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GENERATORS OF MARKOFFIAN SIGNALS WITH A FINITE NUMBER OF DISCRETE SYMBOLS

(ANALYSIS OF A DESIGN METHOD)

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ABSTRACT

A design method for Markov generators is described using as design parameters the elements of the Markov matrix defining the process which characterizes the output signal. Constructing the generator as a probabilistic automaton, the formal analogy between the logical equation system of the automaton and the algebraic equation system of the corresponding Markov process is used to express algebraic conditions for probabilities of the signals in terms of logical conditions for the signals themselves. It is shown, that by establishing an adequate correspondence between the elements of the Markov matrix and the elements of the excitation matrix of the automaton, the equation system and the circuit diagram of a generator corresponding to an arbitrary chosen Markov matrix can be deduced.

RESUMEN

Se describe un método de diseño de generadores Markoffianos usando como parámetros de diseño los elementos de la matriz de Markov que caracteriza la señal de salida del generador. El generador se construye en forma de un autómata probabilístico, aprovechando la analogía formal existente entre el sistema de equaciones lógicas del autómata y el sistema de equaciones algebraicas del proceso de Markov correspondiente, para expresar las condiciones probabilísticas de las señales, en forma de condiciones lógicas válidas para las mismas.

Se demuestra que, estipulando asignaciones adecuadas entre los elementos de la matriz de Markov y los elementos de la matriz de excitación del autómata, el sistema de ecuaciones y el diagrama del circuito del generador pueden deducirse a partir de una matriz de Markov arbitraria.

In the last years the usefulness of signal generators for stochastic signals with well defined statistical characteristics has been recognised in various research centers in different countries. Apart from the already well known method of computer generation of stochastic signals, the construction of special laboratory devices has been undertaken, first for well defined Poissonian signals with adjustable probabilities, afterwards for different classes of continuous and discrete type Markoffian signals[1,2,3,4] where the methods of design differ, in general, as widely as the purposes, for which these instruments have been constructed.

The design method developed in the Laboratorio de Cibernética de la Comisión Nacional de Energía Nuclear is based on the general mathematical theory of finite Markoffian processes[5,6], and the theory of finite automata of A. Medina[7], as well as on the generalisation of this latter for probabilistic automata[8]. Having been applied, some years ago, to a special case of a 3 symbol generator[2], its application to the general case of Markov signals of n discrete symbols characterized by an arbitrary Markov matrix, will be discussed in this paper. The description of a device, realized in our laboratory, will illustrate a possible form of use of this design method.

The generator is considered as an automaton characterized by a system of logical equations which determine its transitions between its internal states. Making use of the formal analogy which exists between this equation system and the system of algebraic equations which determines the transitions of a stationary Markov process of the same finite number n of discrete states, it is possible to deduce the design parameters of the automaton from the characteristic parameters of the Markov process in such a way, that the output signal of the automaton will be a statistical replica of the Markov process.

The correspondence is complete, that is: Starting from a given Markov matrix, one can obtain a system of logical equations of an automaton of the same number of states, whose output signal follows the process determined by the Markov matrix. And, starting from a system of logical equations as characterizing a given automaton, one can obtain the Markov matrix corresponding to its output signal. The circuit diagram of the generator may, finally, be deduced from its system of logical equations by well known methods of logical design.

It will be shown that the correspondence between the design parameters of the automaton and the elements of the Markov matrix can be established in a quite general manner, introducing only the general conditions holding for every finite automaton, as well as the corresponding general conditions holding for every stationary Markov process with the same number of discrete states.

THE STRUCTURE OF THE AUTOMATON

The system of canonical equations of the automaton has the form

$$y(t+1) = F\,[\,y(t),\, x(t)\,] \qquad (1)$$

$$z(t) = G\,[\,y(t)\,] \qquad (2)$$

where y are the state variables at time t and $t+1$ respectively, x the stimuli and z the output variables. Equation system (1) represents the state variables at every quantized time interval (or "moment") t, $t+1$, $t+2$, ... as functions of the state variables of the immediately preceding moment and the stimuli occurring at it, whereas in (2) the output variables are expressed as functions of the state variables occurring at the same moment.

