INTERACTION BETWEEN POTENTIAL RESONANCES AND COMPOUND STATES WITH STRONG COUPLING+

F. Prats
George Washington University, Washington, D.C.

M. Bauer*
Instituto de Física, Universidad Nacional de México
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ABSTRACT

Previous work on the strong coupling between potential resonances and compound nucleus states suggested the possibility od appearance of anomalies in the resulting phase shifts. Such anomalies could result in a modification of the shape of the resonances, or even give rise to additional peaks in the cross-section In this paper this possibility is explored in the frame work of the Dirac model of resonance scattering.

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* Technical Consultant to the Comisión Nacional de Energía Nuclear, México, D.F.
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RESUMEN

En un trabajo previo sobre el acoplamiento fuerte entre resonancias de potencial y estados de núcleo compuesto se sugirió la posibilidad de que aparecieran anomalías en los corrimientos de fase resultantes. Estas anomalías podrían dar lugar a modificaciones en la forma de las resonancias y más aún hacer aparecer picos adicionales en la sección de dispersión. En este trabajo se explora esta posibilidad dentro del marco de la teoría de dispersión resonante de Dirac.

INTRODUCTION

Extended shell model calculations include very often single particle configurations whose energy is larger than the particle emission threshold in the corresponding channel. These configurations are either potential resonances such as $1p_{1/2}^{-1}\,1d_{3/2}$ in O^{16}, or virtual bound states such as $1p_{3/2}^{-1}\,1s_{1/2}$ in O^{16}. They are nevertheless treated as discrete states instead of resonances in the continuum spectrum. In a previous paper[1] it has been shown, using a schematic Brown model, that indeed the positions of the actual resonances may be obtained by an "equivalent" bound state problem, justifying therefore the usual shell model procedure.

In the course of that work it was noticed that the exact treatment of the continuum allows for the possibility of appearance of anomalies in the phase shifts due to the strong coupling between potential resonances and actual bound states. Such anomalies could result in a modification of the shape of the resonances from the Breit-Wigner form, or even give rise to additional peaks in the cross-section.

With the purpose of understanding how these different types of configurations interact, we have considered a simple model where this situation takes place. This model is the Dirac model[2] of resonance scattering.

We consider the case when there is only one scattering channel and one bound state, and restrict ourselves to some eigenvalue of the total angular momentum of the system so that the scattering is described by a phase shift.

1. F. Prats and M. Bauer, Nuclear Physics (in press).
2. P.A.M. Dirac, The Principles of Quantum Mechanics, Oxford University Press, 4th Edition, 1958, p. 201.

2. DIRAC'S MODEL OF RESONANCE SCATTERING WITH STRONG COUPLING.

A particle is scattered by a target in its ground state, this situation being described by a state vector $\psi_E^{(+)}$, where E, the total energy is positive. The particle may also form a bound state with some excited state of the target at some $\epsilon_1 > 0$, described by a state vector φ. The states $\psi_E^{(+)}$ and φ are assumed to be orthogonal: $(\psi_E^{(+)}, \varphi) = 0$.

If some interaction V acts between the states ψ_E and φ, the state of the system becomes $\Psi_E^{(+)}$, which is a solution of the equation

$$\Psi_E^{(+)} = \psi_E^{(+)} + \frac{1}{E + i\varepsilon - H_0} V\Psi_E^{(+)} \tag{1}$$

In Dirac's model ψ_E, φ, H_0 and V are chosen such that the only non-vanishing matrix element of V is $(\psi_E^{(+)}, V\varphi)$. Then this equation can be solved immediately and one finds

$$\Psi_E^{(+)} = \psi_E^{(+)} + \left\{ \varphi + \int dE' \psi_{E'}^{(+)} \frac{1}{E + i\varepsilon - E'} \left(\psi_{E'}^{(+)}, V\varphi\right) \right\} \frac{(\varphi, V\psi_E^{(+)})}{d(E)} \tag{2}$$

where

$$d(E) = E - \epsilon_1 - \Delta(E) + i\,\frac{1}{2}\,\Gamma(E) \tag{3}$$

with the definitions

$$\Delta(E) = P\int dE' \frac{|(\varphi_1 V\psi_{E'}^{(+)})|^2}{E - E'}$$

$$\Gamma(E) = 2\pi\, |(\varphi_1 V\psi_{E'}^{(+)})|^2 \tag{4}$$

and we have assumed the normalization

$$\left(\psi_{E'}^{(+)},\ \psi_E^{(+)}\right) = \delta(E' - E) \qquad (\varphi,\ \varphi) = 1 \tag{5}$$

