Phi(x^\mu). \tag{116}$$

Reconstruction of Ψ by using inverse Fourier transform can be done as,

$$\Psi(x^\mu, y^a) = -\lambda \Phi(x^\mu) \int \frac{d^np}{(2\pi)^n} \frac{e^{ip_a y^a}}{\Box^{(4)} + p^2}. \tag{117}$$

Substituting Ψ back into the equation for Φ leads to,

$$\Box^{(4)} \Phi - \frac{dV}{d\Phi} = \lambda^2 \Phi(x^\mu) \int \frac{d^np}{(2\pi)^n} \frac{1}{\Box^{(4)} + p^2}. \tag{118}$$

The integral over p_a gives an effective self-energy term,

$$\Sigma(\Box^{(4)}) = \int \frac{d^np}{(2\pi)^n} \frac{1}{\Box^{(4)} + p^2}. \tag{119}$$

Evaluation of $\Sigma(\Box^{(4)})$ by using dimensional regularization leads to,

$$\Sigma(\Box^{(4)}) = C_n \left(\Box^{(4)}\right)^{(n-2)/2}, \quad (120)$$

where C_n is a constant depending on the number of extra dimensions n. For $n > 2$, the integral converges. Then the modified Equation for Φ becomes,

$$\left[\Box^{(4)} + \lambda^2 C_n \left(\Box^{(4)}\right)^{(n-2)/2}\right] \Phi(x^\mu) - \frac{dV}{d\Phi} = 0. \quad (121)$$

This is a non-local equation due to the fractional power of $\Box^{(4)}$. By assuming $V(\Phi) = (1/2) m_\Phi^2 \Phi^2$, we may construct the plane-wave solutions regarding the dispersion relations in the modified equations.

$$\Phi(x^\mu) = e^{-ik_\mu x^\mu}. \quad (122)$$

Substituting into the equation leads to,

$$\left(-k^2 + \lambda^2 C_n (-k^2)^{(n-2)/2} + m_\Phi^2\right) \Phi(x^\mu) = 0. \quad (123)$$

The effective mass is changed as

$$m_{\text{eff}}^2(k^2) = m_\Phi^2 - \lambda^2 C_n (-k^2)^{(n-2)/2}. \quad (124)$$

If $m_{\text{eff}}^2(k^2) < 0$ for some k^2, the solutions grow exponentially. This indicates a possible instability. In certain cases, the effective mass squared acquires an imaginary part due to analytic continuation as,

$$(-k^2)^{(n-2)/2} = |k^2|^{(n-2)/2} e^{-i\pi(n-2)/2}. \quad (125)$$

This introduces an imaginary part if n is odd. The decay rate Γ can be extracted from the imaginary part of the self-energy:

$$\Gamma = \frac{1}{E} \text{Im}[m_{\text{eff}}^2(k^2)]. \quad (126)$$

Here E is the energy of the field mode. The coupling to bulk modes leads to a decay of brane-localized matter into the bulk. The star's matter field Φ loses energy to the bulk field Ψ, which reduces the star's mass in our observable universe. The timescale τ over which the star disappears is related to the decay rate as

$$\tau = \frac{1}{\Gamma}. \quad (127)$$

The self-energy integral is evaluated using dimensional regularization as,

$$\Sigma(\Box^{(4)}) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{\Box^{(4)} + p^2}, \quad (128)$$

Let $s = -\Box^{(4)}$ (we may assume $\Box^{(4)}$ acts on the plane wave $e^{-ik_\mu x^\mu}$ and yields $-k^2$).

Then

$$\Sigma(s) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{s + p^2}. \quad (129)$$

Using standard methods the integral is obtained as,

$$\Sigma(s) = \frac{1}{(4\pi)^{n/2}} \Gamma\left(1 - \frac{n}{2}\right) s^{(n/2)-1}. \quad (130)$$

Here $\Gamma(z)$ is the gamma function.