Equation system (1) can always be written as

$$E^1 = X \circ E \tag{1a}$$

where E^1 and E are logical state vectors at time $t+1$ and t resp., whose components are the internal states of the automaton at the corresponding times, while X, the so-called excitation matrix, is a $n \times n$ logical matrix formed by the stimuli governing the transitions between the n states. The operation "o" of symbolic matrix multiplication is defined by the following rule: Proceed as with ordinary matrix multiplication, but substitute every product by the corresponding logical conjunction, and every sum by the corresponding logical alternation.[8]

As an illustration of (1a) suppose an automaton of n binary states $(A_1 A_2 \ldots A_n)$, whose transitions $A_i \rightarrow A_j$ are produced by a set of binary stimuli x_{ji} $(i, j = 1, 2 \ldots n)$. (1a) can then be written formally as

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}_{t+1} = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & & & \vdots \\ x_{n1} & x_{n2} & \ldots & x_{nn} \end{bmatrix} \circ \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}_{t} \tag{3}$$

where the state vector at the left side corresponds to E^1, that of the right side to E of equation (1a). if we designate the components of E^1 with $A_1^1, A_2^1 \ldots A_n^1$, the components of E with $A_1, A_2 \ldots A_n$, equations (3) are written out by application of the rule of symbolic matrix multiplication "o" in the form:

$$A_j^1 = x_{j1} \cdot A_1 \quad \vee \quad x_{j2} \cdot A_2 \quad \vee = \ldots \vee \quad x_{jn} \cdot A_n \tag{4}$$

$$(j = 1, 2 \ldots n)$$

meaning that the automaton will be found at time $t+1$ in state A_j whenever it was at time t, one moment before: In state A_1 *and* being stimulated by x_{j1}, or in state A_2 and stimulated by x_{j2}, or ...., in state A_n, being stimulated by x_{jn} (*Note 1*)

This statement is illustrated by the flow-diagram of Fig. 1.

The general invariance conditions[7], holding at every time interval for every finite automaton, take the form:

$$A_1 \vee A_2 \vee \ldots \vee A_n = \curlyvee$$

$$A_i \cdot A_k = \curlywedge \text{ for } i \neq k \; (i, k = 1, 2 \ldots n) \tag{5}$$

which means that in every quantized time interval the automaton has to be found in *some* state, and can not be *in two different states* at the same time.

If we apply conditions (5) to the states at $t+1$, and substitute from (4) we get analogous conditions for the columns of the excitation matrix X, that is, for the transitions which are possible, starting from the same state A_i at moment t, $(i = 1, 2 \ldots n)$:

$$x_{1i} \vee x_{2i} \vee \ldots \vee x_{ni} = \curlyvee$$

$$x_{ji} \cdot x_{ki} = \curlywedge \text{ for } j \neq k \; (j, k = 1, 2 \ldots n) \tag{6}$$

expressing the above conditions in a dynamic form: Starting from A_i, the automaton has to go in the next moment to some of its states and cannot go to two different states at the same time.

As to the output equations (2), we introduce the following convention: Suppose an output signal composed of n simbols a_1, a_2 ... a_n. Whenever the automaton is in state A_i, the symbol a_i appears in its output $(i = 1, 2 \ldots n)$. In

Note 1: Logical conjunction is designated here by ".", logical alternation (inclusive "or") by "$\vee$", tautology by "$\curlyvee$", antitautology by "$\curlywedge$", whereas the truth values of binary variables are written simply: 0, 1.

this manner the output sequence follows the state sequence and need not to be treated separately.

GENERALISATION FOR PROBABILISTIC AUTOMATA

Equations (1a) and (3), valid for deterministic automata can now be written formally in an identical way as the system of ***algebraic*** equations, which determine the transitions of a stationary Markov process of a finite number n of discrete states, or the symbol sequences of a stationary Markov signal with a finite number n of discrete symbols $a_1, a_2 \dots a_n$. The difference between the probabilistic and the logic equation system – apart from the different algebra involved in each case – lies in the fact, that the elements of the X matrix are in the probabilistic case the conditional probabilities of the transitions $A_i \to A_j$, caused by the stimuli x_{ji}.

The Markov signal of n symbols $a_1, a_2 \dots a_n$ is governed by equation system

$$E^1 = X E \tag{1b}$$

where the operation to be performed between X and E is, this time, ordinary matrix multiplication, and the elements of the X matrix are the conditional probabilities $p(a_j/a_i)$ of occurrence of symbol a_j, when symbol a_i has occurred at the moment immediately before. The components of the state vectors are here the absolute probabilities $p(a_i)$ of the individual symbols, independent of time for the stationary case.

