GRAVITATIONAL RADIATION AND ABERRATED CENTRIPETAL FORCE REACTIONS IN RELATIVITY THEORY

PART 2. RETARDED COHESIVE FORCES*

James C. Keith
University of Detroit, Detroit, Mich.

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ABSTRACT

We show from the standpoint of relativistic gravitational theories, that centripetal as well as gravitational aberrated force reactions can contribute to damping of a freely spinning system of accelerating masses. The new result predicts that a "Freely" spinning mass system will dissipate energy at a rate directly proportional to its kinetic energy and the factor $-KK'\left(\frac{a\omega}{c}\right)^3 \omega \sec^{-1}$, ω *being the angular velocity, a the maximum spin radius, c the velocity of light in vacuo,* $K \sim 1$ *a dimensionless ratio of the system's moments, and* $K' = \frac{2\gamma M}{P c^2} = 1$ *for reaction of the accelerating system with the universe.*

For a small rotor magnetically suspended and spinning freely in ultrahigh

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vacuum, the power loss can be detected by measuring the decay of the rotor's frequency. For $a = 10^{-2}$ cm and $f = \frac{\omega}{2\pi} = 5 \times 10^5 \frac{cycle}{sec.}$ the predicted decay would be on the order of 1/6 cycle/sec. each day.

I. RETARDED FORCE REACTION IN FREELY SPINNING MASSES

Introduction

The mutual internal gravitational interaction between each element of mass and the rest of a laboratory-sized rotating body in relativistic gravitational theories is extremely small for most physical situations. This is due to the negligible size of the constants γ/Nc^3 and γ/c^5 which enter the retarded internal gravitational reaction[1], N being the number of point particles in the rotating body, γ the gravitational constant, and c the speed of light in vacuo.

Here we shall extend a previous analysis[1] and examine the likelihood of additional internal and external retarded force reactions affecting the free motion of a bound system of accelerating masses. In particular, we examine the relative importance of retardation of binding and inertial and remote gravitational interaction forces within a freely rotating body, and estimate the power loss due to the retarded reaction.

I.A *Gravitational interaction with remote external mass.*

Let us explore the interaction of each mass element of a rotor with a large external mass. This problem is very difficult to tackle in general relativity (see for example, Das, Florides and Synge, Proc.Roy.Soc. A. 263 pp 451-472, [1961]), and so to get an idea of its magnitude and characteristics we shall restric our discussion to flat space-time gravitational theories. Among these, the one we shall use will be Birkhoff's theory of gravitation. We expect though that the order of magnitude of our results will still hold in any other relativistic gravitational theory. On the proper frame of the mass element m_i, $\vec{u}_i = 0$, $u_i^1 = 1/[1 - (u/c)^2]^{1/2} = 1$, and the constitutive Birkhoff gravitational force equations reduce to[2]:

$$\vec{f}_i^{\,g}/m_i = \vec{\nabla} h_{11} + 1/c\ \partial \vec{h}/\partial t \tag{1}$$

Here h_{11} is the time component of the gravitational potential of a large external mass M and equals $[2(v^1)^2 - 1]\ \underline{\gamma} M/R$ while $\vec{h} = 2(v^1)^2\ \underline{\gamma} M/R\,(\vec{v}/c)$, where $\vec{v}$ is the velocity and R the distance from m_i to M. If M is so remote that forces of $1/R^2$ and higher order are negligible compared with those of $1/R$ order, $\underline{\gamma} M/R$ can be taken as virtually constant in space and time over the extent of a small rotating body. For a rotor mass element m_i, (v_i^1) is constant, and the Birkhoff equations (1) reduce to:

$$\vec{f}_i = 2(v_i^1)^2\ (\underline{\gamma} M m_i/R c^2)\ d\vec{v}/dt \tag{2}$$

$R \gg a_i$ the maximum dimension of the accelerated system of masses m_i

$$(v_i^1)^2 = 1/[1 - (v/c)^2]$$

$\vec{v}$ is the relative velocity, $d\vec{v}/dt$ the relative acceleration of M and m_i.

In our own stationary space, barring relative movements of M, the force on the mass point m_i from equation (2) depends only on the instantaneous value of acceleration of m_i. One might term this a gauge-dependent force. In the field of a large remote mass M, m_i is acted on by a force (2) in addition to its usual "inertial" force and proportional to its acceleration. In fact, using an established result of Birkhoff Theory[3], $(\underline{\gamma} M/R)_{\text{universe}} = 1/2\ c^2$, and equation (2) is itself found to generate the inertial force, in accordance with Mach's Principle[4].

Because Birkhoff Theory postulates a all forces in a perfect fluid travel at limiting velocity c in solid or rarified matter, the reaction with the rest of the rotor cannot be instantaneous. Since mass elements of the rotor are circularly accelerated, this may lead to aberration of the exchange type binding forces in the equilibrium reaction where the rotor behaves coherently as if it were constituted of perfect and fluid and were nearly rigid (relative positions of particles constituting the rotor in spin equilibrium do not change appreciably in time).

I.B *Internal centripetal force aberration*

Assuming that the equilibrium exchange force reaction is transmited at finite velocity c from m_i to the rest of the rotor and is not affected by the consequent acceleration of m_i, let us examine the force aberration in the frame of the rotor. Consider an arbitrary radial force F_r (the inertial or remote gravitational interaction force of [2]) exerted instantaneously at time t on m_i and let us calculate the reaction force observed by the diametrically opposite mass $m_{i'}$. Due to overall force equilibrium for the rotor as a whole, interactions with all other mass elements of a symmetrical solid rotor cancel[5], leaving only the interaction between m_i and $m_{i'}$ to be considered.

