Vol. XVI, 1 REVISTA MEXICANA DE FISICA 1967

EXCHANGE EFFECTS IN ODD-ODD NUCLEI IN THE 2s-1d SHELL

M. Berrondo* and J. Pineda*
Instituto de Física, Universidad Nacional de México
(Recibido: 14 octubre 1966)

ABSTRACT

A model residual interaction consisting of a central long-range term plus a spin-orbit coupling term is considered, including an exchange force of the Rosenfeld type, and applied to even parity states in de 2s-1d shell in the supermultiplet classification, characterized further by $U_6 \supset SU_3 \supset R_3$ *, where we restrict ourselves to the highest irreducible representation of* SU_3*. Energy levels are obtained as functions of the parameter giving the relative strength of the spin-orbit coupling and the long-range term, for* ^{24}Na, ^{24}Al, ^{26}Na, ^{28}Al, ^{28}P, ^{30}Al, ^{32}Cl, ^{32}P, *and* ^{34}P.

RESUMEN

Se considera una interacción residual modelo que consta de un término central de largo alcance y un término de acoplamiento spin-orbita y que incluye una

*Fellows of the Comisión Nacional de Energía Nuclear.

fuerza de intercambio de Rosenfeld. Se aplica esta interacción a estados de paridad positiva en la capa 2s-1d usando la clasificación de supermultiplete y caracterizando los estados, además, por la cadena de grupos $U_6 \supset SU_3 \supset R_3$, *restringiéndonos a la representación irreducible mas alta de* SU_3. *Se encuentran los niveles de energía como funciones de un parámetro que indica la fuerza relativa del acoplamiento spin-órbita y el término de largo alcance, para los núcleos* ^{24}Na, ^{24}Al, ^{26}Na, ^{28}Al, ^{28}P, ^{30}Al, ^{32}Cl, ^{32}P *y* ^{34}P.

I. THEORETICAL ANALYSIS

In an earlier paper[2], using techniques developed by Moshinsky[1], we calculated energy levels of odd-odd nuclei in the 2s-1d shell, without including any exchange forces.

In order to carry out these calculations, we used the supermultiplet classification[3] of states, using the chain of groups

$$\mathbf{U}_{24} \supset U_6 \times U_4 ; \begin{cases} U_6 \supset D^{(2\,0)}(SU_3) \supset \begin{pmatrix} D^{(2)}(R_3) & 0 \\ 0 & D^{(0)}(R_3) \end{pmatrix} \\ U_4 \supset SU_2^{(\sigma)} \times SU_2^{(\tau)} \end{cases}$$

where we take the completely antisymmetric irreducible representation (IR) of $\mathbf{U}_{24}$, $[1^N]$, as we are dealing with a system of N fermions in the 2s-1d shell. The group $\mathbf{U}_{24}$ is the group of unitary transformations in 24 dimensions, i.e., the number of single-particle states in the 2s-1d shell of the harmonic oscillator; furthermore, we classify the states by a subgroup of $\mathbf{U}_{24}$, which is the direct product of U_6 and U_4, the groups of unitary transformations in 6 and 4 dimensions respectively. The U_6 is connected with the orbital space, while U_4 is related to the spin-isospin space. This subgroup $U_6 \times U_4$ clearly leads to the supermultiplet classification of states[3], and as the IR of $\mathbf{U}_{24}$ is $[1^N]$, the IR of U_6 and U_4 are labeled by conjugate partitions of the number of particles N.

Of these N-nucleon states, we will take only those corresponding to the IR of U_6 lowest in energy, that is, the most symmetric Young partition compatible with the lowest possible value of the isospin of the nucleus; this agrees with Hund's rule when applied to nuclear forces, which are essentially attractive.

We complete our classification using the subgroup $SU_2^{(\sigma)} \times SU_2^{(\tau)}$, the direct product of two unitary unimodular transformation groups in two dimensions, related to the spin and isospin spaces respectively.

Finally, we consider the subgroup of U_6, corresponding to the IR (2 0) of SU_3, associated with one-particle states in 2s-1d shell, and the IR $I = 0,2$ of R_3, the rotation group in three dimensions.

As first approximation, following Elliott[4], we restrict ourselves to the highest IR of SU_3 contained in the corresponding Young partition of U_6.

