Numerical modeling of spin-dependent phase shifts in nucleon-nucleon scattering

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Scattering phase shifts for quantum mechanical potential scattering can be derived directly from the scattering amplitude, bypassing the need to solve the Schrödinger equation explicitly. This can be achieved by numerically integrating the phase equation from the origin to the asymptotic limit. The Phase Function Method (PFM) is considered an effective approach for computing scattering phase shifts in quantum mechanics. The PFM is applied to analyze the Pöschl-Teller potential along with spin-orbit interactions. The strategy includes employing a five-parameter potential model to fit the calculated scattering phase shift curves to the established data for elastic scattering phase shifts. Numerical scattering phase shifts for nucleon-nucleon systems align well with prior research results.

Keywords: Phase function method; Pöschl-Teller plus spin-orbit interaction; scattering phase shifts; nucleon-nucleon systems.

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1. Introduction

Researchers have been investigating how to accurately predict the magic numbers by identifying necessary corrections to existing nuclear models. This involves refining potential energy calculations, adjusting interaction strengths among nucleons, and incorporating effects like deformation and pairing interactions. Correcting these potentials helps improve the predictions for the stability and structure of nuclei, especially those far from stability, such as superheavy elements. Advances in theoretical models, along with experimental data from facilities studying exotic nuclei, are crucial for achieving a more comprehensive understanding of nuclear structure and the underlying forces at play. Ultimately, in 1949, Mayer [1], Haxel [2], and their colleagues demonstrated that incorporating a spin-orbit potential could effectively separate the subshells as required. Initially, the concept of spin-orbit coupling was employed by atomic physicists to account for the fine structure observed in spectral lines. This phenomenon occurs as a result of the electromagnetic interaction between the magnetic field produced by electrons orbiting the nucleus and the magnetic moment of the electrons themselves. While electromagnetic spin-orbit effects [3] are generally quite minor, on the order of one part in 10^5 in atomic level spacing, they are insufficient to produce the significant changes in nuclear level spacing needed to explain the observed magic numbers. Consequently, a similar framework is employed to the nuclear potential as used in the atomic spin-orbit concept. One-gluon exchange (OGE) [4] serves as a plausible origin for velocity-dependent potentials, such as spin-orbit coupling, which is evidenced by the significant polarization observed in nucleon-nucleus scattering and the notable splitting between doublet levels explained within the shell model framework. The exact dynamical origin of the strong nuclear spin-orbit force remains unclear [5], but parallels drawn from atomic physics suggest it may be a relativistic effect. Empirical evidence indicates that static (velocity-independent) nuclear potentials predominantly govern interactions at relatively low energies for the incoming particles. However, at elevated energies, the forces are influenced by the relative momentum $P = P_1 - P_2$ of the nucleons. In this context, P_1 and P_2 represent the momenta of the individual nucleons, and based on Galilean invariance, the interaction is solely dictated by their relative motion, rather than their absolute motion at any specific moment during the interaction. Therefore, for two body interaction, the only viable spin-orbit coupling is $V_{SO}(s)((\vec{s}\times\vec{P})\cdot\vec{S})$. In this context, $V_{SO}(s)$ refers to the spin-orbit potential, characterized by a form factor similar to that of the central potential but with distinct parameters, while 's' represents the relative distance between the nucleons. The expectation value of the spin-orbit interaction is proportional to $2 < \vec{L}\cdot\vec{S} >= J(J+1) - L(L+1) - S(S+1)$ where the notations L, S and J stand for the orbital angular momentum, spin angular momentum and the total angular momentum, respectively. Although the spin-orbit potential is traditionally treated as a surface term, being proportional to $\frac{1}{s}\frac{dV}{ds}$ [6–8], its effect is strongest at the nuclear surface because the spin density vanishes inside the nucleus. However, we adapt the Pöschl-Teller [9–15] like spin-orbit term, added with the central Pöschl-Teller potential, hypothesizing that it will take care of the varying effects of the interior and surface of the nucleus with suitable parameterization of the potential. It is also well known that the form of the potential in spin-orbit term $V_{SO}(s)(\vec{L}\cdot\vec{S})$ is not that vital, but it is the $(\vec{L}\cdot\vec{S})$ factor which contributes significantly to reordering the levels. A significant spin-orbit coupling potential appears to be advantageous in explaining high-energy data.