Assume F_r is transmitted at limiting velocity c a distance $2a/c$ across the rotor to $m_{i'}$, and let us calculate the reaction in the frame moving with constant velocity of m_i at time t. We wish to find F_r as seen by $m_{i'}$, at time $t + 2a/c$. Since the relative velocity of m_i and $m_{i'}$, at time t, is zero on the rotating body, Lagrange's Expansion gives for the differential velocity of $m_{i'}$, at $t + 2a/c$ in terms of its acceleration at time t:[6]

$$\{ \delta \vec{v}_{i'} \}_{t + 2a/c} = \frac{2a}{c} \frac{d\vec{v}}{dt} + \frac{2a^2}{c^2} \frac{d^2\vec{v}}{dt^2} + \dots$$

$$= -2 \frac{(a\omega)^2}{c} \hat{a}_{i'} - 2 \frac{(a\omega)^3}{c^2} \hat{\theta}_{i'} + \dots \quad (3)$$

From special relativity the aberration angle of F_r seen by $m_{i'}$ is therefore[7]

$$\cos\theta' = \frac{\cos\theta + v/c}{1 + v/c \cos\theta} = \delta v^{\theta}_{i'}/c \simeq -2(a\omega/c)^3 \quad (4)$$

and the tangential component of F_r seen by $m_{i'}$ is just

$$F_{\theta'} = F_r \cos\theta' = -2(a\omega/c)^3 F_r \tag{5}$$

One can calculate this result using special relativistic force transformation equations[8]. Consider that $m_{i'}$ is accelerating in a frame moving with constant velocity $\vec{v}_\theta = a\omega\hat{\theta}_o$ relative to our stationary space at time t. The force transformation equations are[8]:

$$F_{\theta'} = F_\theta^o + \frac{\delta v_r' \, v_\theta}{c^2 + \delta v_\theta' \, v_\theta} F_r^o + \ldots$$

$$\simeq 0 + -v_\theta/c^2 \, (F_r^o \, \delta v_r') + 0 \tag{6}$$

Using (3) to find $\delta v_r'$ and noting that F_r^o is the centripetal force $-m_{i'} a\omega^2$ on $m_{i'}$ the last equation yields a tangential force on $m_{i'}$:

$$\vec{F}_{\theta'} = -2\,|F_r|\,(a\omega/c)^3\,\hat{\theta}' \tag{7}$$

In equations (5,7), $F_{\theta'}$ appears as a residual tangential component of the otherwise nearly radial retarded exchange force arising in the equilibrium interaction of all other mass elements of a rotating body with one element $m_{i'}$. The main ***inertial*** force on $m_{i'}$ by equation (2) is ***instantaneously*** radial. The residual tangential component $F_{\theta'}$ therefore must be balanced by a residual inertial force tending to decelerate $m_{i'}$. This gives rise to a Laplace type drag effect of third order in v/c.[9]

The tangential component of reaction force (5,7) acts against the velocity $v_{\theta'}$ of $m_{i'}$ causing kinetic energy K to decay at a rate:

$$F_{\theta'} \, v_{\theta'} = -2\,|F_r|\,(a\omega/c)^3\, a\omega = dK'/dt \tag{8}$$

By symmetry, a similar relation holds for each mass element of a rotating body. $F_{\theta'}\, v_{\theta'}$ thus will be always in the same direction independent of $\hat{\theta}$.

Physically, the term $F_r^0\, \delta v_r'$ in equation (6) might be looked on as time rate of change of $m_{i'}$. For the associated $\hat{\theta}$ component of momentum to be conserved, a continuous increase in $m_{i'}$ must be accompanied by a continuous decrease of $v_{\theta'}$ so that

$$v_{\theta'}\, \delta m_{i'}/\delta t + f_{\theta'} = 0$$

$$f_{\theta'} = - v_{\theta'}\, \delta m_{i'}/\delta t \qquad (9)$$

While the power loss $F_r^0\, \delta v_r'$ is nearly compensated by the force reaction $- F_r'\, \delta v_r'$, $f_{\theta'}$ is generated locally as a real net force opposing the motion. Hence, the net power dissipation neglecting $(F_r^0 - F_r')\, \delta v_r' \sim$ (higher than $1/c^5$ order terms) is

$$f_{\theta'}\, v_{\theta'} \cong - v_{\theta'}^2\, \delta m_{i'}/\delta t = - v_{\theta'}^2\, F_r^0\, \delta v_r'/c^2$$

$$\cong - 2 v_{\theta'}^2/c^2\, |F_r^0\, v_{\theta'}^2/c| = - 2\, |F_r^0|\, v_{\theta'}^4/c^3 \qquad (10)$$

in agreement with (8).

