TENSOR FORCE IN THE NUCLEUS F^{18}.

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ABSTRACT

The energy levels of F^{18} *are calculated when one adds a tensor force to a central interaction with and without exchange. No significant improvement of the levels is obtained which suggests that a purely central interaction, plus spin orbit coupling, could account for the properties of nuclei in the* 2s-1d *shell.*

INTRODUCTION

In a recent series of papers, Moshinsky and his collaborators[1,2] have developed powerful group-theoretical methods for dealing with problems of nuclear structure in the 2s-1d shell. In these methods, tensor forces have not been taken into account and before extending the group theoretical techniques to cover this type of forces it is convenient to investigate the importance of the tensor inter-

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actions in the 2s-1d shell. With this purpose we discuss in this paper the effect of a tensor force on the energy levels and states of F^{18}, i.e. the problem of two particles in the 2s-1d shell with isospin $T = 0$.

We consider the Hamiltonian:

$$\sum_i \frac{p_i^2}{2m} + \frac{m\omega^2}{2} \sum_i r_i^2 + V_C(\alpha) + V_T(\beta) + \xi \sum_i l_i \cdot s_i + \eta \sum_i l_i^2 \tag{1}$$

where the second term in the sum is the common harmonic oscilator potential assumed in the shell model, and $V_C(\alpha)$ is the residual central internucleon potential. For mathematical simplicity it was assumed of Gaussian shape with range α, which from low-energy scattering data is known to have a value between 1.5 fm. and 2.0fm. We can write $V_C(\alpha)$ in the form:

$$V_C(\alpha) = V_C \exp\left[-\left(\frac{r}{a}\right)^2\right] \tag{2}$$

where $r = \sqrt{\frac{m\omega}{2\hbar}}\, r_{12}$ and $a = \sqrt{\frac{m\omega}{2\hbar}}\, \alpha$ are dimensionless quantities, and ω is evaluated using Moszkowski's[3] relation,

$$\hbar\omega \doteq 41\, A^{-1/3}\ \text{MeV}$$

so that for $A = 18$ we have $\alpha = 2.29\, a$ fm. The potential well depth V_C is known to be in the range -20MeV to -35MeV. In the following analysis we shall consider both an ordinary and a Serber exchange central force.

The tensor force term $V_T(\beta)$ was chosen to have the form

$$V_T(\beta) = V_T \exp\left[-\left(\frac{r}{b}\right)^2\right] S_{12} \tag{3}$$

with the potential well taken as Gaussian in shape, where the range β is $\beta = 2.29$ b fm. The operator S_{12} is

$$S_{12} = 3(\mathbf{s}_1 \cdot \mathbf{e}_{12})(\mathbf{s}_2 \cdot \mathbf{e}_{12}) - (\mathbf{s}_1 \cdot \mathbf{s}_2),\ \mathbf{r}_{12} = r_{12}\ \mathbf{e}_{12}$$

The fifth term in the Hamiltonian (1) is the spin-orbit coupling, and for the 2s-1d shell ξ is known from O^{17} experimental data to be -2.0MeV. The last term in (1) represents the departure from the harmonic oscillator potential (fig. 1) and its particular value for the 2s-1d shell, taken from the O^{17} spectrum, is $\eta = .189$ MeV.

Exchange forces are introduced only for the residual central force term. The Serber mixture makes the expectation value of $V_C(\alpha)$ vanish between states of zero spin, in the L-S scheme.

2. The states of two nucleons in the 2s-1d shell could be written either in the L-S or $j-j$ coupled forms respectively

$$\left| n_1 l_1, n_2 l_2, L, \frac{1}{2}\ \frac{1}{2}\ S; J \right> \tag{4a}$$

$$\left| n_1 l_1 \frac{1}{2}\ j_1, n_2 l_2 \frac{1}{2}\ j_2, J \right> \tag{4b}$$

related by $9-j$ coefficients.