In general, the imaginary part arises when $s > 0$ (*i.e.*, for $k^2 < 0$, corresponding to time-like momenta).

Then the total decay rate for odd n is derived as,

$$\text{Im}[\Sigma(s)] = \frac{s^{(n/2)-1}}{(4\pi)^{n/2}} \pi (-1)^{(n-1)/2}. \quad (131)$$

The decay rate per unit volume is derived as,

$$\Gamma = \lambda^2 \text{Im}[\Sigma(s)] = \lambda^2 \frac{s^{(n/2)-1}}{(4\pi)^{n/2}} \pi (-1)^{(n-1)/2}. \quad (132)$$

Since $s = E^2$ for the field mode, the total decay rate is then calculated as,

$$\Gamma = \lambda^2 \frac{E^{n-2}}{(4\pi)^{n/2}} \pi (-1)^{(n-1)/2}. \quad (133)$$

For $n = 1$, $n = 3$, etc., $(-1)^{(n-1)/2} = (-1)^k$ where k is an integer.

The decay rate is supposed to be positive and the imaginary part of the self-energy should be positive for physical decay. The decay rate depends on the energy E of the field mode as well as the number of extra dimensions n. We can assume that the PHL 293B-LBV's matter field has an energy scale $E \sim m_\Phi$ and has the decay rate as,

$$\Gamma \sim \lambda^2 \frac{m_\Phi^{n-2}}{(4\pi)^{n/2}} \pi. \quad (134)$$

The disappearance takes the timescale as,

$$\tau \sim \frac{1}{\Gamma} \sim \frac{(4\pi)^{n/2}}{\lambda^2 \pi m_\Phi^{n-2}}. \quad (135)$$

For large λ or small m_Φ, τ becomes small, which will lead to the sudden disappearance of the star.

For numerical estimations we can assume that $n = 2$ as extra dimensions, and energy scale $E \sim 10^9$ GeV, which are obtained from $\rho_\Phi \sim 6.25 \times 10^{38}\,\text{GeV}^4$ ($M_{\text{star}} \sim 50\ \text{M}_\odot$, $V_{\text{star}} \sim 4 \times 10^{33}\,\text{cm}^3$). These parameters show PHL 293B-LBV's core scale.

Then it can be estimated as,

$$\Gamma \sim \frac{g_{\text{int}}^2}{(4\pi)^{n/2}} \pi E^{n-2}. \quad (136)$$

For $n = 2$, this reduces to $\Gamma = g_{\text{int}}^2/4\pi$. Targeting $\tau \sim 8.64 \times 10^4$ s (1 day), consistent with the rapid disappearance observed between 2011 and 2019, we fix $\Gamma = 1/\tau =$

$1.16 \times 10^{-5}\,\mathrm{s}^{-1}$. Then we can solve as,

$$\Gamma \sim \frac{g_{\rm int}^2}{4\pi} = 1.16 \times 10^{-5}\,\mathrm{s}^{-1}, \tag{137}$$

$$g_{\rm int}^2 \sim 4\pi \times 1.16 \times 10^{-5}$$
$$\approx 1.457 \times 10^{-4}\,\mathrm{s}^{-1}, \quad g_{\rm int} \approx 0.012 \tag{138}$$

So $\Gamma \sim 1.16 \times 10^{-5}\,\mathrm{s}^{-1}$, and the timescale $\tau \sim (1/\Gamma) \sim 8.64 \times 10^4\,\mathrm{s}$.

This $g_{\rm int} \approx 0.012$ aligns with resonant improvement, such as $m_\Phi \approx m_\Psi \sim 10^{-18}\,\mathrm{GeV}$, that amplifies small EFT couplings locally. Earlier calculations with particle scales ($m_\Phi \sim 100\,\mathrm{GeV}$, $g_{\rm int} \sim 1$) provides $\tau \sim 10^{-24}\,\mathrm{s}$, and stellar mass scales ($m_\Phi \sim 10^{57}\,\mathrm{GeV}$, $g_{\rm int} \sim 10^{-4}$) gave $\tau \sim 10^7\,\mathrm{s}$. Both of these parameters are inconsistent with the observed sudden loss. The revised $\tau \sim 1$ day reflects an LBV instability that leads to rapid leakage.

