

DISTRIBUCION DE VALORES ABSOLUTOS DE LOS ANGULOS PROYECTADOS  
EN LA REGION DE DISPERSION PLURAL.

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ABSTRACT

*Keil, Zeitler and Zinn<sup>1</sup> have calculated and tabulated the angular distribution of particle trajectories in collision with matter when the average number of collisions is between 0 and 20. We make an extension of these calculations to cover the distribution of absolute values of projected angles.*

RESUMEN

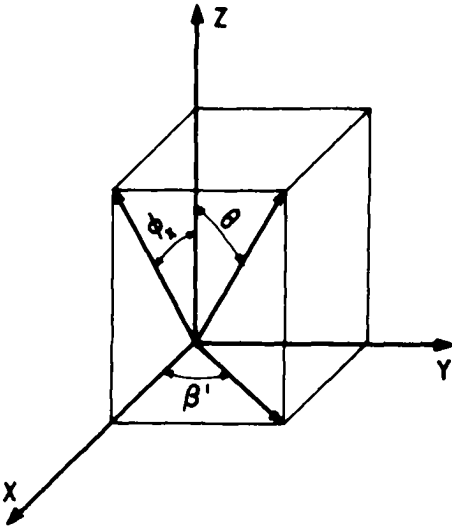
*Keil, Zeitler y Zinn<sup>1</sup> han calculado y tabulado la distribución angular de las trayectorias de las partículas en colisión con materia, cuando el número pro-*

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medio de colisiones está entre 0 y 20. Nosotros hacemos una extensión de estos cálculos cubriendo la distribución de valores absolutos de los ángulos proyectados.



Cuando un haz de partículas cargadas, inicialmente sin dispersión angular, incide sobre materia, las partículas son desviadas de su dirección original (eje  $z$ ) y adquieren una distribución angular de sus trayectorias  $F(\theta, \beta', t)$  que es función de la profundidad  $t$  que han recorrido dentro del material. En las mas de las colisiones que sufren estas partículas, su trayectoria es desviada ligeramente, por lo que es útil tratar este problema en descripción de ángulo pequeño (ver Scott<sup>2</sup>):

$$\begin{aligned} \text{sen } \theta &\simeq \theta ; & \text{sen } \phi_x &\simeq \phi_x \\ \text{cos } \theta &\simeq 1 ; & \text{cos } \phi_x &\simeq 1 \\ 0 < \theta < \infty ; & & 0 < |\phi_x| \equiv \phi < \infty \end{aligned} \tag{1}$$

Nos interesarán las distribuciones de ángulos proyectados  $F_p(\phi_x, m)$  y  $f(\phi, m)$  de valores absolutos de ángulos proyectados.  $F_p(\phi_x, m) d\phi_x$  da la fracción del número de partículas que habiendo sufrido un número promedio  $m$  de colisiones ( $m$  no es necesariamente entero), se mueven en direcciones que se proyectan en el intervalo  $(\phi_x, \phi_x + d\phi_x)$  en el plano  $(x, z)$ .  $f(\phi, m)$  se define mediante:

$$f(\phi, m) \equiv F_p(\phi, m) + F_p(-\phi, m) \tag{2}$$

O en el caso de simetría axial que será el de interés:

$$f(\phi, m) = 2F_p(\phi, m) \quad (3)$$

con

$$0 < \phi < \infty$$

La sección eficaz diferencial para una colisión contra un núcleo apantallado, usada por Keil, Zeitler y Zinn (KZZ) y nosotros es la de Molière<sup>3</sup>:

$$\sigma(\theta) = \frac{4a^2}{k_z^2} (\theta^2 + \chi_\alpha)^{-2} \quad (4)$$

donde  $k_z = p/\hbar$ ,  $a = zZ/137\beta$  el parámetro de Bohr, siendo  $p$  el momento lineal de la partícula incidente,  $z$  su carga,  $v = \beta c$  su velocidad y  $Z$  la carga del núcleo dispersor.  $\chi_\alpha$  es un parámetro de la teoría de Molière (función de la energía de la partícula) que puede ser calculado mediante las expresiones originales de Molière o la modificación de Scott (pág. 288)<sup>2</sup>.