We shall use the basis (4a) and determine the matrix elements of the central and tensor forces with respect to this basis with the help of the Brashinskets[4], i.e. the transformation brackets that take us into states given in terms of the center of mass and relative coordinates. The matrix elements are then functions of the Talmi integrals[5] I_p which for the Gaussian potential take the value[5]

$$I_p = V_C \left(\frac{a^2}{1 + a^2} \right)^{P + 3/2}$$

3. On introducing tensor forces, we expect the following effects on the energy levels:
First, a central-like part due to the $\langle nl | V_T | nl \rangle$ matrix elements, replaceable by a pure central interaction which by the Wigner-Eckart theorem can join only states with like l. Second, an effect peculiar to tensor force, which appears in the non-zero $\langle nl | V_T | nl' \rangle$ elements, where the rank two tensor character of the S_{12} operator in the relative-coordinate space gives the selection rule $l' = l \pm 2$. This effect is expected to shift the levels in a way not achievable by central forces alone.

It is well known that tensor forces are necessary to explain the non-vanishing quadrupole moment in the deuteron. It has been also demonstrated by a procedure similar to the present one that tensor forces were required to put a previously inverted pair of levels in correct order, in the nuclear energy spectrum of Bismuth 210[7]. Work has also been done regarding heavy nuclei with two nucleons beyond a closed shell[8].

Standard techniques[9] have been employed to determine the effect of central residual forcer in F^{18}; the results, however, show (Fig. 2) that the pair of levels with $J = 1$ and $J = 2$ is inverted compared to experiment and that the spacing of the levels is not predicted very accurately. This was also observed by de Llano et al.,whose results are shown in Fig. 2. In Figs. 2b, 2c α was given the reasonable value $\alpha = 1.6$fm, and V_C was adjusted to give the best fit to the levels, which happened when $V_C = 35$ MeV.

Taking these data for the central potential wa shall now explore the effect of a tensor force. First we want to see the effect of the range of the force. We can achieve this by taking a definite value of X_T and diagonalizing the matrices for different values of β. In Fig. 3 we show the energy levels for $X_T = .5$ as function of β for $0 < b < 2.3$ fm. In the lower part of the graph we indicate the mean deviation as function of β. We see that the fit of the results to experiment is rather insensitive to the change in the range of the tensor forces.

We now want to see the effect of tensoı force strength, which we can achieve if we fix the range and diagonalize the matrices for different values of X_T. In Fig. 4 we show the energy levels for $\beta = .92$ fm. i.e. for $b = 0.4$, and for values of X_T in the range $0 \leq X_T \leq 2$. In the lower part of the graph we indicate the mean deviation as a function X_T. We see that the fit of the results to the experiment is rather insensitive to X_T.

We conclude that no appreciable improvement of the fit to the experimental energy levels of F^{18} is achieved by the introduction of tensor forces[10]. Although caution should be used if we want to extend these results to other 2s-1d nuclei, simplicity of calculation makes it worthwhile to ignore tensor forces.

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REFERENCES

1. J. Flores, E. Chacón, P.A. Mello, M. de Llano
   Nuclear Physics (in press)
2. M. Moshinsky "Group Theory and the Many Body Problem"
   To be published in "Physics of Many Particle Systems"
   edited by E. Meron (Gordon and Breach, New York 1965)
3. S.A. Moszkowsky in "Handbuch der physik" B.XXXIX
   (Springer Verlag, Berlin 1957) p. 469
4. T.A. Brody, M. Moshinsky, "Tablas de Paréntesis de Ttransformación"
   Monografías del Instituto de Física (México 1960)
5. I. Talmi, Helvetica Physica Acta 25 (1952) p. 185
6. R. Thieberger, Nuclear Physics 2 (1956/57) p. 533
7. J. Flores and P.A. Mello, Nuclear Physics 47 (1963) p. 177
8. Y.E. Kim and J.O. Rasmussen, Nuclear Physics 47 (1963) p. 190
9. T. Inoue, T. Sebe, H. Hagiwara and A. Arima "The Structure of the S-D Shell
   Nuclei O^{18} F^{18} O^{19} F^{19} and Ne^{20}" (To be published)
10. W.J.S. Young Nuclear Physics 55 (1964) p. 84

