Dimensional consistency of derivatives with non-singular kernels

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The Caputo-Fabrizio (CF) and Atangana-Baleanu (AB) derivatives were proposed as alternatives to the usual Caputo and Riemann-Liouville fractional derivatives. An advantage of CF and AB derivatives is that they are defined in terms of non-singular kernels, such as the exponential function and the Mittag-Leffler function, respectively. However, the use and application of fractional derivatives with non-singular kernels has resulted in many models in terms of fractional differential equations that do not comply with the principle of dimensional homogeneity and therefore their solutions are not entirely satisfactory. In this work, we propose New Derivatives of the Caputo-Fabrizio (NDCF) and Atangana-Baleanu (NDAB) types, derive their Laplace transforms and apply them to the solution of an RC circuit as an illustrative example. Furthermore, we confirm that the price to pay for attempting to avoid the singularity of the Caputo derivative is that the initial condition is not satisfied for kernels without singularities. The initial condition depends on the order of the fractional derivative γ and only reduces to the initial condition when $\gamma = 1$. To provide a more realistic response, a shift in the dynamic variable was introduced, yielding physical solutions consistent with the dimensionality, the initial condition, and the steady state.

Keywords: Caputo fractional derivative; Caputo-Fabrizio derivative; Atangana-Baleanu derivative; RC circuit.

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1. Introduction

Fractional calculus (FC), which might be considered a generalization of ordinary calculus, deals with derivatives and integrals of non-integer order [1-3]. Although fractional derivatives and integrals appeared almost at the same time as ordinary derivatives, it was not until about 40 years ago that they gained wide acceptance in the scientific community, particularly in applied sciences. Solutions obtained from fractional differential equations have been shown to fit experimental data better [4-10]. Despite the efforts to have a physical and geometrical interpretation of fractional derivatives and integrals, today there is no fully accepted answer to this issue [11, 12]. A further aspect that requires attention and is overlooked in many works is the dimensional homogeneity of equations, particularly when physical processes are modeled. It is very common to find, in the literature, equations that do not comply with the principle of homogeneity, which states that the dimensions of each of the terms of a dimensional equation on both sides should be the same [13,14]. The

standard procedure for converting an integer-order model into its corresponding fractional-order model consists of replacing the classical differential operator(s) with those of non-integer order. This method, however, unbalances the units in the equations. One possibility to overcome this difficulty is to change the ordinary derivative d/dt by the fractional one $1/\sigma^{1-\gamma}d^{\gamma}/dt^{\gamma}$, introducing a parameter, commonly denoted as σ, first introduced in Refs. [15-17]. The units of this parameter depend on the variable to which the derivative is applied (time or space). Thus, if we are dealing with a dynamic process, where time derivative is required, σ must have units of time $[T]$. The above strategy has yielded dimensionally consistent results in models of different processes in the fields of electric circuits, physics, biology and engineering, among others [18-25]. Because the fractional Riemann-Liouville and Caputo derivatives have a weak singularity at $t = \tau$, in recent times the Caputo-Fabrizio derivative (CFD) [26] and the Atangana-Baleanu derivative (ABD) [27], were proposed as alternatives to solve this problem. However, it has been observed that, by starting from a dimensionally consistent fractional differential equation the resulting solutions are not dimensionally consistent [6, 28, 29]. This does not occur when applying the fractional Caputo derivative, in this case, dimensionally consistent solutions are obtained [6, 29]. The growing interest in these non-singular operators motivates a deeper examination of their properties, including the issue of dimensional consistency addressed in the present work.

2. Basic concepts

Definition 1: Given the function $f \in C^m[a, b]$ and γ a positive real parameter, let $n \in \mathbb{N}$ be such that $\gamma \in (n-1, n)$, then the Caputo fractional derivative of f of order γ and $n = 1$, is defined by the integral [1],

$$ {}^{C}D^{\gamma}f(t) = \frac{1}{\Gamma(1-\gamma)}\int_0^t \frac{f'(\tau)}{(t-\tau)^{\gamma}}d\tau, \tag{1}$$

where, $f' = df/d\tau$, and, $\Gamma(x) = \int_0^{\infty} e^{-t}t^{x-1}dt$, are the ordinary derivative and the Gamma function, respectively.