The dissipated power might be thought of physically as arising in connection with successive transmission of centripetal forces between mass points of a spinning system. To show this, take a particular force equilibrium mechanism where simultaneous emission and adsorbtion of quanta ΔE are responsible for transmission of reaction (exchange or other binding) forces at speed c between point particles of the rotor. The emitted ΔE goes into a frame instantaneously at rest with respect to the mass m, while by the time the absorbed ΔE travels from the opposite mass, the absorbing point m has accelerated in the direction of $\hat{\theta}$ acquiring a velocity given similarly to equation (3) by:

$$\delta \vec{v}_\theta = \tfrac{1}{2}\, \dot{\vec{v}}_\theta\, (\delta t)^2 = - 2\, v_\theta^2/c^2\, \hat{\theta} \qquad (11)$$

By hypothesis, absorbtion of the quantum ΔE gives rise to the centripetal force:

$$\vec{F}_r = - \vec{\nabla} \Delta E = 1/c (\partial \Delta E / \partial t) \hat{r}$$

$$1/c^2 \, \partial \Delta E / \partial t = \delta m / \delta t = |F_r / c| \tag{12}$$

The quantum absorbed by m at a time $2a/c$ after it was emitted however must be boosted to a $\hat{\theta}$ velocity $\delta \vec{v}_\theta$ given by (3), (11). This gives a tangential force on m of

$$\vec{T}_\theta = \delta \vec{v}_\theta \, \delta m / \delta t = - 2 \, |F_r| / c \; v_\theta^3 / c^2 \, \hat{\theta} \tag{13}$$

with a corresponding power dissipation

$$\vec{v}_\theta \, \vec{T}_\theta = - 2 \, v_\theta^4 / c^3 \; |F_r| \tag{14}$$

which is identical with (8) and (10). This $1/c^3$ order effect in this way arises from a combined effect of acceleration of the mass point and finite speed c of the equilibrium exchange force reaction. By symmetry, the effect is additive for each mass point of the spinning rotor.

I.C *Integration of the retarded force reaction over the entire rotor.*

Let us use the interaction symmetry to integrate equations (8), (10), or (14) to obtain the net effect of retarded equilibrium force reaction on the whole rotor. Replacing 'a' by 'r' and noting that $r^4 |r| = |r^5|$, we can write the above equations in integrable form:

$$\delta dK/dt = - 2 \, |F_r / r \, \delta m| \; \omega^4 \, |r^5 / c^3| \, \delta m \tag{15}$$

where the brackets $|\;|$ indicate the absolute value is to be taken.

If we assume F_r equals some constant k' (to be determined later) multiplied by $m_{i'}\, r\omega^2$ and that for an extended body of manv points the differential element δm replaces $m_{i'}$, the net kinetic energy lost from a rotating body can be obtained by direct integration from equation (15):

$$dK/dt = -2k'\,\omega^6/c^3 \int_m |r^5|\,\delta m = -k'\,kK(a\omega/c)^3\omega \qquad (16)$$

Here r is the spin radius of δm and 'a' the maximum spin radius. k is a dimensionless ratio of geometric moments of the rotating body, and from (16) is:

$$k = 4\,\frac{\int_m |r^5|\,\delta m}{a^3 \int_m r^2\,\delta m} \qquad (17)$$

for a spinning rod $k = 2$. For a solid rotating right cylinder, $k = 16/7$, while for a sphere, $k_s = 75\pi/128$.

I.D *Interatomic aberrated reaction forces in a spinning solid.*

Physically speaking, a point particle or atom in a rotating body is generally subject to tensional forces on the order of $Nm_0 = M$ times $a\omega^2$. Because of the limiting velocity of coherent force propagation in matter, it takes a time A/c for such forces to be transmitted to and from neighboring atoms, where A is the atomic spacing. The fact that the atom or molecule cannot react instantaneously to such forces gives rise to the power dissipated by the aberration effect in (16) above.

Equation (6) leads to a method of calculating aberrated tangential components of radial centripetal force acting on each mass element of a symmetrical spinning body. Because interparticles forces are transmitted at limiting velocity c in Birkhoff Theory, an atom cannot react instantaneously to strain forces exerted by neighboring particles. Using the approximate special relativistic force transformation equation (6)

$$F_{\theta'} \cong F_\theta^0 + u_r\, v/c^2\, F_r \qquad (6a)$$

we can find the last term by noting $u_r = -A/c\, r\omega^2$ while F_r is now considered as the total strain force, A being the interatomic spacing.

For brevity, let us consider only the special case of a spinning rod consisting of a string of atoms. The centrifugal force on an atom m_i a distance r from the origin is

$$F_r \cong \int_r^a \rho r\, \omega^2 dr = 1/4\, Ma\omega^2 (1 - r^2/a^2) \qquad (18)$$

where 'a' is the maximum radius and M the total mass. Assuming the effect on m_i is doubled because of its reaction with atoms on both sides, the total reaction over one half the rod due to the last term in (6a) is found by summing

$$\sum_0^a F_{\theta'}\, v \cong - \sum_0^a 2A/c^3\, r^3 \omega^6\, 1/4\, Ma(1 - r^2/a^2) = - 1/24\, Ma^5 \omega^6/c^3 \qquad (19)$$

so that the proportional energy decay for the entire rod from this term becomes;

$$dK/dt/K \approx - 1/2\, (a\omega/c)^3 = - 1/4\, k(a\omega/c)^3 \omega \qquad (20)$$

This is the contribution due only to interatomic aberration. The first term on the right of (6a) gives initial tangential forces on neighboring atoms due to the retarded interaction of the latter. This result should be compared with equation (16).

The net kinetic energy dissipation has the important property that it is proportional to the total kinetic energy of the rotating body and does not vanish as the number of point particles or atoms approaches infinity. In fact it does not depend on the point-particle or continuous structure of the rotor as such, only on the limiting velocity c of the binding forces and the acceleration.