Application of the formula of Watson[3] gives the scattering amplitude

$$T(E) = e^{i\delta} \sin \delta - \frac{1}{2}\ e^{2i\delta}\ \frac{\Gamma(E)}{d(E)} = e^{i\eta} \sin \eta \tag{6}$$

where δ is the phase shift associated with the state $\psi_E^{(+)}$ and $\eta = \delta + \delta'$, with

$$\operatorname{tg} \delta' = - \frac{1/2\,\Gamma(E)}{E - \epsilon_1 - \Delta(E)} \tag{7}$$

is the new phase shift, associated with the state $\Psi_E^{(+)}$.

These are well known results and have been used previously in the situation when one assumes that Δ and Γ are nearly independent[4] of E.

We are now interested in the situation when there is a resonance in the uncoupled energy continuum, that is, when the phase shift δ goes through $\pi/2$ at, say $E = \epsilon_0$. Then the pole at $E = \epsilon_0 - i\frac{\omega_0}{2}$ of the scattering amplitude $e^{i\delta}\sin\delta$ is also a pole of the matrix element $(\varphi,\ V\psi_E^{(+)})$ and consequently $\Gamma(E)$ will show a maximum at, or near, $E = \epsilon_0$ and $\Delta(E)$ will oscillate in that region as shown in Fig. 1a, b. For purposes of illustration, we have assumed that the interaction V and the states φ, ψ_E are such that

$$\left|(\varphi,\ V\psi_E^{(+)})\right|^2 = \frac{1}{\pi}\ \sqrt{E}\quad \frac{\sqrt{\beta}}{\left(\frac{E - \epsilon_0}{\omega_{0/2}}\right)^2 + 1} \tag{8}$$

3. M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953).
4. See for instance B. Zumino, Research Report No .CX-23, Institute of Mathematical Sciences, New York University, N. Y. (1956); U. Fano, Phys. Rev. 124, 1866 (1961).

(The $\frac{1}{\pi}\sqrt{E}$ factor comes from the normalization of $\psi_E^{(+)}$. The remaining factor is only a function of E). One can then find out easily that

$$\Delta(E) = \sqrt{\beta}\,\frac{\omega_0^2}{4}\;\frac{1}{\sin\{\lambda\}}\;\frac{E - |\lambda|^2}{(E-\epsilon_0)^2 + \frac{\omega_0}{2}^{\,2}} \tag{9}$$

where

$$\lambda = + \qquad \epsilon_0 + i\,\frac{\omega_0}{2}$$

The width function, $\Gamma(E)$, and level displacement function, $\Delta(E)$ are plotted in Fig. 1a, b for the values of the parameters

$$\epsilon_0 = 2.6 \text{ MeV}$$

$$\omega_0 = 1.0 \text{ MeV}$$

$$\beta = 6.5 \text{ MeV}$$

In Fig. 2a, b, c, d, the phase correction δ' obtained from Eq. (7) is shown for the values of β indicated as well as the total phase shift η obtained, assuming the same "potential" phase shift δ in all cases. In cases a, b, c, $\epsilon_1 = 6.6$ MeV, in case d, $\epsilon_1 = 5.0$ MeV.

Figs. 3, a, b, c, d show the respective curves for $\sin^2\eta$, which is proportional to E times the corresponding partial cross-section. One can observe one or more maxima between the two resonance maxima.

In Fig. 4 a case is shown when the compound state is below the potential resonance and the interaction is strong enough to push it into a negative energy E_1, becoming a bound state.

One can obtain different sets of curves by changing the shape of the potential phase shift δ, while letting it go through $\pi/2$ at the same point ϵ_0 and with the same slope $\omega_0/2$. They exhibit the same features as the curves shown.

It should be remarked that the possibility of non-resonance peaks in cross sections between two resonance peaks has been already suggested by Teichmann from the Wigner-Eisenbud theory[5].

3. DISCUSSION

The examples presented in the figures are typical of the different situations possible. We can start a classification of these different cases by studying the function

$$R(E) = E - \epsilon_1 - \Delta(E) \qquad (10)$$

which has the behavior shown in Fig. 1b. It has always a zero at some value E and has the straight line $E - \epsilon$ as asymptote. Changes in the interaction strength while maintaining ω_0 constant merely change the amplitude of its oscillation around ϵ_0. For weak interaction the oscillation is of small amplitude and $R(E)$ has only the zero at E_1. This implies that the phase correction δ' goes through $\pi/2$ only once, at E_1. This is the situation in the first case, Figs. 2a, 3a.