Definition 2: The Caputo-Fabrizio derivaitive is obtained from (1) by replacing the power law kernel $(t-\tau)^{-\gamma}$ with the exponential function $\exp(-[\gamma/(1-\gamma)]t)$ and $1/\Gamma(1-\gamma)$ with $M(\gamma)/[1-\gamma]$, then, we have [26],

$$ {}^{CF}D^{\gamma}f(t) = \frac{M(\gamma)}{1-\gamma}\int_0^t f'(\tau)\exp\left(-\frac{\gamma(t-\tau)}{1-\gamma}\right)d\tau, \tag{2}$$

where $M(\gamma)$ is a normalization function, such that, $M(0) = M(1) = 1$. The above is to ensure that, when $\gamma \to 1$, the Caputo-Fabrizio derivative should be reduced to a first-order ordinary derivative.

Definition 3: The Atangana-Baleanu (AB) derivative is defined in terms of a Mittag-Leffler function as a kernel, such as [27],

$$ {}^{AB}D^{\gamma}f(t) = \frac{B(\gamma)}{1-\gamma}\int_0^t f'(\tau)E_{\gamma}\Big[-\frac{\gamma(t-\tau)^{\gamma}}{1-\gamma}\Big]d\tau, \tag{3}$$

where $B(\gamma)$ has the same properties as in the CF case, and $E_{\gamma}(\cdot)$, is the one-parameter Mittag-Leffler function, defined in Ref. [1],

$$ E_{\gamma}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\gamma n+1)}. \tag{4}$$

The advantage of the CF and AB derivatives is that, we do not have to deal with singular kernels at $t = \tau$, as is the case of (1).

Definition 4: The Laplace transform of a function $f(t) : [0, \infty) \to \mathbb{R}$ is the function $F(s)$, defined by

$$ F(s) = L[f(t)] = \int_0^{\infty} f(t)e^{-st}dt. \tag{5}$$

The Laplace transform in the case of the Caputo, CF and AB derivatives, are, respectively:

$$ \mathcal{L}[{}^{C}D^{\gamma}f(t)] = s^{\gamma}F(s) - s^{\gamma-1}f(0), \tag{6}$$

$$ \mathcal{L}[{}^{CF}D^{\gamma}f(t)] = \frac{sF(s)-f(0)}{s+\gamma(1-s)}, \tag{7}$$

$$ \mathcal{L}[{}^{AB}D^{\gamma}f(t)] = \frac{1}{1-\gamma}\frac{s^{\gamma}F(s)-s^{\gamma-1}f(0)}{s^{\gamma}+\frac{\gamma}{1-\gamma}}, \tag{8}$$

where $f(0)$ represents the initial condition at $t = 0$, and we have taken $M(\gamma) = B(\gamma) = 1$. In this work, we will only analyze the case where the order of the derivative satisfies the inequality $0 < \gamma \leq 1$. It is clear that γ is a dimensionless quantity.

3. Dimensional analysis of fractional derivatives

First, note that an ordinary derivative has the physical units

$$ \Big[\frac{df}{dt}\Big] = \Big[\frac{f}{T}\Big]. \tag{9}$$

If f represents position and is measured in length $[L]$ and t has units of time $[T]$, then (9) represents the velocity of an object at a given point. Every ordinary or partial differential equation that represents a physical system must comply with the principle of dimensional homogeneity [13, 14].

On the other hand, the Caputo fractional derivative, as defined in Eq. (1), has the following physical units, [28, 30],

$$ \Big[{}^{C}D^{\gamma}f(t)\Big] = \Big[\frac{1}{T^{\gamma}}\times\frac{f}{T}\times T\Big] = \Big[\frac{f}{T^{\gamma}}\Big] \quad \Longrightarrow $$
$$ \Big[\frac{f}{T^{\gamma}}\Big] = \Big[\frac{f}{T^{\gamma}}\Big]. \tag{10}$$

This expression complies with the principle of dimensional homogeneity [13, 14]. If f represents position and t represents time, then the fractional derivative would represent

"fractional" velocity. But, what would the physical interpretation be? This question still lacks a satisfactory and universal answer. On the other hand, the Caputo-Fabrizio, Eq. (2), and Atangana-Baleanu, Eq. (3), derivatives do not satisfy the principle of dimensional homogeneity, because the kernels; exponential function and Mittag-Leffler function do not have well-defined dimensions [28].