When particles undergo scattering, the manner in which they deflect provides valuable insight into the underlying interaction mechanisms. The outgoing particle effectively encodes information about the forces it has experienced. However, achieving a precise and universally accepted description of the nuclear force remains a significant challenge in

nuclear physics. To reconcile theoretical predictions with experimental observations, many researchers have developed phenomenological models that incorporate various aspects of nucleon-nucleon interactions. The investigation of three-nucleon systems plays a crucial role in advancing our understanding of nuclear forces. Consequently, considerable theoretical and experimental attention has been devoted over the past several decades to studying such systems [16–20]. Extensive efforts have aimed at extracting reliable information about nuclear interactions and accurately describing scattering observables [21–27]. In particular, the inclusion of the Coulomb interaction in three-body calculations has proven to be technically demanding, especially in proton-deuteron scattering. To address this issue, different computational approaches and potential models have been proposed to consistently incorporate electromagnetic effects while preserving the accuracy of nuclear force descriptions. A central quantity in scattering theory is the differential cross-section, which represents the likelihood of scattering into a specific solid angle at a given energy. By examining its angular dependence, along with observables such as analysing powers, one can deduce important characteristics of the interaction, including whether the scattering process is isotropic or exhibits directional preferences. More recently, significant progress has been achieved through the development of nuclear forces based on chiral effective field theory. In particular, work by Epelbaum *et al*. [28] has introduced a new generation of interactions employing semi-local regularization schemes. These studies provide a clear conceptual framework for chiral effective field theory and demonstrate its applicability to two- and few-nucleon systems.

In the realm of non-relativistic quantum scattering theory [29], exploring the energy spectra and wave functions of quantum systems influenced by different potentials is a captivating area of study, as it provides essential insights into the behavior of the system. The Pöschl-Teller [9–15] diatomic molecular potential, introduced by Pöschl and Teller, was designed to model the vibrational excitations in molecular systems. In this study, the Pöschl-Teller potential is employed in the nuclear context to investigate nucleon-nucleon interactions with spin-orbit effect, utilizing the PFM [30] effectively. Several authors [9–15, 29–33] have explored the Pöschl-Teller potential in various quantum mechanical contexts, applying several standard approximation methods to find solutions in both relativistic and non-relativistic frameworks. Using this potential, Dong *et al*. [12] derived the exact solutions of the Schrödinger equation through conventional methods, while Infield *et al*. [34] employed the factorization method for their analysis. Recently, Aktas and Sever [35] as well as Diaz *et al*. [36] explored the supersymmetric solutions of the modified Pöschl-Teller potential. Therefore, the Pöschl-Teller potential presents a compelling option for theoretical physicists, particularly in its applications to molecular dynamics [9–13, 31–33]. Numerous studies have been conducted in this area using the PFM [15, 37–40]. In a relatively recent study, Behera *et al*. [40] investigated nucleon-nucleon and alpha-nucleon elastic scattering within the Manning-Rosen potential, while Sahoo *et al*. [39] examined nucleon-nucleon scattering for F- and G-partial waves using the Hulthén potential, both employing the Phase Function Method effectively. This text focuses on the analysis of nucleon-nucleon phase shifts using the variable phase method [30], which is briefly outlined in Sec. 2. Section 3 presents the results and discussions, while Sec. 4 offers the concluding remarks.