Haciendo uso de (4), KZZ calculan  $F(\theta, m)$  bajo el siguiente procedimiento:

En términos de su transformada de Hankel  $\tilde{F}(\xi, m)$ :

$$F(\theta, m) = \int_0^\infty \xi d\xi J_0(\xi\theta) \tilde{F}(\xi, m) \quad (5)$$

$\tilde{F}(\xi, m)$  es calculable usando el método de Wentzel<sup>4</sup>.

Usando las expresiones reducidas:

$$F_{red}(U, m) \cup dU = F(\theta, m) \theta d\theta \quad (6)$$

$$U = \theta/\chi_\alpha, \quad (s = \xi\chi_\alpha)$$

$\tilde{F}(s, m)$  se expresa:

$$\tilde{F}(s, m) = \tilde{F}(s)^m \quad (7)$$

con 
$$\tilde{F}(s) = \exp [sK_1(s) - 1] \quad (8)$$

siendo  $K_1(s)$  la función modificada de Hankel de orden uno. Esta expresión es obtenida bajo las condiciones:

$$F(\theta, 0) = \delta_s(\theta)$$

$$\int_0^\infty \delta_s(\theta) \theta d\theta = 1 \quad (9)$$

$$\int_0^\infty F(\theta, m) \theta d\theta = 1 \quad (10)$$

Con objeto de tener una expresión analítica manejable de  $F_{red}(U, m)$ , KZZ aproximan la expresión (8) mediante:

$$\tilde{F}(s) \simeq e^{-1} (1 + b_1 e^{-c_1 s} + b_2 e^{-c_2 s}) \quad (11)$$

donde

$$b_1 = 2.10667, \quad c_1 = 0.935 \quad (12)$$

$$b_2 = -0.388388, \quad c_2 = 5.000$$

La aproximación es válida para toda  $s$  dentro de un .002 .

Las expresiones anteriores son substituidas en (5) y  $F_{red}(U, m)$  calculada.

En nuestro caso, la expresi3n que nos interesa es  $f(\phi, m)$  defini3ndose la distribuci3n reducida mediante:

$$f_{red}(\varphi, m) d\varphi = f(\phi, m) d\phi \quad (13)$$

$$\varphi = \phi / X_a$$

con las restricciones an3logas a (9) y (10):

$$f_{red}(\varphi, 0) = \delta(\varphi)$$

$$\int_0^\infty \delta(\varphi) d\varphi = 1 \quad (14)$$

$$\int_0^\infty f_{red}(\varphi, m) d\varphi = 1$$

La expresi3n para  $f_{red}$  puede ser encontrada de la relaci3n entre  $F$  y  $f$ :

$$f(\phi, m) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\phi_x F[(\phi^2 + \phi_x^2)^{1/2}, m] \quad (16)$$

y tambi3n, usando su transformada de Fourier:

$$f_{red}(\varphi, m) = \frac{2}{\pi} \int_0^\infty ds \cos(s\varphi) \tilde{f}(s, m) \quad (17)$$

y el hecho (Scott, f3rmula 2.35):

$$\tilde{f}(s, m) = \tilde{F}(s, m) \quad (18)$$

de este modo encontramos:

$$f_{\text{red}}(\varphi, m) = e^{-m} \delta(\varphi) + \frac{2}{\pi} e^{-m} \sum_{\lambda=0}^m \sum_{l=0}^{m-\lambda} \binom{m}{\lambda} \binom{m-\lambda}{l} \frac{b_1^\lambda b_2^l (\lambda c_1 + l c_2)}{(\lambda c_1 + l c_2)^2 + \varphi^2} \quad (19)$$

expresión válida para valores de  $m$  enteros. Esta fórmula también puede obtenerse de la expresión (22) de KZZ, usando (16).

El término  $e^{-m} \delta(\varphi)$  representa a la fracción de partículas de la distribución original que no han sido dispersadas. La prima en las sumas significa que el término  $\lambda = l = 0$ , no se incluye.