It is clear that we cannot use the definition of a fractional derivative in an ordinary differential equation simply by changing the ordinary derivative to a fractional derivative, since, as we have seen, they have different units. One way to go from an ordinary derivative to a fractional derivative was first presented in [15-17], as follows

$$\frac{df}{dt} \neq \frac{d^\gamma f}{dt^\gamma} \quad \Longrightarrow \quad \frac{df}{dt} = \frac{1}{\sigma^{1-\gamma}}\frac{d^\gamma f}{dt^\gamma} \quad \Longrightarrow$$
$$\Big[\frac{f}{T}\Big] = \Big[\frac{1}{T^{1-\gamma}}\cdot\frac{f}{T^\gamma}\Big] = \Big[\frac{f}{T}\Big], \tag{11}$$

provided that σ has units of time $[T]$. As can be seen in (11), the principle of dimensional homogeneity [13, 14] is fulfilled. In (11), the right-hand side corresponds to a fractional derivative which can be any of the known ones, such as Caputo, Caputo-Fabrizio, Atangana-Baleanu, conformable derivative [31], etc.

Taking into account the expressions (11) and (1), we have

$$\frac{df}{dt} = \frac{1}{\sigma^{1-\gamma}}\,{}^{C}D^\gamma f(t)$$
$$= \frac{1}{\sigma^{1-\gamma}}\frac{1}{\Gamma(1-\gamma)}\int_0^t \frac{f'(\tau)}{(t-\tau)^\gamma}d\tau. \tag{12}$$

It is easy to see that the expression (12), has the following dimensionality,

$$\Big[\frac{f}{T}\Big] = \Big[\frac{1}{T^{1-\gamma}}\times\frac{f}{T}\times\frac{1}{T^\gamma}\times T\Big] = \Big[\frac{f}{T}\Big]. \tag{13}$$

This method of converting an ordinary derivative into a fractional derivative, using the Caputo derivative, has been used in different physical systems, resulting in a more general solution that is homogeneously consistent with the physical units of the system [15-17]. As we can see, starting from an ordinary derivative of time or space, by means of the transformation (11), we can obtain dimensionally homogeneous fractional differential equations, and therefore their solutions will be consistent with the physical units.

4. A new derivative of the Caputo-Fabrizio type

For the relation (12) to hold, in the case of the Caputo-Fabrizio derivative, we define a New Derivative of the Caputo-Fabrizio type (NDCF),

$${}^{NDCF}Df(t) = \frac{1}{(1-\gamma)}\frac{1}{\sigma}\int_0^t \frac{df}{d\tau}$$
$$\times\exp\Big[-\frac{\gamma}{1-\gamma}\Big(\frac{t-\tau}{\sigma}\Big)\Big]d\tau. \tag{14}$$

Unlike the Caputo-Fabrizio derivative, in this expression the exponential is dimensionless. The dimensionality of this new derivative is the same as in (13), that is, it is seen as an ordinary derivative (9). Its corresponding Laplace transform has the form

$$\mathcal{L}\{{}^{NDCF}D(t)\} = \frac{1}{\sigma(1-\gamma)s+\gamma}\Big(sF(s)-f(0)\Big). \tag{15}$$

Due to the dimensionality of the new derivative (14), the direct transformation of an ordinary differential equation into a fractional differential equation can be performed, thanks to its construction. To verify this, let us look at a simple example, such as the RC electrical circuit.