On increasing the interaction sufficiently the maximum of $R(E)$ touches the E-axis and we have a double zero at the point of contact. The phase correction $\pi/2$ at that point. This corresponds to the second case, Fig. 2b, 3b. Further increase of the interaction makes $R(E)$ have three zeros, the one at about ϵ_0 with negative slope. Correspondingly δ' goes through $\pi/2$ three times and at about ϵ_0 with negative slope. This is the situation in the third and fourth cases, Fig. 2c, d and 3c, d.

The different cases illustrated above are entirely analogous to those discussed by Fonda and Newton[6]. As in their work, the resonance at E_0 in Fig. 2b, 3b is not of the Breit-Wigner type.

5. T. Teichmann, Phys. Rev. 77, 506 (1950).
6. L. Fonda and R.G. Newton, Annals of Phys. 10, 490 (1960).

It is also possible to increase the interaction so that the lowest of the three zeros moves into the negative energy region yielding a bound state of the system.

An index for classifying the interaction according to strength is provided by the ratio

$$\rho = 2 \sqrt{\int dE \left|(\varphi, V\psi_E)\right|^2} / \epsilon_1 - \epsilon_0 \approx \sqrt{\Gamma_{max}\, \omega_0} / \epsilon_1 - \epsilon_0$$

$\rho < 1$ corresponds to weak, $\rho > 1$ to strong interaction. Thus for the cases a, b, c, d illustrated above, ρ is respectively equal to 0.4, 1.0, 1.7, 2.9.

As in the case of the interaction between two discrete levels, the levels "repel" each other, this "repulsion" increasing with the strength of the interaction as a glance at Figs. 3a, b, c, d shows. In going from c to d the effective strength has been increased by moving the level ϵ_1 down from 6.6 to 5 MeV.

These figures show also a narrowing of the lower resonance as the interaction increases. This is simply due to the level repulsion pushing the lower resonance closer to become a bound state and therefore the phase shift η tending to become π at $\epsilon = 0$, just as in potential scattering. This effect has been observed by Lemmer and Shakin[7] in their work on O^{16}.

It is interesting to note that in all cases the scattering amplitude has two poles, but that whereas for weak interaction one is the pole of the potential $T_p = e^{i\delta} \sin \delta$ and the other the pole of the resonant term (from the higher root of $R(E)$), for strong interactions both poles come from the resonant term. In the latter case the root of $R(E)$ at, or near, ϵ_0 does not yield a pole of the scattering matrix but, as Figs. 2c, d show, makes the cancellation of the original pole in the potential term.

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7. R.H. Lemmer and C.H. Shakin, Annals of Physics 27, 13 (1964).