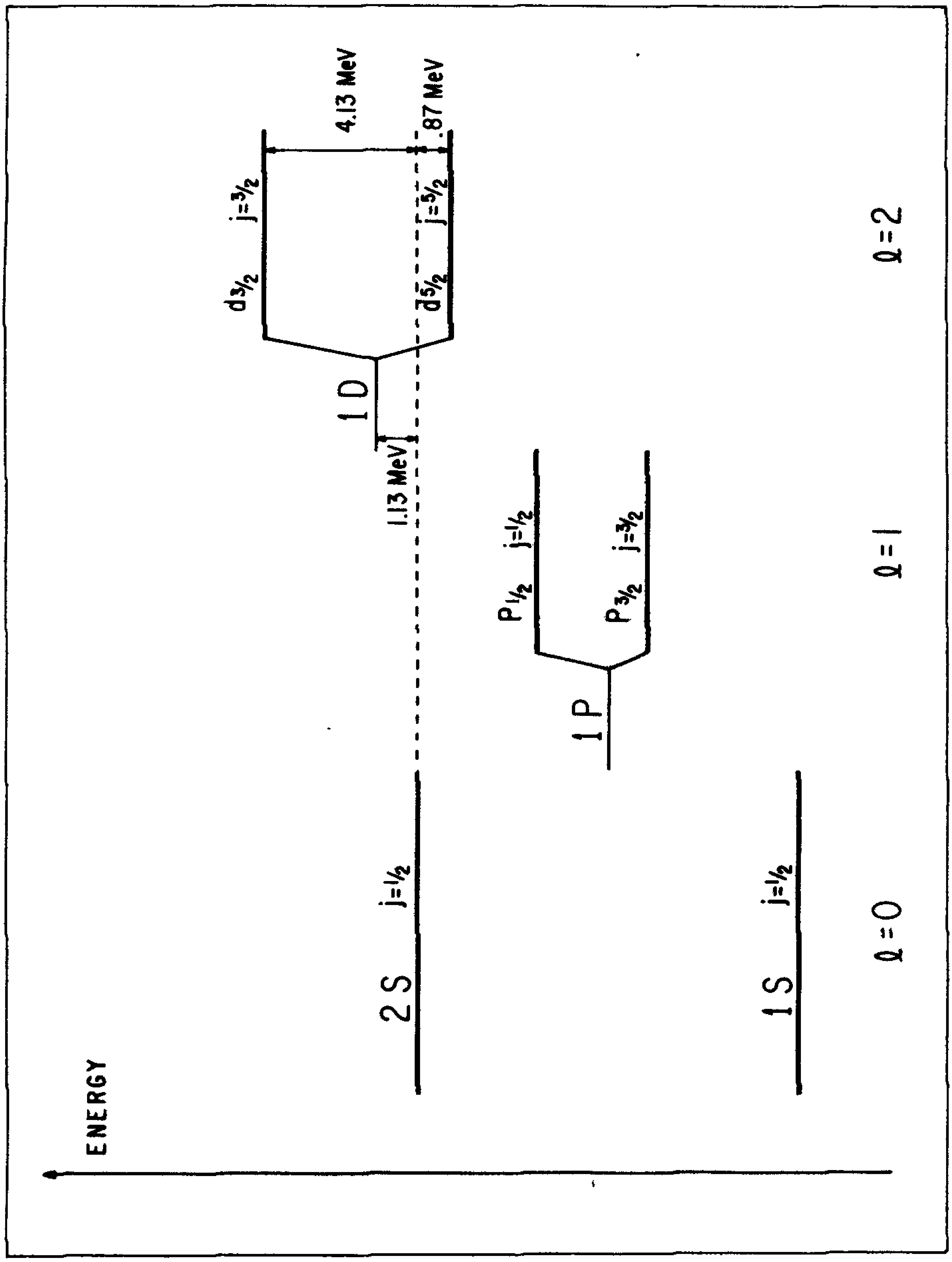

Fig. 1. Energy levels up to the 2s 1d shell, according to shell-theory labeling.