I.E ***Interaction of a freely spinning mass with external masses***

Let us substitute for F_r in (15) the force on a spinning mass element due to its interaction with the gravitational potential of an external mass M (equation (2)) to determine k' in (16):

$$k' = \frac{F_r}{\dot{m}\,|dv/dt|} = 2\gamma M/Rc^2 \qquad (21)$$

Here R is the distance to M. k' therefore does not depend on rotor geometry. Using this result, equation (16) transforms into:

$$dK/dt = -\,kK(2\gamma M/Rc^2)\,(a\omega/c)^3\omega \qquad (22)$$

Again, it should be pointed out that the effect from all mass elements of the rotor are additive by symmetry and have a direct dependence on the mass rather than on the square of the mass. The existence of the effect does not depend on the shape of the rotating body, but only on the sum of individual, noncoherent contributions (linear instead of quadratic in m).

Nevertheless, it is interesting to compare the result for coherent gravitational quadrupole radiation-reaction from a spinning dumbbell[10]:

$$dK/dt = +\,44/15\,\gamma m^2/a\,(a\omega/c)^5\omega \qquad (23)$$

with the force aberration reaction (22)

$$dK/dt = -\,4\gamma M\,(2m)/R\,(a\omega/c)^5\omega \qquad (24)$$

where for the dumbbell $K = ma^2\omega^2$ and $k = 4$, the mass of each pole being m. Both external and internal reactions are basically $1/c^5$ order effects. However, the *mutual* gravitational energy in (23) is replaced in (24) by the external interaction energy.

I.F $1/c^3$ *order centripetal force reaction effect: interaction with the universe*

In a Machian type universe, $\gamma M/Rc^2$ in equations (2,21,22,24) is on the order of one, and the $1/c^5$ is reduced to a $1/c^3$ order effect. Several astronomical observations tend to bear out the Machian cosmological model and it therefore seems valid to consider the present consequences of this assumption.

In Birkhoff Theory, $2\gamma M/Rc^2 = 1$. This represents the sum of the potentials for all matter in the universe[3]. Equation (22) is then

$$dK/dt = - kK(a\omega/c)^3\omega \qquad (25)$$

There would be an analog to this situation in Einstein Theory. There k_s' is just twice that given by equation (21) while $\gamma M/Rc^2 = \pi/2$[11] so that in the Einstein analog, the last equation is augmented by a factor of 2π.

The result (25) may be looked on therefore as a direct gravitational reaction with the rest of universe. But it need not be! The same result is obtained by taking F_r in equation (8) as the centrifugal force $m_i r\omega^2$, in which case k' in (16) is one, and that equation reduces inmediately to (25). The alternative approach of calculating the reaction of a spinning rotor with external masses interestingly leads to (25) in the degenerate case that those masses constitute the totality of the universe.

The two views can be combined by ascribing the centrifugal forces to a gravitational interaction with the universe. Physically, the power dissipated in such a way appears to depend on the steady acceleration and force interaction of a roter's mass elements and is not characterized by any particular frequency or spatial distribution in the far field. It is as if entire reaction force on the universe, a universe which cannot itself react to forces or torques inertially, acted back on the freely spinning mass system causing a real slowing down.

II. FAR FIELD G-INTERACTION OF A SPINNING SYSTEM OF MASS POINTS WITH A LARGE REMOTE MASS

II.A *Far field source equations*

The slowing down of a freely spinning rotor caused by interaction with a large remote mass or set of masses must be compensated by the far-field reaction in which energy and/or angular momentum of the rotor are eventually transferred to the remote mass. Just as with direct gravitational force disturbances which propagate between bodies at fundamental velocity c, the transmitted energy is not characterized by any frequency dependence, but can be linked with a DC flow of energy or mass away from the rotor. This is associated with the energy necessary to sustain the force field reaction being transmitted through space at velocity c eventually to interact with the remote mass itself.

The question arises in how far the reaction force power dissipation of equation (22) is also compatible with the far field interaction between the remote mass M (now taken as the field mass) and the mass points m_π of the spinning rotor (treated as the sources of the reaction force field).

Let us calculate the force as seen by a remote mass M. To do this we note again that M does not have any velocity relative to itself, and therefore the Birkhoff force-field equations (1) reduce in the frame of M to:

$$\vec{G} = \vec{\nabla} b_{11} + 1/c\, \partial\vec{b}/\partial t \qquad (26)$$

$M\vec{G}$ will be the force on M at M, $\vec{b}$ being generated by the m_π of the spinning rotor. $\partial/\partial t$ and $\vec{\nabla}$ refer to the field point M which is at a time r/c removed from the source of the field. It is therefore necessary, since the b's are given in terms of the time at $t - r/c = t'$ rather than at t, to transform the operators as well as r which appears in the denominator of the potential. This method has been carried out previously for the electromagnetic field and reference is made to standard texts[14]. Using this we can easily find the result for circular motion of a source mass point in the Birkhoff theory is[13]:

$$\frac{\vec{G}}{\gamma m_*} = -\frac{\vec{r}_p}{s^3}\,[1-(v/c)^2] - \frac{2}{c^2 s^3}\,[\vec{r}\times(\vec{r}_p + d\vec{v}/dt)]$$

$$+\frac{1-(v/c)^2}{c^2\, s^3}\,[\vec{r}\,(\vec{r}\cdot d\vec{v}/dt) + \vec{v}\times(\vec{v}\times\vec{r})]^* \qquad (27)$$

This field as calculated from the Birkhoff Theory with the appropriate retardation transformation is much like that obtained in Maxwell Theory except for the numerical factor of 2 multiplying the standard radiation field and the residual term in the direction of r. This latter term shows that the far ($1/r$ order) force field has a longitudinal component in addition to the purely transverse components found in Maxwell Theory (see Table 1).