In conclusion, our states are represented by kets

$$|[h_1 \ldots h_6](k_1\, k_2)\, \omega\, L; \{v_1, \ldots, v_4\}\, ST; JM\rangle \ldots \qquad (1)$$

where $[h_1 \ldots h_6]$ and $\{v_1 \ldots v_4\}$ are the conjugate partitions of U_6 and U_4, and (k_1, k_2), L, S, T label the IR of SU_3, R_3, $SU_2^{(\sigma)}$ $SU_2^{(\tau)}$ respectively; ω is used to distinguish between repeated L values contained in (k_1, k_2); and we have coupled the L and S values to a total angular momentum J and projection M.

In this paper, we use as the residual interaction a model interaction which is a linear combination of two terms:

a) long-range correlations including a quadrupole-quadrupole interaction $\mathcal{Q}^2$, multiplied by an exchange force $\mathcal{J}$, and

b) a single-particle spin-orbit interaction W_{so}, i.e.,

$$-\left[x\,\frac{\mathcal{Q}^2}{100} + (1-x)\,W_{so}\right] \ldots \qquad (2)$$

where x is a parameter which runs from 0 to 1, and which indicates the relative

strength of the two terms; as we are interested only in obtaining relative energies, there is no need to consider a total-strength parameter.

The spin-orbit coupling term for the highest IR of SU_3 is very easily obtained from a closed formula[1,2].

On the other hand, due to the fact that we are taking only a long-range term as the central part, we can approximate the matrix elements of $\mathcal{Q}^2$ between kets (1), as a product of two matrix elements[5,6], namely, the exchange term calculated between states characterized by $U_4 \supset SU_2^{(\sigma)} \times SU_2^{(\tau)}$, and the corresponding quadrupole-quadrupole matrix element calculated between states classified by the orbital subchain of groups $U_6 \supset SU_3 \supset R_3$. For these states, both matrices are diagonal[6,2], with eigenvalues

$$I = \frac{1}{2} W N(N-1) + \frac{1}{4} (B-H) N(N-4) + BS(S+1) - HT(T+1) +$$

$$+ \frac{1}{2} M \sum_{\mu=1}^{6} b_\mu (b_\mu - 2\mu + 1) \qquad \text{and}$$

$$E_L^{(k_1 k_2)} = \frac{2}{3} (k_1 + k_2)^2 - 2k_1 (k_2 - 1) - \frac{1}{2} L(L+1)$$

respectively, where W, B, H and M are the coefficients corresponding to the Wigner, Bartlett, Heisenberg and Majorana exchange terms respectively. In our case, using a Rosenfeld[7] mixture, the numerical values are:

$$W = -0.13, \quad B = 0.46, \quad H = -0.26 \quad \text{and} \quad M = 0.93.$$

For a given nucleus, I is of the form $\alpha + \beta S(S+1)$, with α and β constants, so only nuclei with two or more different values of the total spin S compatible with the lowest possible isospin, and contained in the U_4 given partition, could yield results different from the ones we obtained previously[2] without considering an ex-

change term. The only odd-odd nuclei in the 2s-1d shell with more than four particles or four holes outside the core are the ones mentioned in the abstract. Nuclei with four particles or less have been already studied elsewhere[8].

We calculated with the aid of a computer* the corresponding matrices for these nuclei, and made the corresponding linear combinations, varying the parameter value from 0 to 1 in steps of 0.05. Diagonalizing the resulting matrices, we obtained the energy eigenvalues, which we plotted as a function of the parameter x for convenience, we used the experimental ground state as our energy origin, and we divided the $\mathcal{Q}^2$ matrix elements by 100, in order to have comparable energy spectra for the W_{so} and the $\mathcal{Q}^2$ parts of the graph.

II. COMPARISON WITH EXPERIMENT

^{24}Na, ^{24}Al (Fig. 1)

As these two nuclei are mirror nuclei, the theoretical results obtained are the same for both of them. In the case of ^{24}Na, there is no possibility to obtain the correct experimental ground state 4^+ for any value of the parameter x. The same could be said for ^{24}Al, if the ground state is 4^+, as preliminary experiments do indicate. There is no relevant change when they are compared with the results obtained using no exchange term. Using the $U_6 \supset R_6$ chain of groups for the orbital part, the 4^+ state is obtained as ground state[9].

^{26}Na (Fig. 2)

For this nucleus, preliminary experiments indicate 2^+ or 3^+ as ground state. In our analysis, a 2^+ state is obtained as the lowest state for a strong quadrupole-quadrupole interaction, while the 3^+ state does not appear as ground state.

