AN AMPLIFIER FOR FAST AND LONG PULSES

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ABSTRACT

A new low and high-frequency compensating circuit for R-C amplifiers which offers several advantages over other available circuits is presented. Among these advantages are very high compensating factors —of the order of several tens—; either monotonic or at least non-oscillatory response; ability to cancel the overshoot when it is produced by overcompensation; and circuit simplicity. Two forms of the circuit are considered: the "ideal" and the simplest one. An approximate analysis for each case is given, which provides a reasonable prediction of the results. Some experimental figures are also given to verify the validity of the approximations.

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RESUMEN

En este trabajo se propone un nuevo circuito de compensación tanto de altas como de bajas frecuencias para amplificadores RC, el cual posee varias ventajas respecto de otros métodos conocidos. Entre estas ventajas, se pueden señalar las siguientes: obtención de factores de compensación muy altos –frecuentemente del orden de varias decenas–; la respuesta a tan elevadas compensaciones se puede mantener monótona o, al menos, no oscilante; en el caso de que se produzcan respuestas no monótonas por sobrecompensación, es posible restablecer la monotonía perdida con el empleo de métodos simples; finalmente, el circuito es sumamente sencillo de construir y ajustar.

Se analizan en forma aproximada dos formas del circuito: la ideal y la más elemental posible. Se presentan, asímismo, algunos resultados experimentales.

1. INTRODUCTION

At present, there is an increasing need for amplifiers with very high figure of merit and with extreme requirements upon its low-frequency or flat-top response. By "figure of merit" we will understand in this paper either the harmonic figure of merit, defined as the gain-bandwidth product, or the transient figure of merit, defined as the gain-risetime ratio, unless otherwise noted. It is well known that the figure of merit is limited by the amplifying valve itself and by the circuit configuration. The conventional compensating circuits, which lead to the so-called video amplifiers, can introduce, at most, factors in the figure of merit ot the order of 2 for two-terminal circuits[1] and of the order of 4 to 5 for four-terminal circuits[2] and generally with considerable overshoot. For negligible overshoot with four-terminal circuits the best known arrangement is that of Dietzold[3] with associated factors approximately equal to 2.5. Other circuits can improve the figure of merit only at the expense of simplicity and robustness as in the case of the feedback chain[4] and the distributed amplifier[5], or are very specialized, as for example the neutralized input capacity amplifier[6]. Besides, feedback in its simpler forms is very ineffective in the improvement of the figure of merit; yet in more elaborated versions, as the previously cited feedback chain or its active-error form[7], it can serve this purpose but with much increased complexity[8]. Finally, some circuits have been proposed that are capable of yielding high figures of merit, such as that of Golding and White[9], characterized by relative complexity and some other undesirable features.

On the other hand, only one circuit is known for the improvement of the flat-top response, namely, the decoupling RC circuit in series with the plate load, but it is insufficient for the simultaneous compensation of the several factors affecting the low-frequency response: the R-C coupling, and the cathode and screen grid impedances.

The object of this paper is to present a new circuit, capable of introducing very high compensating factors simultaneously in both extremes of the pass-band, which avoids almost all of the mentioned difficulties.

A block diagramme of the basic idea is presented in fig. 1. In this figure, A represents the conventional amplifier to be compensated (by conventional amplifier we understand in this paper an R-C coupled amplifier with its cathode and screen and suppresor grids earthed for all signal components); for the compensation of the edge response we added a differentiator D and an associated conventional amplifier A_D, connected as shown in figure 1. For the compensation of the flat-top response we added a second circuit, dual in a sense to the last mentioned, which consists of an integrator I and another conventional amplifier A_I, as shown in figure 1. As the complete circuit is formed by an integrator, a differentiator and the main amplifier, the circuit is called, briefly, the IDA unit.

In this work we consider two forms of the IDA unit. In the first one, the exact IDA unit, the mathematical operations of integration and differentiation are supposed to be performed exactly or ideally. The second one performs these operations with the aid of the simplest possible circuits and can be named, therefore, the elementary IDA unit.