Considerando como distribución exacta a aquella en que  $\tilde{F}(s)$  está dada por (8), el error en  $f_{\text{red}}$  será:

$$\begin{aligned} \Delta[\tilde{F}(s)]^m &= m \tilde{F}(s)^{m-1} \Delta\tilde{F}(s) \\ \Delta f_{\text{red}}(\varphi, m) &= m \int_0^\infty ds [\Delta\tilde{F}(s)] [\tilde{F}(s)]^{m-1} \cos(s\varphi) \\ &\leq 0.002 m f_{\text{red}}(\varphi, m-1) \end{aligned} \quad (20)$$

Como hace notar Scott (pág. 289), el error aumenta con  $m$ , mientras que en el cálculo original de Molière el error disminuye al aumentar  $m$ , siendo  $m = 20$  un buen punto de separación entre ambos métodos.

Los valores numéricos de la parte no singular de la expresión (19):

$$g(\varphi, m) \equiv f_{\text{red}}(\varphi, m) - e^{-m} \delta(\varphi) \quad (21)$$

se reportan en la tabla I.

En la tabla II se dan los valores de la integral de  $g$ :

$$\begin{aligned} \hat{g}(\varphi, m) &\equiv \int_{\varphi}^{\infty} dx g(x, m) = \\ &= \frac{2}{\pi} e^{-m} \sum_{\lambda=0}^m \sum_{l=0}^{m-\lambda} \binom{m}{\lambda} \binom{m-\lambda}{l} b_1^{\lambda} b_2^l \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{\varphi}{\lambda c_1 + l c_2} \right) \right] \end{aligned} \quad (22)$$

Los valores de  $b_1$  y  $b_2$  son tales que

$$e^{-m} + \hat{g}(0, m) = 1 \quad (23)$$

cumpliéndose (15).

Las fórmulas (19) y (22) son válidas para  $m$  entero entre 1 y 20, pudiéndose utilizar las tablas para interpolar valores de  $m$  no enteros.

Si el interés principal se encuentra en la región  $0 \leq m < 1$ , es conveniente modificar el cálculo. KZZ utilizan los 3 primeros términos de la suma de Wentzel, que equivale a considerar a aquellas partículas que han sufrido exactamente cero, una ó dos colisiones.

La expresión para  $f_{\text{red}}(\varphi, m)$  resulta:

$$f_{\text{red}}(\varphi, m) \simeq e^{-m} \left[ \delta(\varphi) + m f_1(\varphi) + \frac{1}{2} m^2 f_2(\varphi) \right] \quad (24)$$

donde:  $f_1(\varphi) = (1 + \varphi^2)^{-3/2}$

$$f_2(\varphi) = \frac{1}{2} \int_{-\infty}^{\infty} (1 + x^2)^{-3/2} [1 + (\varphi - x)^2]^{-3/2} dx \quad (25)$$

$f_2$  puede expresarse como:

$$f_2(0) = \frac{3\pi}{16}$$

$$f_2(\varphi) = \frac{\omega}{2(1-\omega^4) \left[ 1 + \frac{\varphi^2}{2} \right]} \left\{ 2(1-3\omega^4) [K(k) - F(\phi'; k)] - \right.$$

$$\left. \omega^{-4}(\omega^4 - 6\omega^2 + 1) [E(k) - E(\phi'; k)] + (1 - k^2 \operatorname{sen}^2 \phi')^{-\frac{1}{2}} (k^2 \operatorname{sen} \phi' \cos \phi') \right]$$

(26)

Expresión válida si  $\varphi \neq 0$ , siendo las 2 funciones  $E$  y  $K$  y  $F$  las integrales elípticas, completa e incompleta de segunda y primera clase respectivamente<sup>5</sup>.

Además:

$$k^2 = 1 - \omega^4 \quad (27)$$

$$\omega = \left[ \frac{[1 + (\varphi/2)^2]^{\frac{1}{2}} - (\varphi/2)}{[1 + (\varphi/2)^2]^{\frac{1}{2}} + (\varphi/2)} \right]^{\frac{1}{2}} \quad (28)$$

$$\phi' = \operatorname{sen}^{-1} \left[ \omega (1 + \omega^4)^{-\frac{1}{2}} \right] \quad (29)$$

En las tablas III y IV se tabulan las funciones

$$g'(\varphi, m) = e^{-m} \left[ m f_1(\varphi) + \frac{1}{2} m^2 f_2(\varphi) \right] \quad (30)$$

y  $f_2(\varphi)$ , habiendo sido esta última, calculada directamente de (25), por integración numérica.



