WEAK INTERACTIONS AND HIGHER SYMMETRIES

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(Recibido: Noviembre 27, 1964)

ABSTRACT

Assuming the weak currents belong to the $T^1 = 1$ *(rotated basis in U-space) components of the regular representation of* SU_3 *and* R_8 *we find that the* $\Delta T = \frac{1}{2}$, $\Delta T = \frac{3}{2}$ *relative phase may be nearly* 90° *without predicting a* $\Delta T = \frac{3}{2}$ *rate incompatible with the experimental evidence. We also find that even a rather small* $\Delta T = \frac{3}{2}$ *contribution may give us a sizeable* CP *violation in the* Σ, Λ *leptonic decays.*

INTRODUCTION

The unitary symmetry octet model[1] has been applied lately in order to explain certain features of the weak leptonic decays with fair success[2]. The weak

*Work supported by the Comisión Nacional de Energía Nuclear. México.

hadronic current is generally assumed to belong to the regular representation of SU_3 and therefore, the model allows the leptonic decays with $\Delta T = 1$; $\Delta T = \frac{1}{2}$, $\Delta S = 1$. Although the presence of $\Delta T = \frac{3}{2}$ leptonic processes is questionable (if there exist they are an order to magnitude smaller that the $\Delta T = \frac{1}{2}$ ones[3]), if they are actually observed it would lead to consider a higher symmetry group which a) contains SU_3/Z_3 (octet model) as a subgroup b) contains both $\Delta T = \frac{1}{2}$, $\Delta T = \frac{3}{2}$. Y. Ne'eman[4] and specially Gourdin[5] have considered this problem and show that the smaller rank simple Lie group which obeys these conditions is R_8 (the group of rotations in 8 dimensions).

On the other hand, the recent experimental evidence about the possible violations of CP conservation[6] (the process $K_2 \to \pi^+ + \pi^-$ has been observed) has been explained assuming the $\Delta T = \frac{3}{2}$ weak amplitude to have a relative phase of nearly 90° with respect to the $\Delta T = \frac{1}{2}$[7]. In such a case the CP violation would be due to the interference term $Im\ A_{1/2}\ A^*_{3/2}$.

The purpose of this work is to show that if the vector current transforms as a combination of the regular representation of SU_3 and R_8 (as $R_8 \supset SU_3/Z_3$ under SU_3 the vector current will belong to the 8, 10,, $\overline{10}$ representations), the 10-amplitude may have a relative phase of nearly 90° degrees with respect to the 8-amplitude without predicting a $\Delta T = 3/2$ rate incompatible with the experiments.

We also show that even a rather small $\Delta T = \frac{3}{2}$ contribution may give us a sizeable CP violation in the Σ, Λ leptonic decays.

II. MODEL OF CABIBBO

This model rests on the following three assumptions about the structure of the weak currents:

a) The weak current J_μ behaves under SU_3 transformations as a sum of octet members.

b) The vector part $V_\mu^{\cdot}$ is connected to the isovector part of the electromagnetic current through a rotation in unitary space.

c) The linear combination $T(\theta, \Delta S = 1)\, J_\mu^{\Delta S = 0} + T(\theta, \Delta S = 1)\, J_\mu^{\Delta S = 1}$ couples universally. Here $T(\theta, 0) = \cos\theta$ and $T(\theta, 1) = \sin\theta$ both for $\Delta Q = 1$.

For the decay $A \to B + e + \nu$ (A, B baryons) we have the matrix element:

$$\mathcal{L} = \frac{G}{\sqrt{2}} J_\mu^{\cdot}(\text{lept}) <B| V_\mu^{i} + A_\mu^{i}|A> = \frac{G}{\sqrt{2}} J_\mu^{\cdot}(\text{lept}) \times$$

$$\times\ T(\theta, \Delta S)\, [f^{iBA}\, \gamma_\mu^{\cdot} + (f^{iBA}\, F + d^{iBA}\, D)\, \gamma_\mu^{\cdot}\, \gamma_5] \qquad (1)$$

where f^{iBA} (d^{iBA}) is the coupling coefficient for antisymmetric (symmetric) octet coupling of baryons B and A.

The model shows a reasonable concordance with the experimental data[2] with $\theta \simeq 0.26$, $F \simeq 0.44$, $D \simeq 0.74$.

However the model does not allow decays with $\Delta T = \frac{3}{2}$, $\Delta S = 2$.