The exact IDA unit works as follows. Assume that the input signal is a unit step; we consider first the edge response. The (linear) conventional amplifier A, that is, the amplifier whose response is to be compensated, gives as

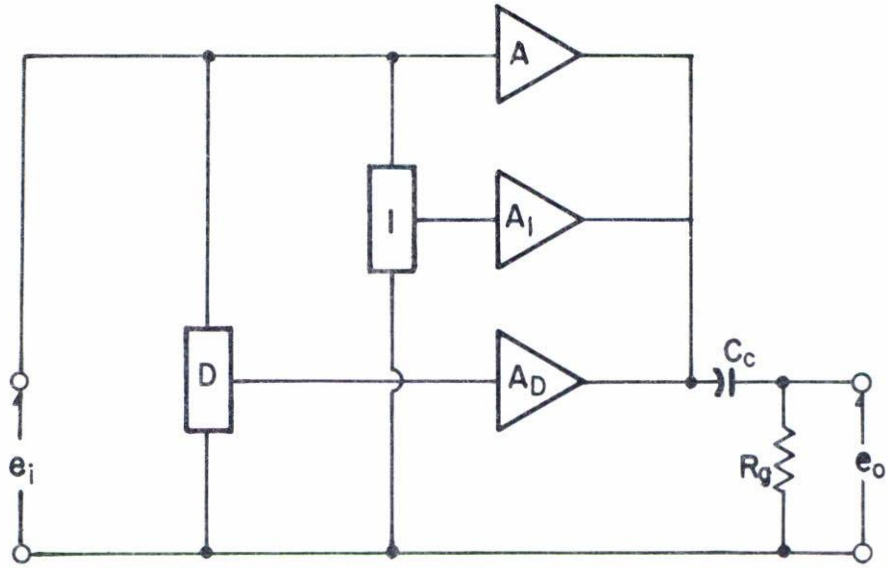

Fig. 1. Block diagramme of IDA unit. D and I are the differentiator and integrator, respectively; A, A_D and A_I are conventional amplifiers.

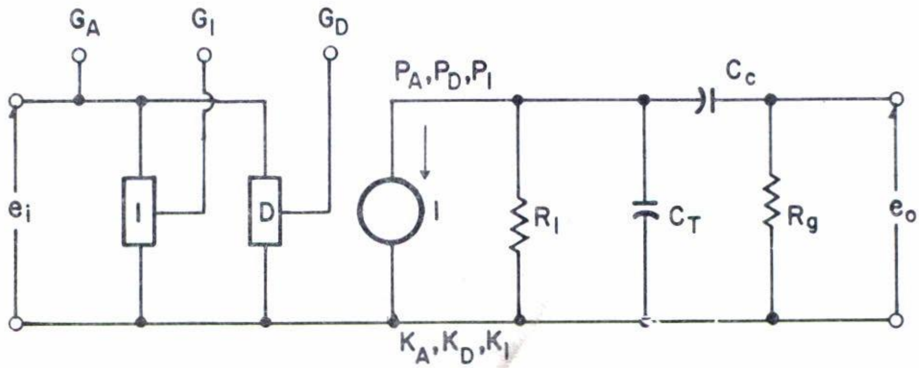

Fig. 2. A proposed equivalent plate circuit of the IDA unit. The C_{gp} capacitance was omitted.

output an exponentially rising step; simultaneously the input signal is applied to the exact differentiator D whose output, an impulse, is amplified by A_D ; as both amplifiers, A and A_D , have a common output terminal, the two previously mentioned voltages are summed; because the anode time constant is the same for both amplifiers, the amplified impulse has an exponentially decaying form of exactly the same time constant as the output of the main amplifier; therefore we can find a value for the gain of A_D such as to compensate exactly the edge response; as the impulse given by D has zero build-up time and sufficient amplitude, the final output voltage will be a perfect step. These operations are pointed out in the first part of fig. 4. That theoretically this is the case will be demonstrated later.

Second, we consider the flat-top response. The input signal step is exactly integrated by the ideal integrator I and the resulting ramp is amplified by A_I : this type of excitation produces an exponentially growing signal at the output terminals, due to the action of the *R-C* coupling; as the output terminals of amplifiers A and A_I are tied together, the last cited voltage can compensate exactly the sag of the main amplifier A if the gain of the A_I stage is adequately adjusted. If this is the case, the bass response of the conventional amplifier will be perfect, as is demonstrated in the next paragraph.