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TABLA DE  $g(\varphi, m)$ . TABLA I

| $\varphi \setminus m$ | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0                   | .50949   | .55688   | .49217   | .41681   | .35461   | .30733   | .27182   | .24476   | .22365   | .20676   |
| 0.2                   | .48643   | .53763   | .47979   | .40944   | .35030   | .30476   | .27023   | .24373   | .22294   | .20624   |
| 0.4                   | .42797   | .48806   | .44728   | .38977   | .33857   | .29764   | .26574   | .24075   | .22086   | .20472   |
| 0.6                   | .35583   | .42498   | .40453   | .36293   | .32197   | .28720   | .25894   | .23612   | .21756   | .20226   |
| 0.8                   | .28691   | .36190   | .35971   | .33341   | .30284   | .27463   | .25043   | .23015   | .21321   | .19897   |
| 1.0                   | .22864   | .30542   | .31723   | .30387   | .28270   | .26079   | .24871   | .22312   | .20797   | .19494   |
| 1.4                   | .14589   | .21715   | .24469   | .24917   | .24262   | .23150   | .21905   | .20681   | .19540   | .18505   |
| 1.8                   | .96021-1 | .15629   | .18858   | .20248   | .20542   | .20232   | .19619   | .18875   | .18098   | .17336   |
| 2.2                   | .65489-1 | .11430   | .14587   | .16387   | .17242   | .17486   | .17358   | .17017   | .16563   | .16059   |
| 2.6                   | .46107-1 | .84923-1 | .11346   | .13247   | .14395   | .14994   | .14217   | .15191   | .15009   | .14732   |
| 3.0                   | .33342-1 | .64020-1 | .88810-1 | .10717   | .11982   | .12788   | .13247   | .13457   | .13491   | .13406   |
| 3.4                   | .24660-1 | .48912-1 | .69992-1 | .86885-1 | .99607-1 | .10867   | .11473   | .11847   | .12045   | .12115   |
| 3.8                   | .18592-1 | .37830-1 | .55545-1 | .70643-1 | .82797-1 | .92132-1 | .98996-1 | .10380   | .10697   | .10885   |
| 4.2                   | .14251-1 | .29590-1 | .44384-1 | .57636-1 | .68880-1 | .78024-1 | .85194-1 | .90624-1 | .94583-1 | .97338-1 |
| 4.6                   | .11083-1 | .23386-1 | .35708-1 | .47204-1 | .57389-1 | .66056-1 | .73193-1 | .78902-1 | .83343-1 | .86698-1 |
| 5.0                   | .87330-2 | .18662-1 | .28919-1 | .38820-1 | .47912-1 | .55944-1 | .62823-1 | .68563-1 | .73242-1 | .76972-1 |
| 7.0                   | .31035-2 | .68412-2 | .11060-1 | .15586-1 | .20245-1 | .24886-1 | .29385-1 | .33652-1 | .37622-1 | .41261-1 |
| 9.0                   | .13438-2 | .29694-2 | .48581-2 | .69760-2 | .92788-2 | .11717-1 | .14239-1 | .16798-1 | .19349-1 | .21855-1 |
| 12.0                  | .49309-3 | .10744-2 | .17479-2 | .25144-2 | .33717-2 | .43147-2 | .53360-2 | .64259-2 | .75739-2 | .87683-2 |
| 16.0                  | .17735-3 | .37774-3 | .60319-3 | .85546-3 | .11360-2 | .14459-2 | .17856-2 | .21554-2 | .25549-2 | .29832-2 |
| 20.0                  | .8064 -4 | .16874-3 | .26496-3 | .36997-3 | .48442-3 | .60892-3 | .74403-3 | .89026-3 | .10480-2 | .12177-2 |

.16874-3 significa  $.16874 \times 10^{-3}$ , etc.