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5. T. Teichmann, Phys. Rev. **77**, 506 (1950).
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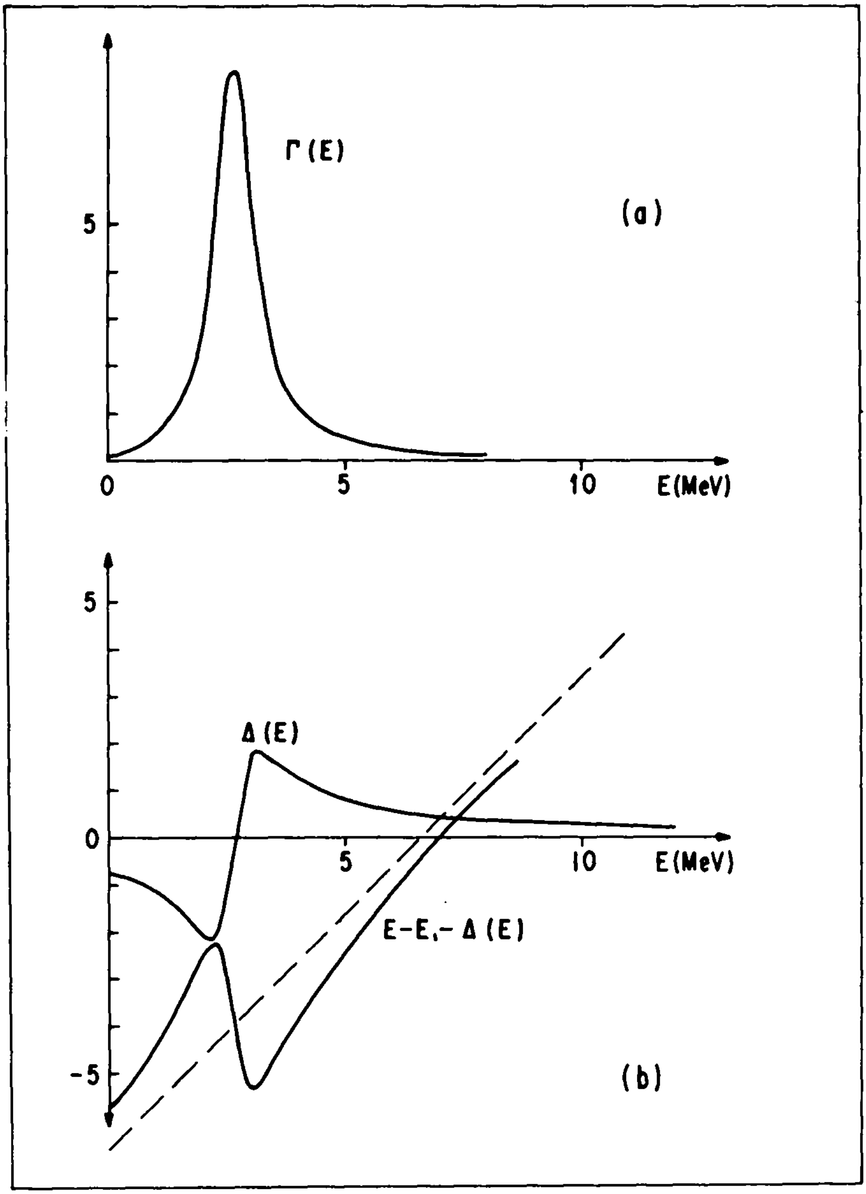

Figure 1. a) The width function $\Gamma(E)$ and b) the level displacement function $\Delta(E)$ for $\epsilon_0 = 2.6\,\mathrm{MeV}$, $\omega_0 = 1.0\,\mathrm{MeV}$, $\beta = 6.5\,\mathrm{MeV}$. In b) it is also shown $E - \epsilon_1 - \Delta(E)$ for $\epsilon_1 = 6.6\,\mathrm{MeV}$.