^{28}Al, ^{28}P (Fig. 3)

For ^{28}Al, the ground state 3^+ is obtained for a rather strong spin-orbit coupling; a first excited state 2^+ is also obtained as part of a quadruplet formed by 3^+, 2^+, 2^+, 3^+ states for $x = 0.47$, which is not in contradiction with the order of the lowest known states. The appearance of doublets in our graphs is due to the fact that we are taking into account two SU_3 IR's corresponding to the same lowest energy. It is very interesting to note the appearance of doublets in the experimental spectrum also.

*We used the CDC G-20 computing machine of the Centro de Cálculo Electrónico of the UNAM, to obtain these results. We would like to express our thanks to the personnel of the Centro that helped us in this problem.

For ^{28}P, only the ground state 3^+ is tentatively known, which is not in contradiction with our theoretical results.

^{30}Al (Fig. 4)

As in the case of ^{26}Na, experiment yields 2^+ or 3^+ as a possible ground state; theoretical results determine 2^+ as the ground state and eliminate 3^+ as a possible lowest state.

^{32}Cl, ^{32}P (Fig. 5)

For ^{32}Cl the correct 2^+ ground state is obtained, which is the only one known.

For ^{32}P, the experimental ground state 1^+ does not appear as the lowest state. The results are similar to those obtained without exchange[2].

^{34}P (Fig. 6)

The only experimental data we have is the ground state 1^+, which is not obtained for any value of the parameter x, with and without an exchange term.

CONCLUSIONS

Except for the ^{28}Al, no interesting change is obtained when we include the Rosenfeld type exchange force in the model interaction.

Calculations including a short-range correlation term are being carried out for nuclei in 2s-1d shell, and we hope to obtain some relevant changes.

ACKNOWLEDGEMENTS

We are deeply grateful, to Dr. Marcos Moshinsky for his constant help during the development of this work.

REFERENCES

1. M. Moshinsky, "Group Theory and the Many Body Problem" in *The Many Particle Systems*, Ed. by Gordon & Breach, 1966.
2. M. Moshinsky, M. Berrondo and J. Pineda, "Group Theory & Nuclear Structure in the 2s-1d Shell", in Second Symposium on the Structure of Low-Medium Nuclei, Ed. by P. Goldhammer, University of Kansas, 1966.
3. E. P. Wigner, Phys. Rev., 51 (1937) 106, and also M. Moshinsky and J. G. Nagel, Phys. Letters, 5 (1963) 173.
4. J. P. Elliott, Proc. Roy. Soc., A245 (1958) 128, 562.

5. M. de Llano, Ph.D. Thesis, Cath. University of America, Washington D.C., 1965.
6. E. Chacón and M. de Llano, Rev. Mex. Fís., **12** (1965) 87.
7. L. Rosenfeld, *Nuclear Forces*, Interscience, 1948.
8. M. de Llano, P.A. Mello, E. Chacón and J. Flores, Nuc. Phys., **72** (1965) 379.
9. E. Chacón, Ph.D. Thesis, University of Mexico, 1967.