2. ANALYSIS OF THE EXACT IDA UNIT

For the analysis of the exact IDA unit, we will consider fig. 2 to represent one of its possible equivalent circuits; in that figure, R_1 is the parallel combination of the three different plate resistances and the common load resistance; all the other components have obvious meanings and, for simplicity, we have omitted the grid-to-plate and the grid-to-cathode capacitances. Then we can write for the transfer impedance of the unit[10]

$$Z_m(s) = \frac{R}{1 + R C_T s + \dfrac{1}{(R_1 + R_g) C_c s} + \dfrac{R C_T}{R_g C_c}} \approx \frac{\omega_2 R s}{(s + \omega_1)(s + \omega_2)}$$

where R stands for the parallel combination of R_1 and R_g; for this approximation we assumed that the relation $R_g >> R_1$ holds, as is generally true in practical conditions, and we used the definition of low and high cut-off frequencies

$$\omega_1 = \frac{1}{(R_1 + R_g)\, C_c} \quad ; \qquad \omega_2 = \frac{1}{R\, C_T}$$

respectively. Then the output voltage is $-IZ_m$, where I stands for the current of the equivalent generator. Now let us apply a unit step voltage signal $u(t)$ at the input terminals at $t = 0$, with all initial conditions being zero. We obtain for the Laplace transform of the output voltage

$$-\mathcal{L}e_0(t) = \frac{g_{m\,\mathrm{I}} R}{T_{\mathrm{I}}\omega_1} \cdot \frac{1}{s} + \frac{\omega_2}{B} R \left(g_{m\,\mathrm{A}} - \frac{g_{m\,\mathrm{I}}}{T_{\mathrm{I}}\,\omega_1} - g_{m\,\mathrm{D}}\, T_{\mathrm{D}}\, \omega_1\right) \frac{1}{s + \omega_1} -$$

$$- \frac{\omega_2}{B} R \left(g_{m\,\mathrm{A}} - \frac{g_{m\,\mathrm{I}}}{T_{\mathrm{I}}\,\omega_2} - g_{m\,\mathrm{D}}\, T_{\mathrm{D}}\, \omega_2\right) \frac{1}{s + \omega_2} \qquad (1)$$

B being the band-width, $B = \omega_2 - \omega_1$, the g_m's are the transconductances of the respective tubes and T_{I} and T_{D} the time constants which characterize the integrator and differentiator, respectively. As the Laplace transform of an ideal step is proportional to s^{-1}, we shall obtain an exact step for the output, of amplitude $g_{m\,\mathrm{I}} R / T_{\mathrm{I}}\,\omega_1$, if we cancel the two coefficients within the brackets in (1). These two independent conditions give us two independent relations which can be used as design conditions:

$$T_{\mathrm{I}} = \frac{g_{m\,\mathrm{I}}}{g_{m\,\mathrm{A}}} \cdot \frac{\omega_1 + \omega_2}{\omega_1\,\omega_2} \quad ; \quad T_{\mathrm{D}} = \frac{g_{m\,\mathrm{A}}}{g_{m\,\mathrm{D}}} \cdot \frac{1}{\omega_1 + \omega_2} \qquad (2a)$$

from these we obtain, taking into account that $\omega_2 >> \omega_1$ in almost every case,

$$\frac{g_{m\mathrm{A}}}{g_{m\mathrm{I}}} = \frac{1}{\omega_1 T_{\mathrm{I}}} \equiv \frac{T_1}{T_{\mathrm{I}}} \quad ; \quad \frac{g_{m\mathrm{D}}}{g_{m\mathrm{A}}} = \frac{1}{\omega_2 T_{\mathrm{D}}} \equiv \frac{T_2}{T_{\mathrm{D}}} \qquad (2b)$$

If we adjust the circuit to satisfy (2), the reference gain A_r, that is, the absolute value of the mid-band gain, turns out to be

$$A_r = \frac{g_{m\mathrm{I}} R}{T_{\mathrm{I}}\omega_1} = g_{m\mathrm{A}} R \frac{\omega_2}{\omega_1 + \omega_2} \approx g_{m\mathrm{A}} R = -A_m \qquad (3)$$

In (3), A_m represents the (mid-band) gain of the main amplifier because $R = R_L \| r_{p\mathrm{A}} \| r_{p\mathrm{D}} \| r_{p\mathrm{I}} \approx R_L$ with sufficient accuracy if we use pentodes as is customary in pulse amplifiers.

Thus we can say that the exact IDA unit compensates exactly, to within the simplifying assumptions made, the harmonic and transient responses of its conventional amplifier and without any alteration in its reference gain. We include here the harmonic response because zero rise-time is associated with infinite high cut-off frequency.