TABLA DE  $g(\varphi, m)$ . TABLA I

| $\varphi \setminus m$ | 11       | 12       | 13       | 14       | 15       | 16       | 17       | 18       | 19       | 20       |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0                   | .19291   | .18133   | .17147   | .16295   | .15550   | .14892   | .14305   | .13778   | .13301   | .12867   |
| 0.2                   | .19252   | .18102   | .17122   | .16274   | .15533   | .14877   | .14292   | .13766   | .13291   | .12858   |
| 0.4                   | .19135   | .18010   | .17047   | .16212   | .15480   | .14831   | .14252   | .13732   | .13260   | .12830   |
| 0.6                   | .18946   | .17859   | .16923   | .16109   | .15392   | .14756   | .14187   | .13674   | .13209   | .12784   |
| 0.8                   | .18689   | .17652   | .16754   | .15967   | .15271   | .14652   | .14096   | .13594   | .13137   | .12720   |
| 1.0                   | .18370   | .17395   | .16541   | .15787   | .15118   | .14519   | .13980   | .13491   | .13046   | .12639   |
| 1.4                   | .17575   | .16742   | .15996   | .15326   | .14723   | .14176   | .13679   | .13225   | .12808   | .12425   |
| 1.8                   | .16614   | .15941   | .15319   | .14747   | .14221   | .13738   | .13292   | .12881   | .12501   | .12149   |
| 2.2                   | .15541   | .15030   | .14539   | .14072   | .13632   | .13219   | .12832   | .12471   | .12132   | .11815   |
| 2.6                   | .14403   | .14049   | .13686   | .13325   | .12974   | .12636   | .12312   | .12003   | .11710   | .11432   |
| 3.0                   | .13243   | .13030   | .12788   | .12531   | .12267   | .12003   | .11743   | .11490   | .11244   | .11007   |
| 3.4                   | .12092   | .12004   | .11872   | .11710   | .11529   | .11337   | .11140   | .10941   | .10743   | .10549   |
| 3.8                   | .10976   | .10993   | .10956   | .10880   | .10775   | .10651   | .10514   | .10368   | .10217   | .10064   |
| 4.2                   | .99127-1 | .10015   | .10059   | .10057   | .10020   | .99577-1 | .98759-1 | .97800-1 | .96743-1 | .95617-1 |
| 4.6                   | .89142-1 | .90840-1 | .91932-1 | .92540-1 | .92762-1 | .92682-1 | .92366-1 | .91868-1 | .91229-1 | .90485-1 |
| 5.0                   | .79875-1 | .82075-1 | .83685-1 | .84805-1 | .85524-1 | .85916-1 | .86043-1 | .85958-1 | .85702-1 | .85312-1 |
| 7.0                   | .44549-1 | .47489-1 | .50089-1 | .52368-1 | .54347-1 | .56052-1 | .57507-1 | .58738-1 | .59769-1 | .60622-1 |
| 9.0                   | .24288-1 | .26621-1 | .28838-1 | .30928-1 | .32882-1 | .34699-1 | .36377-1 | .37920-1 | .39332-1 | .40619-1 |
| 12.0                  | .99974-2 | .11249-1 | .12514-1 | .13779-1 | .15038-1 | .16280-1 | .17500-1 | .18690-1 | .19847-1 | .20967-1 |
| 16.0                  | .34393-2 | .39214-2 | .44276-2 | .49559-2 | .55037-2 | .60687-2 | .66482-2 | .72396-2 | .78403-2 | .84479-2 |
| 20.0                  | .13994-2 | .15935-2 | .17998-2 | .20183-2 | .22488-2 | .24911-2 | .27448-2 | .30094-2 | .32844-2 | .35693-2 |

.13994-2 significa  $.13994 \times 10^{-2}$ , etc.