The ordinary differential equation that describes the behavior of an RC circuit with a constant source V_0, is given by

$$RC\frac{dv}{dt} + v(t) = V_0, \qquad v(0) = 0. \tag{16}$$

Now, we change the derivative in (16) by the NDCF (14), then, the corresponding new differential equation is given by

$${}^{NDCF}Dv(t) + \frac{1}{RC}v(t) = \frac{1}{RC}V_0. \tag{17}$$

Applying the Laplace transform (15),

$$\mathcal{L}[\,{}^{NDCF}Dv(t)] + \frac{1}{RC}\mathcal{L}[v(t)] = \frac{V_0}{RC}\mathcal{L}[1],$$
$$v(0) = 0, \tag{18}$$

we get,

$$\frac{1}{\sigma(1-\gamma)s+\gamma}sV(s) + \frac{1}{RC}V(s) = \frac{V_0}{RC}\frac{1}{s}. \tag{19}$$

Factoring out $V(s)$

$$V(s) = \frac{V_0}{s}\frac{\sigma(1-\gamma)s+\gamma}{[RC+\sigma(1-\gamma)]s+\gamma}. \tag{20}$$

Now, applying the inverse Laplace transform, we get

$$v(t;\gamma)_{NDCF} = V_0\Bigg[1 - \frac{RC}{RC+\sigma(1-\gamma)}$$
$$\times\exp\Big(-\frac{\gamma}{RC+\sigma(1-\gamma)}t\Big)\Bigg]. \tag{21}$$

The solution (21) is dimensionally consistent, since σ has units of time. However, when $t\to 0$, we have

$$v(0;\gamma)_{NDCF} = \Big[\frac{\sigma(1-\gamma)}{RC+\sigma(1-\gamma)}\Big]V_0 \neq 0. \tag{22}$$

In other words, the initial condition is not satisfied. Initial condition is satisfied only when $\gamma = 1$. However, when $t\to\infty$, from (21), we have $V(t\to\infty) = V_0$. This fact had already been observed in Refs. [6, 29], however, it differs a little from this work, since the dimensionality of the CF derivative was not defined. That is, the definition (2) was used there. It can easily be verified that, when $\gamma = 1$ the expression (21) reduces to the well-known solution.

5. A new derivative of the Atangana-Baleanu type

Because the Atangana-Baleanu derivative is a generalization of the Caputo-Fabrizio derivative, then we can define a New Derivative of the Atangana-Baleanu type (NDAB), as follows,

$$ {}^{NDAB}Df(t) = \frac{1}{(1-\gamma)}\frac{1}{\sigma}\int_0^t \frac{df}{d\tau} \times E_\gamma\Big[-\frac{\gamma}{1-\gamma}\Big(\frac{t-\tau}{\sigma}\Big)^\gamma\Big]d\tau. \tag{23}$$

Its dimensionality is the same as in (13). Therefore, the Laplace transform in this case, is

$$\mathcal{L}\{{}^{NDAB}Df(t)\} = \frac{1}{\sigma(1-\gamma)s^\gamma + \gamma\sigma^{1-\gamma}} \times \Big(s^\gamma F(s) - s^{\gamma-1}f(0)\Big). \tag{24}$$

Now, applying the Laplace transform (24) to the same RC electrical circuit (16), we have

$$\frac{1}{as^\gamma + b}s^\gamma V(s) + \frac{1}{RC}V(s) = \frac{V_0}{RC}\frac{1}{s}, \tag{25}$$

where $a = \sigma(1-\gamma)$ and $b = \gamma\sigma^{1-\gamma}$. Solving for $V(s)$, we have

$$V(s) = \frac{V_0 a\Big(s^\gamma + \frac{b}{a}\Big)}{(RC+a)s\Big(s^\gamma + \frac{b}{RC+a}\Big)}. \tag{26}$$

Taking the inverse Laplace transform, we can write the solution as follows

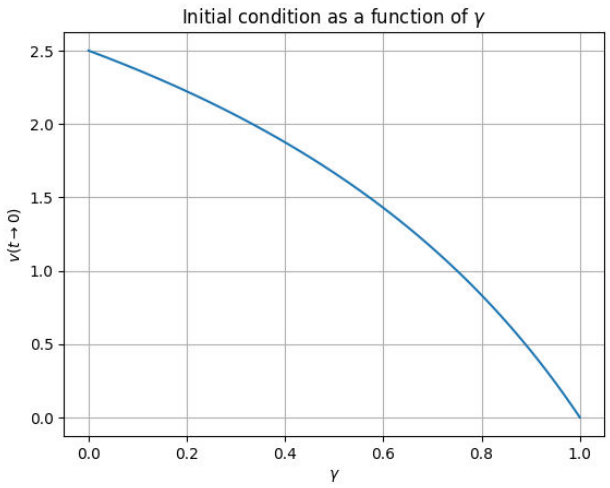


FIGURE 1. This graph represents the initial condition (22), for certain values of γ, with the following physical parameter values: $R = 1000, \Omega$; $C = 1 \times e^{-3}$, F; $\sigma = 1$, s; $V_0 = 5$, volts.