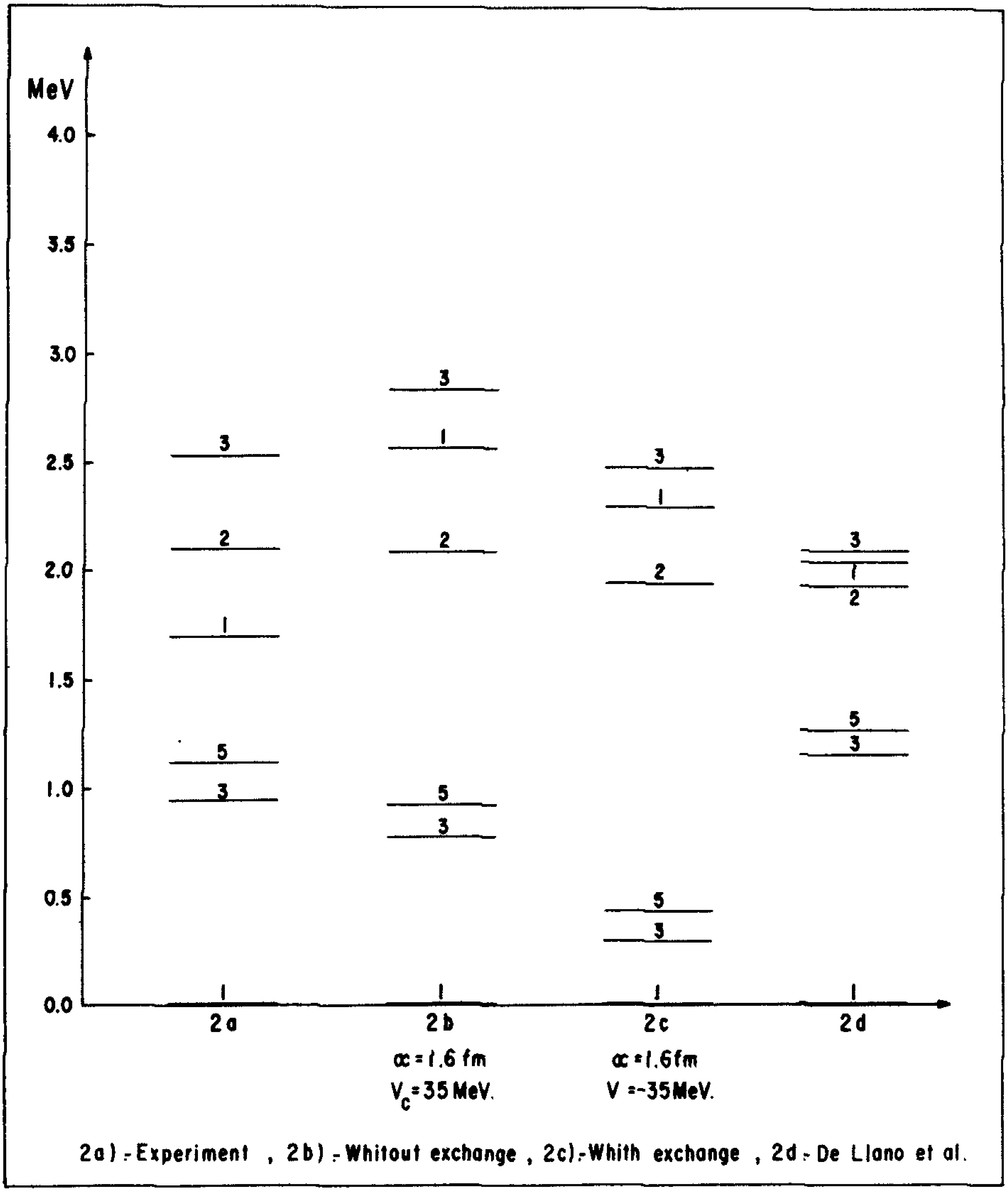

Fig. 2. Comparison with experiment of the various results using purely central residual internucleon force.