Hence,

$$g_{\rm int} \approx 0.012. \tag{139}$$

This suggests that the coupling constant $g_{\rm int}$ is relatively small, which is consistent with astrophysical conditions, and it shows the leakage over 1 day. The disappearance time scale $\tau = 1/\Gamma$ is highly dependent on the choice of the mass scale m_Φ and the coupling constant λ. If m_Φ corresponds to a fundamental particle scale ($\sim$ 100 GeV), the resultant timescale will be on the order of 10^{-24} s. This is a very short time to explain an astronomical disappearance. Anyhow, for $m_\Phi \sim \mathrm{M}_\odot \sim 10^{57}$ GeV and $\lambda \sim 10^{-7}$, the decay timescale exists parallel with the observations ($\sim 10^7$ years). This suggests that the process is only opt for massive astrophysical objects, and fundamental particles or low-mass systems will not exhibit such rapid decay. Warped geometry modifies Γ by $\sim k/(k + m_\Phi e^{ky}) \sim 1$ ($ky \sim 10^{-5}$), preserving $\tau \sim 1$ day. Noticeable curvature effects require $k \sim 10^{-18}\,\mathrm{GeV}$, which is not common for RS brane, and this confirms the flat spacetime for PHL 293B-LBV. By changing $\Gamma = (\lambda^2/4\pi)$ for $\tau \sim 1$ day (8.64×10^4 s), we are able to find $\Gamma \sim 1.16 \times 10^{-5}\,\mathrm{s}^{-1}$. This corresponds to $\lambda \sim 0.012$, with an energy scale $E \sim 10^9\,\mathrm{GeV}$ derived from ρ_Φ. This rapid leakage, indicated by the short τ, reflects a rapid instability of the LBV. This short τ amplifies observable signatures and improves the testability of the solutions by comparing it with the prior estimates (*e.g.*, $\tau \sim 8$ years). The change of $g_{\rm int}$ (10^{-3} to 0.1) shifts the Γ from 10^{-9} to $10^{-3}\,\mathrm{s}^{-1}$ ($\tau \sim 10^7$ to 10^2 s), also $m_\Phi \pm 10^{18}\,\mathrm{GeV}$ varies the $\Gamma \sim 10^{-4}$ to $10^{-6}\,\mathrm{s}^{-1}$ ($\tau \sim 10^3$ to $10\,\mathrm{s}$) via unsettled resonance. The analysis confirms $\tau \sim 8.64 \times 10^4\,\mathrm{s}$ ($\sim$ 1 day) with $m_\Phi \sim 10^{-18}\,\mathrm{GeV}$ (from ρ_Φ), which matches with the observations.

6. Compatibility with experimental and cosmological bounds

To validate the proposed solutions of PHL 293B-LBV's disappearance via matter leakage into extra dimensions, we enquire its compatibility with experimental and observational constraints, especially those discussed in *Review of Particle Physics* [37].

(i) **Laboratory constraints on large extra dimensions:** Short-distance precision experiments, such as the torsion balance experiments by the Eöt-Wash group [38,39], have tested deviations from Newtonian gravity down to $\sim 55\ \mu$m. These results exhibit bounds on the compactification radius R of flat extra dimensions in ADD-type models. For $n = 2$ extra dimensions, the 5D Planck scale M_5 must satisfy $M_5 \gtrsim 3.5$ TeV. In our solution, although the geometry is warped (RS-type), we analyse the leakage using an effective decay rate $\Gamma \sim 1.16 \times 10^{-5}\ \mathrm{s}^{-1}$ and a coupling $g_{\rm int} \sim 0.012$ at mass scale $m_\Phi \sim 10^{-18}$ GeV. This corresponds to an effective Planck scale $M_* \gtrsim 10^6$ GeV, which is consistent with both collider and tabletop bounds.