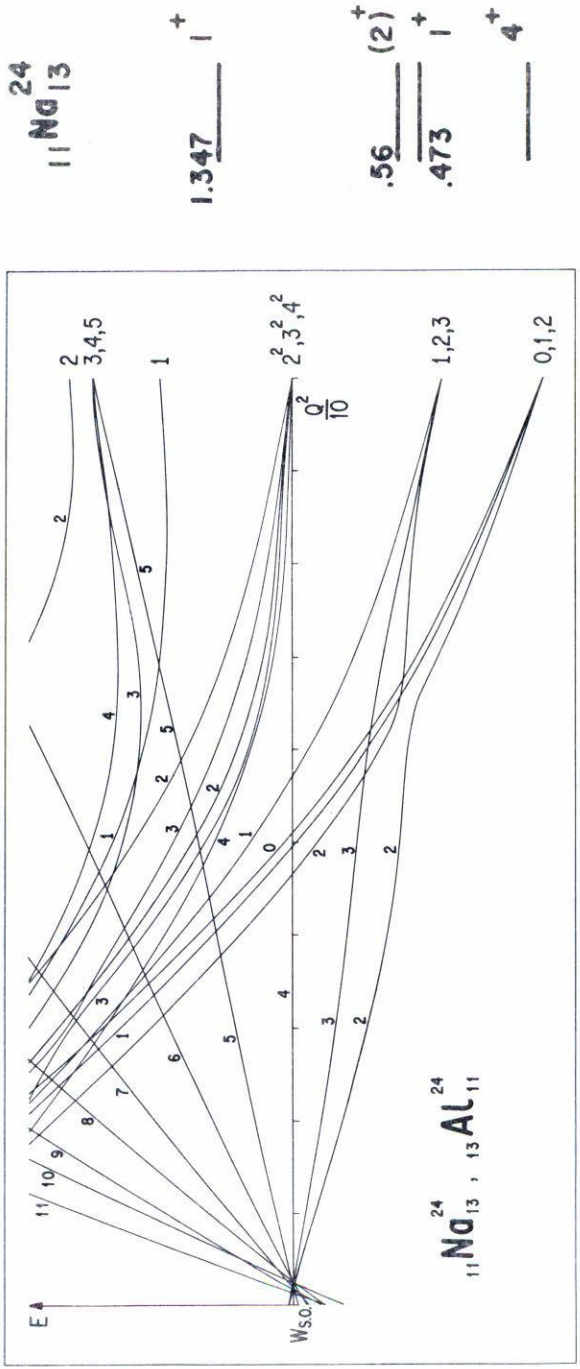

Fig. 1. Arbitrary scale has been used in both experimental data and theoretical results.

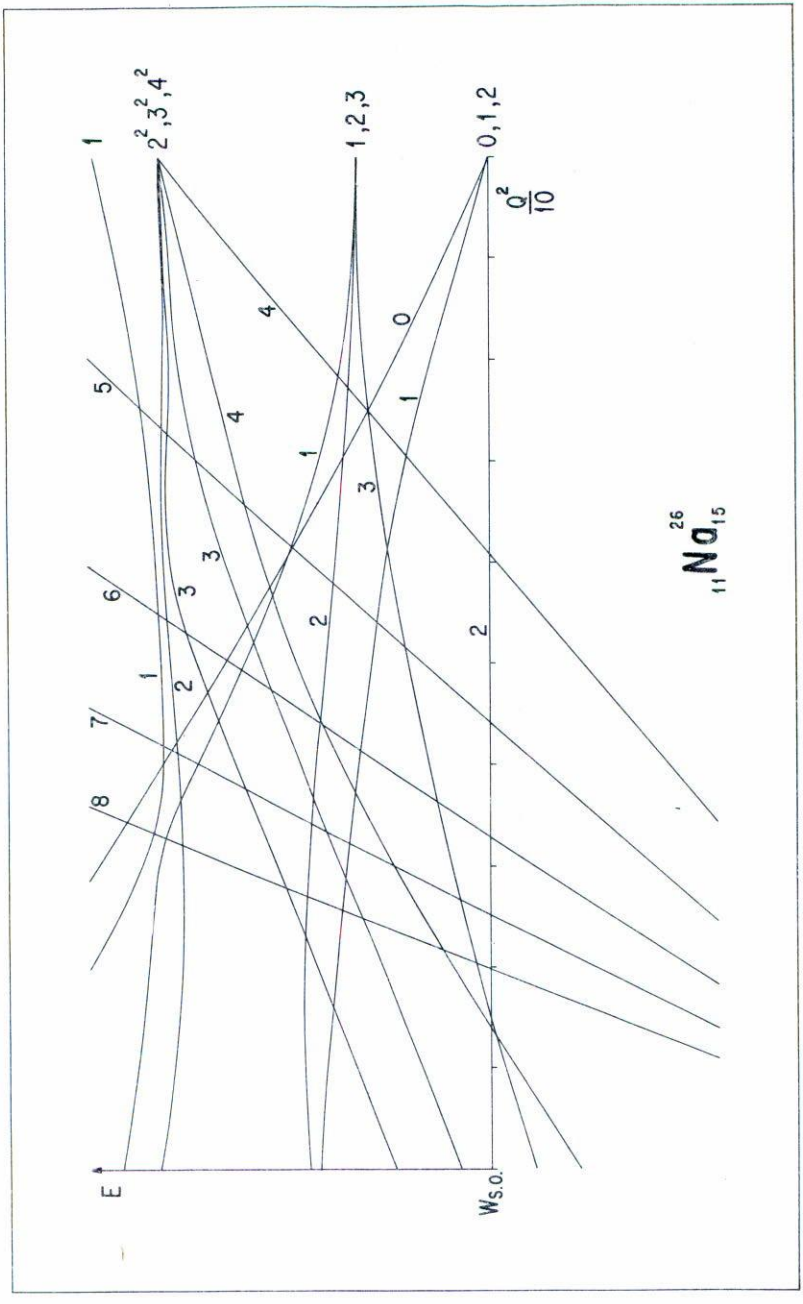

Fig. 2.