If we assign now to every stimulus x_{ji} a certain probability of assuming the value 1, $p(x_{ji} = 1)$, as well as a certain probability of assuming the value 0, $p(x_{ji} = 0) = 1 - p(x_{ji} = 1)$, we can establish a 1 : 1 correspondence between the stimuli x_{ji}, which induce in the output of the automaton the sequence $a_i \to a_j$, and the elements of the Markov matrix $p(a_j/a_i)$, setting

$$\begin{aligned} p(x_{ji} = 1) &= p(a_j/a_i) \\ p(x_{ji} = 0) &= 1 - p(a_j/a_i) \end{aligned} \tag{7}$$

Writing, for brevity

$$p(x_{ji} = 1) = p(x_{ji})$$
$$p(x_{ji} = 0) = p(x'_{ji}) \qquad (8)$$

where x'_{ji} is the negation of x_{ji}, (7) becomes

$$p(x_{ji}) = p(a_j/a_i)$$
$$p(x'_{ji}) = 1 - p(a_j/a_i) \qquad (7a)$$

Assignment (7a) introduces for the stimuli x_{ji} a series of new conditions to be satisfied, in addition to conditions (6) deduced from general automata theory. It can be shown that this restriction leads to no contradiction. Moreover, the x_{ji} can be selected in such a way, that conditions deduced from the theory of Markov processes are automatically fulfilled, if the x_{ji} satisfy conditions (6), deduced from general automata theory.

CONDITIONS FOR THE INPUT SIGNALS

By definition of the Markov process we have

$$p(a_j/a_1\, a_2 \ldots a_i) = p(a_j/a_i) \qquad (9)$$

for any sequence of symbols a_1, a_2 ... a_i of any length $i > 1$. That is, the conditional probability of the occurrence of symbol a_j depends only of the symbol a_i, which had occurred immediately before, and not of the past history of the latter. The so-defined Markov process is called also "first order Markov process".

Moreover, the elements of the Markov matrix have to fulfill the so-called marginal conditions:

$$\sum_j p(a_j/a_i) = 1 \quad (i = 1,2 \ldots . n) \tag{10}$$

To satisfy condition (9) the n^2 stimuli x_{ji} $(j, i = 1,2 \ldots n)$ have been organized so that every stimulus acts only on one single state of the automaton and has no influence on transitions starting from any other state. The n^2 stimuli of the X matrix are partitioned in n sets of n stimuli each, every set $(x_{1i}, x_{2i} \ldots x_{ni})$ being assigned to one state A_i $(i = 1,2 \ldots n)$, and are effective only when the automaton is in state A_i. Figs. 2 and 3 illustrate this distribution of the x_{ji} for $n = 3$ and $n = 4$. This way, the transition $A_i \rightarrow A_j$ depends of A_i, but not on the manner, how the automaton entered in state A_i.

As to the marginal conditions (10) they are fulfilled automatically in this case, due to conditions (6) holding for the stimuli assigned to the same state A_i. Since for every state A_i one and only one of the stimuli x_{ji} $(j = 1,2 \ldots n)$ is acting at any time, we have

$$p(x_{1i} \vee x_{2i} \vee \ldots \vee x_{ni}) = \sum_j p(x_{ji}) \tag{11}$$

so that

$$\sum_j p(x_{ji}) = \sum_j p(a_j/a_i) = 1 \tag{10a}$$

Besides, to be able to equalize the probabilities $p(x_{ji})$ to the corresponding elements of any given Markov Matrix, the $p(x_{ji})$ have to be constant and adjustable in the interval $0 \leqslant p(x_{ji}) \leqslant 1$. The device, constructed to this purpose, will be described in a later chapter of this paper. For the moment, let us assume, that this has been achieved.

Having thus transformed every condition to be satisfied by probabilities of signals into the equivalent logical condition to be fulfilled by the signals themselves, the design problem is reduced to that of a special automaton and can be solved by standard methods of logical design.

The so introduced probabilistic X matrix can be considered, in a way, as a generalization of the logical excitation matrix in the sense, that the latter has elements capable of assuming only the values 0 and 1, whereas the former can, within the marginal restrictions, assume any value between 0 and 1.

CONSTRUCTION OF THE X MATRIX

Every set of n stimuli x_{ji} $(j = 1,2 \ldots n)$, assigned to the same state A_i, will be formed by the terms of the Alternative Canonic Form (ACF), the so-called "minterms" of a certain number r of *binary, independent, poissonian* signals $m_1, m_2 \ldots m_r$. The minterms t_q are logical conjunctions of the m_k or their negation

$$t_q = \overline{m}_1 \cdot \overline{m}_2 \ldots . \overline{m}_r \tag{12}$$

where $\overline{m}_k$ stands for m_k or m_k'.