III. $SU_3 + R_8$ MODEL

We will assume the vector current belongs to the regular representations of SU_3 (dim = 8) and R_8 (dim = 28). The axial current will belong to the representations 8 of SU_3 and 8_v of R_8 (8_v is the vector-representation of R_8). In this we assume the vector current is conserved and the axial current is dominated by the meson pole (Goldberger-Treiman relations). As in the model of Cabibbo, the weak current will be obtained by rotating the $T = 1$ current an angle θ around the 2nd axis of U-spin space. (In the present model we have $T = 1$ contributions not only in the 8-representation of SU_3 but also in the 10, $\overline{10}$ representations contained in in the regular representation of R_8). We will suppose the θ rotation is not necessarily valid for the 8_v representation (R_8) which contributes to the axial current. Therefore, in an obvious notation we have:

$$J_\mu^{\cdot} = V_\mu^{\cdot} + A_\mu^{\cdot} \qquad (2)$$

$$V_\mu^{\cdot} = V_\mu^{\cdot}(8) + V_\mu^{\cdot}(28) \qquad (3)$$

$$A_\mu^{\cdot} = A_\mu^{\cdot}(8) + A_\mu^{\cdot}(8_v) \qquad (4)$$

The V_μ (8), $A_\mu^{\cdot}$(8) are the same as in Cabibbo model.

For the decay $A \to B + l + \nu$ we have the contribution:

$$\mathcal{L} = \frac{G}{\sqrt{2}}\, j_\mu^{\cdot}\,(\text{lept}) <B|\, V_\mu^i + A_\mu^{i\cdot} |\, A> = \frac{G}{\sqrt{2}}\, j_\mu^{\cdot}(\text{lept})\ \times$$

$$\times \{ [T(\theta,8,\Delta S)(1+c)f^{iBA} + T(\theta,10,\Delta S)\,[iBA]_{10}(a+ib) + T(\theta,\overline{10},\Delta S)\,[iBA]_{\overline{10}}\,(a-ib)\,]\,\gamma_\mu^{\cdot}$$

$$+ [T(\theta,8,\Delta S)(Ff^{iBA} + Dd^{iBA}) + T(\gamma,8_v,\Delta S)\,M\,[iBA]_{8_v}\,]\,\gamma_\mu\,\gamma_5 \}$$

$$(5)$$

where f^{iBA}, d^{iBA}, $[iBA]x$, are coupling coefficients and a, b, c, M are real number which we expect to be small (as the R_8 mixture is expected to be small).

The $T(\theta, X, \Delta S)$ are obtained from the $T = 1$ component when we rotate in the U space. The products of these T_s by the coupling coefficients are shown explicitly in the appendix. We don't assume in general $\cos\gamma = \cos\theta$

The results for different leptonic decays are shown in Table I. When we compare with the experimental data we need M small. The decays with $\Delta S = 2\,(\Xi^- \overline{N})$ are an order of magnitude smaller than the $\Delta T = \frac{3}{2}$; as we have an extra factor of $\sin^2\theta$ in the transition probability.

TABLE I

$A\,\overline{B}$	Amplitude
$n\,\overline{p}$	$-\frac{1}{\sqrt{3}}\{[1+c-2a]\gamma_\mu + [F+D^1]\gamma_\mu\gamma_5\}\cos\theta$
$\Sigma^-\,\overline{n}$	$\frac{1}{\sqrt{3}}\{[1+c-a+ib]\gamma_\mu + [-F+D^1+M(\frac{\sin\gamma}{\sin\theta}-\frac{\cos\gamma}{\cos\theta})]\gamma_\mu\gamma_5\}\sin\theta$
$\Lambda\,\overline{p}$	$-\frac{1}{\sqrt{2}}\{[1+c+2a+2ib]\gamma_\mu + [-F-\frac{D^1}{3}+M(\frac{1}{\sqrt{3}}\frac{\sin\gamma}{\sin\theta}+\frac{1}{3}\frac{\cos\gamma}{\cos\theta})]\gamma_\mu\gamma_5\}\sin\theta$
$\Xi^-\,\overline{\Lambda}$	$-\frac{1}{\sqrt{2}}\{[1+c+2a+2ib]\gamma_\mu + [F-\frac{D^1}{3}+M(\frac{1}{\sqrt{3}}\frac{\sin\gamma}{\sin\theta}+\frac{1}{3}\frac{\cos\gamma}{\cos\theta})]\gamma_\mu\gamma_5]\}\sin\theta$
$\Sigma^0\,\overline{p}$	$-\frac{1}{\sqrt{6}}\{[1+c-4a-3\sqrt{2}ib]\gamma_\mu + [-F-\frac{D^1}{3}+M(\frac{1}{\sqrt{3}}\frac{\sin\gamma}{\sin\theta}+\frac{1}{3}\frac{\cos\gamma}{\cos\theta})]\gamma_\mu\gamma_5\}\sin\theta$
$\Sigma^-\,\overline{\Lambda}$	$\sqrt{2}\{ib\gamma_\mu + \frac{1}{3}(D^1-0.13M\frac{\cos\gamma}{\cos\theta})\gamma_\mu\gamma_5\}\cos\theta$
$\Sigma^+\,\overline{n}$	$-\sqrt{3}[a-ib]\gamma_\mu\sin\theta$
$\Xi^-\,\overline{n}$	$-\sqrt{3}[a-ib]\gamma_\mu\sin^2\theta$