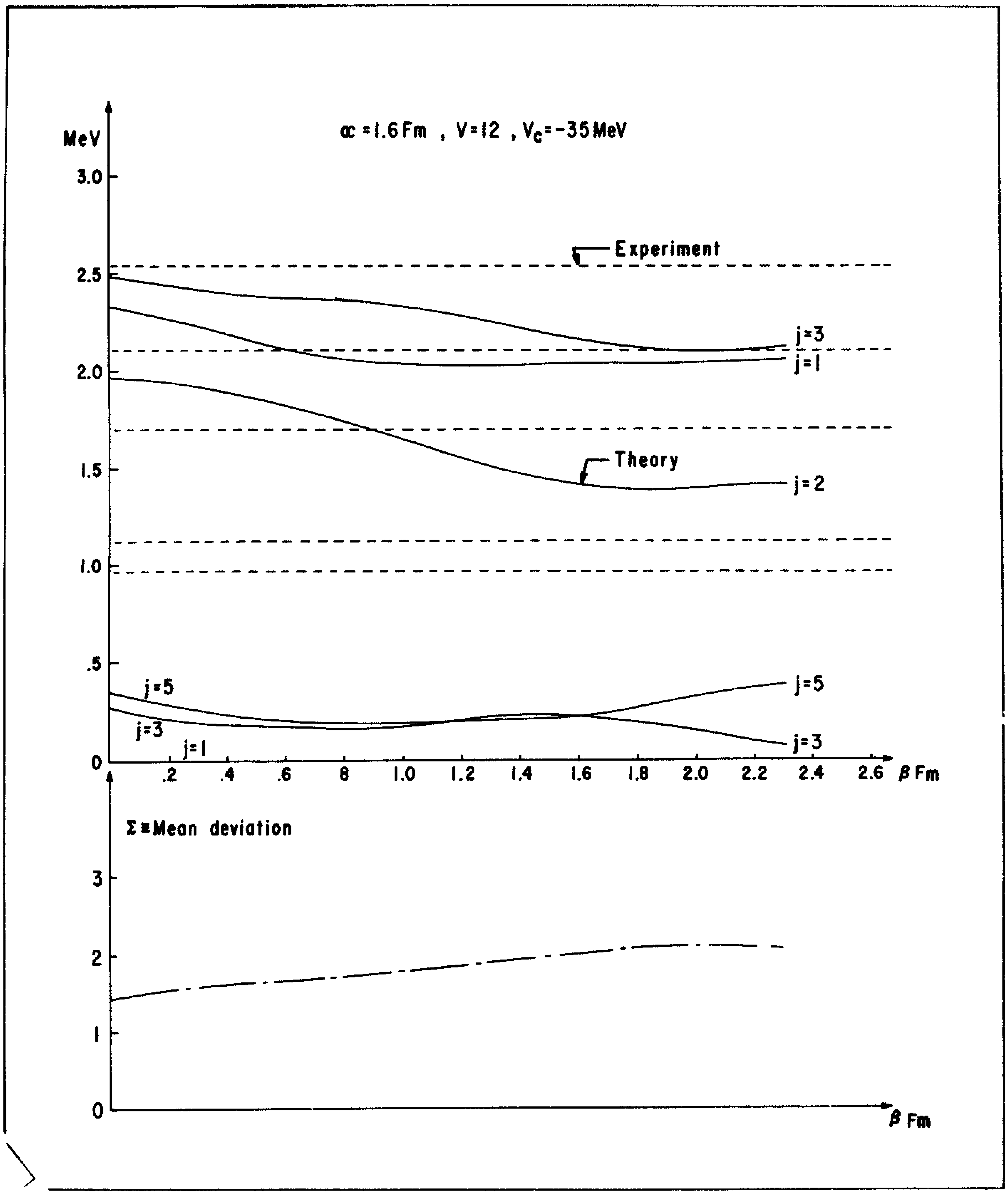

Fig. 3

Fig. 3. Behavior of the energy levels with tensor force range: $\beta = 2.29b$. The $J = 1$ level coincides, as lowest level, with the abscissa axis.