The 2^r minterms of the $m_k (k = 1,2 \ldots r)$ are mutually exclusive and exhaustive, so that

$$t_q \cdot t_p = 0 \quad \text{if} \quad q \neq p$$

and

$$t_1 \vee t_2 \vee \ldots \vee t_{2^r} = 1 \tag{13}$$

It can be shown that for $2^r \geq n$, the n stimuli x_{ji} can always be represented by combinations of the minterms so that conditions (6) are satisfied. To this purpose the t_q, or logical alternations of some of them, have to be distributed over the set of the x_{ji} in such a way, that these latter are exclusive two by two and contain, in conjoint, *all* the minterms of the m_k. This problem can, in general, be solved by various possible distributions of the t_q, all of them fulfilling conditions (6) for the x_{ji}, but not all of them being adequate to reproduce with the probabilities of the x_{ji} the elements of an arbitrary given Markov matrix.

To this end, the number of independent signals m_k, cannot be smaller than the number of independent parameters contained in each column of a Markov matrix of order n. With n elements in every column, satisfying 1 equation derived from the marginal restrictions, the number of independent parameters is $n-1$, for every column.

$$r = n-1 \tag{14}$$

will thus be the smallest number of independent signals m_k necessary, to reproduce the stimuli of the ith column of an X matrix, which corresponds to an arbitrary chosen Markov matrix. For the automaton this means a minimum of $n(n-1)$ input signals m_{ki} $(k = 1,2 \ldots . n-1;\ i = 1,2 \ldots . n)$. It can be seen, that it is always easy to find one or more distributions of the minterms t_q over the column elements x_{ji} which satisfy the logical conditions (6). But not all these possible distributions permit to reproduce the x_{ji} with the probability values determined by an arbitrary Markov matrix.

The signals m_{ki} being poissonian signals with probability $p(m_{ki} = 1) = p(m_{ki})$, their 0 and 1 values are uniformly distributed and the $p(m_{ki})$ can be considered as constants. Because of their independence, no extra correlation is introduced by them in the generator signal apart from that determined by the Markov matrix. Their values have to be deduced from a system of n algebraic equations for each set of $n-1$ signals m_{ki} assigned to the elements x_{ji} $(j = 1,2 \ldots n)$ of column i of the X matrix. These equations are non linear in the general case and will not always have a solution for *real, rational* and *positive* values of the $p(m_{ki})$.

Remembering that the m_{ki} are independent by assumption, it follows that the $p(x_{ji})$ will have the form of products of factors $p(\overline{m}_{ki})$, where $p(\overline{m}_{ki}) = p(\overline{m}_{ki})$ for $\overline{m}_{ki} \neq m_{ki}$, and $p(\overline{m}_{ki}) \neq 1 - p(m_{ki})$ for $\overline{m}_{ki} \neq m'_{ki}$. The number of factors of every product depends of the particular combination of minterms chosen for every x_{ji}. The system of n algebraic equations for the $n-1$ unknown $p(m_{ki})$ will have the form

$$p(x_{ji}) = \prod_k p(\overline{m}_{ki}) \quad (j = 1,2 \ldots n)$$

To solve it for acceptable values of the $p(m_{ki})$ the following combination rule has been adopted for the x_{ji}, making use of the circumstance, that the x_{ji} of every column of the X matrix are exclusive and exhaustive, so that at every moment one and only one of them can assume the value 1:

$$
\begin{aligned}
x_{1i} &= m_{1i} \\
x_{2i} &= m'_{1i} \cdot m_{2i} \\
x_{3i} &= m'_{1i} \cdot m'_{2i} \cdot m_{3i} \\
&\;\;\vdots \\
x_{ni} &= m'_{1i} \cdot m'_{2i} \cdot m'_{3i} \cdots\cdots m'_{n-1,i}
\end{aligned}
\qquad (15) \quad \text{(Note 2)}
$$

For the probabilities this makes:

$$
\begin{aligned}
p(x_{1i}) &= p(m_{1i}) \\
p(x_{2i}) &= [1 - p(m_{1i})]\, p(m_{2i}) \\
p(x_{3i}) &= [1 - p(m_{1i})][1 - p(m_{2i})]\, p(m_{3i}) \\
p(x_{n-1,i}) &= [1 - p(m_{1i})][1 - p(m_{2i})] \cdots\cdots [1 - p(m_{n-2,i})]\, p(m_{n-1,i})
\end{aligned}
\qquad (16)
$$

the equation for $p(x_{ni})$ being satisfied due to the marginal condition $\sum_j p(x_{ji}) = 1$.