TABLA DE  $\hat{g}(\varphi, m)$ . TABLA II

| $\varphi \setminus m$ | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0                   | .63212   | .86467   | .95022   | .98169   | .99327   | .99752   | .99909   | .99967   | .99988   | .99996   |
| 0.2                   | .53179   | .75459   | .85262   | .89882   | .92263   | .93623   | .94483   | .95078   | .95520   | .95864   |
| 0.4                   | .43994   | .65166   | .75966   | .81874   | .85364   | .87592   | .89119   | .90230   | .91079   | .91753   |
| 0.6                   | .36150   | .56026   | .67439   | .74339   | .78753   | .81739   | .83869   | .85459   | .86693   | .87682   |
| 0.8                   | .29737   | .48165   | .59798   | .67374   | .72502   | .76118   | .78773   | .80795   | .82384   | .83668   |
| 1.0                   | .24601   | .41505   | .53034   | .61002   | .66646   | .70763   | .73860   | .76260   | .78171   | .79728   |
| 1.4                   | .17247   | .31160   | .41854   | .49966   | .56145   | .60913   | .64658   | .67653   | .70095   | .72120   |
| 1.8                   | .12492   | .23767   | .33239   | .40961   | .47197   | .52241   | .56352   | .59739   | .62563   | .64948   |
| 2.2                   | .93100-1 | .18406   | .26589   | .33660   | .39655   | .44705   | .48959   | .52560   | .55629   | .58266   |
| 2.6                   | .71061-1 | .14456   | .21432   | .27755   | .33343   | .38218   | .42449   | .46121   | .49315   | .52107   |
| 3.0                   | .55341-1 | .11500   | .17409   | .22981   | .28081   | .32671   | .36762   | .40395   | .43617   | .46480   |
| 3.4                   | .43848-1 | .92573-1 | .14250   | .19115   | .23705   | .27949   | .31825   | .35339   | .38513   | .41378   |
| 3.8                   | .35267-1 | .75336-1 | .11752   | .15976   | .20067   | .23942   | .27557   | .30898   | .33968   | .36780   |
| 4.2                   | .28745-1 | .61931-1 | .97628-1 | .13420   | .17043   | .20546   | .23879   | .27014   | .29941   | .32659   |
| 4.6                   | .23711-1 | .51394-1 | .81681-1 | .11331   | .14525   | .17671   | .20717   | .23629   | .26386   | .28981   |
| 5.0                   | .19770-1 | .43026-1 | .68810-1 | .96166-1 | .12425   | .15237   | .18002   | .20684   | .23258   | .25710   |
| 7.0                   | .90972-2 | .19835-1 | .32104-1 | .45705-1 | .60408-1 | .75968-1 | .92142-1 | .10871   | .12546   | .14224   |
| 9.0                   | .49492-2 | .10672-1 | .17184-1 | .24471-1 | .32494-1 | .41193-1 | .50496-1 | .60322-1 | .70583-1 | .81196-1 |
| 12.0                  | .24562-2 | .51912-2 | .82257-2 | .11575-1 | .15250-1 | .19251-1 | .23587-1 | .28240-1 | .33202-1 | .38456-1 |
| 16.0                  | .12562-2 | .25886-2 | .40232-2 | .55646-2 | .72107-2 | .89767-2 | .10865-1 | .12881-1 | .15026-1 | .17305-1 |
| 20.0                  | .76693-3 | .15677-2 | .24054-2 | .32830-2 | .42032-2 | .51689-2 | .61828-2 | .72473-2 | .83632-2 | .95386-2 |

.76693-3 significa  $.76693 \times 10^{-3}$ , etc.