Retaining only terms of $1/r$ order, (27) reduces to:

$$\frac{\Delta\vec{G}}{\gamma m_*} \xrightarrow[\text{1/r order terms only}]{} -\frac{1}{s^3c^2}\,\vec{r}(\vec{r}\cdot d\vec{v}dt)(1+(v/c)^2) + \frac{2r^2}{s^3c^2}\,d\vec{v}/dt$$

$$+\frac{2}{s^3c^2}\,\frac{r}{c}\,(\vec{r}\times(\vec{v}\times d\vec{v}/dt)) \qquad (28)$$

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* $\vec{\nabla} \rightarrow \vec{\nabla}_1 - \frac{\vec{r}}{sc}\,\partial/\partial t'$ $\qquad \partial/\partial t = \frac{r}{s}\,\partial/\partial t'$

$s = r - (\vec{r}\cdot\vec{v})/c$ $\qquad b_{11} \xrightarrow[v^2\,=\,\text{cons.}]{} \frac{\gamma m_*}{s}\,[1+(v/c)^2]$

$\vec{b} = 2\,\frac{\gamma m_*}{s}\,(\vec{v}/c)$ $\qquad m_* = \frac{m_*^0}{(1-(v/c)^2)^{1/2}}$

$\vec{r}_p = \vec{r}_{\text{present}} = \vec{r} - r/c\,\vec{v}$; $\vec{r}, \vec{v}$ taken at $t' = t - r/c$

Table 1. GRAVITATIONAL THEORIES AND THEIR ELECTROMAGNETIC ANALOGUES***

	Retarded Field of a Source in Circular Motion*	Poisson's Equation
Newton	$\vec{G} = \vec{\nabla}_{bo}$	$\Box^2_{bo} = 4\pi\underline{\gamma}\rho$
	$\vec{G}/\underline{\gamma} M = - \frac{(\vec{r} - \frac{r}{c}\vec{v})^{**}}{s^3} - \frac{1}{s^3 c^2} [\vec{r}(\vec{r}\cdot\frac{d\vec{v}}{dt}) + \vec{v}\times(\vec{v}\times\vec{r})]$	
Birkhoff	$\vec{G} = \vec{\nabla}_{g_{11}} + \frac{1}{c}\frac{\partial\vec{b}^{a}}{\partial t}$	$\Box^2 b_{ij} = 8\pi\underline{\gamma}\rho(u_i u_j - \frac{1}{2}\eta_{ij})$
	$\vec{G}/\underline{\gamma}M = -\frac{(\vec{r}-\frac{r}{c}\vec{v})[1-(\frac{v}{c})^2]}{s^3} - \frac{2}{s^3c^2}[\vec{r}\times\{(\vec{r}-\frac{r}{c}\vec{v})\times\frac{d\vec{v}}{dt}\}] + \frac{1-(\frac{v}{c})^2}{s^3c^2}[\vec{r}(\vec{r}\cdot\frac{d\vec{v}}{dt}) + \vec{v}\times(\vec{v}\times\vec{r})]$	
Einstein	$\vec{G} = \frac{1}{2}\vec{\nabla}_{g_{11}} + \frac{1}{c}\frac{\partial b^{a}}{\partial t}$	$\Box^2 g_{ij} \cong 16\pi\underline{\gamma}\rho(u_i u_j - \frac{1}{2}\eta_{ij})$
	$\vec{G}/\underline{\gamma}M = -\frac{(\vec{r}-\frac{r}{c}\vec{v})[1-3(\frac{v}{c})^2]}{s^3} - \frac{4}{s^3c^2}[\vec{r}\times\{(\vec{r}-\frac{r}{c}\vec{v})\times\frac{d\vec{v}}{dt}\}] + \frac{3-(\frac{v}{c})^2}{s^3c^2}[\vec{r}(\vec{r}\cdot\frac{d\vec{v}}{dt}) + \vec{v}\times(\vec{v}\times\vec{r})]$	
Maxwell	$\vec{E} = -\nabla\phi_1 - \frac{1}{c}\frac{\partial\phi^{a}}{\partial t}$	$\Box^2\phi_i = -4\pi\rho u_i$
	$\vec{E}/e = \frac{(\vec{r}-\frac{r}{c}\vec{v})[1-(\frac{u}{c})^2]}{s^3} + \frac{1}{s^3c^2}[\vec{r}\times\{(\vec{r}-\frac{r}{c}\vec{v})\times\frac{d\vec{v}}{dt}\}]$	

*Taken in the rest frame of the observed; series approximations good to $1/c^3$ order.

**The retarded radius vector $\vec{r}$, velocity $\vec{v}$, and acceleration $\frac{d\vec{v}}{dt}$ of the source are taken at a time $(t_{obs} - \frac{r}{c})$. $s \equiv r - \vec{r}\cdot\vec{v}/c$

II.B *Compatibility between far field and internal force reaction pictures*

To investigate compatibility of these terms with the previous result for decay of spin energy of the source system, let us use (5) to find $(dv/dt)_{total}$ and substitute in (28). Then $((d\vec{v}/dt)_{total} = -\vec{a}\omega^2 - 2(a\omega/c)^3 a\omega^2 \hat{\theta}$, and

$$\frac{\Delta \vec{G}}{\underline{\gamma} m_n} = \frac{1}{s^3 c^2} \vec{r}(\vec{r} \cdot \vec{a})\,\omega^2 (1 + (v/c)^2) - \frac{2r^2}{s^3 c^2}\,\omega^2 \vec{a}$$

$$+ \frac{1}{s^3 c^2} \vec{r}(\vec{r} \cdot \hat{\theta})\; 2(a\omega/c)^3 a\omega^2 (1 + (v/c)^2) -$$

$$- \frac{2r^2}{s^3 c^2}\; 2\,(a\omega/c)^3 a\omega^2\, \hat{\theta} + \frac{2}{s^3 c^2}\;\frac{r}{c}\,(\vec{r} \times \hat{k})\, a^2 \omega^3 \qquad (29)$$