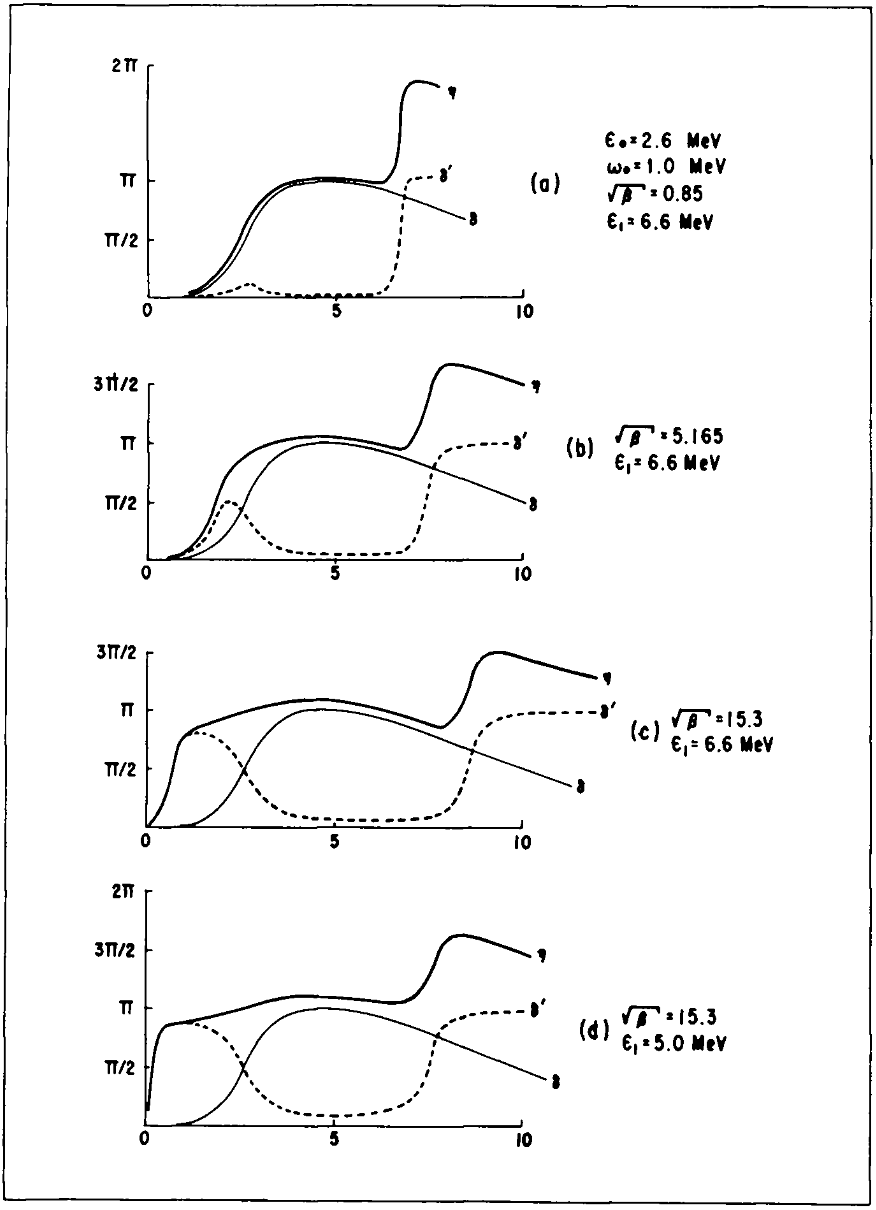

Figure 2a, b, c, d. The "potential" phase shift δ, the phase correction δ' and the total phase shift η for various values of β.