2. Methodology

PFM [30] is a valuable numerical approach that provides a strong alternative to the standard methods used to solve the Schrödinger equation. The methodology relies on the potential transformation of second-order linear homogeneous equations into first-order nonlinear Riccati equations or phase equations, as illustrated below. The function $\delta_\ell(k,r)$, referred to as the phase function, represents the accumulated phase shift at a distance r, effectively modeling the scattering caused by a truncated version of the potential up to that range. If the potential is completely cut off, no phase shift occurs, implying $\delta_\ell(k,r)=0$. This framework transforms the original second-order linear differential equation into a first-order nonlinear equation-commonly a Riccati-type equation-thereby simplifying the computational treatment of the scattering problem. The radial Schrödinger equation for a given potential $V(r)$ is given as:

$$\frac{d^2}{dr^2}u_l(k,r)+\left[k^2-\frac{l(l+1)}{r^2}\right]u_l(k,r) = V(r)\,u_l(k,r). \tag{1}$$

We Solve Eq. (1) using PFM, as it is a useful special method to solve the radial Schrödinger equation.

The regular free particle Green's function reads as,

$$G_l(r,r')=\begin{cases}-\frac{1}{k}\left[\hat{j}_l(kr)\,\hat{\eta}_l(kr')-\hat{j}_l(kr')\,\hat{\eta}_l(kr)\right], & r'<r,\\ 0, & r'>r.\end{cases} \tag{2}$$

Using Eq. (2), the solution to Eq. (1) is given by

$$u_l(k,r)=\hat{j}_l(kr)-\frac{1}{k}\,\hat{j}_l(kr)\int_0^r dr'\,\hat{\eta}_l(kr')\,V(r')\,u_l(k,r'), + \frac{1}{k}\,\hat{\eta}_l(kr)\int_0^r dr'\,\hat{j}_l(kr')\,V(r')\,u_l(k,r'). \tag{3}$$

Let

$$S_l(r)=-\frac{1}{k}\int_0^r dr'\,\hat{j}_l(kr')\,V(r')\,u_l(k,r'), \tag{4}$$

$$C_l(r)=1-\frac{1}{k}\int_0^r dr'\,\hat{\eta}_l(kr')\,V(r')\,u_l(k,r'). \tag{5}$$

Also, $S_l(r) \xrightarrow{r\to 0} 0$ and $C_l(r) \xrightarrow{r\to 0} 1$. Using Eqs. (4) and (5), Eq. (3) becomes

$$u_l(k,r) = \hat{j}_l(kr)\, C_l(r) - \hat{\eta}_l(kr)\, S_l(r). \tag{6}$$

As

$$\hat{j}_l(kr) \xrightarrow[r\to\infty]{} \sin\left(kr - \frac{l\pi}{2}\right), \quad kr \gg l, \tag{7}$$

$$\hat{\eta}_l(kr) \xrightarrow[r\to\infty]{} -\cos\left(kr - \frac{l\pi}{2}\right), \quad kr \gg l, \tag{8}$$

$C_l(r) \xrightarrow{r\to\infty} C_l(\infty)$ and $S_l(r) \xrightarrow{r\to\infty} S_l(\infty)$.

$$\therefore\ u_l(k,r) \xrightarrow{r\to\infty} C_l(\infty)\, \sin\left(kr - \frac{l\pi}{2}\right) + S_l(\infty)\, \cos\left(kr - \frac{l\pi}{2}\right). \tag{9}$$

However,

$$u_l(k,r) \xrightarrow{r\to\infty} \text{Const.} \times \sin\left(kr - \frac{l\pi}{2} + \delta_l\right) = A\left[\sin\left(kr - \frac{l\pi}{2}\right)\cos\delta_l + \cos\left(kr - \frac{l\pi}{2}\right)\sin\delta_l\right]. \tag{10}$$

From Eqs. (9) and (10)

$$A\left[\sin\left(kr - \frac{l\pi}{2}\right)\cos\delta_l + \cos\left(kr - \frac{l\pi}{2}\right)\sin\delta_l\right] = C_l(\infty)\, \sin\left(kr - \frac{l\pi}{2}\right) + S_l(\infty)\, \cos\left(kr - \frac{l\pi}{2}\right). \tag{11}$$