From (16) the $p(m_{ki})$ can easily be calculated from the known values of $p(x_{ji})$. Progressing gradually from $j = 1$ to $j = n - 1$, only some simple algebraic operations have to be performed, as every following equation contains only one unknown variable more than the preceding one.

Note 2: The Veitch and Karnaugh diagram of Fig. 4 illustrates the combination rule (15) and show that the resulting x_{ji} satisfy conditions (6). Using ordinary logic circuitry, the x_{ji} can be mecanized from (15).

Making use of the marginal condition the $p(m_{ki})$ take the form:

$$p(m_{ki}) = \frac{p(x_{ki})}{p(x_{ki}) + p(x_{k+1,i}) + \dots + p(x_{ni})} \tag{17}$$

It can easily be seen that each $p(m_{ki})$ so obtained, is a real rational and positive number between 0 and 1.

Simplified Models for Special Markov matrices.

In special cases the number of independent signals m_{ki} can be reduced. Introducing, f.i., the extra condition, that the automaton can make only two allowed transitions from every state, these can be realized with a single signal and its negation, x_i and x_i' $(i = 1, 2 \dots n)$, reducing the total number of independent input signals from $n(n-1)$ to n. Conditions (6) are reduced in this case to

$$x_i \vee x_i' = 1 \qquad x_i \cdot x_i' = 0 \tag{6a}$$

which is always true.

This special case has been discussed in an earlier work[2].

Considering the rapid growth of the number of elements with the number of symbols n, the practical limitations of the method are quite serious, especially as long as only traditional electronic circuit elements are used for construction. But even the use of integrated circuits would not alliviate the difficulties of the high number of input signals necessary in the general case, so that realizations for simplified special Markov matrices are to be taken into account whenever the number of symbol is higher than 3 or 4. These cases have been solved in our laboratory.

Input Signals with Constant and Adjustable Probabilities.

For the independent input signals m_{ki} we used poissonian pulse trains, each one derived from the output of a Geigercounter system mounted around a radioactive sample adequately shielded, in order to obtain only a single kind of radiation.

Each one of these signals was fed into a device designed to achieve probability adjustment in steps of 1/10 between 0 and 1.

The device consists mainly of a 10 stage ringcounter and a bistable multivibrator. The ringcounter stages are switched "on" one by one, as the pulses of the Geigercounters reach the input. The sequence of the "on" stages follows thus a poissonian time series. The outputs of the 10 ringcounterstages are connected to the multivibrator inputs in such a way, that the multivibrator triggers to its "1" state, whenever the ringcounter has reached the first of its 10 stages, while the multivibrator triggers to its "0" state with the one of the remaining 9 stages of the ringcounter, which has been selected by a rotary switch placed in the instrument panel, and moved by hand. In this manner the relative time interval, during which the signal assumes the value 1, is adjusted on the instrument panel, where it can be read off from the switch scale. The readings of all the switch scales permit to evaluate the elements of the Markov matrix of the generated signal or - for the simplified case - coincide with them. Block diagram and panel view of the device are shown in Figs. 5 and 6.

This simple solution of a rather complicated problem has been proposed by Ing. Fernando Camarena of our laboratory.

Design process for $n = 3$

As an example the design method will be illustrated for the case $n = 3$. The flow diagram of the automaton is shown in Fig. 7. The automaton has 3 states A_1, A_2, A_3 and an output signal of 3 symbols a_1, a_2, a_3, corresponding to the states designed with the same letter.

To each state A_i 3 stimuli are assigned: x_{1i}, x_{2i}, x_{3i} $(i = 1,2,3)$

The 3 stimuli assigned to the same state A_i are built as combinations of

$$n - 1 = 2 \tag{14'}$$

independent binary poissonian signals: m_{1i}, m_{2i} $(i = 1,2,3)$, so that we need 6 input signals for the automaton.

Applying combination rule (15) we get:

$$x_{1i} = m_{1i}$$

$$x_{2i} = m'_{1i} \cdot m_{2i} \tag{15'}$$

$$x_{3i} = m'_{1i} \cdot m'_{2i} \qquad (15')$$

The probabilities of the stimuli $p(x_{ji})$, equal to the corresponding elements of a given Markov matrix are substituted in equations (17) to obtain the probabilities $p(m_{ki})$ for the adjustments of the 6 input signals :

$$p(m_{1i}) = p(x_{1i}) \qquad p(m_{2i}) = \frac{p(x_{2i})}{p(x_{2i}) + p(x_{3i})} \tag{17'}$$

$$(i = 1,2,3)$$

The remaining steps correspond to ordinary logical design methods:

From (3) and (4) we get in our case:

$$A_1^{(1)} = x_{11} \cdot A_1 \lor x_{12} \cdot A_2 \lor x_{13} \cdot A_3$$

$$A_2^{(1)} = x_{21} \cdot A_1 \lor x_{22} \cdot A_2 \lor x_{23} \cdot A_3$$

$$A_3^{(1)} = x_{31} \cdot A_1 \lor x_{32} \cdot A_2 \lor x_{33} \cdot A_3 \tag{4'}$$

where A_i represents state i at the moment t, $A_i^{(1)}$ the same state at $t+1$. The 3 states are represented now, in the usual way, by 2 binary state variables, called q_1 and q_2 for the states at moment t, Q_1 and Q_2 for the states at $t+1$. Combinations of the state variables are assignated to the 3 automata states as follows:

$$A_1 = q_1' \cdot q_2 \qquad A_1^{(1)} = Q_1' \cdot Q_2$$

$$A_2 = q_1 \cdot q_2 \qquad A_2^{(1)} = Q_1 \cdot Q_2 \qquad (18)$$

$$A_3 = q_1 \cdot q_2' \qquad A_3^{(1)} = Q_1 \cdot Q_2'$$

while the combination $q_1' \cdot q_2'$ and $Q_1' \cdot Q_2'$ resp. does not occur so that

$$\vartheta = q_1' \cdot q_2' \qquad \vartheta = Q_1' \cdot Q_2'$$

Substituting (18) in equations (4') and resolving the resulting logical equations for Q_1 and Q_2, we get

$$Q_1 = (x_{21} \vee x_{31}) \cdot q_1' \cdot q_2 \vee (x_{22} \vee x_{32}) \cdot q_1 \cdot q_2 \vee (x_{23} \vee x_{33}) \cdot q_2' \cdot q_1$$

$$Q_2 = (x_{11} \vee x_{21}) \cdot q_1' \cdot q_2 \vee (x_{12} \vee x_{22}) \cdot q_1 \cdot q_2 \vee (x_{13} \vee x_{23}) \cdot q_2' \cdot q_1$$

If the state variables are represented now (see Fig. 8) by $j-k$ memories, we get:

$$Q_1 : j_1 = (x_{13} \vee x_{23}) \cdot t$$

$$k_1 = (q_2 \cdot x_{32} \vee q_2' \cdot x_{31}) \cdot t$$

$$Q_2 : j_2 = (x_{21} \vee x_{31}) \cdot t$$

$$k_2 = (q_1 \cdot x_{12} \vee q_1' \cdot x_{13}) \cdot t \qquad (19)$$

where t are clockpulses fixing the duration of the symbols. Substituting from (15) we get the equations for the $j-k$ memories in terms of the input variables m_{ki}, the state variables q and the clockpulses t, from where the logical circuit diagram (Fig. 9) may be drawn:

$$Q_1 : j_1 = (m_{13} \vee m_{23}) \cdot t \qquad k_1 = (q_2 \cdot m_{12}' \cdot m_{22}' \vee q_2' \cdot m_{11}' \cdot m_{21}') \cdot t$$

$$Q_2 : j_2 = m_{11}' \cdot t \qquad k_2 = (q_1 \cdot m_{12} \vee q_1' \cdot m_{13}) \cdot t$$

$$(19a)$$

For the *correlation function* practically the same result has been obtained as in the simplified case analized in a former work[2]. Its form depends exclusively of the Eigenvalues of the Markov matrix which can be controlled with the adjustable probabilities of the input signals, corresponding to Fig. 10 if the Eigenvalues are real, and to Fig. 11 if two of them are imaginary. In case of two complex Eigenvalues, the more or less oscillatory character of the function depends of the relation between the real and imaginary part of the complex roots. Correlation is appreciable up to sequences of 4 symbols.

BIBLIOGRAPHY

1. Mealy G.H., A Method for Synthesizing Sequential Circuits. Bell Syst. Techn. J. **34**, (1955)
2. Gertrudis Kurz de Delara y Antonio Elguézabal Buchanan, Diseño y Construcción de un Generador Markoffiano. Rev. Mex. Fís. XI., 4, 1962.