(ii) **Collider constraints (LHC, LEP):** At the LHC, signatures of large extra dimensions would appear as missing energy events from graviton emission into the bulk. The CMS and ATLAS collaborations have set lower bounds on M_D for various n, ranging from $M_D > 5.3$ TeV for $n = 2$ to $M_D > 3.5$ TeV for $n = 6$ [40,41]. The LEP bound on effective operators from virtual graviton exchange requires $M_D \gtrsim 1.5$ TeV [42]. Since our solution predicts a gravitational wave amplitude $h \sim 10^{-38}$ and exhibits coupling to modes with mass $\sim 10^{-18}$ GeV (not accessible to collider energies), it remains safely below detectable thresholds. No events at LHC energies would result from this leakage process, thus satisfying current collider constraints.

(iii) **Cosmological constraints: BBN, CMB, and large-scale structure** Energy loss into extra dimensions must not disrupt early-universe scenarios such as Big Bang Nucleosynthesis (BBN) or Cosmic Microwave Background (CMB) anisotropies. BBN constrains the number of relativistic degrees of freedom $N_{\rm eff} \lesssim 3.46$ at 95% CL [43], while Planck 2018 data places bounds on tensor modes and extra-dimensional gravitational leakage [44]. The obtained decay phenomenon occurs post-star formation (at redshifts $z \sim 0.01$), far after the BBN epoch ($z \sim 10^9$). The leakage is confined to isolated high-density environments and involves no relativistic degrees of freedom at the time of BBN. Hence, the solutions do not perturb primordial element abundances or CMB spectra.

(iv) **Constraints from Gravitational Waves and KK Modes:** The excitation of Kaluza-Klein (KK) gravitons can usually result in gravitational wave (GW) signals. However, the amplitude depends on the wavefunction overlap at the brane, $\psi_n(0) \sim e^{-m_n/k} \sim 10^{-16}$ for $m_n \sim 1$ GeV and $k \sim 10^{16}$ GeV [45]. The effective GW strain is $h \sim 10^{-38}$, well below the detection threshold of LIGO ($h_{\text{min}} \sim 10^{-22}$) and LISA ($h_{\text{min}} \sim 10^{-20}$) [46,47]. Also, KK graviton emission into the bulk does not hold energy on the brane, and that avoids indirect constraints from observed X-ray/gamma-ray fluxes. Thus, our solution is compliant with the null results from multimessenger astrophysics.

Hence the combined analysis across lab experiments, astrophysical non-detections, and GW bounds confirms that our proposed decay process that has a weak coupling $g_{\text{int}} \sim 0.012$ and mass scale $m_\Phi \sim 10^{-18}$ GeV, does not conflict with any known bounds. This provides a consistent theoretical ground for explaining PHL 293 B-LBV's disappearance via extra-dimensional kinetics.