$$v(t;\gamma)_{NDAB} = V_0\Big[1 - \frac{RC}{RC+\sigma(1-\gamma)} \times E_\gamma\Big(-\frac{\gamma\sigma^{1-\gamma}}{RC+\sigma(1-\gamma)}t^\gamma\Big)\Big]. \tag{27}$$

It was expected that in this case the initial condition would not be satisfied, when $t = 0$, as in the case of the Caputo-Fabrizio derivative (22). From Eq. (22) and Fig. 1, we can see that the initial condition is also a function of γ. We can see that, when $\gamma = 1$, the expression (27) reduces to the ordinary solution.

$$v(t) = V_0\Big[1 - \exp\Big(-\frac{t}{RC}\Big)\Big]. \tag{28}$$

6. Applying the Caputo fractional derivative

To analyze and compare the different derivatives, we will solve the same RC circuit problem using Caputo's derivative (1) and its corresponding Laplace transform (6), we have

$$\frac{d^\gamma v}{dt^\gamma} + \frac{\sigma^{1-\gamma}}{RC}v(t) = \frac{\sigma^{1-\gamma}}{RC}V_0. \tag{29}$$

Applying the Laplace transform and its inverse, we finally obtain the solution

$$v(t,\gamma)_C = V_0\Big[1 - E_\gamma\Big(-\frac{\sigma^{1-\gamma}}{RC}t^\gamma\Big)\Big]. \tag{30}$$

In this case the conditions, $v(0,\gamma)_C = 0$, $v(\infty;\gamma)_C = V_0$ are satisfied.

7. Discussion

So far, starting from the transformation (11), the new derivatives NDCF (14) and NDAB (23) have been defined, which are dimensionally consistent and also preserve the dimensionality of the ordinary time derivative. Their corresponding Laplace transforms, (15) and (24) were also obtained and applied to a simple example, the RC circuit with a constant source, yielding the solutions (21), (27) and for the Caputo fractional derivative (30), which are dimensionally consistent, unlike those obtained in Ref. [6, 29]. However, only the Caputo derivative satisfies the conditions $v(0;\gamma) = 0$ and $v(\infty,\gamma) = V_0$, as can be seen in Fig. 2.

Figure 2 clearly shows that the initial condition only holds in the case of Caputo's derivative; in the other two cases, a jump occurs. Furthermore, from Fig. 2, we can observe that the NDCF has a behavior similar to that of the ordinary integer derivative, which implies that this derivative does not contribute anything new to the description of the system, in total agreement with [32]. In particular, the decay pattern follows an exponential trend, indicating that the memory introduced by the NDCF is effectively finite and rapidly attenuated, and the behavior of the system is more like that of an integer-order RC circuit. Unlike the NDCF, the NDAB uses

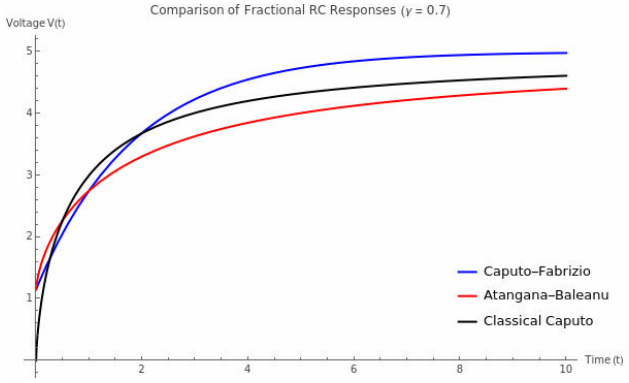


FIGURE 2. This graph represents the different solutions given by the equations (21), (27) and (30), respectively. Here, $R = 1\,\Omega\text{hm}$, $C = 1\,\text{F}$, $\sigma = 1\,\text{s}$ and $V_0 = 5\,\text{V}$.

the Mittag-Leffler kernel, specifically designed to overcome the short-memory limitations of exponential kernels. The log-log representation in Fig. 3, makes this distinction explicit: while NDCF curves bend downward due to exponential decay, both Caputo and NDAB maintain nearly linear behavior in the asymptotic region, revealing algebraic relaxation. This confirms that NDAB preserves genuine long-memory effects comparable to those of Caputo.