with $D^1 = D + M\frac{\cos\gamma}{\cos\theta}$

For the effective $n\bar{p}$ coupling constant we have $G \cos\theta\,[1 + c - 2a]$. The transitions probability is proportional to

$$G^2 \cos^2\theta\,[1 + c - 2a] \simeq G^2\,[1 - \frac{1}{16} + 2c - 4a]$$

Experimentally[8] we need $2c - 4a \simeq 0.02$. If $c = 0$ or $c = a$

$$\frac{\Sigma^+ \bar{n}}{\Sigma^- \bar{n}} \simeq \frac{q\,(a^2 + b^2)}{1.2} = 0.0002 + \frac{q}{1.2}\,b^2 < 0.1 \quad \text{(Experimental value}^3\text{)}$$

$$\therefore\ b^2 < 0.012,\ b < 0.11$$

If $b > 0.02$ which gives us $\frac{\Sigma^+ \bar{n}}{\Sigma^- \bar{n}} \sim 0.003$ we obtain

$\phi = \text{arctag}\ \frac{v}{a} > 76°$ a value not very far from $90°$.

In Table II we show the relative $G_V - G_A$ phase (a measure of T violation and if CPT is good a measure of CP violation) for some processes. We see that the β decay $(n\bar{p})$ does not violate T, however other leptonic decays show a sizeable T violation. In the $\Sigma\bar{\Lambda}$ process the relative phase is $90°$; although as b is small the T violating terms will be small.

TABLE II

$A\overline{B}$	Relative phase	If $0.02 < b < 0.1$
$n\overline{p}$	0	0
$\Sigma^-\overline{n}$	arctag $3b$	$17° > \phi < 4°$
$\Lambda\overline{p}$, $\Xi^-\overline{\Lambda}$	$-$ arctag $2b$	$-11° < \phi < -2.2°$
$\Sigma^0\overline{p}$	$-$ arctag $4.2b$	$-25° < \phi < 5°$
$\Sigma^-\Lambda$	90°	90°

IV DISCUSSION

When we assume the weak current transforms as a linear combination of members of the regular representations for SU_3 and R_8, such that they have $T^1 = 1$ (isospin in the rotated basis) we have:

1) The R_8 mixture is small.
2) The data are compatible with $Im\ A_{1/2}\ A^*_{3/2}$ large.
3) The processes with $\Delta S = 2$ are allowed but they are an order of magnitude weaker.
4) We do not predict T violation β decay but this violation may be sizeable in the decays:

$$\Sigma^- \to n + e + \overline{\nu},\ \Lambda \to p + e + \overline{\nu},\ \Xi^- \to \Lambda + e + \overline{\nu}$$

$$\Sigma^0 \to p + e + \overline{\nu},\ \Sigma^- \to \Lambda + e + \overline{\nu}$$

APPENDIX

In this appendix we write the results when we rotate the T = 1 components θ degrees in the unitary space.

In the rotated basis we have:

$$\pi'^{\pm} = \cos\theta\,\pi^{\pm} + \sin\theta\,K^{\pm}$$

$$K'^{\pm} = -\sin\theta\,\pi^{\pm} + \cos\theta\,K^{\pm}$$

$$K'^{0} = \frac{1}{\sqrt{2}}\left\{\cos 2\theta(\overline{K}_0 - K_0) - (\overline{K}_0 - K_0) + \frac{1}{\sqrt{2}}\sin 2\theta(\pi^0 - \sqrt{3}\eta)\right\}$$

$$\pi'^{0} = \frac{1}{4}\left\{\pi^0(3 + \cos 2\theta) + \sqrt{3}\eta(1 - \cos 2\theta) - \sqrt{2}\sin 2\theta(\overline{K}_0 + K_0)\right\}$$

$$\eta' = \frac{1}{4}\left\{\eta(1 + 3\cos 2\theta) + \sqrt{3}\,\pi^0(1 - \cos 2\theta) + 6^{1/2}\sin 2\theta(\overline{K}_0 + K_0)\right\}$$

The $\Delta Q = 1$ currents are:

Octet

$$-\frac{1}{\sqrt{3}}K'^{-}K'^{0} + \sqrt{2/3}\,\pi'^{-}\pi'^{0} = \cos\theta\left[-\frac{1}{\sqrt{3}}K^{-}K^{0} + \sqrt{2/3}\,\pi^{-}\pi^{0}\right]$$

$$+\sin\theta\left[\frac{1}{\sqrt{3}}\pi^{-}\overline{K}^{0} - \frac{1}{\sqrt{2}}(K^{-}\pi^{0} - \sqrt{3}K^{-}\eta)\right] =$$

$$= \cos\theta\, J_\mu^{\cdot}(\Delta S = 0) + \sin\theta\, J_\mu^{\cdot}(\Delta S = 1)$$

Decuplet

$$\frac{1}{\sqrt{2}}\left\{ \sqrt{2/3}\; K'^- K'^0 + \frac{1}{\sqrt{3}}\; \pi'^- \pi^0 \right\} \pm \frac{1}{\sqrt{2}}\; \eta' \pi'^- =$$

$$= \frac{1}{\sqrt{2}}\left\{ \pi^- K^0 \,[-\sqrt{2/3}\, \sin\theta \cos^2\theta] + \pi^- \bar{K}^0\, [2\sin^3\theta - \sin\theta\cos^2\theta]\frac{1}{\sqrt{6}} \right.$$

$$+ \pi^- \pi^0\, \frac{-2\sin^2\theta\cos\theta + \cos\theta(1+\cos^2\theta)}{2\sqrt{3}} + \pi^- \eta \frac{3}{2}\, \sin^2\theta\cos\theta$$

$$+ K^- K^0\, [\sqrt{2/3}\, \cos^3\theta - \frac{1}{\sqrt{6}}\; \sin^2\theta\cos\theta] + K^- \bar{K}^0\, [-2\cos\theta\sin^2\theta - \sin^2\theta\cos\theta]\frac{1}{\sqrt{6}}$$

$$\left. + K^- \pi^0\, [3\cos^2\theta\sin\theta + \sin\theta] \times \frac{1}{2\sqrt{3}} + K^- \eta\, [-\cos^2\theta\sin\theta + \frac{1}{2}\sin^3\theta] \right\}$$

$$\pm \frac{1}{\sqrt{2}}\left\{ \pi^- \eta\, [\frac{\cos\theta\,[2-3\sin^2\theta]}{2} + \pi^- \pi^0\, \frac{\sqrt{3}\cos\theta\sin^2\theta}{2} + \frac{6^{1/2}\cos^2\theta\sin\theta}{2}\, \pi^- \bar{K}^0 \right.$$

$$+ \pi^- K^0\, \frac{6^{1/2}\cos^2\theta\sin\theta}{2} + K^- \eta\, \frac{\sin\theta\,[2-3\sin^2\theta]}{2} + K^- \pi^0\, \frac{\sqrt{3}\sin^3\theta}{2}$$

$$\left. + K^- \bar{K}^0\, \frac{6^{1/2}\cos\theta\sin^2\theta}{2} + K^- K^0\, \frac{6^{1/2}\cos\theta\sin^2\theta}{2} \right\} \simeq$$

$$\simeq \frac{1}{\sqrt{2}} \Big\{ \pi^- K^0 \left[-\sqrt{3/2}\,\sin\theta\right] - \pi^- \overline{K}^0 \frac{\sin\theta}{\sqrt{6}} + \frac{1}{\sqrt{3}} \cos\theta\, \pi^- \pi^0 + \sqrt{2/3}\, \cos\theta\, K^- K^0$$

$$+ K^- \pi^0 \frac{2}{\sqrt{3}} \sin\theta - \sin\theta\, K^- \eta - K^- \overline{K}^0 \sqrt{3/2}\, \sin^2\theta \cos\theta + 0\,(\sin^2\theta) \Big\}$$

$$\pm \frac{1}{\sqrt{2}} \Big\{ \pi^- \eta \cos\theta + \pi^- \overline{K}^0 \sqrt{3/2}\, \sin\theta + \pi^- K^0 \sqrt{3/2}\, \sin\theta + K^- \eta \sin\theta$$

$$+ \sqrt{3/2}\, \cos\theta \sin^2\theta\, K^- \overline{K}^0 + 0\,(\sin^2\theta) \Big\}$$

For the baryonic decays

$$\pi \to \Sigma$$

$$\eta \to \Lambda$$

$$pn \to K^+, K^0$$

$$\Xi^0, \Xi^- \to \overline{K}^0, K^-$$

The process with $\Delta S = 2$ ($K^- \overline{K}^0$) are multiplied by an extra factor $\sin\theta$ ($\sin^2\theta$ in the transition probability) and therefore they are an order of magnitude weaker than the $\Delta T = \frac{3}{2}$ decay.

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