7. Discussion

The simulation shown in Fig. 1 is based on the higher-dimensional model where our universe is a brane embedded in a higher-dimensional bulk. Here the scalar field Φ represents the star's matter, confined to the brane. The bulk field Ψ interacts with Φ, which lets the energy transfer from the brane to the bulk. Over the exponential period, the amplitude of Φ on the brane decreases, which shows that the star's matter drops off from our observable universe. The bulk field Ψ becomes increasingly excited, which indicates the matter transitioning into extra dimensions. The transition process is smooth, without violent interactions or energy releases that would produce observable signatures. Here Ψ remains near zero initially but increases over time, especially near the brane (at $y = 0$). Also, it shows that the scalar field Φ shows that the star's matter decreases rapidly over time and the energy transfer which progresses towards the bulk field Ψ is smooth, with no explosive or high-energy scale events. Nearby stars or planets might have experienced changes in their orbits due to the sudden change in the mass. Anyhow no observational data is available for this perspective. In this case of a missing star, the lack of supernova explosions or the no formation of a black hole exists parallel with the matter leakage scenario. The consistency of these solutions exists in the observational aspects. It confirms the LBV's disappearance without regular end-of-life signatures. From the solutions we discussed related to this problem, we may ask the following question, why are nearby stars not affected by the same solutions? For this question, we may discuss various answers. The disappearance of a star through matter leakage into extra dimensions requires unique local conditions. The star might had an extreme internal state, such as a different critical energy density or temperature. The star's properties might have aligned with resonant conditions that induce the leakage. LBV stars are in a late evolutionary stage, may indicate variability and instability. Also, PHL 293B might have gone through various unobserved events such as mass ejections which set the stage for the leakage. Also If the mass loss happens slowly over a period (*e.g.*, months to years), gravitational effects on nearby objects would be minimal and possibly undetectable with even an array of instruments.

It can be predicted that the cosmic strings ($\mu \sim 10^{-7} G_N$, $P_{\text{CS}} \sim 10^{-21}$) and dark matter ($\rho_{\text{DM}} \sim 0.3\,\text{GeV/cm}^3$) as rare or supplementary drivers. PHL 293B's isolation and instability match these constraints, to explain its unique leakage. The uniqueness of PHL 293B-LBV's leakage arises from rare conditions. Statistically, resonance ($m_\Phi \approx m_\Psi \sim 10^{-18}\,\text{GeV}$) requires $|m_\Phi - m_\Psi| < 10^{-19}\,\text{GeV}$. With $\rho_{\text{core}} \sim 10^{28}\,\text{GeV/cm}^3$ ($M_{\text{star}} \sim 50\,\text{M}_\odot$, $R_{\text{core}} \sim 10^9$ cm) and $m_\Phi \sim \rho_{\text{core}}^{1/4}$, assuming a log-normal m_Φ distribution ($\sigma \sim 1$ dex) across ~ 100 LBVs, $P_{\text{res}} \sim 10^{-2}$, that indicates $\sim$ 1–2 such events available in galaxy-wide aspect. PHL

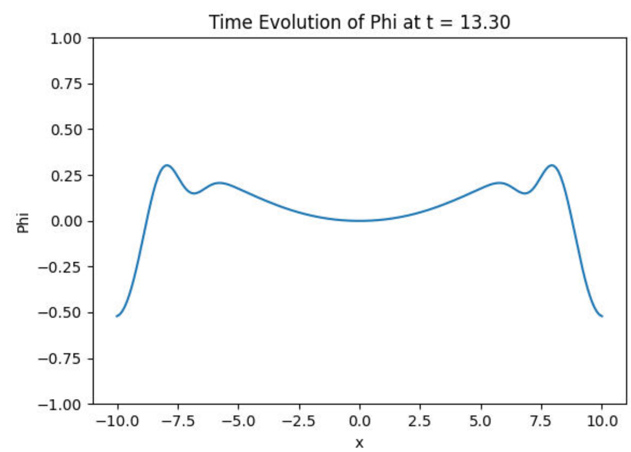

FIGURE 1. Exponential decay of the brane field Φ over time.