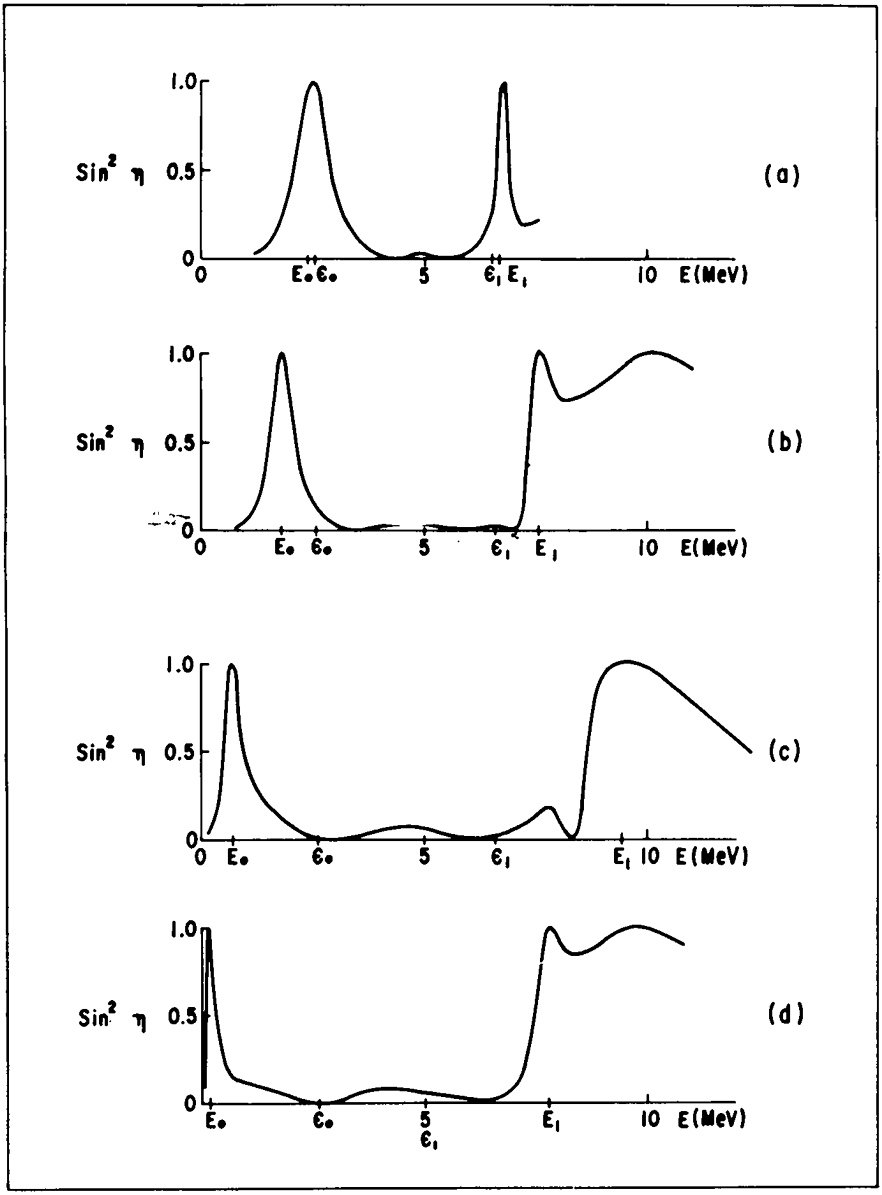

Figure 3a, b, c, d. $\sin^2\eta$ for the same parameters as in Fig. 2.

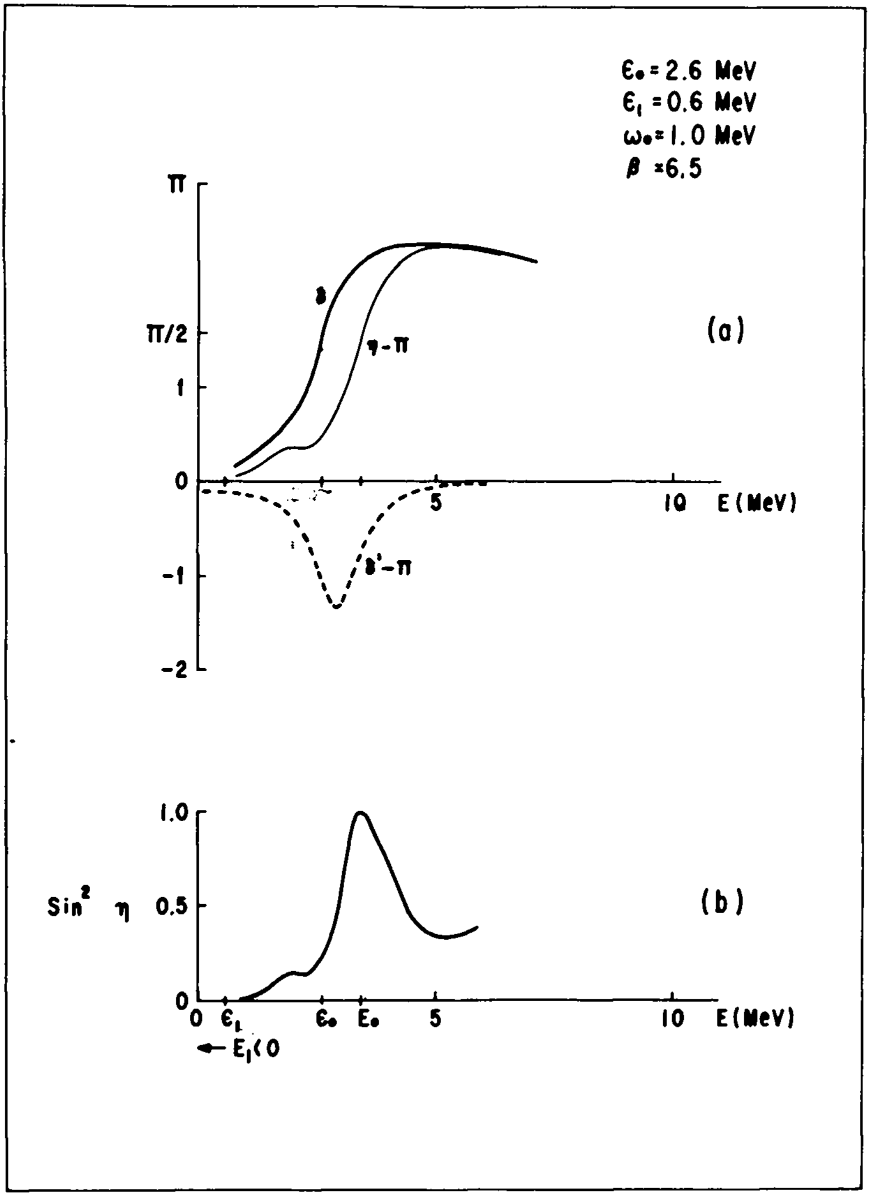

Figure 4. The compound state below the potential resonance, $\epsilon_1 < \epsilon_0$.