3. Swoboda J., Elektrische Erzeugung von Zufallsprozessen mit vorgegebenen statistischen Eigenschaften. A.E.Ue., 16, 1962.
4. H. Donko, A. Grassl, W. Wehrmann, H. Weinrichter, "Experimentelle Stochastische Prozesse." Forschungsberichte des Instituts f. Niederfrequenztechnik (Vst. Ing. Dr. G. Kraus), Technische Hochschule Wien, 1965-1966.
5. A. Blanc-Lapierre et R. Fortét, Théorie des Fonctions Aléatoires. Masson, Paris, 1953.
6. A.N. Kolmogorov, Foundations of the Theory of Probability. Chelsea, N.Y., 1956.
7. A. Medina, Logical Analysis of Organization in Finite Automata. Rev. Mex. Fís., XV., 1, 1966.
8. A. Medina, Time Dependent Logic. Laboratory Report 1962, Comisión Nacional de Energía Nuclear. México.

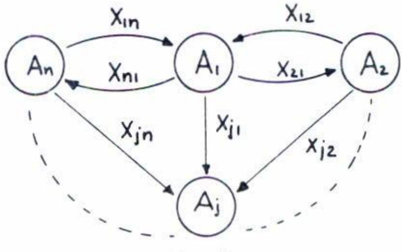

FIG. 1

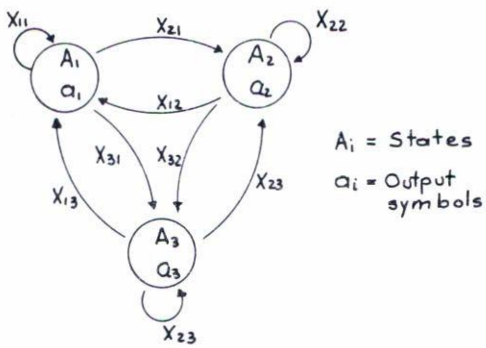

FIG. 2

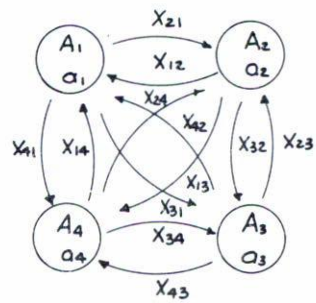

FIG. 3

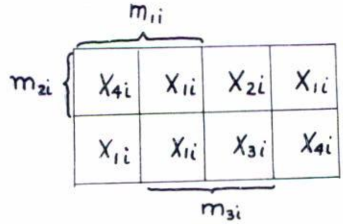

FIG. 4

Veitch & Karnaugh diagram for $n=4$

a.- Input (Poissonian train)

R.- Ring Counter.
M.- Multivibrator
SW.-Probability Adjust.
α, α'.- Output.