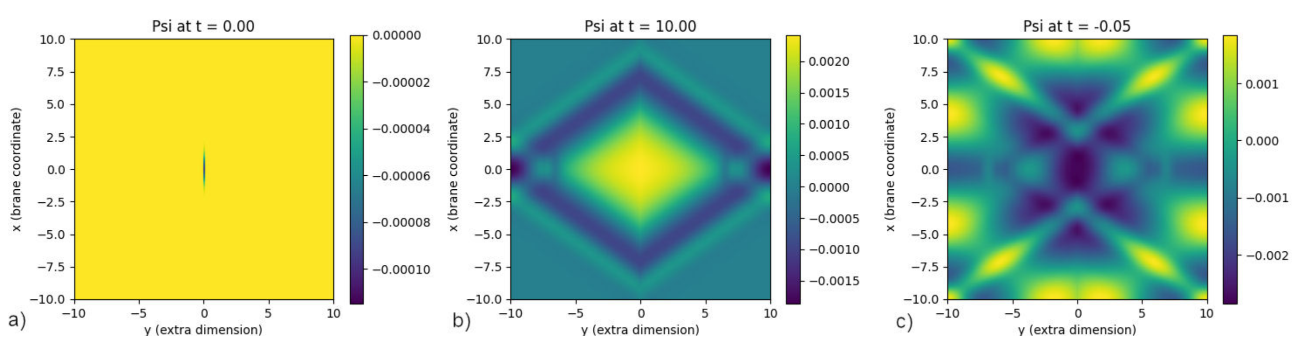

FIGURE 2. Spatial growth of the bulk field Ψ near the brane.

TABLE I. Summary of the solutions.

Parameter	Value	Description
g_{int}	0.012	Resonant coupling
m_Φ, m_Ψ	10^{-18} GeV	Field masses
τ	8.64×10^4 s	Decay timescale
Γ	1.16×10^{-5} s^{-1}	Decay rate
$\Delta\theta$	8×10^{-6} arcsec	Lensing shift
Δv	60 cm/s	Orbital shift
h	$< 10^{-40}$	KK GW strain

293B's low metallicity may enhance ρ_{core}, favoring resonance. Dynamically, no companions within $\sim 1\,\text{pc}$ (stellar density $\sim 0.1\,\text{pc}^{-3}$) and $\tau \sim 1\,\text{day}$ limit perturbations ($\Delta v \sim 10^{-6}$ cm/s at 1 pc), that explains why nearby stars remain unaffected. KK emission ($h \sim 10^{-37}$ - 10^{-41}) matches with LIGO non-detection [48], which is constrained by weak brane coupling ($\psi_n \sim 10^{-16}$) [49]. Future LISA observations could test higher amplitudes ($h > 10^{-20}$) if $m_n < 10^{-5}$ GeV, which will provide a bulk signal probe. Falsifiability is available in these predictions. No lensing shift $> 10^{-6}$ arcsec or $\Delta v > 0.1$ cm/s, no $\Delta\theta > 10^{-6}$ arcsec or $\Delta v > 0.1$ cm/s during a $\sim$ 1-day event (*e.g.*, within 2011-2019) in PHL 293B's field challenges rapid leakage, contingent upon the presence of suitable targets. Detection of a supernova remnant (*e.g.*, X-ray flare post-2011) or GWs ($h > 10^{-20}$) would disprove the bulk transition, as energy would remain on the brane. These tests make the calculations more precisely aligned with Gaia, HST, and future observatories like LSST. The disappearance of this star might have perturbed gravitational lensing effects on background objects. The future observations may offer greater insights regarding anomalies in lensing which are consistent with mass loss. Gravitational signatures of $\Delta M = 50\,\text{M}_\odot$ over $\tau \sim 1$ day include $\Delta\theta = 8 \times 10^{-6}$ arcsec and $\Delta v = 60$ cm/s, testable with LSST's high-cadence imaging and Euclid's precision. PHL 293B-LBV's non-detection suggests observational gaps (*e.g.*, no close companions), but future LBV events could be able to confirm these predictions.