Let us define a shifted physical variable as

$$\tilde{v}(t) = v(t) - v(0). \tag{31}$$

Then, using the NDCF operator, the initial value problem (16) is described by

$${}^{NDCF}D[v(t) - v(0)] + \frac{1}{RC}[v(t) - v(0)] = \frac{V_0}{RC}. \tag{32}$$

Taking into account the initial condition $v(0) = 0$, we get

$${}^{NDCF}D[v(t)] + \frac{1}{RC}v(t) = \frac{V_0}{RC}. \tag{33}$$

This equation is the same as in (17), therefore the solution is given by (21) with the initial condition (22). Now, taking the relation (31), we have

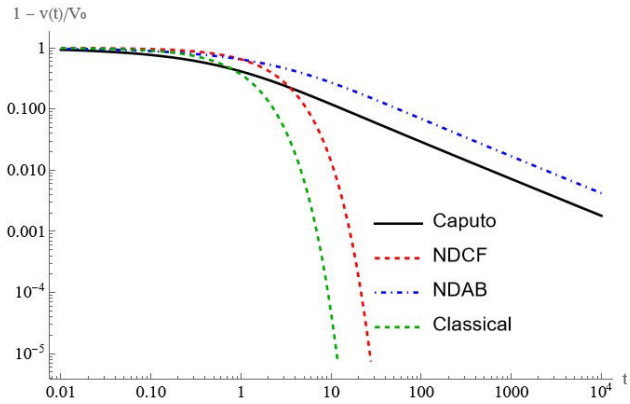


FIGURE 3. Log-log comparison of the asymptotic residual $1 - V/V_0$ for the fractional RC circuit $\gamma = 0.6$. The NDCF model exhibits exponential decay, indicating a short memory.

$$\begin{aligned}\tilde{v}(t;\gamma)_{NDCF} &= V_0\Big[1 - \frac{RC}{RC + \sigma(1-\gamma)} \\ &\times \exp\Big(-\frac{\gamma}{RC + \sigma(1-\gamma)}t\Big)\Big] \\ &\times -\Big[\frac{\sigma(1-\gamma)}{RC + \sigma(1-\gamma)}\Big]V_0 = \frac{RC}{RC + \sigma(1-\gamma)} \\ &\times \Big[1 - \exp\Big(-\frac{\gamma}{RC + \sigma(1-\gamma)}t\Big)\Big]V_0. \end{aligned} \tag{34}$$

In this case, when $t = 0$, we have

$$\tilde{v}(0,\gamma)_{NDCF} = 0, \tag{35}$$

and the initial condition holds. But, in the limit $t \to \infty$, we get

$$\lim_{t\to\infty} \tilde{v}(\infty,\gamma)_{NDCF} = \frac{RC}{RC + \sigma(1-\gamma)}V_0. \tag{36}$$

If we choose

$$N_{NDCF} = \frac{RC + \sigma(1-\gamma)}{RC}, \tag{37}$$

as the normalization parameter, then the physical voltage $\tilde{v}^{phys}(t;\gamma)_{NDCF} = N_{NDCF} \cdot \tilde{v}(t;\gamma)$, will be given by

$$\begin{aligned}&\tilde{v}^{phys}(t;\gamma)_{NDCF} \\ &= V_0\Big[1 - \exp\Big(-\frac{\gamma}{RC + \sigma(1-\gamma)}t\Big)\Big]. \end{aligned} \tag{38}$$

In this case, the conditions

$$\begin{aligned}\tilde{v}^{phys}(0;\gamma)_{NDCF} &= 0, \\ \tilde{v}^{phys}(\infty;\gamma)_{NDCF} &= V_0. \end{aligned} \tag{39}$$

are hold, and, when $\gamma = 1$ the solution (38) becomes (28). In the case of the NDAB, the system is described by

$${}^{NDAB}D[v(t) - v(0)] + \frac{1}{RC}[v(t) - v(0)] = \frac{V_0}{RC}. \tag{40}$$