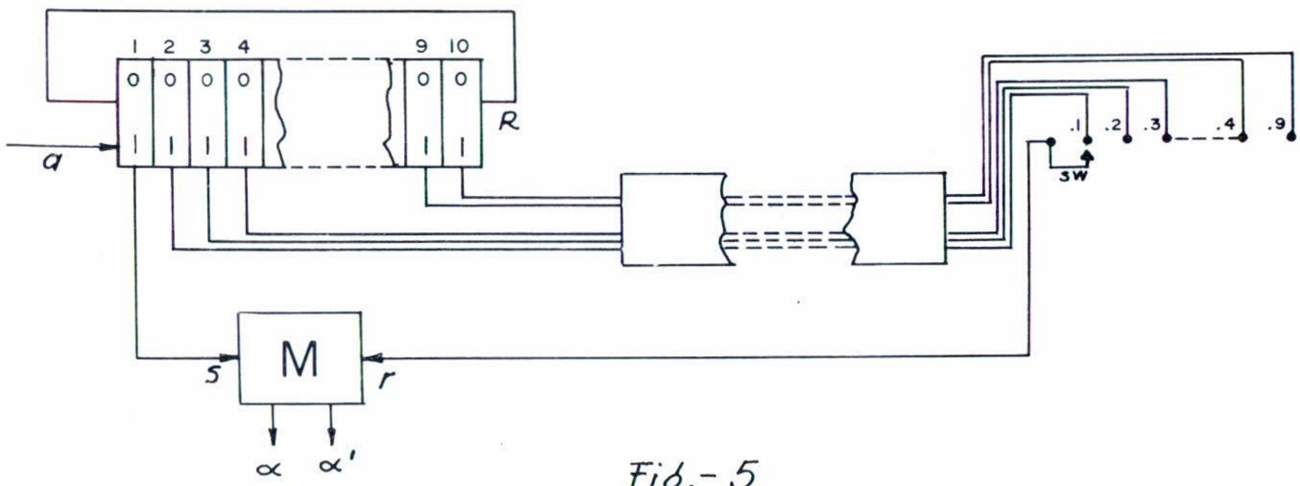

Fig.- 5

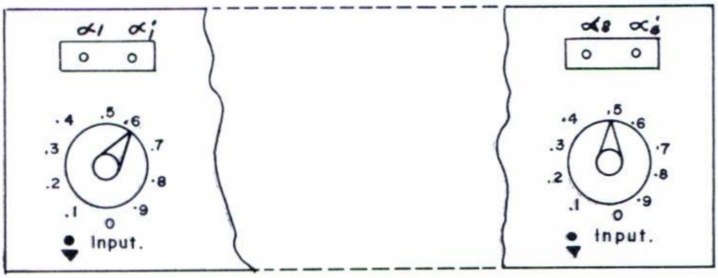

Fig. 6

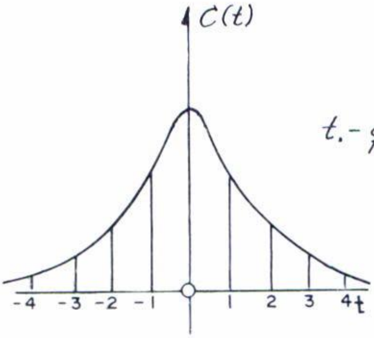

Fig.-8

t.- quantized times.

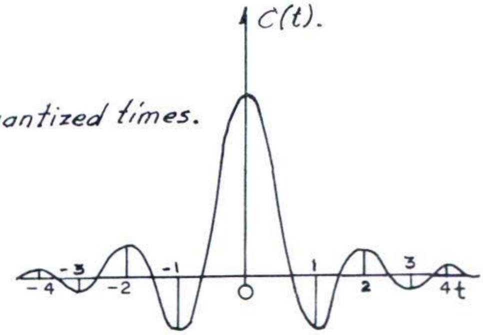

Fig.-9

$$J_1 = (\cdot m_{13} \vee m_{23}) \circ t$$

$$K_1 = (q_2 \circ m'_{12} \circ m'_{23} \vee q'_2 \circ m'_{11} \circ m'_{21}) \circ t.$$

$$J_2 = m'_{11} \circ t$$

$$K_2 = (q_1 \circ m_{12} \vee q'_1 \circ m_{13}) \circ t.$$

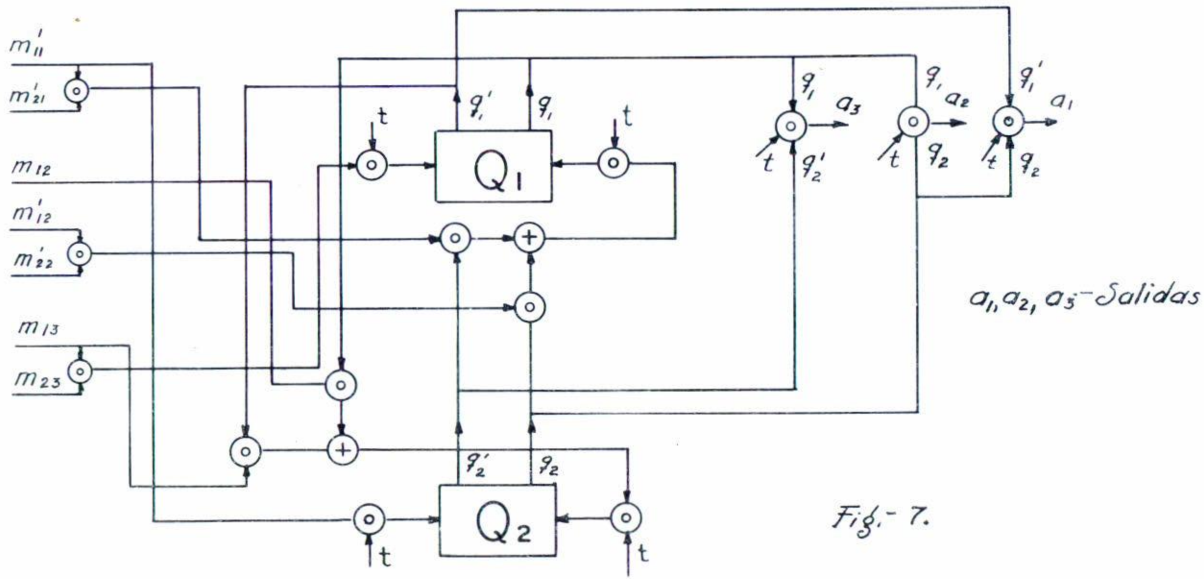

Fig. - 7.