For a more precise description of the circuit, we must remove the simplifying assumptions made; of them the most important refers to the differentiating and integrating networks: only with very complex circuitry can we realize the postulated operating conditions with the accuracy required to consider valid the previous analysis. For practical applications, we must direct our attention to the simplest of all possible circuits. This is the subject of the following paragraphs.

3. THE IDA UNIT WITH AN ELEMENTARY DIFFERENTIATOR

In this section we will analyze the IDA unit when it is constructed with the elementary series R-C differentiator and here and in the next section we will maintain the supposition of exact integration, that is, of perfect amplification down to dc.

The straightforward analysis of the proposed circuit (without considering the previously omitted inter-electrode capacitances) gives for the normalized transfer function $a(s)$ of the circuit

$$a(s) = (k_a + 1)\,\frac{\omega_2\left(s + \dfrac{\omega_D}{k_a + 1}\right)}{(s + \omega_2)(s + \omega_D)} \quad ; \quad k_a = \frac{g_{mD}}{g_{mA}} \tag{4}$$

and for the output voltage when $e_i = u(t)$ with zero initial conditions

$$\frac{e_0}{E_0} = 1 - \left(1 - \frac{k_a}{k_t - 1}\right)\exp\,(-\,t/T_2) - \frac{k_a}{k_t - 1}\,\exp\,(-\,k_t\,t/T_2) \tag{5}$$

where we have introduced

$$E_0 = e_0\,(t \to \infty) \;;\quad k_t = \frac{T_2}{T_D}$$

As eq. (5) can have one maximum at most, its time derivate can have one sign inversion at most. If the response given by (5) is to be monotonic, its time derivate must maintain its sign for all positive values of the independent variable t. The introduction of this condition gives one non-trivial relation, namely,

$$k_t \geq k_a + 1 \tag{6}$$

Now let us rewrite eq. (4) in the form

$$\alpha(s) = \frac{1 + (k_a + 1)\, T_D\, s}{1 + (T_D + T_2)\, s + T_2\, T_D\, s^2}$$

From this expression, we can write for Elmore's rise time[11] T_F

$$T_F^2 = 2\pi\ T_D^2 \left[1 + k_t^2 - (k_a + 1)^2\right] \tag{7}$$

which applies only as long as (6) does. Therefore, for minimum rise-time with zero overshoot, $(k_a + 1)^2$ must have the maximum possible value consistent with (6); but from (6) this maximum is k_t^2. Then Elmore's minimum possible rise-time consistent with the condition of monotonicity is $\sqrt{2\pi}\ T_D$; if we introduce now the relation so derived

$$k_a + 1 = k_t \tag{8}$$

into the transfer function (4), this reduces to

$$\alpha(s) = \frac{\omega_D}{s + \omega_D} \tag{9}$$

As (9) has the same form as the normalized transfer function of a conventional amplifier with perfect bass response, we see directly that the new high cut-off frequency is $\omega_D = k_t\, \omega_2$ and that the new 10-90 percent rise-time is reduced by a factor of k_t^{-1}. An analogous treatment may be carried out for frequency response; the best monotonic response is characterized by an almost identical relation $k_t^2 = k_a\,(k_a + 2)$ and the factor in the harmonic figure of merit is approximately $k_a + 1$. Consequently if we adjust the circuit according to the procedure just described we obtain $k_a + 1$ as the maximum factor in the figures of merit with monotonic responses.

4. FURTHER REDUCTION OF THE RISE-TIME WITHOUT OVERSHOOT

We have seen that if we adjust the circuit in its proposed elementary form, according to eq. (8), we improve $k_a + 1$ times the figure of merit of the main amplifier; the circuit so adjusted will be named "correctly compensated" and the terms "subcompensated" will refer to the same circuit but with k_a (i.e., with g_{mA}) smaller or greater than the value given by (8), respectively.