Following the similar procedure, we get the solution

$$\begin{aligned}&\tilde{v}^{phys}(t;\gamma)_{NDAB} \\ &= V_0\Big[1 - E_\gamma\Big(-\frac{\gamma\sigma^{1-\gamma}}{RC + \sigma(1-\gamma)}t^\gamma\Big)\Big]. \end{aligned} \tag{41}$$

The solution (41) satisfies the conditions

$$\begin{aligned}\tilde{v}^{phys}(0;\gamma)_{NDAB} &= 0, \\ \tilde{v}^{phys}(\infty;\gamma)_{NDAB} &= V_0. \end{aligned} \tag{42}$$

and, in the case $\gamma = 1$ it becomes (28).

Even after enforcing structural normalization to guarantee physical consistency (39) and (42), the intrinsic asymptotic nature of each operator remains unchanged, Fig. 4.

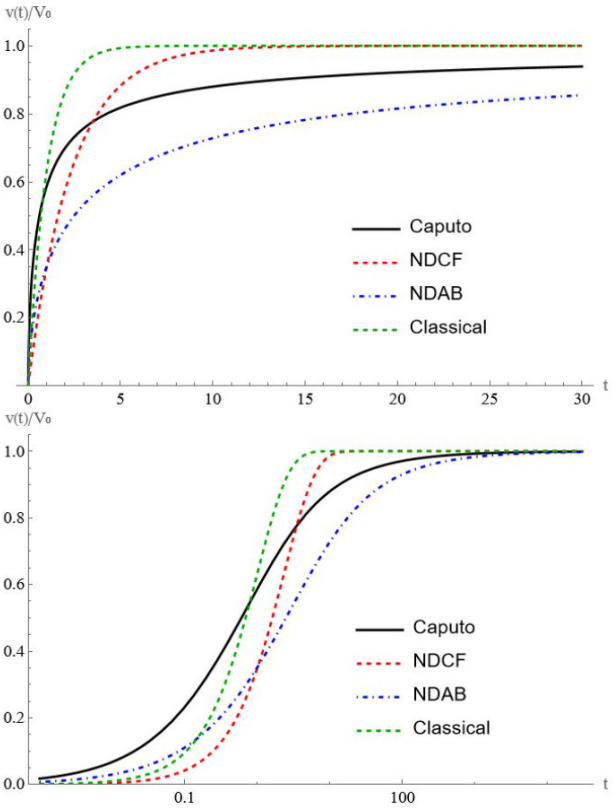


FIGURE 4. Time-domain response $v(t)/V_0$ of the fractional RC circuit corresponding to solutions (30), (38) and (41). After structural normalization, the solutions (38) and (41) satisfy the physical initial condition, $\tilde{v}^{phys}(0;\gamma) = 0$, as can be seen in the first graph, and in the second it can be seen that it converges to the steady-state value $\tilde{v}^{phys}(\infty;\gamma) = V_0$. In the second graph, a logarithmic scale is used on the horizontal axis to clearly illustrate convergence toward the steady state.

NDCF modifies the effective time scale but does not alter the exponential character of the relaxation. In contrast, Caputo and NDAB maintain heavy-tailed decay, which is the defining structural property of fractional-order systems.

In summary, while Caputo is suitable for modeling systems with persistent memory, NDAB refines this behavior through a non-singular Mittag-Leffler kernel, NDCF behaves as a short-memory approximation. From a dynamical perspective, only Caputo and NDAB generate true nonlocal fractional behavior in the RC circuit, whereas Caputo-Fabrizio reduces effectively to a time-rescaled integer-order model.

8. Conclusions

In this work, starting from the transformation (11), the New Derivatives of the Caputo-Fabrizio type NDCF (14) and of Atangana-Baleanu NDAB (23) were defined, as well as their corresponding Laplace transforms. Because these new derivatives have units of $\left[f/T\right]$, just like an ordinary derivative, they can be used directly in physical problems without altering their physical units. This was verified in the RC system, where the solutions are dimensionally consistent; however, the initial condition results as a function of the fractional order of the derivative. From a physical point of view, the above is incorrect, since the capacitor was initially discharged. To obtain a more physically realistic response, a shift was made in the dynamic variable (31) and the physical solutions are obtained. The solutions (38) and (41) are now dimensionally consistent and the initial conditions are maintained at $t \to 0$, as well as the steady state, when $t \to \infty$.