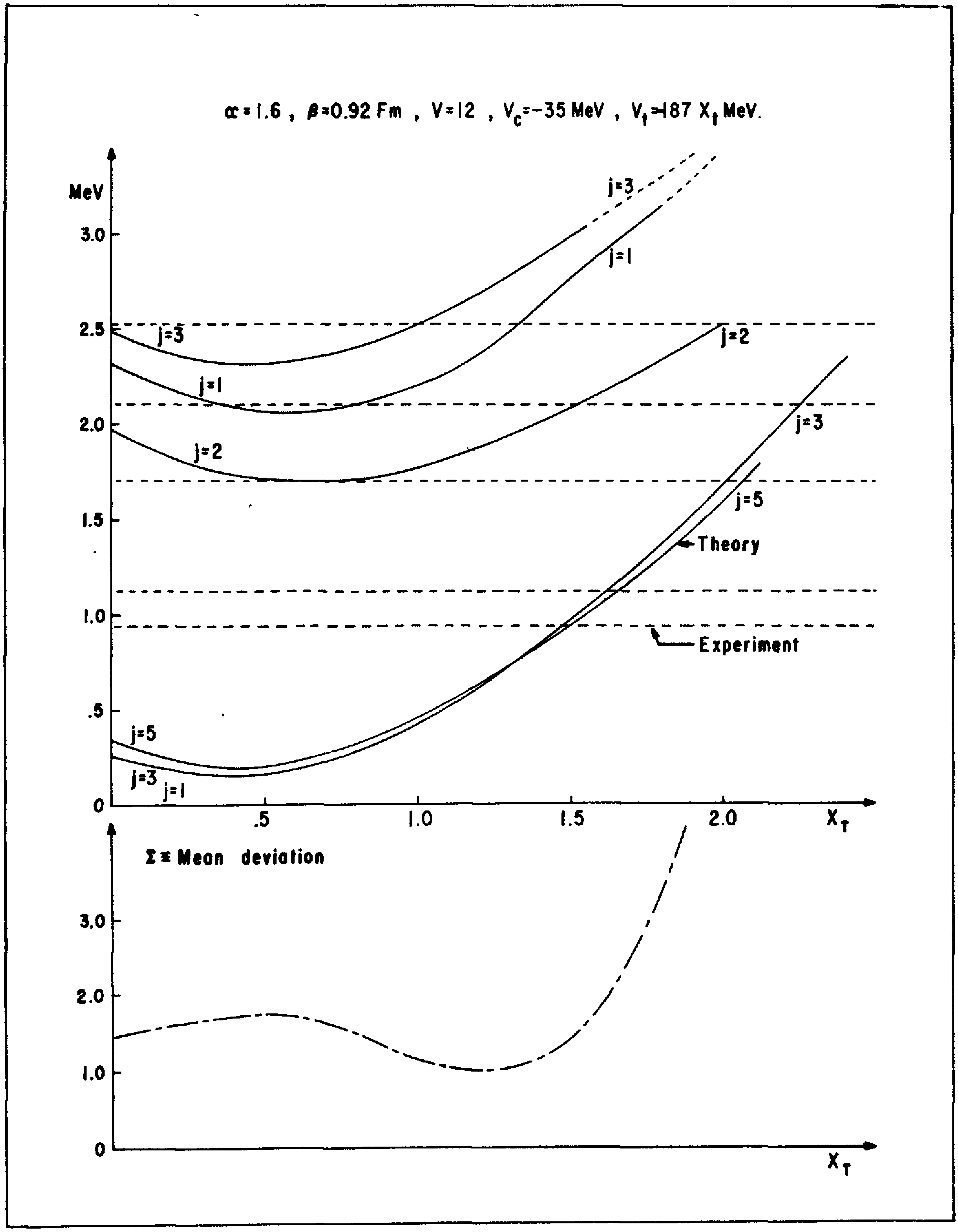

Fig. 4. Behavior of the energy levels with tensor force well depth. For $\alpha = 1.6$fm, V_C was found to be -35 MeV, and thus $V_T = -187\ X_T$ MeV.