Equating the coefficient of $\sin(kr - [l\pi/2])$ and $\cos(kr - [l\pi/2])$ on both sides, one gets

$$A\cos\delta_l = C_l(\infty), \tag{12}$$

$$A\sin\delta_l = S_l(\infty), \tag{13}$$

which gives

$$\tan\delta_l = \frac{S_l(\infty)}{C_l(\infty)}. \tag{14}$$

Hence the knowledge of the asymptotic limit of $S_l(\infty)$ and $C_l(\infty)$ gives an idea about the tangent of scattering phase shifts. Let us define a new function

$$t_l(r) = \frac{S_l(r)}{C_l(r)}. \tag{15}$$

From Eqs. (4) and (5), it is clear that $t_l(r)$ vanishes at $r \longrightarrow 0$. At the same time asymptotic behaviour of $t_l(r)$ gives the value of tangent of scattering phase shifts

$$\lim_{r\to\infty} t_l(r) = t_l(\infty) = \tan\delta_l. \tag{16}$$

From Eq. (4)

$$S_l'(r) = -\frac{1}{k}\, V(r)\, \hat{j}_l(kr)\, u_l(k,r), \tag{17}$$

and

$$C_l'(r) = -\frac{1}{k}\, V(r)\, \hat{\eta}_l(kr)\, u_l(k,r). \tag{18}$$

Differentiating Eq. (15) and substituting Eqs. (17) and (18),

$$\frac{d}{dr}\left(\frac{S_l(r)}{C_l(r)}\right) = \frac{C_l(r)\, S_l'(r) - S_l(r)\, C_l'(r)}{C_l^2(r)}, \tag{19}$$

we get,

$$t_l'(r) = -\frac{1}{k}\, V(r)\left[\hat{j}_l(kr) - t_l(r)\, \hat{\eta}_l(kr)\right]^2. \tag{20}$$

Again,

$$t_l(r) = \tan\delta_l. \tag{21}$$

Differentiating Eq. (21), we get,

$$t_l'(r) = \frac{\delta_l'(r)}{\cos^2\delta_l(r)}. \tag{22}$$

Substituting Eq. (22) in Eq. (20), we get,

$$\delta_\ell'(k,r) = -k^{-1}V(r) \times [\cos\delta_\ell(k,r)\hat{j}_\ell(kr) - \sin\delta_\ell(k,r)\hat{\eta}_\ell(kr)]^2, \tag{23}$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati-Bessel functions [41]. The resulting phase equations [30, 41] for $\ell = 0, 1$ and 2 read as

$$\delta_0'(k,r) = -k^{-1}V(r)[\sin(\delta_0(k,r) + kr)]^2, \tag{24}$$

$$\delta_1'(k,r) = -\frac{V(r)}{k^3r^2}\big[\sin(\delta_1(k,r) + kr) - kr\cos(\delta_1(k,r) + kr)\big]^2, \tag{25}$$

and

$$\delta_2'(k,r) = -k^{-1}V(r)\big[(\frac{3}{k^2r^2} - 1)\sin(\delta_2(k,r) + kr) - \frac{3}{kr}\cos(\delta_2(k,r) + kr)\big]^2. \tag{26}$$

Here, k represents the center of mass momentum, which is related to the center of mass energy E as $k = \sqrt{2\mu E/\hbar^2}$. The term $\delta_\ell(k,r)$ is referred to as the phase function, which satisfies the phase equation described in Eq. (1). This represents the phase shift of the wave function as it scatters off the potential at each point, ultimately truncated at a distance 'r'. With the initial condition provided, the phase equations described by Eqs. (2-4) are solved numerically for the potential under consideration to align the scattering phase shifts across various states with the established data.