When $\vec{v}$ and its derivatives are interpreted as the relative velocity of source point and observer, the Birkhoff force equations become symmetrical under interchange of source anc observer. Under these conditions, the above force (29) equals the force exerted on the source mass point. The power dissipated bv the source is $\vec{f}_g \cdot \vec{v}$ and for each mass point this is equal to:

$$\frac{dK_n}{dt} = \vec{f}_g \cdot \vec{v} = \frac{\underline{\gamma} M m_n}{s^3 c^2}\,(\vec{r} \cdot \vec{v})(\vec{r} \cdot \vec{a})\,\omega^2 (1 + (v/c)^2) - \frac{2\underline{\gamma} M m_n}{s^3 c^2}\, r^2 \omega^2 (\vec{v} \cdot \vec{a})$$

$$+ \frac{2\underline{\gamma} M m_n}{s^3 c^2}\,\frac{r}{c}\;\vec{v} \cdot (\vec{r} \times \vec{k})\, a^2 \omega^3 + \frac{\underline{\gamma} M m_n}{s^3 c^2}\,(\vec{r} \cdot \hat{\theta})^2\, 2\,\frac{a^5 \omega^6}{c^3}\,(1 + (v/c)\;) -$$

$$- \frac{2\underline{\gamma} M m_n r^2}{s^3 c^2}\; 2\,\frac{a^5 \omega^6}{c^3} \qquad (30)$$

Averaging over the extension of the source mass or over a full revolution of the rotor in time, the first three terms of (30) average to zero while the last terms give

$$\sum_{n} \frac{dK_n}{dt} \approx - kK \frac{2\gamma M}{rc^2} \quad (a\omega/c)^3 \omega \,(1 - 1/4 \sin^2 \theta) \tag{31}$$

and where k and K are as defined in equation (22). Equations (31) and (22) differ only by the additional term containing $\sin^2 \theta$.

There are two important features to notice in (31). First, if the potential $\gamma M/r$ is set up by an isotropic distribution of masses in the universe, $(b_{11})_{universe}$ would equal $c^2/2$ with no preferred direction of $\hat{\theta}$. The $\sin^2 \theta$ term average then to 1/2 and the total power dissipation for the mass ring is

$$< \sum_{n} \frac{dK}{dt} > \; = - kK(a\omega/c)^3 \omega \;\; (1 - 1/8) \tag{32}$$

which seems compatible with the initial assumption $(d\vec{v}/dt)_{total} = - \omega^2 \vec{a} - 2(a\omega/c)^3 a\omega^2 \hat{\theta}$

Secondly, for interaction with a single remote mass M, the energy dissipated in (31) depends slightly on the square of the sine of the colatitude angle θ If a spinning rotor on Earth were to interact with the sun, the power dissipated by the rotor should therefore have both a DC and an AC component.

For electric dipole source, an effect similar to (32) does not take place since opposite contributions to dK/dt cancel. Further, since there is no known Mach's principle for electric charges there is no comparative electromagnetic interaction.

II.C *Graviton exchange*

The power dissipated by a spinning mass system has beeh calculated in the frame of a large remote mass causing the dissipation. Other terms in (28) suggest that a spinning mass ring or other symmetrical rotating body can exert steady, unidirectional, $1/r$ order residual forces, transverse and radial, on the remote mass.

Equation (28) gives a far field which generally has a component along the radius vector from the source point of the rotor to M. Let us assume the instantaneous configuration of mass points in a spinning ring is symmetrical such that for each mass point on one there is a mass point directly opposite along a diameter. The opposite mass point therefore almost, but not quite, because of retardation cancels the first mass point's longitudinal field component and:

$$< M\vec{G} > \cong \frac{\gamma M}{b^3 c^2} \sum_n m_n \vec{r} \left[\vec{r} \cdot (d\vec{v}_n/dt + d\vec{v}_{n'}/dt) \right]$$

$$= -8/3 \sum_n m_n a\omega^2 \frac{\gamma M}{bc^2} (a\omega/c)^2 \sin^3 \theta \, \hat{b} = \vec{f} \qquad (33)$$

Integration of this term over the entire ring of mass leaves a time average steady component of the radial far field.

Whether such non-vanishing forces can explain the energy degradation of a rotor (32) above to some extent on the assumed physical nature of these forces. If we assume a continuous stream of gravitational energy (graviton) leaves the spinning mass ring and at a time r/c later becomes absorbed by M causing net average force, and a continuous stream at the same time enters from the far field of M and is absorbed by the mass ring, the graviton exchange may be accompanied by a fractional exchange of energy or angular momentum.

It is helpful to examine this idea for the far field net radial force component obtained by use of the last equation, (33). One assumes the amount of gravitational energy in an interval $dr = c\,dt$ is dE so that $1/c\, dE/dt$ equals the net radial component of force on M. dE/dt is the amount of energy per second going off as gravitons from the spinning rotor and this equals $\vec{f} \cdot \vec{c}$. These gravitons can be considered to carry off a mass $dm'/dt = 1/c^2 \vec{f} \cdot \vec{c}$ from the rotor each second.