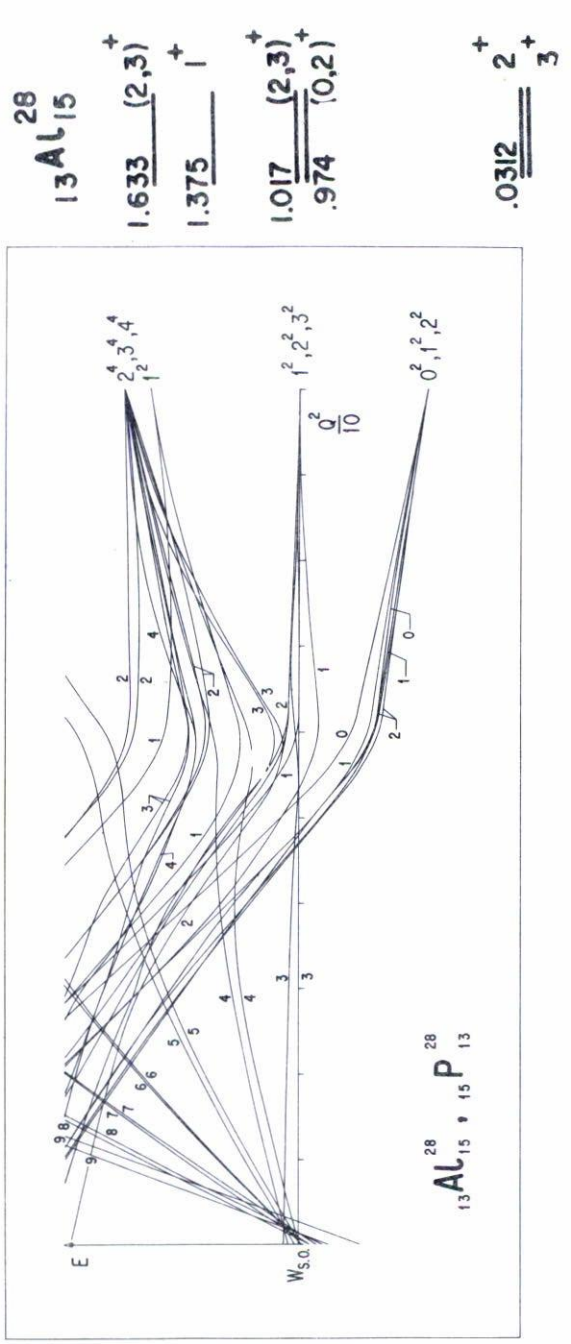

Fig. 3. Arbitrary scale has been used in both experimental data and theoretical results.