The effective Pöschl-Teller potential [9–15] in all partial waves is written as

$$V_{PT}(r) = \frac{V_1 - V_2\cosh(\alpha r)}{\sinh^2(\alpha r)} + \frac{\ell(\ell+1)}{r^2}, \tag{27}$$

and the effective spin orbit coupling is taken as

$$V_{SO}(r) = \frac{V_1' - V_2'\cosh(\alpha r)}{\sinh^2(\alpha r)}(\vec{L}\cdot\vec{S}). \tag{28}$$

(29)

The effective equivalent nuclear potential under investigation for uncharged hadrons is thus

$$V(r) = V_{\rm PT}(r) + V_{\rm SO}(r). \tag{30}$$

(31)

The strength parameters V_1, V_2, V_1', V_2' of the total potential $V(r)$ have the unit of fm^{-2} and α is the inverse range parameter with dimension of fm^{-1}. To address charged hadron systems, it is necessary to incorporate an electromagnetic interaction alongside the nuclear potential

$$V(r) = V_{\rm PT}(r) + V_{\rm SO}(r) + V_C(r). \tag{32}$$

(33)

The pure Coulomb potential theoretically allows for an infinite range of interaction. However, in practice, it becomes screened beyond a certain distance, rendering the Coulomb potential negligible after a finite range. Motivated by this the electromagnetic potential is taken as screened atomic Hulthén potential [42–45]

$$V_C(r) = V_0\frac{\exp(-r/a)}{[1-\exp(-r/a)]}, \tag{34}$$

with V_0, the strength and a , the screening radius of the potential with $V_0 a = 0.03472 fm^{-1}$ for nucleon-nucleon systems.

3. Results and discussion

The parameterization of the nuclear Pöschl-Teller potential, including the spin-orbit term as outlined in Eqs. (29) and (30), to match the available standard phase parameters [46] for various states of the (n-p) and (p-p) systems through the numerical solution of the differential equations (24-26). The best fitted parameters for the (n-p) and (p-p) systems across various states is presented in Table I.

TABLE I. List of parameters for (n-p) system.

States	α	V_1	V_2	V_1'	V_2'
$^3P_0$	0.92	9.02	11.25	0.50	2.70
$^3P_1$	2.24	4.35	0.15	0.08	15.60
$^3P_2$	0.96	9.25	8.89	0.51	1.27
$^3D_1$	2.03	8.65	2.05	1.85	6.70
$^3D_2$	0.96	2.031	32.065	8.05	18.27
$^3D_3$	1.26	6.36	8.65	18.69	31.35

TABLE II. List of parameters for (p-p) system.

States	α	V_1	V_2	V_1'	V_2'
$^3P_0$	0.86	9.05	8.92	0.5	1.70
$^3P_1$	2.46	4.34	0.17	0.08	15.50
$^3P_2$	0.82	10.08	0.82	2.02	2.29

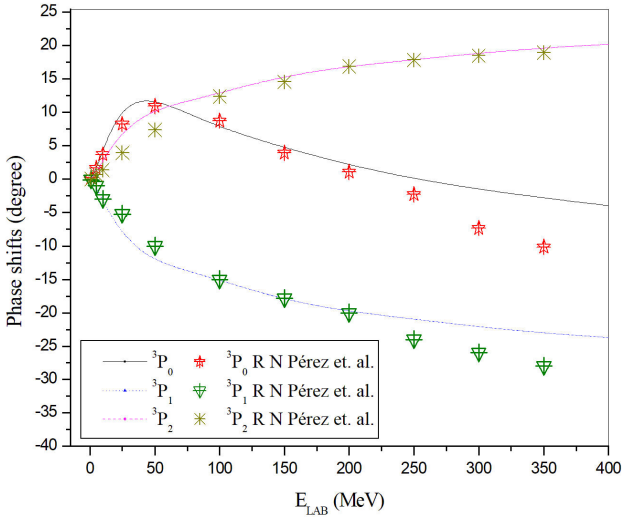


FIGURE 1. Variation of (n-p) P-wave scattering phase shifts with laboratory energy.