Because the rotor and the mass M experience net equal and opposite $1/r$ order forces (by inverting (q) regarding the velocity and acceleration as relative parameters), the number of gravitons given off by the rotor will equal the number absorbed in a given time. But each time a graviton is given off by a mass point of a spinning ring, it carries away additional energy $v^2/2\ dm'/dt$ and when a graviton is absorbed by the ring, that much energy must be imparted to it. Thereby arises a net degradation of the kinetic energy of the rotor. For the component in (33) this is found to be

$$\frac{dK}{dt} = (v/c)^2(\vec{f}\cdot\vec{c}) = -8/3\ \sum_n m_n(a\omega)^2\ \frac{\gamma M}{bc^2}\ \sin^3\ \theta\,(a\omega/c)^3\omega \qquad (34)$$

The slowing down of a spinning symmetrical rotor then will eventually be compensated by a gain in the energy of the remote mass M. In the case of interaction between the mass points of the rotor and the remote universe this becomes a kind of entropy degradation of energy.

In calculating the complete retarded interaction of a spinning rotor of neutral mass points with a large remote mass M, one must ascertain the "true" (present) configuration of mass points. To do this consistently, the inertial electromagnetic and inertial-gravitational force equilibrium equations first have to be solved to get the retarded stress-strain configuration within the spinning rotor.

This means an accurate far field calculation may hinge on the retarded force equilibrium solution within the rotor.

The problem has been approached first by obtaining an internal picture of rotor equilibrium and drag forces and then checking the degree of compatibility with the external far field view rather than doing things the other way around.

III. EXPERIMENTAL MAGNITUDE OF EFFECTS

III.A *Comparison of retarded binding force and direct gravitational force reactions*

Birkhoff Theory predicts that retarded internal gravitational reaction in cohesively bound rotating bodies will cause a time rate of change of free rotational kinetic energy. For the rotating rod[14]

$$\frac{dK}{dt} = -\ 5/9\ \frac{\gamma M^2}{Na}\ (a^3\omega^4/c^3)\ +\ 44/15\ \frac{\gamma M^2}{9a}\ (a^5\omega^6/c^5) \tag{35}$$

where $M = Nm_o$, m_o being the mass of each point particle.

On the other hand, the retarded binding force reaction in the spinning rod from equations (16) and (17) is found to be:

$$\frac{dK}{dt} = -\ 1/3\ (2\gamma m/R)_{\text{external}}\ M\ a^5\omega^6/c^5 \tag{36}$$

As the number of point particles in the rod becomes infinite, the first term in (35), the incoherent gravitational reaction, goes to zero, leaving only the last coherent "quadrupole" reaction term to be considered. The power dissipated due to the binding force reaction (36) also is not affected by the number of point particles in the rod, and more generally in any rotating body.

Taking the ratio of (36) with the last term of (35), we find:

$$\frac{(dK/dt)^{\text{rod}}_{\text{binding force reaction}}}{(dK/dt)^{\text{rod}}_{\text{coherent quadrupole reaction}}} = +\ 45/44\ \frac{(2\gamma m/R)_{\text{external}}}{(\gamma M/a)_{\text{internal}}} \tag{37}$$

The internal g-potential $\gamma M/a$ is big only for large astronomical systems and possibly, sub-nuclear point masses. $(2\gamma m/R)_{\text{external}}$ however is large for physical systems, approaching c^2 for the universe as a whole. Thus in general (36) has an experimentally more favorable magnitude than the internal gravitational reaction (35).

III. B *Self-reaction of a single particle forced to rotate in a circle*

Previously, we have assumed only mutual interactions between two or more point particles are of importance, and have neglected any interaction which a fundamental particle such as an electron might have with itself. Formally speaking, as long as the particle is truly a point particle, there can be no retarded *internal* reaction, so that our previous assumption is still valid.

To react internally with itself, a particle must have a finite extension so that reaction forces take a finite time to propagate across it[15]

Let us take an electron rotating in a circle of radius 'a' and angular velocity ω. Further let us assume

$$e^2/r_0 = m_0 c^2 \qquad r_0 = 3 \times 10^{-13} \text{ cm.} \tag{38}$$

where r_0 is on *the order of the "true" radius of the electron. Binding forces of* unspecified nature are required to hold together the electronic charge. Because of internal linkages necessary for stability and finite extension, the force instantaneously exerted on a small element of the electron is transmitted to all other elements of the electron. In accordance with Birkhoff Theory, we take the reaction force to propagate through the electron at fundamental velocity c.

Let us examine the equilibrium between an instantaneous external force on a small *element* of a *spherically* symmetric electron and the retarded binding force exerted by the combined effect of all other elements. Say the combined binding force reaction has taken an average time $\delta t = \overline{r}_0/c = k\, r_0/c$ (k on the order of one) to "cross" the electron. At that earlier time, the force instantaneously exerted on the other elements was at an angle

$$\delta\theta = \omega\, \delta t = \omega\, k r_0/c \tag{39}$$

to the instantaneous force on the small element under consideration. By symmetry, the situation is the same for every element.