Suppose now that k_a is so limited by whatever reason might be that it is impossible to satisfy (8) without undue sacrifice of the compensated figure of merit. We can overcome this difficulty and make it possible to get the minimum rise-time without overshoot and without complicating the differentiator by simply adding a new element: a small capacitance C_f between plate and grid of the compensating tube, as is shown in fig. 3. Qualitatively, the action of this negative-differentiated-feed-back capacitor may be described by means of fig. 4. The analysis of the new circuit has been carried out for the case in which C_f is sufficiently small not to alter the transfer impedance appreciably nor to introduce appreciable feedback to the input generator. With these assumptions we obtain, when $e_i = u(t)$ and with all initial conditions being zero, after some algebraic simplifications[12] irrelevant for our purposes, for the output voltage $e_{0f}(t)$, and for $t > 0$,

$$e_{0f}(t) = e_0(t) + \frac{(k_a - k_t + 1)^2}{k_a(k_a + 1)} e^{-s_0 t} u(t) \qquad (10)$$

where $s_0 = A_r \dfrac{k_a(k_a + 1)}{k_a - k_t + 1} \omega_2$ and $e_0(t)$ is the output voltage without C_f. In spite of the crude simplification made, this result gives us sufficient information about the action of C_f. Mathematically, we can describe the essentials of this action as the introduction of a new term in the response, of a structure analogous to that of the overshoot but with opposite sign. In this way we have introduced ample possibilities of control over the rise-time and overshoot by the adjustment of some of the circuit parameters.

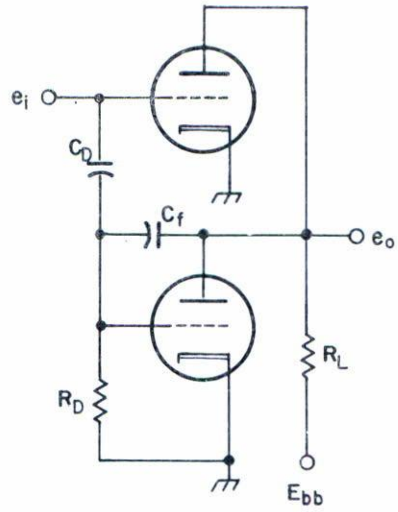

Fig. 3. The modified version of the elementary IDA unit with negative differentiated feedback through C_f.

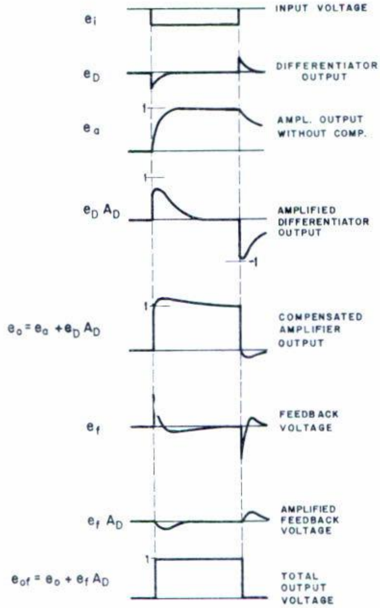

Fig. 4. A qualitative representation of the operation of the elementary IDA unit with negative feedback. The low-frequency response is assumed flat.

5. FLAT-TOP RESPONSE WITH AN ELEMENTARY R-C INTEGRATOR

The analysis of the low-frequency response of the R-C amplifier is a little more complex than the analogous calculation for the high-frequency case, due to the degenerative effects of the cathode and screen-grid circuits. In any case we can reduce the normalized low and medium-frequency transfer function of the circuit to the form

$$a(s) = \frac{s}{s + \omega_1} \cdot \frac{s + \omega_k}{s + \alpha\omega_k} \cdot \frac{s + \omega_s}{s + \beta\omega_s}$$

where α and β (both > 1) are functions of some circuit and tube parameters and of the circuit configuration; the frequencies ω_1, ω_k, ω_s are the conventional cut-off frequencies of coupling, cathode and screen-grid R-C circuits, respectively. Now, if we introduce the conventional $R_d - C_d$ decoupling circuit in the anode line, the transfer function of the circuit will take the form

$$a(s) = \frac{s}{s + \omega_1} \cdot \frac{s + \omega_k}{s + \alpha\omega_k} \cdot \frac{\omega_s}{s + \beta\omega_s} \cdot \frac{s + \omega_L + \omega_d}{s + \omega_d}$$

where ω_d is the cut-off frequency of the decoupling circuit and $\omega_L = (R_L \cdot C_d)^{-1}$. The structure of this transfer function allows us to reduce it by means of an adequate selection of the ω's to the simple, conventional form

$$a(s) = \frac{s}{s + \omega_0}$$

where ω_0 is some frequency generally equal to or a little less than ω_1. As this reduction is always possible, we conclude that this characteristic low-frequency transfer function pertains not only to the conventional amplifier but that it defines too any classical low-frequency-compensated R-C amplifier, so to speak. To the amplifier so compensated we add now a new compensation through the elementary R-C integrator and its associated amplifier as was described in the introduction.