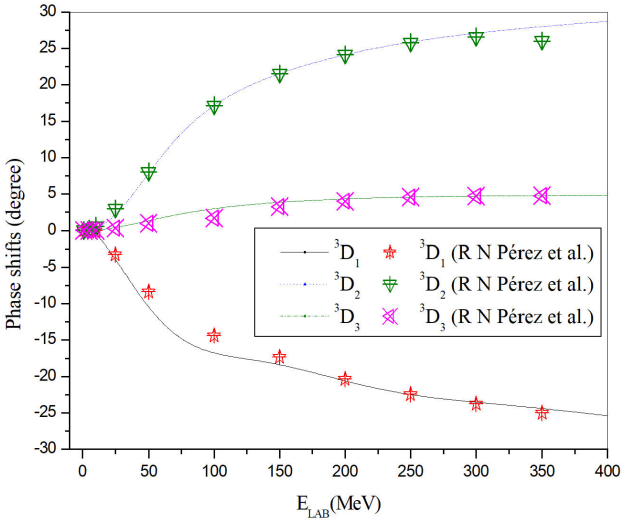


FIGURE 2. Variation of (n-p) D-wave scattering Phase shifts with laboratory energy.

For numerical adjustments of the phase shift results, we have utilized $\hbar^2/2\mu = 41.47$ MeV fm^2 specific to the (n-p) and (p-p) systems, respectively. Here, μ denotes the reduced mass of the corresponding systems. In this article, we examine the scattering phase shifts for neutron-proton (n-p)

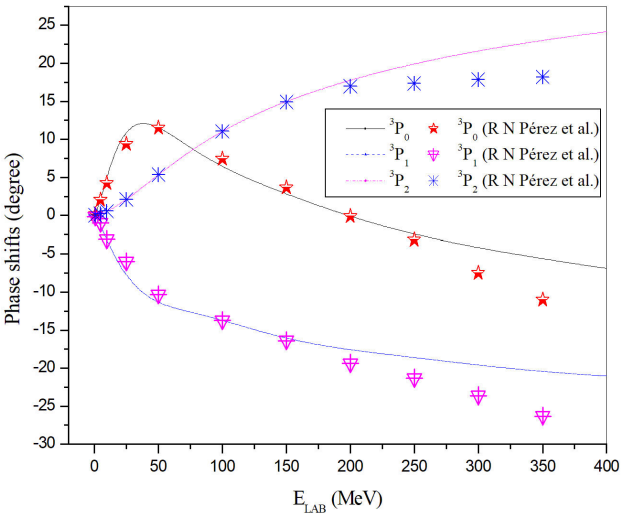


FIGURE 3. Variation of (p-p) P-wave scattering phase shifts with laboratory energy.

and proton-proton (p-p) interactions for angular momenta $\ell = 1\,\&\,2$. Figures 1-3 illustrate the computed phase shift values in comparison with the established results [46] for various (n-p) and (p-p) triplet scattering states, as detailed in Tables I and II. As shown in Figs. 1 and 2, the analysis for the P- and D-wave states in the (n-p) system aligns well with the established phase parameters up to a laboratory energy of 350 MeV, closely matching the findings of Perez *et al*. [46]. The phase parameters for p-p scattering, as depicted in Fig. 3, show strong agreement with the results of Perez *et al*. [46] up to a laboratory energy of 350 MeV. The strength parameters for the p-p triplet states $^3P_0$, $^3P_1$ and $^3P_2$ are kept consistent with those of the n-p states $^3P_0$, $^3P_1$ and $^3P_2$. A slight adjustment has been introduced for the inverse range parameter 'α' leading to minor effects on both the central nuclear and spin-orbit terms, but the changes are not considerable. The strength values remain unchanged because the nuclear force treats n-p and p-p interactions similarly, considering both neutrons and protons as equivalent nucleons. Therefore, it is reasonable to expect analogous parameterization of the strong nuclear environment for two-nucleon interactions in similar states, regardless of their charge. However, the Coulomb force is relevant for p-p interactions, and we have addressed this by incorporating additional Coulomb terms into the existing nuclear and spin-orbit terms, as shown in Eq. (8). As the nuclear potential is greatly influenced by spin and varies with different states, it is necessary to individually parameterize each state of the P and D waves in n-p interactions and the P-wave states in p-p interactions to accurately reflect the real inter-nucleon interaction field.