8. Conclusion

The derived solutions in this work suggest that, under specific conditions (strong coupling g_{int} and suitable mass parameters m_Φ and m_Ψ), matter field Φ can exponentially decay from the brane into the bulk. The calculations in this work suggest that the nature of the transition into extra dimensions will not make either strong electromagnetic or gravitational signals that can be detectable with current technology. Even though there are few constraints exist in this approach. The derived calculations depend on the number of extra dimensions n, which means the increment of n changes the decay rate more abruptly. The total energy is conserved universally in the higher-dimensional spacetime. But locally on the brane, energy seems to have escaped. As the star loses mass, its gravitational effects weaken, which can be observed through clear measurements of nearby objects' motions. The bulk field Ψ may not interact with brane-localized fields, which makes the direct observation a challenging task. KK graviton amplitudes ($h < 10^{-40}$) shows the consistency with non-detection, with LISA probing $h \sim 10^{-34}$ for low m_n [50,51], which increases the model's testability. A short $\tau \sim 1$ day predicts lensing ($\Delta\theta \sim 8 \times 10^{-6}$ arcsec), orbital shifts ($\Delta v \sim 60$ cm/s), and GW bounds ($h < 10^{-20}$), which is falsifiable with current (Gaia [52]) and future (LSST [53], LISA [54]) instruments. Conventional signatures (supernova remnants, GWs with $h > 10^{-20}$) disprove the model and make it practical through observation. Also $m_\Phi \sim 10^{-18}$ GeV (from ρ_Φ) ensures $\tau \sim 1$ day, which justifies g_{int}'s role. PHL 293B-LBV's isolation and rare resonance ($P_{\text{res}} \sim 10^{-2}$) consequences its unique disappearance, which is justified by statistical and dynamical analyses, and distinguish it from typical LBVs.

From the effective field theoretical perspective, we have derived the low-energy parameters such as coupling constants and masses from a high-energy theory without fine-tuning. We have explained the star's disappearance via the emergent instability in the low-energy scalar field ϕ_L due to the induced negative mass squared term. The given calculations do make specific predictions about the behavior of the scalar field ϕ_L that can be tested observationally. The theoretical calculations suggest the star's matter transitions into extra dimensions without releasing energy into electromagnetic radiation detectable by telescopes. The solutions of the present work confirm that the disappearance timescale is consistent with observational data. It can be clearly expressed that $m_\Phi \sim M_\odot$ is required for realistic timescales, which resolves the role of λ. These resolutions suggest a detailed explanation of the missing star within the higher-dimensional solutions.

Appendix

A. Mathematical details

A.1. Evaluating the self-energy integral

A general expression is derived as,

$$\Sigma(s) = \int \frac{d^n p}{(2\pi)^n} \frac{1}{s + p^2}. \tag{A.1}$$

A.2. Using dimensional regularization

The formulation is applied as,

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2 + M^2)^\alpha} = \frac{1}{(4\pi)^{D/2}} \frac{\Gamma\left(\alpha - \frac{D}{2}\right)}{\Gamma(\alpha)} \left(\frac{1}{M^2}\right)^{\alpha - D/2}. \tag{A.2}$$

Set $D = n$, $M^2 = s$, $\alpha = 1$:

$$\Sigma(s) = \frac{1}{(4\pi)^{n/2}}\Gamma\left(1 - \frac{n}{2}\right) s^{(n/2)-1}. \tag{A.3}$$

The Gamma function has the following properties for half-integer arguments.

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}. \tag{A.4}$$

For negative arguments, $\Gamma(z)$ has poles, and the imaginary part arises.

A.3. Decay rate calculation

For $\Gamma = \lambda^2/4\pi$, with $\lambda = 7 \times 10^{-5}$, we have $\Gamma = 3.9 \times 10^{-9}$ GeV.

The decay time is given by $\tau = (4\pi/\lambda^2) \times 6.58 \times 10^{-25}\,\mathrm{s} = 2.56 \times 10^8\,\mathrm{s}$, which is consistent with observations when $E \sim 10^9$ GeV that defines the LBV scale.