To investigate the behaviour of the circuit let us suppose that $e_i = u(t)$ with all initial conditions being zero; then we obtain for the output voltage and for $t > 0$,

$$\frac{e_0(s)}{E_0} = \frac{s + (k_a + 1)\,\omega_I}{(s + \omega_0)\,(s + \omega_I)} \qquad (11)$$

We can, then, adjust the circuit for two types of response. We show how to do this in the next paragraphs.

a) CONVENTIONAL COMPENSATION. If in (11) we introduce

$$\frac{T_I}{T_0} = k_a + 1 \;; \quad T_0\,\omega_0 \equiv 1 \qquad (12)$$

the output voltage reduces to $\dfrac{e_0(t)}{E_0} = e^{-\omega_I t}$ for $t > 0$. This flat-top response has a fractional sag Γ, for $\omega_I T \ll 1$;(T being some value of t), given by

$$\Gamma = \omega_I T = \frac{\omega_0 T}{k_a + 1} = \frac{\Gamma_0}{k_a + 1} \quad ;$$

here Γ_0 stands for the sag of the amplifier being compensated. Let f represent the sag factor, defined as the ratio Γ_0/Γ. Then $f = k_a + 1$; we conclude that the elementary integrator introduces a sag factor of several units or tens of units at least in the first case.

b) OPTIMUM COMPENSATION. The optimum compensation is defined[13] as the adjustment of the circuit constants to produce a flat-top response with zero initial slope. The introduction of this condition in (11) gives the design relations

$$k_a = k_t = k \; ; \; k_t = \frac{T_I}{T_0} \tag{13}$$

so the equation for the output voltage reduces to the form

$$\frac{e_0(t)}{E_0} = \frac{k}{k-1} e^{-\omega_I t} - \frac{1}{k-1} e^{-k\omega_I t}$$

From this last result we obtain for the sag factor associated with this second adjustment $f \approx \frac{2k}{\Gamma_0}$, which may attain very high values, of the order of some hundreds if correctly designed. As an example, if the original sag is 10% for certain value of t, and $k = 10$, we have $f = 200$, that is, the sag is reduced to 1/20 of 1% for the same value of t.

6. SOME EXPERIMENTAL RESULTS

With the purpose of checking some of the previous results, particularly those referring to the suppression of the overshoot, we constructed a very simple version of the elementary IDA unit. The diagramme of the experimental circuit is presented in fig. 5; the variable cathode resistor and capacitors allowed us to adjust the circuit to a wide variety of working conditions. In fig. 6 we have plotted some of our results without employing C_f and in fig. 7 we have an example of the action of C_f. In fig. 7a we have plotted the response of the overcompensated amplifier ($k_a = 25$, $\rho = 43$); in this case we did not introduced C_f and the overshoot attained a very high value, of the order 20-25%; in fig. 7b we introduced C_f and readjusted C_D for the same ρ but with almost no overshoot; this process was repeated in fig. 7c and 7d until we finally reduced the rise-time 52 times but with inappreciable overshoot. In 7e we show the best response that we could get; in this figure it is easy to see that, in spite of the overcompensation, the output was

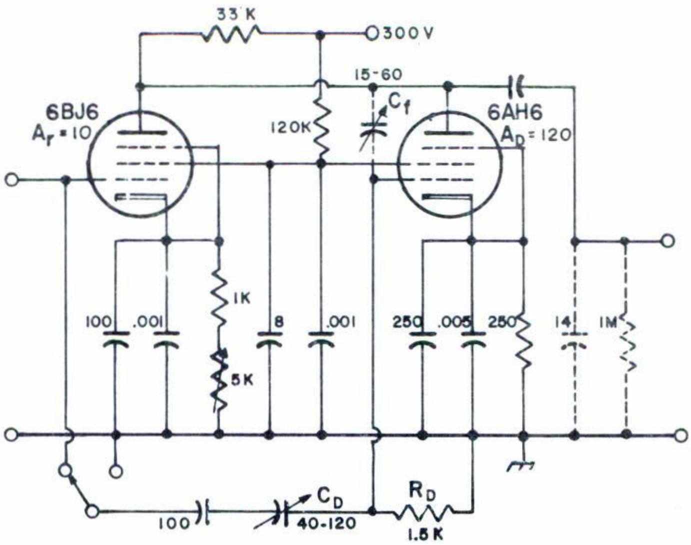

Fig. 5. Diagramme of the experimental amplifier constructed to verify the action of the differentiator $R_D - C_D$ and of the capacitor C_f for the cancellation of the overshoot.