We hope that these New Derivatives of the Caputo-Fabrizio and Atangana-Baleanu type, will find applications in various physical problems, where dimensionality is important.

Appendix A

In this appendix we will explicitly obtain the Laplace transform of the Atangana-Baleanu derivative (23). Expression (23) can be written as a convolution

$$ {}^{NDAB}Df(t) = \frac{1}{\sigma(1-\gamma)} \cdot \Big[f'(t) * E_\gamma\Big(-\frac{\gamma}{(1-\gamma)\sigma^\gamma}t^\gamma\Big)\Big]. \quad \text{(A.1)}$$

Then, applying the Laplace transform, we have

$$ \mathcal{L}\{{}^{NDAB}Df(t)\} = \frac{1}{\sigma(1-\gamma)}\mathcal{L}[f'(t)] \cdot \mathcal{L}\Big[E_\gamma\Big(-\frac{\gamma}{(1-\gamma)\sigma^\gamma}t^\gamma\Big)\Big]. \quad \text{(A.2)}$$

Taking into account the formulas

$$ \mathcal{L}\Big[E_\gamma(-at^\gamma)\Big] = \frac{s^{\gamma-1}}{s^\gamma + a}, \qquad \mathcal{L}[f'(t)] = sF(s) - f(0). \quad \text{(A.3)}$$

We get

$$ \mathcal{L}\{{}^{NDAB}Df(t)\} = \frac{1}{\sigma(1-\gamma)}\Big[sF(s) - f(0)\Big] \cdot \frac{s^{\gamma-1}}{s^\gamma + \frac{\gamma}{(1-\gamma)\sigma^\gamma}} = \frac{1}{\sigma(1-\gamma)s^\gamma + \gamma\sigma^{1-\gamma}} \times \Big(s^\gamma F(s) - s^{\gamma-1}f(0)\Big), \quad \text{(A.4)}$$

which is the expression (24). The process for obtaining the Laplace transform of the NDCF (15) is similar to that shown here.

Appendix B

In this appendix we will explicitly obtain the Laplace transform of the Atangana-Baleanu derivative (23). Expression

(23) can be written as a convolution

$$^{NDAB}Df(t) = \frac{1}{\sigma(1-\gamma)} \cdot \Big[f'(t) * E_\gamma\Big(-\frac{\gamma}{(1-\gamma)\sigma^\gamma}t^\gamma\Big)\Big]. \quad \text{(B.1)}$$

Then, applying the Laplace transform, we have

$$\mathcal{L}\{^{NDAB}Df(t)\} = \frac{1}{\sigma(1-\gamma)}\mathcal{L}[f'(t)] \cdot \mathcal{L}\Big[E_\gamma\Big(-\frac{\gamma}{(1-\gamma)\sigma^\gamma}t^\gamma\Big)\Big]. \quad \text{(B.2)}$$

Taking into account the formulas

$$\mathcal{L}\Big[E_\gamma(-at^\gamma)\Big] = \frac{s^{\gamma-1}}{s^\gamma + a}, \qquad \mathcal{L}[f'(t)] = sF(s) - f(0). \quad \text{(B.3)}$$

We get

$$\mathcal{L}\{^{NDAB}Df(t)\} = \frac{1}{\sigma(1-\gamma)}\Big[sF(s) - f(0)\Big] \cdot \frac{s^{\gamma-1}}{s^\gamma + \frac{\gamma}{(1-\gamma)\sigma^\gamma}} = \frac{1}{\sigma(1-\gamma)s^\gamma + \gamma\sigma^{1-\gamma}} \times \Big(s^\gamma F(s) - s^{\gamma-1}f(0)\Big), \quad \text{(B.4)}$$

which is the expression (24). The process for obtaining the Laplace transform of the NDCF (15) is similar to that shown here.

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Author contributions

Each of the authors contributed to each part of this study equally and read and approved the final version of the manuscript.

Conflict of interest

The authors declare no potential conflict of interests.

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