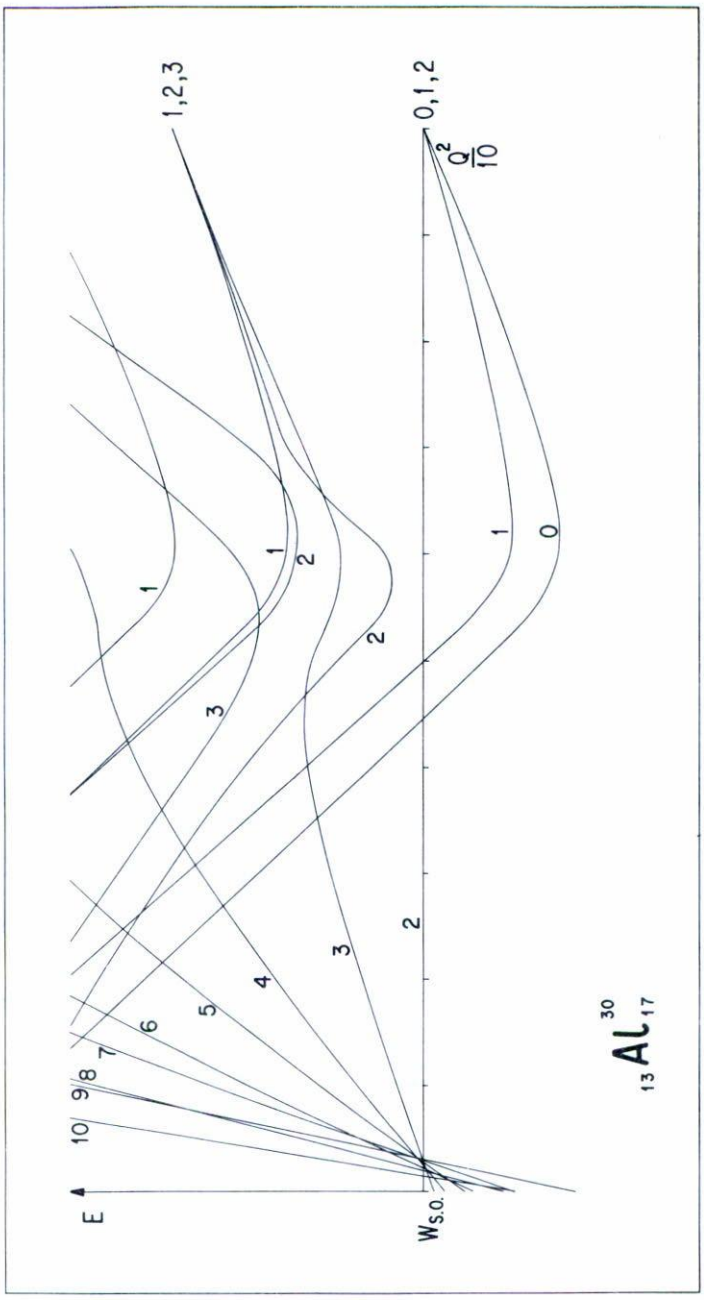

Fig. 4.

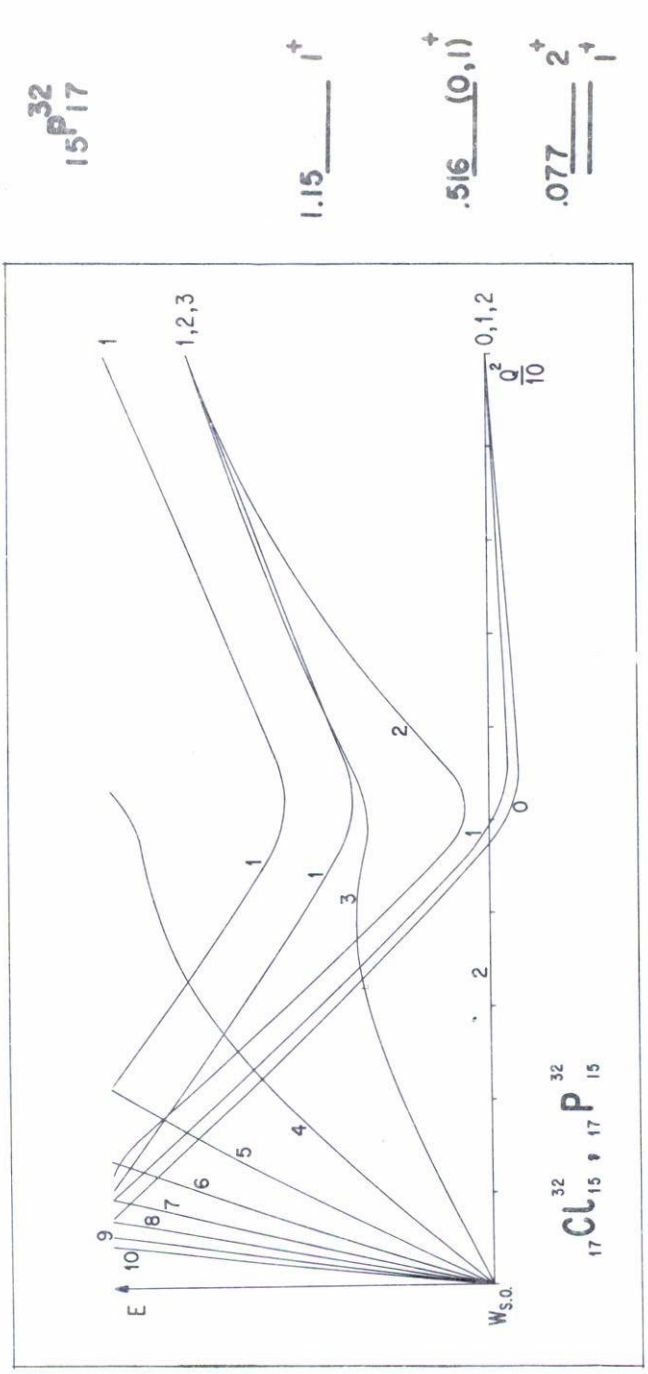

Fig. 5. Arbitrary scale has been used in both experimental data and theoretical results.

Fig. 6.