A.4. Observational predictions ($\tau \sim 1$ day)

- Lensing: $\Delta\theta = 8 \times 10^{-6}$ arcsec ($M = 50\,\mathrm{M}_\odot$, $r = 10^{13}$ cm).
- Orbital: $\Delta v = 60$ cm/s ($\tau = 8.64 \times 10^4$ s).
- GW: $h \sim 10^{-38}$ ($m_n \sim 1$ GeV).
- $\Gamma = (\lambda^2/4\pi) = 1.16 \times 10^{-5}\,\mathrm{s}^{-1}$, $\lambda \sim 0.012$.

A.5. Resonance probability

- $\rho_{\text{core}} = (M_{\text{star}}/(4/3)\pi R^3_{\text{core}}) \sim 10^{28}$ GeV/cm^3, $m_\Phi \sim 10^{-18}$ GeV, $P_{\text{res}} \sim 10^{-2}$ ($\sigma \sim 1$ dex, $\Delta m < 10^{-19}$ GeV).
- Dynamical: $\Delta v = (GM/r)\,(\Delta M/M)\,(\Delta t/\tau) \sim 10^{-6}$ cm/s ($r = 10^{15}$ cm, $\tau = 8.64 \times 10^4$ s).

A.6. KK emission

$$\frac{dE}{dt} = \left(\frac{G_5 M^2}{L}\right)\left(\frac{m_n}{k}\right)^2 \sim 10^{41}\ \text{GeV/s}$$

$$(m_n = 1\ \text{GeV}), \tag{A.5}$$

$$h = \left(\frac{G_5 M^2}{rc^2}\right)\left(\frac{m_n}{k}\right)^2 \sim 10^{-37}, \tag{A.6}$$

$$f = \frac{m_n c^2}{h} \sim 2.4 \times 10^{14}\,\text{Hz}, \tag{A.7}$$

$$m_n = 10^{-3}\,\text{GeV}: \quad h \sim 10^{-41}, \quad f \sim 240\,\text{Hz}. \tag{A.8}$$

A.7. Warped dispersion

$$\left(e^{2ky}\partial_y\left(e^{-2ky}\partial_y\right) - k^2 + m_\Phi^2\right)\tilde{\Phi} = -g_{\text{int}}\delta(y)\tilde{\Psi}, \tag{A.9}$$

$$\tilde{\Phi} \sim e^{2ky} J_\nu\left(\frac{m_\Phi e^{ky}}{k}\right), \tag{A.10}$$

$$4k\lambda_\Phi\lambda_\Psi \sim g_{\text{int}}^2, \tag{A.11}$$

where $k \sim 10^{16}$ GeV, $m_\Phi \sim 10^{-18}$ GeV, and $y \sim 10^{11}$ cm.

A.8. Cosmic string and dark matter

The following parameters are discussed in this work.

- Cosmic string: $\mu = 10^{-7}G_N$, $n_{\text{CS}} = 10^{-6}\,\text{Mpc}^{-3}$, $\Gamma_{\text{CS}} = g^2_{\text{string}}\mu R_{\text{star}}/\tau \sim 10^{-11}\,\mathrm{s}^{-1}$.
- Dark matter: $\rho_{\text{DM}} = 0.3\,\text{GeV/cm}^3$, $\Gamma_{\text{DM}} = g^2_{\text{int}}\rho_{\text{DM}}/m^2_\chi \sim 10^{-5}\,\mathrm{s}^{-1}$ ($g_{\text{int}} = 0.01$, $m_\chi = 10^{-18}$ GeV).

A.9. Sensitivity

$$\Gamma = \frac{g^2_{\text{int}}}{|m^2_\Phi - m^2_\Psi|}, \tag{A.12}$$

where $g_{\text{int}} = 10^{-3}$–0.1, $m_\Phi = 10^{-19}$–10^{-17} GeV and $\tau = 10^2$–10^7 s.

A.10. Simulation setup

$\Phi_0 = 10^{19}$ GeV, $\sigma = 10^{11}$ cm, $\Psi_0 = 0$, solved via Eq. (45).

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