TABLA DE  $\hat{g}(\varphi, m)$ . TABLA II

| $\varphi \setminus m$ | 11       | 12       | 13       | 14       | 15       | 16       | 17       | 18       | 19       | 20       |
|-----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.0                   | .99999   | 1.0000   | 1.0000   | 1.0000   | 1.0000   | 1.0000   | 1.0000   | 1.0000   | 1.0000   | 1.0000   |
| 0.2                   | .96143   | .96375   | .96572   | .96743   | .96891   | .97023   | .97140   | .97246   | .97341   | .97428   |
| 0.4                   | .92303   | .92763   | .93155   | .93493   | .93790   | .94052   | .94285   | .94495   | .94685   | .94858   |
| 0.6                   | .88494   | .89175   | .89757   | .90261   | .90702   | .91092   | .91441   | .91754   | .92038   | .92297   |
| 0.8                   | .84729   | .85623   | .86389   | .87052   | .87635   | .88151   | .88612   | .89027   | .89403   | .89746   |
| 1.0                   | .81022   | .82118   | .83058   | .83876   | .84596   | .85234   | .85804   | .86319   | .86785   | .87210   |
| 1.4                   | .73827   | .75285   | .76546   | .77649   | .78624   | .79491   | .80270   | .80973   | .81611   | .82195   |
| 1.8                   | .66984   | .68744   | .70279   | .71631   | .72832   | .73906   | .74873   | .75749   | .76547   | .77278   |
| 2.2                   | .60551   | .62547   | .64305   | .65865   | .67258   | .68512   | .69646   | .70676   | .71619   | .72483   |
| 2.6                   | .54561   | .56729   | .58658   | .60383   | .61935   | .63339   | .64615   | .65780   | .66849   | .67832   |
| 3.0                   | .49031   | .51313   | .53362   | .55211   | .56886   | .58410   | .59803   | .61080   | .62256   | .63343   |
| 3.4                   | .43965   | .46306   | .48430   | .50363   | .52126   | .53741   | .55225   | .56593   | .57858   | .59031   |
| 3.8                   | .39353   | .41708   | .43865   | .45844   | .47665   | .49343   | .50894   | .52331   | .53665   | .54908   |
| 4.2                   | .35177   | .37507   | .39662   | .41657   | .43506   | .45221   | .46816   | .48301   | .49687   | .50982   |
| 4.6                   | .31414   | .33689   | .35813   | .37796   | .39647   | .41376   | .42993   | .44507   | .45927   | .47260   |
| 5.0                   | .28036   | .30232   | .32302   | .34250   | .36082   | .37805   | .39425   | .40951   | .42388   | .43744   |
| 7.0                   | .15890   | .17533   | .19146   | .20720   | .22251   | .23738   | .25177   | .26567   | .27909   | .29204   |
| 9.0                   | .92076-1 | .10315   | .11434   | .12559   | .13685   | .14806   | .15918   | .17019   | .18105   | .19174   |
| 12.0                  | .43985-1 | .49766-1 | .55776-1 | .61991-1 | .68386-1 | .74937-1 | .81620-1 | .88411-1 | .95289-1 | .10223   |
| 16.0                  | .19718-1 | .22265-1 | .24946-1 | .27759-1 | .30701-1 | .33768-1 | .36956-1 | .40260-1 | .43675-1 | .47194-1 |
| 20.0                  | .10770-1 | .12061-1 | .13413-1 | .14828-1 | .16308-1 | .17852-1 | .19462-1 | .21138-1 | .22879-1 | .24687-1 |

.10770-1 significa  $.10770 \times 10^{-1}$ , etc.

TABLA DE  $f_2(\varphi)$ . TABLA III

| $\varphi$ | $f_2(\varphi)$ |
|-----------|----------------|
| 0.0       | 5.8904 -1      |
| 0.1       | 5.8629 -1      |
| 0.2       | 5.7815 -1      |
| 0.3       | 5.6498 -1      |
| 0.4       | 5.4733 -1      |
| 0.5       | 5.2584 -1      |
| 0.6       | 5.0131 -1      |
| 0.7       | 4.7456 -1      |
| 0.8       | 4.4633 -1      |
| 0.9       | 4.1742 -1      |
| 1.0       | 3.8844 -1      |
| 1.2       | 3.3219 -1      |
| 1.4       | 2.8072 -1      |
| 1.6       | 2.3537 -1      |
| 1.8       | 1.9654 -1      |
| 2.0       | 1.6387 -1      |
| 2.2       | 1.3679 -1      |
| 2.4       | 1.1449 -1      |
| 2.6       | 9.5991 -2      |
| 3.0       | 6.8466 -2      |
| 3.4       | 5.0068 -2      |
| 3.8       | 3.7246 -2      |
| 4.2       | 2.8272 -2      |
| 4.6       | 2.1841 -2      |
| 5.0       | 1.7156 -2      |
| 6.0       | 9.8414 -3      |
| 7.0       | 6.3080 -3      |
| 8.0       | 4.1452 -3      |
| 9.0       | 2.9243 -3      |
| 10.0      | 2.1479 -3      |

2.1479-3 significa  $2.1479 \times 10^{-3}$ , etc.