Thus, the combined action of the other elements not only must cancel the *instantaneous radial force* $\delta m_0 a\omega^2$, but *in addition causes a tangential component* in the same direction for every element:

$$f_\theta^{\text{combined binding force reaction}} = f_r\, \delta\theta = f_r\, \overline{r}_0\, \omega/c \tag{40}$$

This tangential force acts againts the velocity of the electron and causes its kinetic energy to be dissipated at a rate:

$$\frac{dK}{dt} = f_\theta v_\theta = - f_r \bar{r}_o \omega/c \, a\omega = - m_o a^2 \omega^2 a\omega^2/c \; \bar{r}_o/a \qquad (41)$$

or since $m_o r_o = e^2/c^2$ from (38), and $\bar{r}_o = k r_o$,

$$\frac{dK}{dt} = - ke^2 (a\omega^2)^2/c^3 \qquad (42)$$

which is the non-relativistic radiation reaction derived by Lorentz from an accelerated charge if $k = 2/3$ in Gaussian units[16]. This radiation must be taken to arise every time an individual electron is accelerated, and can be cancelled only by mutual external interaction with other charges as in the case of the rotating quadrupole[17].

Equation (41) should be compared with (10) for the energy dissipated due to retarded centripetal force reaction, which in the case of the single electron of finite extension derives from (10) with

$$\delta u_r = a\omega^2 \, \delta t = k \, a\omega^2 \; r_o/c$$

$$\frac{dK}{dt} = - k m_o a^2 \omega^2 (a\omega/c)^3 \omega (r_o/a) = - ke^2 (a\omega^2)^2/c^3 (a\omega/c)^2 \qquad (43)$$

and is seen to be formally similar to electric quadrupole radiation[17].

As previously shown, in the case of macrocospic rotating bodies, the retarded centripetal force reaction is additive across the entire body. There, the factor r_o/a in the last equation (43) becomes on the order a/a or one, while m_o is replaced by the total mass of the rotor. Thus, the ratio of energy dissipated by centripetal force reaction to that of electric radiation reaction is from (42) and (43), for $a\omega << c$

$$(dK/dt)_c/(dK/dt)_{em} \sim (a\omega/c)^2 \tag{44}$$

for the single electron, proton, or other fundamental particle (since $m\bar{r} = e^2/c^2$ for all such particle). This is not the case for the spinning laboratory sized mass. If the latter contains an equivalent of N protons, electrons, and neutrons, and the protons and electrons are assumed to radiate singly and additively by equation (42), we find that on comparison with equation (25)

$$(dK/dt)_c/(dK/dt)_{em} \simeq m_p\, av^2/c^2 \simeq 7 \times 10^{-6}\, av^2 \tag{45}$$

and this is stepped up by a factor of N/n if only the number of n free charges on the body undergo incoherent electromagnetic radiation-reaction. The ratio of the two effects is not independent of the frequency at which the rotor spins. For a 0.1 mm radius rotor, the retarded centripetal force reaction effect predominantes above the 100 Kc/s range and grows as the square of the frequency in comparison with any incoherent electric radiation-reaction effects. On the other hand, for particles spun say in an electron synchrotron, the incoherent electromagnetic radiation-reaction effect predominates at both low and high energies, at low energies going from (44) as m_0c^2/K and at high energies ($K >> m_0c^2$) as $(K/m_0c^2)^2$ and at intermediate energies directly as the number of particles in the beam (because of coherence at $a\omega \sim c$). For this reason, the centripetal force reaction effect appears easier to examine in macroscopic bodies, than in high or medium energy particle accelerators.

III. C *Experimental magnitude of predicted $1/c^3$ inertial-gravitational force reaction*

Equation (25) leads to a proportional frequency decay ratio for "freely" spinning rigid rotors of:

$$df/dt/f = \frac{1}{2}\, dK/dt/K = -\frac{1}{2}\, k\,(a\omega/c)^3\,\omega \tag{46}$$

$$\omega = 2\pi f$$

The rotational decay df/dt depends on the fifth power of frequency. Interactions with large external masses as the sun or galaxy depend on the value of $k' = 2\gamma M/Rc^2$ multiplied by (46), and are small fractions (10^{-8}, 10^{-6}) of the effect (46).

It seems feasible to measure such quantities experimentally using a type of magnetic suspension and rotation apparatus developed by Beams[18]. By suspending a rotating mass in ultrahigh vacuum and making use of the axial suspension symmetry, stray frictions are impressively reduced. Moreover, magnetic, air friction, and other residual drags vary proportionally to much lower powers of the frequency[19], making it possible to discriminate out the fourth power frequency dependence in (46).

For a steel rotor spinning near limits of tensile strength, the rim velocity is approximately $a\omega \cong 10^5$ cm/sec. Plastic creep of the metal usually limits the usable rim velocity to about one-half of this. For a spinning steel sphere, $k = 75\pi/128$ from (17) and the decay ratio in (46) becomes numerically

$$df/dt/f = -75\pi/256\,\left(\frac{1}{2}\,10^5/3\times10^{10}\right)^3 f2\pi = -3\times10^{-17} f\ \mathrm{sec}^{-1} \tag{47}$$

while for a spinning rod, $k = 2$ and if $a\omega = \frac{1}{2}\,10^5$ cm/sec

$$df/dt/f = -\left(\frac{1}{2}\,10^5/3\times10^{10}\right)^3 f2\pi \cong -3\times10^{-17} f\ \mathrm{sec}^{-1} \tag{48}$$

These results depend on the fact that the incoherent retarded force reaction (25) does not depend for its existence on the shape of the rotor, the shape entering is 10^6 cycles/sec, the decay of frequency from the spinning rod or sphere is about

df/dt = 3 cycles/sec each day, about 3 parts in 10^6 on freauency. If only the low rate of tree decay desired can be obtained experimentally, the frequency and its rates of change are capable of precise detection by comparison with a good crystal frequency standard[20].

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