We assessed the goodness of fit by computing the χ^2 per degree of freedom (reduced χ^2 values) for different states in the n-p and p-p systems, considering (8-5) = 3 degrees of freedom. The calculated reduced χ^2 values for the $^3P_0$, $^3P_1$, $^3P_2$,$^3D_1$,$^3D_2$,$^3D_3$ states of the n-p system are 0.684, 0.923, 0.527, 0.125, 0.571, and 1.186, respectively. For the p-p system, the reduced χ^2 values for $^3P_0$, $^3P_1$ and $^3P_2$states are 0.812, 0.136, and 1.136, respectively. Overall, these reduced chi-square values are either below or around 1, suggesting that the model provides a good fit to the standard data for all states of the n-p and p-p systems. Also, we have verified the R-square values for different states which are close to unity. Thus, the inclusion of the Pöschl-Teller and Pöschl-Teller like spin-orbit interaction successfully modifies the phase shifts to match the established data [46] for the n-p and p-p systems, demonstrating the robustness and reliability of this approach. As the energy increases, the probability of the projectile accessing the nuclear range of interaction also grows, allowing for a more pronounced spin-orbit effect, which is crucial for accurately modelling the real nuclear phenomenological field. Beyond 200 MeV the phase parameters for $^3P_0$, $^3P_1$ and $^3P_2$ states differ from those of standard values by maximum of 6 degree. It is argued that the discrepancy in phase shifts at higher energies is largely reduced if the short-range part of the LS force in the nucleon-nucleon potential is modified. Since nucleon- nucleon scattering at energies below 15 MeV is almost purely S-wave scattering which arises from the interference between S- and P-waves, is very small. Contrarily, the P-wave nucleon-nucleon interactions are very important in scattering and PJ state contribution [47–49] must be included. The lack of PJ state in the proposed potential employed in this calculation may be the cause of the discrepancies in the phase parameters beyond 200 MeV.

At higher incident energies near 200 MeV, a reversal of the phase shift for the $^3P_0$ state is evident in both (n-p) and (p-p) scattering in Figs. 1 and 3, implying the influence of a repulsive hardcore.

4. Conclusion

This study tests the phase shift analysis of charged and uncharged nucleon-nucleon interactions in the context of the Pöschl-Teller [9–15] plus Pöschl-Teller like spin-orbit interaction field [Eq. 6], focusing on higher incident particle energies up to 350 MeV and higher partial waves using phase function methodology [30]. Using a five-parameter potential, we achieve excellent fitting of the phase shift parameters for all P-wave and D-wave triplet states in n-p scattering, as well as for the P-wave triplet states in p-p scattering, in line with the results of [46]. The reduced χ^2 values suggest that the quality of the adjustment is quite satisfactory. This confirms that the model effectively calibrates the phase shifts for both n-p and p-p systems. The results demonstrate that high-energy scattering data up to 350 MeV can be interpreted if the nuclear interaction exhibits two main traits: a robust spin-orbit coupling possibly in the outer region, similar to the exchange character of pion theoretical potential, along with a hard-core-like repulsive force in the inner region. Encouraged by the success of this work, I am motivated to advance to the calculation of polarization effects for scattered particles with unpolarized incident and target particles. Nonetheless, assessing polarization will involve calculating the phase pa-

rameters for the triplet state ($^3S_1$ state), despite the absence of any spin-orbit contribution for this particular state. I am eager to explore spin-orbit coupling, particularly through the incorporation of second-order momentum approximations.

Data availability statement

The author declares that the data supporting the findings of this study are available within the paper.

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Conflict of interest

The author declares that there exists no competing financial interest or personal relationships that could have appeared to influence the work reported in this paper.

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