TABLA DE  $\hat{g}(\varphi, m)$ . TABLA IV

| $\varphi \backslash m$ | 0.1      | 0.2      | 0.3      | 0.4      | 0.5      | 0.6      |
|------------------------|----------|----------|----------|----------|----------|----------|
| 0.0                    | 9.3149-2 | 1.7339-1 | 2.4188-1 | 2.9972-1 | 3.4792-1 | 3.8748-1 |
| 0.1                    | 9.1796-2 | 1.7092-1 | 2.3850-1 | 2.9560-1 | 3.4322-1 | 3.8233-1 |
| 0.2                    | 8.7930-2 | 1.6386-1 | 2.2882-1 | 2.8381-1 | 3.2977-1 | 3.6759-1 |
| 0.3                    | 8.2068-2 | 1.5314-1 | 2.1413-1 | 2.6591-1 | 3.0933-1 | 3.4517-1 |
| 0.4                    | 7.4900-2 | 1.4003-1 | 1.9613-1 | 2.4396-1 | 2.8423-1 | 3.1763-1 |
| 0.6                    | 5.9319-2 | 1.1145-1 | 1.5684-1 | 1.9594-1 | 2.2922-1 | 2.5714-1 |
| 0.8                    | 4.5102-2 | 8.5275-2 | 1.2070-1 | 1.5160-1 | 1.7824-1 | 2.0088-1 |
| 1.0                    | 3.3748-2 | 6.4254-2 | 9.1525-2 | 1.1563-1 | 1.3667-1 | 1.5479-1 |
| 1.2                    | 2.5243-2 | 4.8402-2 | 6.9385-2 | 8.8163-2 | 1.0475-1 | 1.1921-1 |
| 1.4                    | 1.9038-2 | 3.6750-2 | 5.2999-2 | 6.7704-2 | 8.0833-2 | 9.2391-2 |
| 1.6                    | 1.4536-2 | 2.8232-2 | 4.0933-2 | 5.2540-2 | 6.2994-2 | 7.2274-2 |
| 1.8                    | 1.1253-2 | 2.1973-2 | 3.2008-2 | 4.1250-2 | 4.9636-2 | 5.7131-2 |
| 2.0                    | 8.8345-3 | 1.7329-2 | 2.5341-2 | 3.2770-2 | 3.9549-2 | 4.5641-2 |
| 2.4                    | 5.6661-3 | 1.1191-2 | 1.6462-2 | 2.1395-2 | 2.5935-2 | 3.0045-2 |
| 2.6                    | 4.6201-3 | 9.1468-3 | 1.3481-2 | 1.7551-2 | 2.1307-2 | 2.4715-2 |
| 3.0                    | 3.1711-3 | 6.2992-3 | 9.3105-3 | 1.2150-2 | 1.4781-2 | 1.7176-2 |
| 3.4                    | 2.2593-3 | 4.4985-3 | 6.6620-3 | 8.7086-3 | 1.0609-2 | 1.2344-2 |
| 3.8                    | 1.6599-3 | 3.3089-3 | 4.9049-3 | 6.4168-3 | 7.8225-3 | 9.1069-3 |
| 4.2                    | 1.2523-3 | 2.4976-3 | 3.7041-3 | 4.8478-3 | 5.9118-3 | 6.8846-3 |
| 4.6                    | 9.6621-4 | 1.9273-3 | 2.8586-3 | 3.7416-3 | 4.5631-3 | 5.3142-3 |
| 5.0                    | 7.6013-4 | 1.5160-3 | 2.2483-3 | 2.9425-3 | 3.5882-3 | 4.1785-3 |
| 6.0                    | 4.4656-4 | 8.8871-4 | 1.3156-3 | 1.7191-3 | 2.0936-3 | 2.4353-3 |
| 7.0                    | 2.8447-4 | 5.6644-4 | 8.3889-4 | 1.0967-3 | 1.3360-3 | 1.5545-3 |
| 8.0                    | 1.9142-4 | 3.8034-4 | 5.6228-4 | 7.3394-4 | 8.9298-4 | 1.0378-3 |
| 10.0                   | 9.8861-5 | 1.9649-4 | 2.9056-4 | 3.7934-4 | 4.6162-4 | 5.3659-4 |

9.8861-5 significa  $9.8861 \times 10^{-5}$ , etc.

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