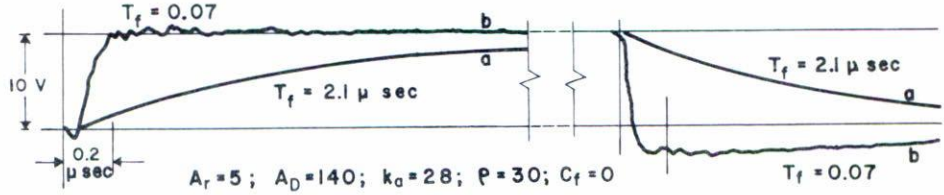

Fig. 6. Transient response of the a) non-compensated and b) correctly compensated amplifier shown in fig. 5. The experimental factor of figure of merit was $\rho = 30$ with no feedback $(C_f = 0)$.

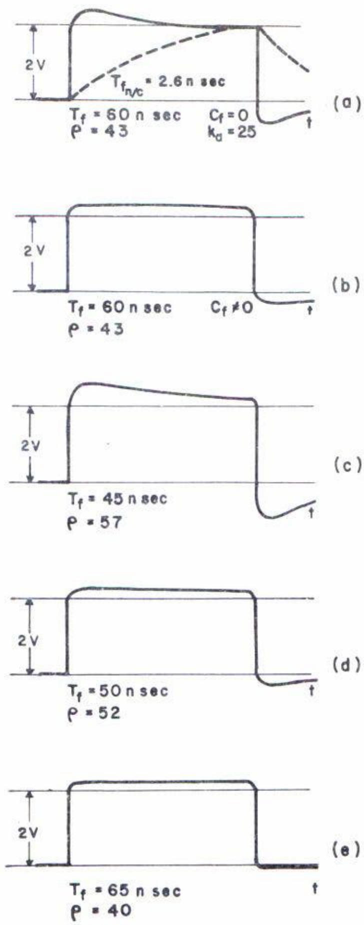

Fig. 7. Some experimental results with varying C_D and C_f and constant gains $\frac{A_D}{A} = \frac{125}{5} = 25$). In a) the amplifier is overcompensated (factor in the figure of merit = 43). The overshoot is 20–25%. No feedback has been used. The response with no compensation is shown by the dashed line; in b) C_f has been introduced and C_D readjusted for the same compensation but without overshoot; in c) and d) the same process has been repeated to increase ρ to 52 with almost no overshoot. Finally, in e) is shown the best response we were able to obtain; in this case, the rise-time was reduced 40 times without any overshoot.

very fair and with a high factor in the figure of merit ($\rho \approx 40$).

7. CONCLUSIONS

The theory and the experimental results just presented allow us to conclude that the IDA unit, even in its most elementary form, that is, constructed by means of the simplest acceptable circuits, may introduce very high factors for the (harmonic and transient) figure of merit and of sag, giving in every case either monotonic or at least non-oscillatory responses. Even more, the characteristic overshoot of the overcompensated circuit may be reduced or entirely removed with only the addition of a small capacitance in the high-frequency compensating circuit. The circuit is very simple and its design does not offer much difficulty.

The introduction of the compensating circuits affects the original edge response only through the added capacitance at the output terminals and introduces no change in the reference gain. By its essential characteristics, the IDA unit is more suitable for compensating low-gain fast-response amplifiers.

Of the different high-frequency compensating circuits published in the technical literature available to us, apparently that of Golding and White previously cited is that which most resembles ours, but the two circuits are not neccessarily interchangeable in every case; when this interchange is permissible, the IDA unit has the advantage over Golding and White's of simplicity and, if the case applies, the additional possibility of low-frequency compensation. It is estimated that with more elaborate differentiators the edge response can be improved by factors exceeding one hundred. In any instance, the parasitic inter-electrode capacitances neglected in the analysis carried out must be kept to their minimum realizable values.

Finally, it is interesting to remark that high and low-frequency compensations are entirely independent of each other and that one of them may be used without regard being had to the other.

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